Laboratory 1

0. Read carefully and try the Maple commands written in the first 3 pages of the tutorial.

1. Evaluate the number using floating point arithmetic: $\frac{1}{2}$; e ; $\sqrt{3}$; π ; $\sin(0.1)$

```
> evalf(1/2); evalf(exp(1)); evalf(sqrt(3)); evalf(Pi); evalf(sin
  (0.1));
```

$$0.5000000000$$
$$2.718281828$$
$$1.732050808$$
$$3.141592654$$
$$0.09983341665 \qquad (1)$$

2. Assign the following expression to a variable and then expand it:

a) $(x^2+2\cdot x-1)^3\cdot(x^2-2)$ b) $(x+n)^5$. Unassign the used variables.

```
> a:=(x^2+2*x-1)^3*(x^2-2); expand(a);
```

$$a := (x^2+2x-1)^3(x^2-2)$$
$$x^8+6x^7+7x^6-16x^5-27x^4+14x^3+17x^2-12x+2 \qquad (2)$$

```
> b:=(x+n)^5; expand(b);
```

$$b := (x+n)^5$$
$$n^5+5n^4x+10n^3x^2+10n^2x^3+5nx^4+x^5 \qquad (3)$$

```
> a:='a'; b:='b';a;b;
```

$$a := a$$
$$b := b$$
$$a$$
$$b \qquad (4)$$

3. Factorize: x^8-1

```
> factor(x^8-1);
```

$$(x-1)(x+1)(x^2+1)(x^4+1) \qquad (5)$$

4. Add the following rational expressions by applying the simplify command: $\frac{2\cdot x^2}{x^3-1}+\frac{3\cdot x}{x^2-1}$

```
> simplify(2*x^2/(x^3-1)+3*x/(x^2-1));
```

$$\frac{(5x^2+5x+3)x}{(x^2-1)(x^2+x+1)} \qquad (6)$$

5. Simplify the expression: $\sin(x)^2+\cos(x)^2$

```
> simplify(sin(x)^2+cos(x)^2);
```

$$1 \qquad (7)$$

6. Evaluate using both subs and eval the expression $e^x+\ln(x)$ in x=1. Use first ?subs and ?eval

```
> subs(x=1,exp(x)+ln(x)); eval(exp(x)+ln(x),x=1);
```

$$e+\ln(1)$$

$$e \quad \textbf{(8)}$$

7. Solve: a) the equation $x^2 - 4 \cdot x + 3 = 0$ where x is the unknown;

```
> solve(x^2-4*x+3=0,x);
```

$$3, 1 \quad \textbf{(9)}$$

```
>
```

b) the equation $x^2 \cdot y + 2 \cdot y - x = 0$ where x is a parameter and y is the unknown;

```
> solve(x^2*y+2*y-x=0,y);
```

$$\frac{x}{x^2+2} \quad \textbf{(10)}$$

```
>
```

c) the equation $x^2 \cdot y + 2 \cdot y - x = 0$ where y is a parameter and x is the unknown;

```
> solve(x^2*y+2*y-x=0,x);
```

$$\frac{1}{2} \frac{1+\sqrt{-8y^2+1}}{y}, -\frac{1}{2} \frac{-1+\sqrt{-8y^2+1}}{y} \quad \textbf{(11)}$$

```
>
```

d) the equation $x - \cos(x) = 0$ where x is the unknown; Remark! We can prove that this equation has a unique solution, which is an irational number. With fsolve we find an approximation of this irrational number.

```
> solve(x-cos(x)=0,x);
```

$$\mathit{RootOf}(\_Z - \cos(\_Z)) \quad \textbf{(12)}$$

```
> fsolve(x-cos(x)=0,x);
```

$$0.7390851332 \quad \textbf{(13)}$$

```
>
```

e) the equation $x^5 - 3 \cdot x^3 - 1 = 0$ where x is the unknown; Below we see that "solve" does not provide us any information. Maybe the roots are real or imaginary involving irrational numbers. The command "fsolve" gives three approximate roots. It seems that this equation has 3 real roots and 2 complex conjugate. After we represent the graph of x^5-3x^3-1, we see that it has 3 real zeros, indeed.

