2.2 General Theory for Filtration
过滤基本理论

Now we turn to the mathematical description of filtration.

（对过滤过程机械描述）

Darcy's law （达西定律）

- **Darcy's law relates the flow rate through a porous bed of solids to the pressure drop causing that flow**

（定律把流速与压降联系起来。）

$$v = \frac{k\Delta p}{\mu l}$$

(式 1)

- **υ directly proportional to $\triangle p$**

（流速正比压降）

- **inversely proportional to l/k**

（反比于l/k）

- **where υ is the velocity of the liquid**
- ***k*: a proportionality constant (usually called the Darcy's law permeability of the bed)**

（*k*比例常数,过滤床渗透性的达西方程参数）

- **$\triangle p$: the pressure drop across the bed of thickness *l***

通过厚度为*l*的床产生的压降

- ***μ*: viscosity of the liquid**

流体粘度

Darcy's law holds only when Re<5

（当R_e <5时达西定律才成立）

$$Re = \frac{dv\rho}{\mu(1-\varepsilon)} \quad \text{(式2)}$$

- ***d*** – particle size（粒径）
- ε – the void fraction in the cake （滤饼空隙率）
- ρ – the liquid density（流体的密度）
- R_e – Reynolds number（雷诺准数）

- **For batch filtration （分批过滤），**
- **the velocity is**

$$v = \frac{1}{A}\frac{dV}{dt} \qquad \textbf{(式3)}$$

where

- **A – filtration area （过滤面积）**
- **V – the total volume of filtrate（滤液总体积）**
- **t – the filtration time（过滤时间）**

so

- $l/k = R_M + R_C$ **(式 4 ）**

- **rewrite the resistance (l/k) to include explicitly the contributions of cake and filter medium.**
- **R_M is the resistance of the filter medium**

  过滤介质的阻力
- **R_C is the resistance of accumulated cake biomass**

  滤饼的阻力
- **Combining Eqs.(1), (3), and (4),**

We find basic differential equation for batch filtration at constant pressure drop .

（分批过滤压降是常数）

$$\frac{1}{A}\frac{dV}{dt}=\frac{\Delta p}{\mu\left(R_M+R_C\right)}$$ (式 5)

R_M is a constant, independent of the cake

（R_M是常数，与滤饼无关）

The cake's resistance R_C varies with the amount filtered volume of filtration.

（ R_C与滤液总体积有关）

The exact nature of this variation depends on whether the cake is incompressible or not.

（R_C的变化取决于滤饼是不可压缩还是可压缩）

1. Incompressible Cakes（不可压缩滤饼）

If the cake is incompressible (rigid), the cake thickness is directly proportional to the filtrate volume and inversely proportional to the filter area.

如果滤饼是不可压缩的，则滤饼的厚度正比于过滤液的体积，反比于过滤面积。

As a result the cake's resistance R_C is described by the equation:

$$R_C=\alpha\rho_0(V/A) \qquad \text{（式 6）}$$

- **where α represents the specific cake resistance**

（滤饼的阻力特性）

- **ρ_0 - the mass of cake solids per volume of filtrate**

（单位体积滤液含固体滤饼量）

- **Substitution of Eq.(6) into Eq.(5). Differential equation for incompressible cake is:**

$$\frac{1}{A}\frac{dV}{dt}=\frac{\Delta p}{\mu\left[\alpha\rho_0\left(V/A\right)+R_M\right]} \qquad \text{(式7)}$$

- **The equation is subject to the initial condition**

  初始条件为

$$t=0 \qquad V=0$$

This condition says that at the start of the experiment, no solution has been filtered.

开始时没有滤液被过滤

Equation(2.2-7) is easily integrated and rearranged to give

代入（式7）积分后整理得

$$\left(\frac{At}{V}\right) = K\left(\frac{V}{A}\right) + B \qquad \textbf{(式8)}$$

where（其中）

$$K = \frac{\mu\alpha\rho_0}{2\Delta p}$$

and

$$B = \frac{\mu R_M}{\Delta p}$$

- **Thus a plot of (At/V) versus (V/A) should be linear.**

  （At/V对 V/A作图）

- **The slope K is a function of the pressure drop $\triangle p$ and of the properties of the cake, represented by α and ρ_0.**

  斜率K是压降$\triangle p$及滤饼特性α、ρ_0的函数。

- **The Intercept B should be independent of the properties of the cake, but it is proportional to the medium's resistance R_M, usually R_M is insignificant.**

  截距B与滤饼特性无关，但它正比于介质阻力R_M 。通常R_M可忽略不计

- **Eq.(8) becomes simpler still:**

$$t = \left(\frac{\mu\alpha\rho_0}{2\Delta p}\right)\left(\frac{V}{A}\right)^2 \qquad \textbf{(式 9)}$$

These equations, valid for incompressible cakes, will be tested in the examples given later in the section.