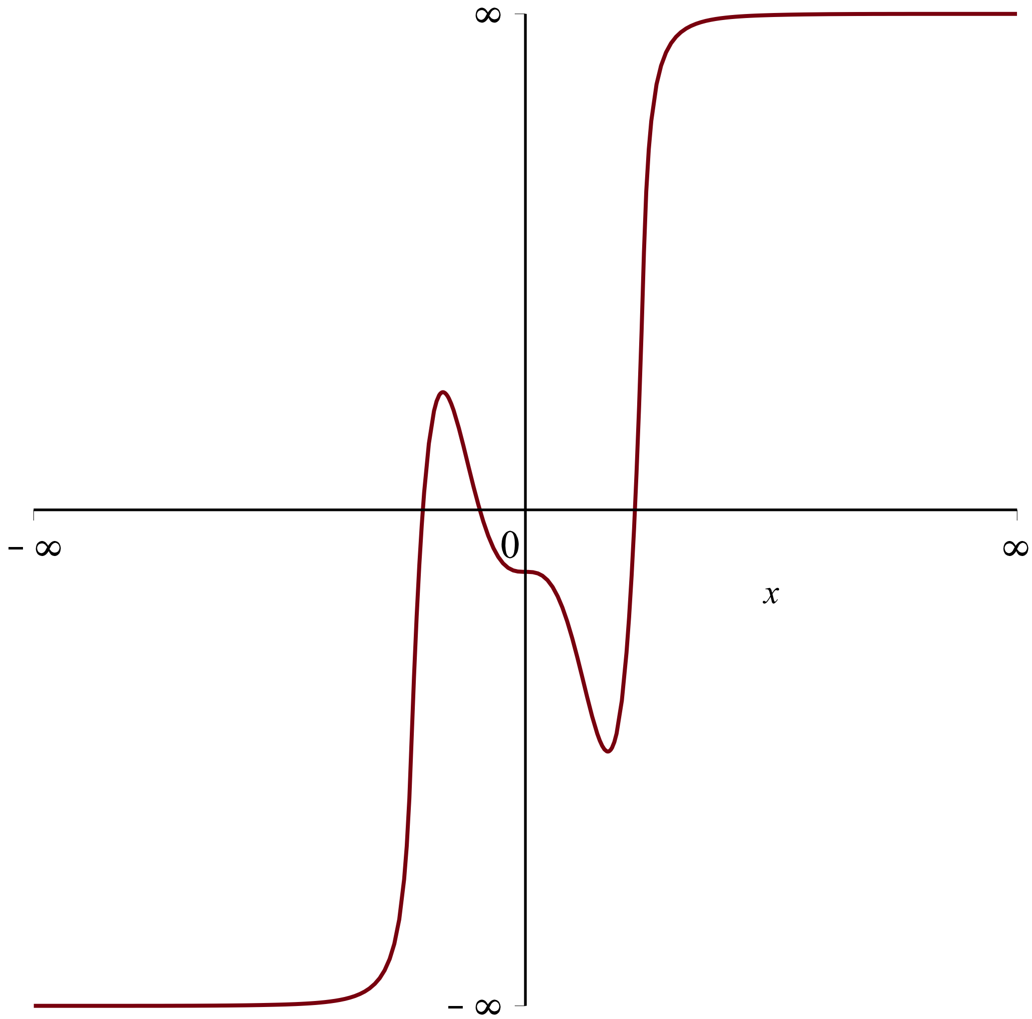
```
> solve(x^5-3*x^3-1,x);
```

$$\mathit{RootOf}(\_Z^5 - 3\,\_Z^3 - 1, \mathit{index}=1), \mathit{RootOf}(\_Z^5 - 3\,\_Z^3 - 1, \mathit{index}=2), \mathit{RootOf}(\_Z^5 - 3\,\_Z^3 - 1, \mathit{index}=3), \mathit{RootOf}(\_Z^5 - 3\,\_Z^3 - 1, \mathit{index}=4), \mathit{RootOf}(\_Z^5 - 3\,\_Z^3 - 1, \mathit{index}=5) \quad \textbf{(14)}$$

```
> fsolve(x^5-3*x^3-1,x);
```

$$-1.668777593, -0.7418139305, 1.782308780 \quad \textbf{(15)}$$

```
> plot(x^5-3*x^3-1,x=-infinity..infinity);
```



>

f) the system of two equations $4x + 3y = 10,\ 3x - y = 1$ where x and y are the unknowns.

```
> solve({4*x+3*y=10,3*x-y=1},{x,y});
```

$$\{x = 1, y = 2\} \tag{16}$$

>

8. Assign to a variable f the function (not the expression) $f : R \to R,\ \ f(x) = e^x - \sin(x)$. Evaluate $f(0), f(-1), f'(0), f'(1)$. Calculate the first and second order derivatives of f (using both D and diff) and a primitive of f. Evaluate $\int_{-1}^{1} f(x)\,dx$. Unassign f. Write the commands together with your comments in your notebook.

```
> f:=x->exp(x)-sin(x);
```

$$f := x \to e^x - \sin(x) \tag{17}$$

```
> f(0); f(-1); D(f)(0); D(f)(1);
```

$$1$$
$$e^{-1} + \sin(1)$$
$$0$$
$$e - \cos(1) \tag{18}$$

```
> D(f);diff(f(x),x);
```

$$x \to e^x - \cos(x)$$

$$e^x - \cos(x) \quad (19)$$

```
> D(D(f)); diff(f(x),x$2);
```

$$x \to e^x + \sin(x)$$

$$e^x + \sin(x) \quad (20)$$

```
> int(f(x),x); int(f(x),x=-1..1);
```

$$e^x + \cos(x)$$

$$-e^{-1} + e \quad (21)$$

```
> f:='f';
```

$$f := f \quad (22)$$

```
>
```

9. Assign to a variable g the expression $e^x - \sin(x)$. Evaluate this expression in $x = 0$. Compute its first order derivative and then evaluate it in $x = 0$. Find a primitive. Evaluate $\int_{-1}^{1} g \, dx$. Using unapply assign to a variable f the second order derivative of g. Evaluate $f(0)$. Write the commands together with your comments in your notebook.

```
> g:=exp(x)-sin(x);
```

$$g := e^x - \sin(x) \quad (23)$$

```
> eval(g,x=0);
```

$$1 \quad (24)$$

```
> gd:=diff(g,x); eval(gd,x=0);
```

$$gd := e^x - \cos(x)$$

$$0 \quad (25)$$

```
> int(g,x); int(g,x=-1..1);
```

$$e^x + \cos(x)$$

$$-e^{-1} + e \quad (26)$$

```
> gdd:=diff(g,x$2); f:=unapply(gdd,x);
```

$$gdd := e^x + \sin(x)$$

$$f := x \to e^x + \sin(x) \quad (27)$$

```
> f(0);
```

$$1 \quad (28)$$

```
>
```

10. Find $\lim_{x \to 0} \frac{\sin(x)}{x}$ and $\lim_{x \to \pi} \frac{\cos(x) + 1}{x - \pi}$.

```
> limit(sin(x)/x,x=0);
```

$$1 \quad (29)$$

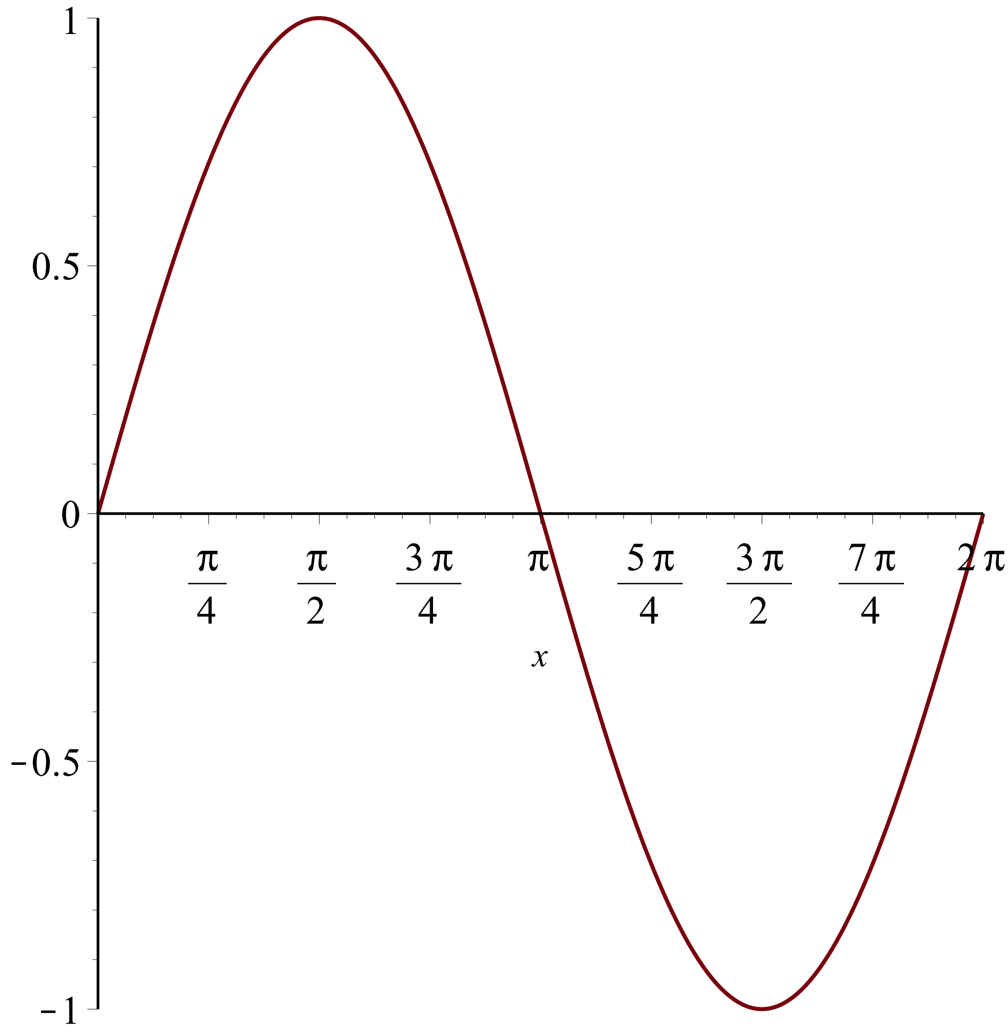
```
> limit((cos(x)+1)/(x-Pi),x=Pi);
```