该式仅适合不可压缩的滤饼

2. Compressible cake（可压缩滤饼）

- **Almost all cakes formed of biological materials are compressible, and so cannot be described with the simple analysis just outlined.**
- **As these cakes compress, filtration rates drop.**

大多数生物滤饼都可压缩，不能仅用作图法描述，滤饼可压缩，则过滤速度降低。

To estimate the effects of compressibility, we assume that the cake resistance *α* is a function of the pressure drop.

为了估计可压缩性的影响，我们假定滤饼阻力*α*是压降的函数来估计可压缩性滤饼的影响

$$\alpha = \alpha'(\triangle p)^s \qquad \text{(式 10)}$$

where

- α'-- a constant related largely to the size and shape of the particles forming the cake.

  α'是一个与滤饼组成、粒子大小和形状相关的常数

- s -- the cake compressibility

  s是压缩系数

s=0 a rigid & highly incompressible cake

s=0 理想的不可压缩

s=1 a highly compressible cake

s=1 高度可压缩

s=0.1～0.8 in practice

实践中，s的变化范围从0.1到0.8

Values of s and α' are most easily determined by plotting the logarithm of αversus the logarithm of $\triangle p$, as shown in Fig.(1)

计算s和α'可用logα对log$\triangle p$作图。

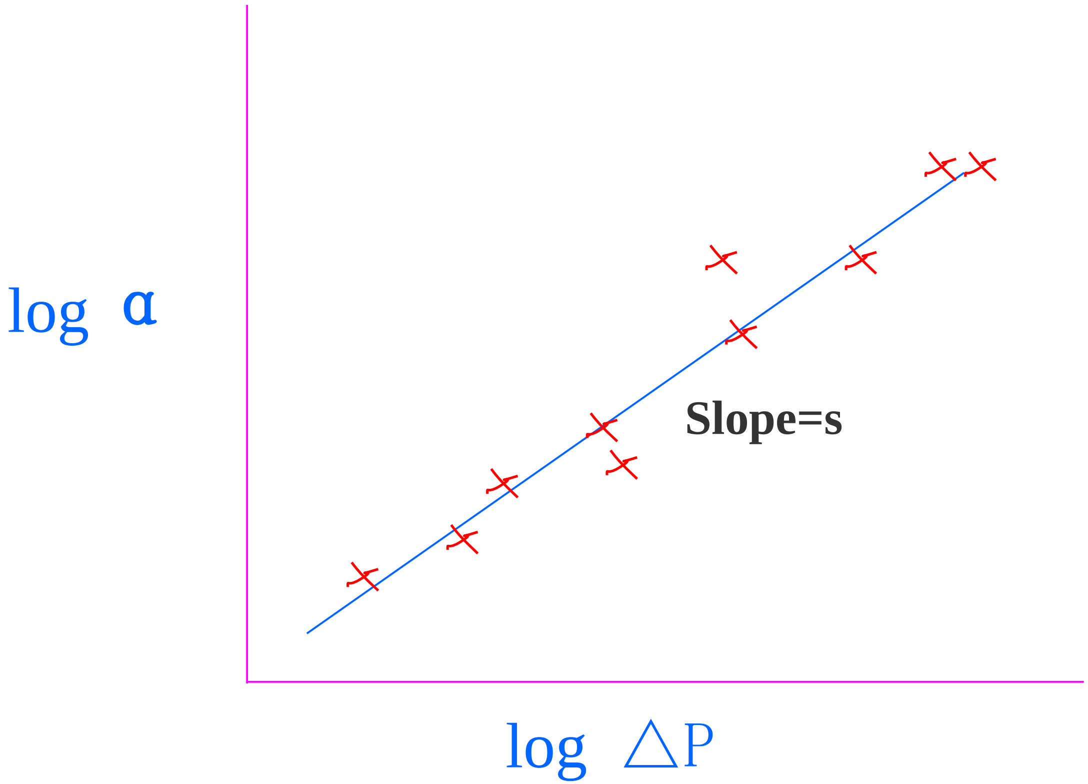

Fig1 Cake resistance versus pressure drop

图1 logα对log△*p*作图

When values of s are high , one should consider pretreating the feed with filter aids, as described in Section 2.1 .

当s的数值很高时，须加入助滤剂对原液进行预处理

The fact that a cake is compressible dose not change the basic result for a constant pressure filtration, given by Eq (10).

可压缩滤饼并不改变式（10）给出的对基本结果

The value of K measured with this equation may be a more complex function of p than first expected. It should not change the value of B.

K转变为关于△P的复杂函数，与压力成正比的B值不变