$$0 \quad (30)$$

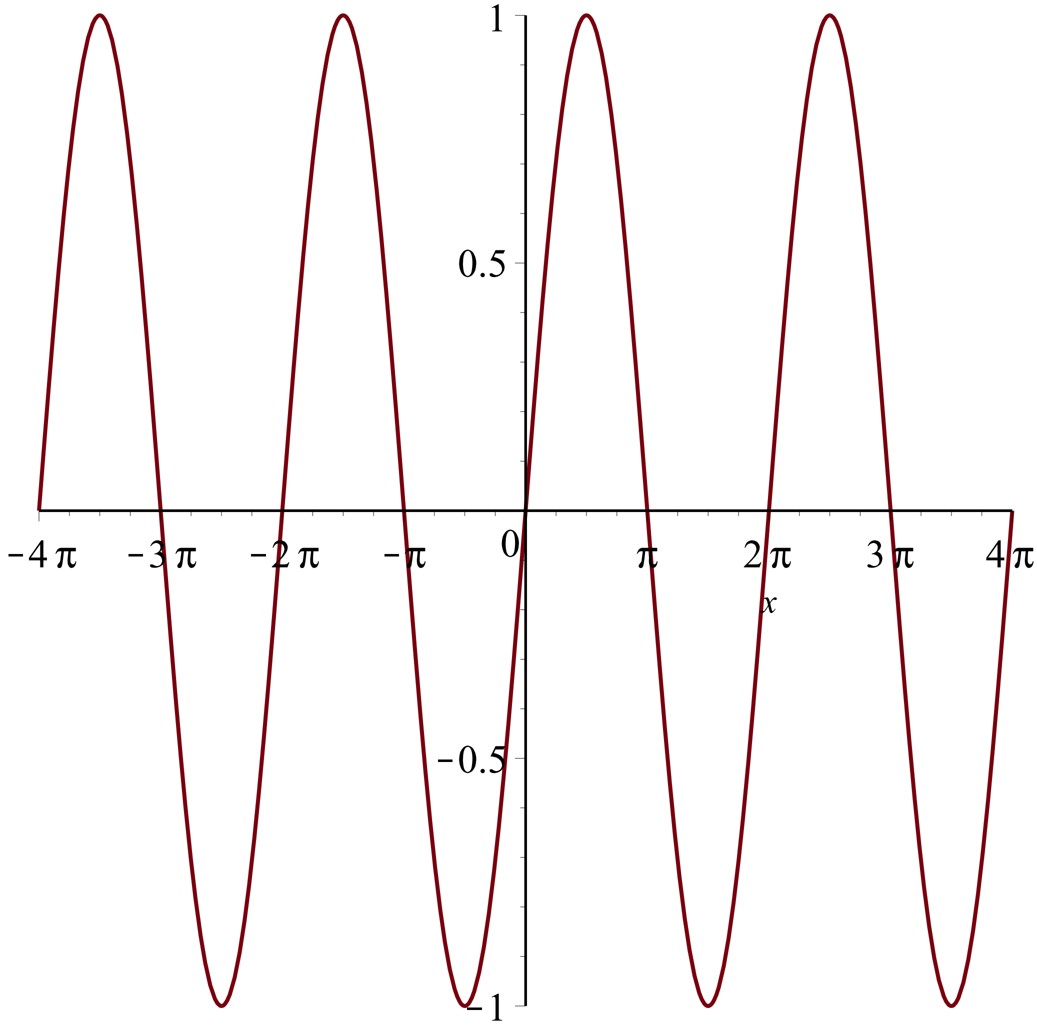
```
>
```

11. First plot the graph of $f(x) = \sin(x)$ in your notebook. Then plot it using Maple, in each of the intervals: $[0, 2\cdot\pi]$, $[-4\,\pi, 4\cdot\pi]$, $[-100, 100]$, $(-\infty, \infty)$. Write your comments in your notebook.

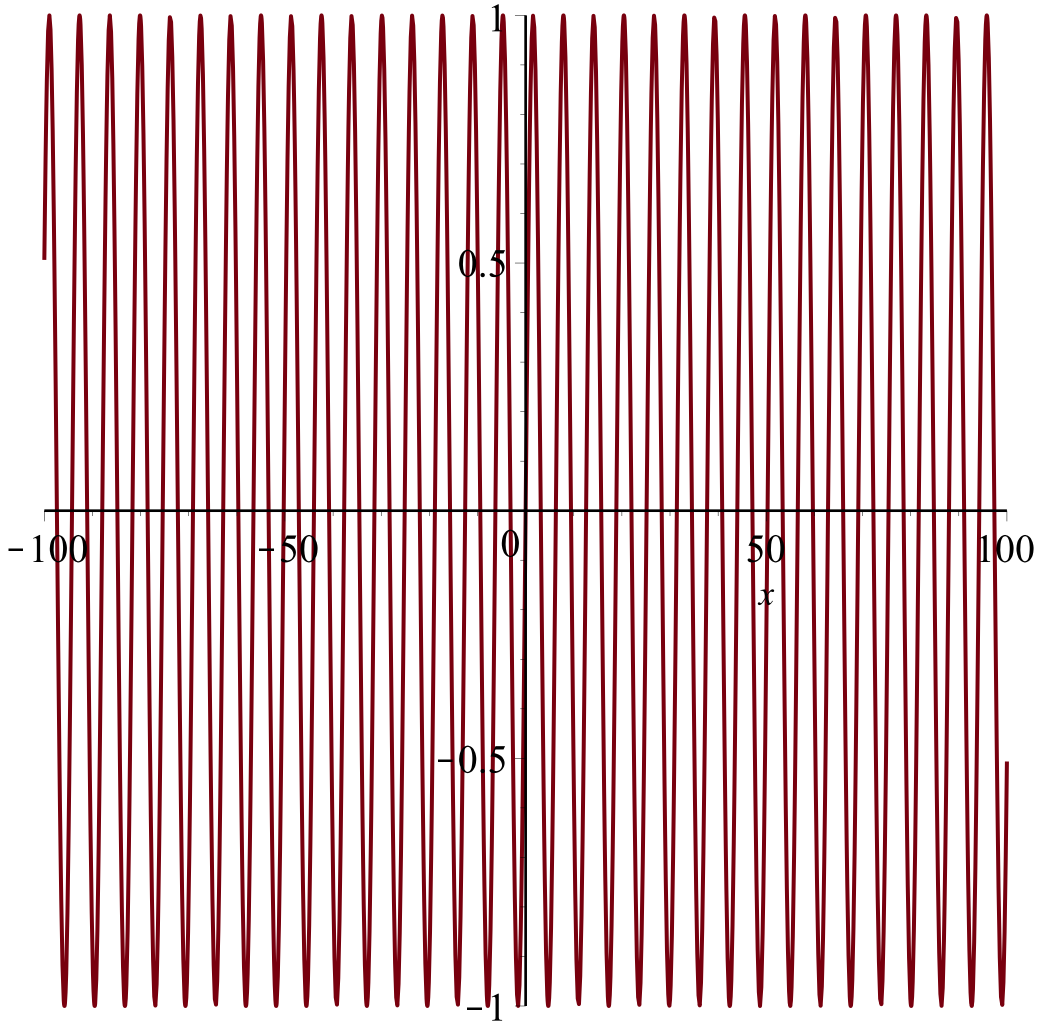
```
> plot(sin(x),x=0..2*Pi);
```



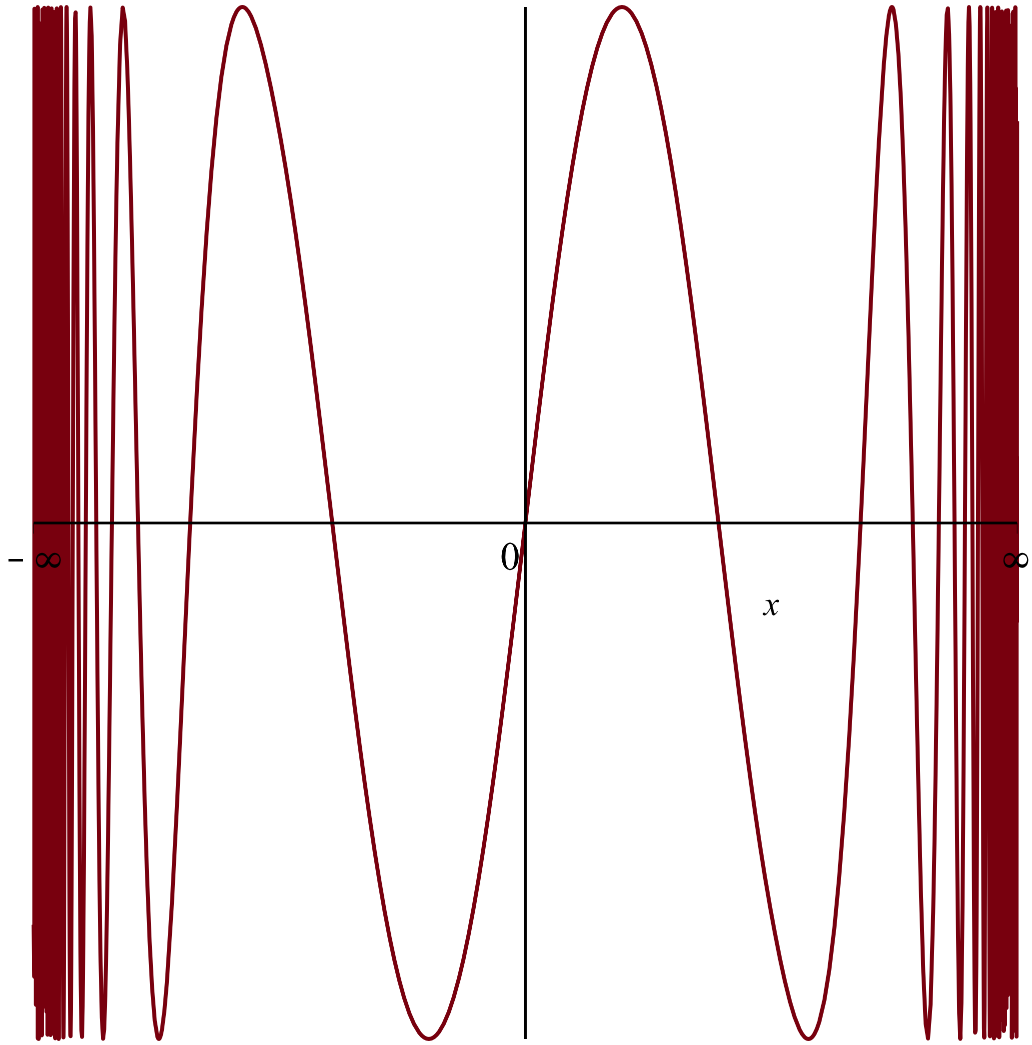

```
> plot(sin(x),x=-4*Pi..4*Pi);
```




```
> plot(sin(x),x=-100..100);
```



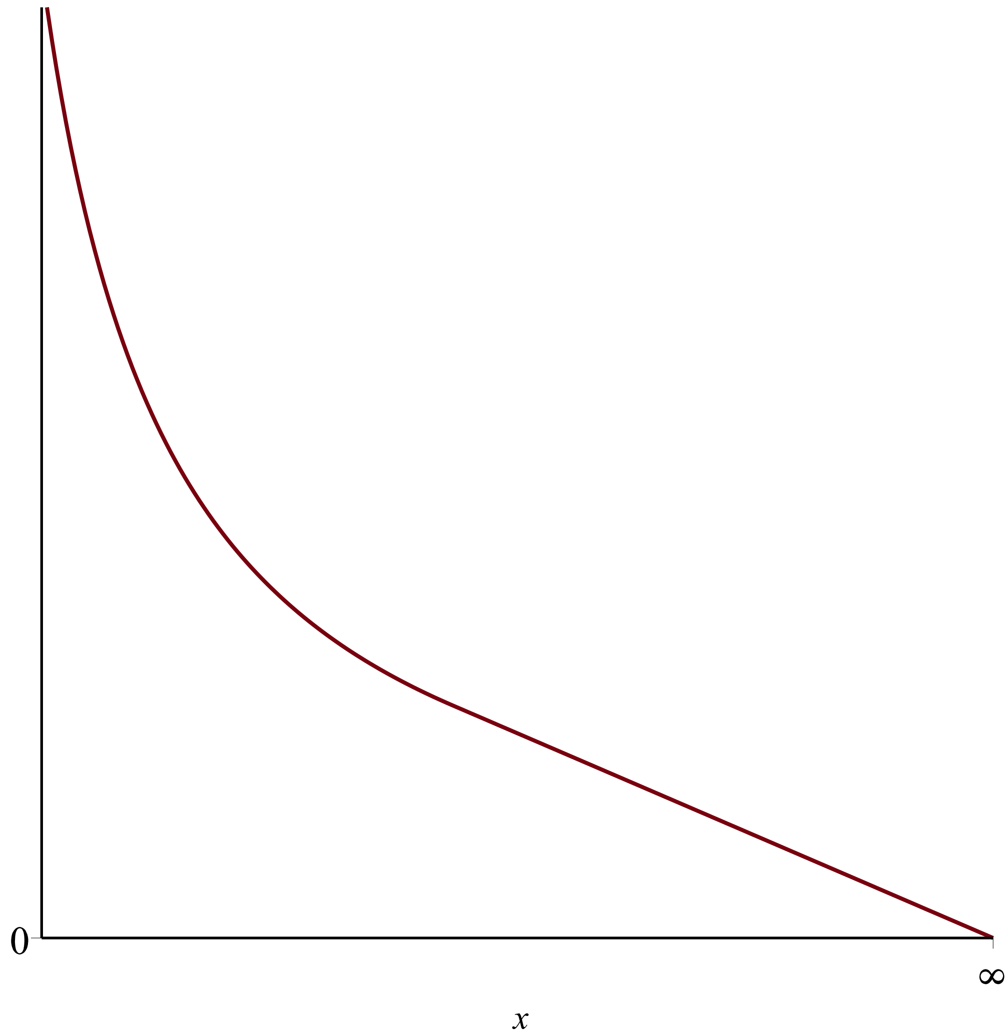

```
> plot(sin(x),x=-infinity..infinity);
```



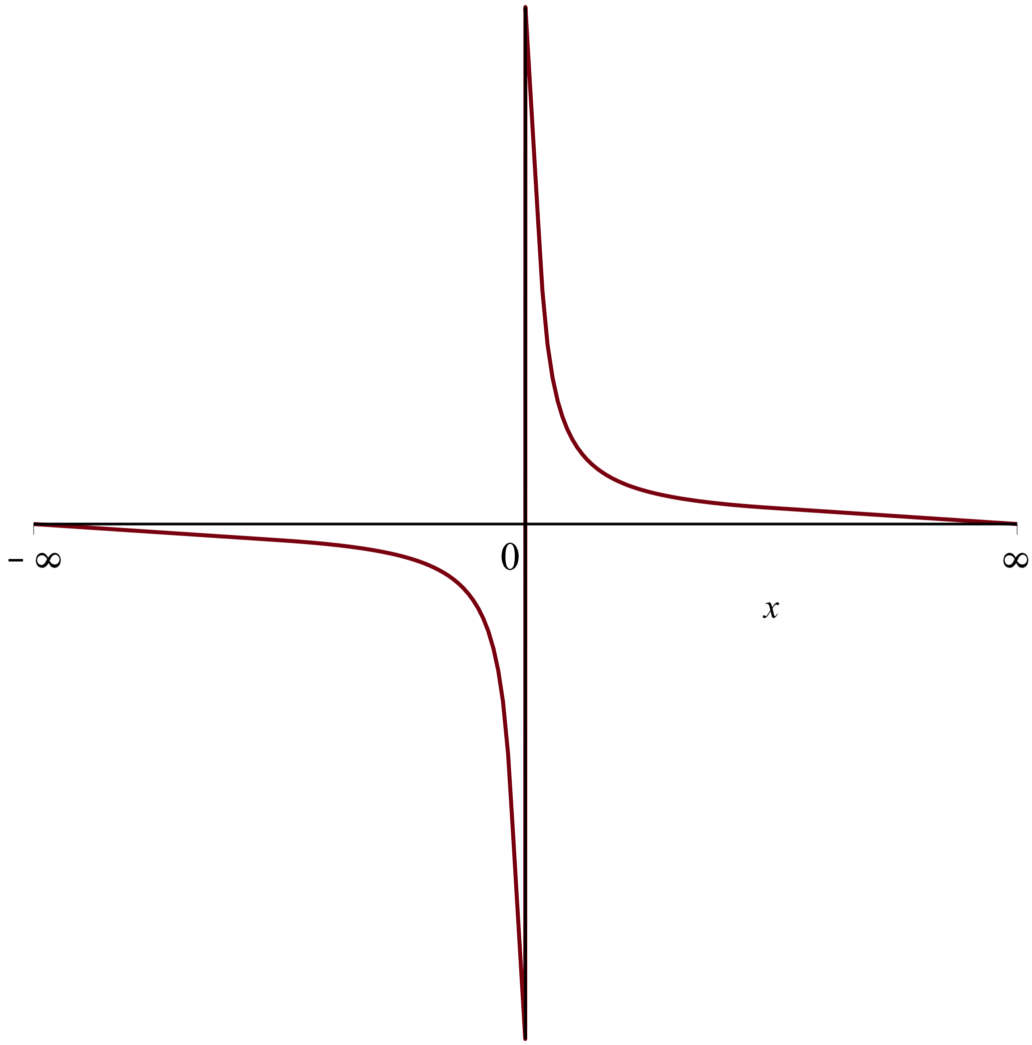
>

12. First plot the graph of $f(x) = \dfrac{1}{x}$ in your notebook. Then plot it using Maple, in intervals at your choice. Write your comments in your notebook.

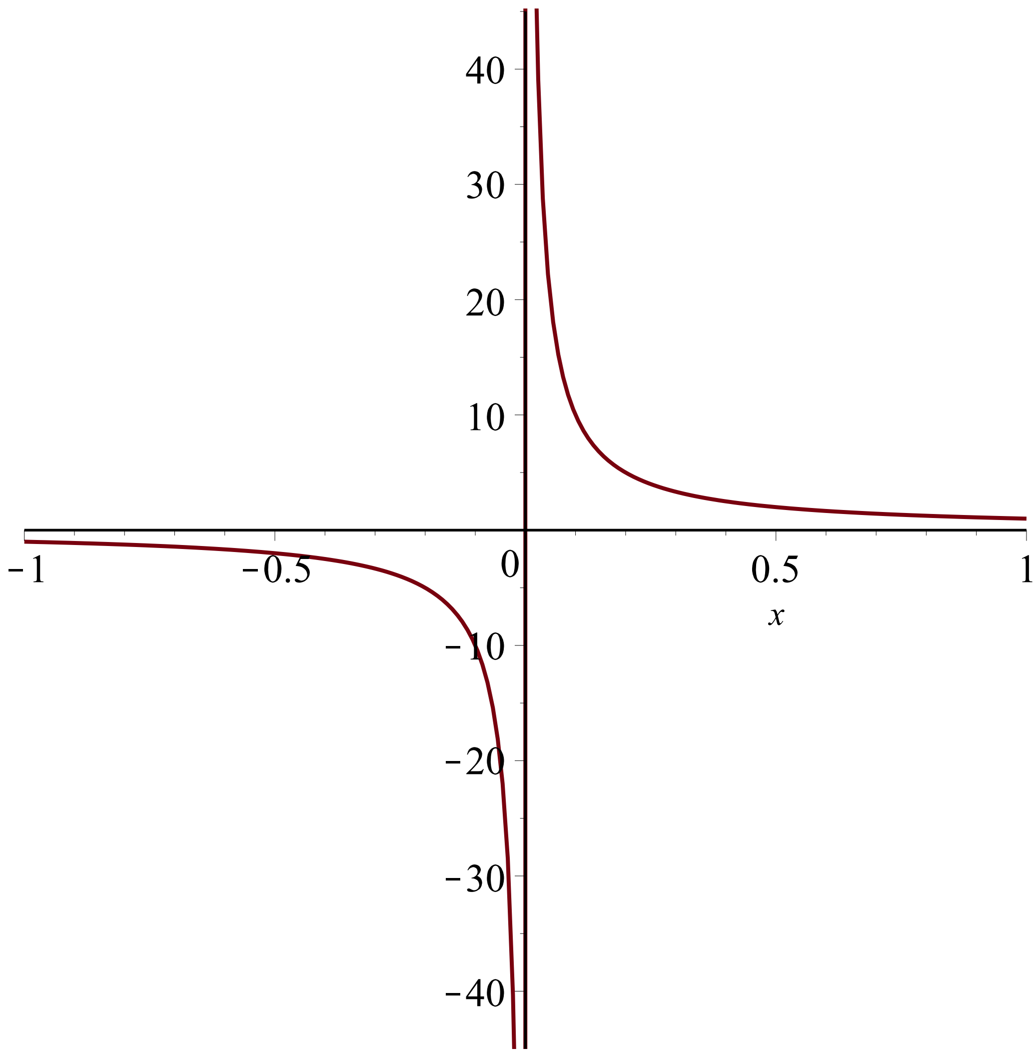
```
> plot(1/x,x=1..infinity);
```




```
> plot(1/x,x=-infinity..infinity);
```



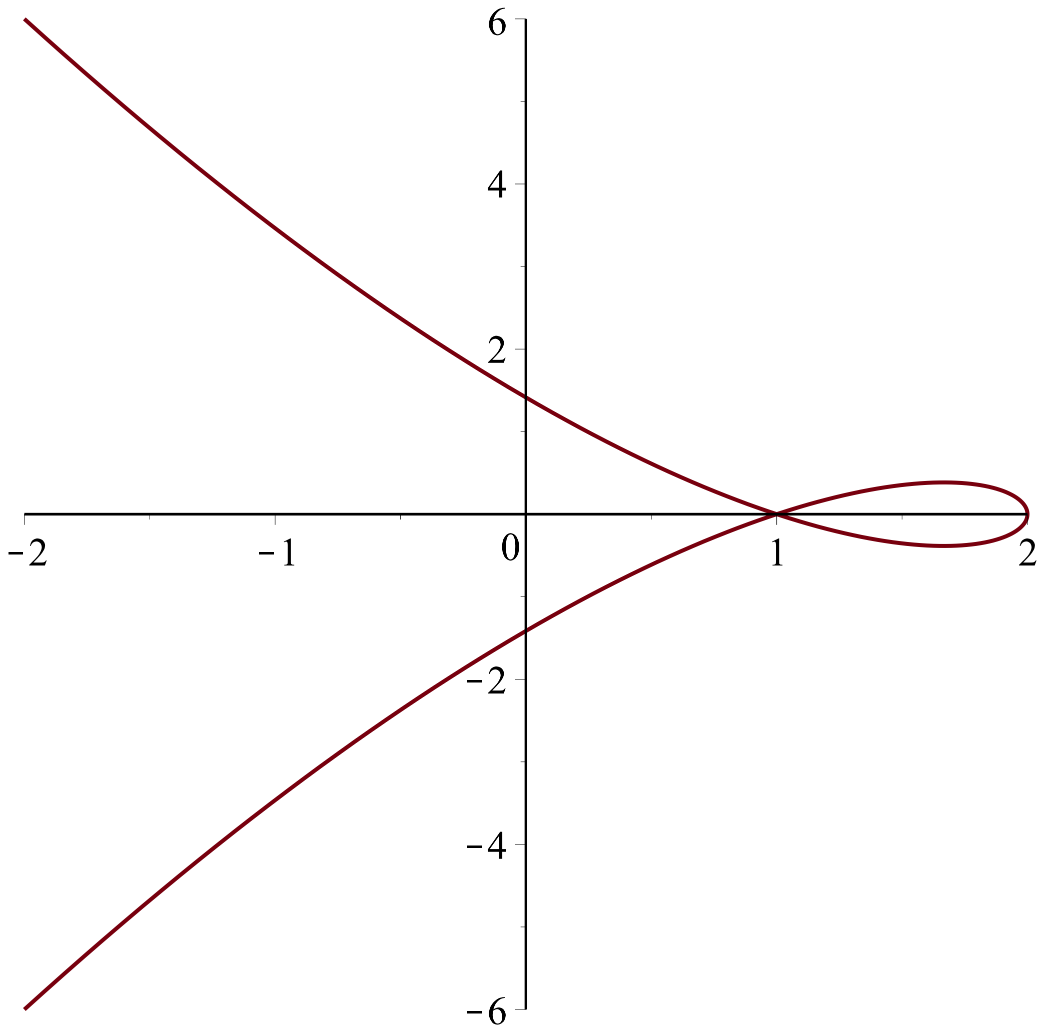

```
> plot(1/x,x=-1..1);
```




```
>
```

13. Plot the planar curve of parametric equations $x = 2 - t^2$, $y = t - t^3$, $t \in [-2, 2]$.

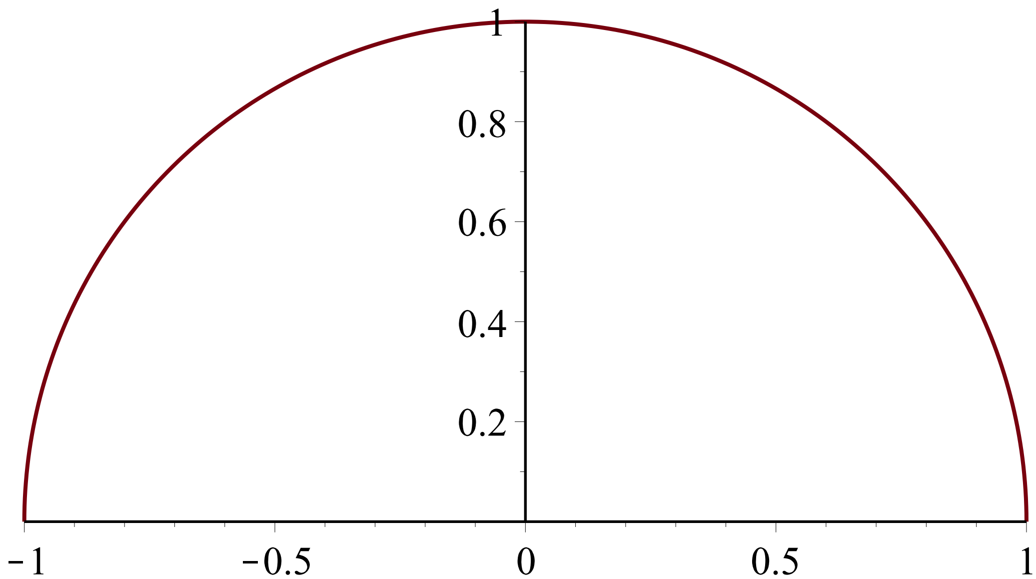
```
> plot([2-t^2,t-t^3,t=-2..2]);
```



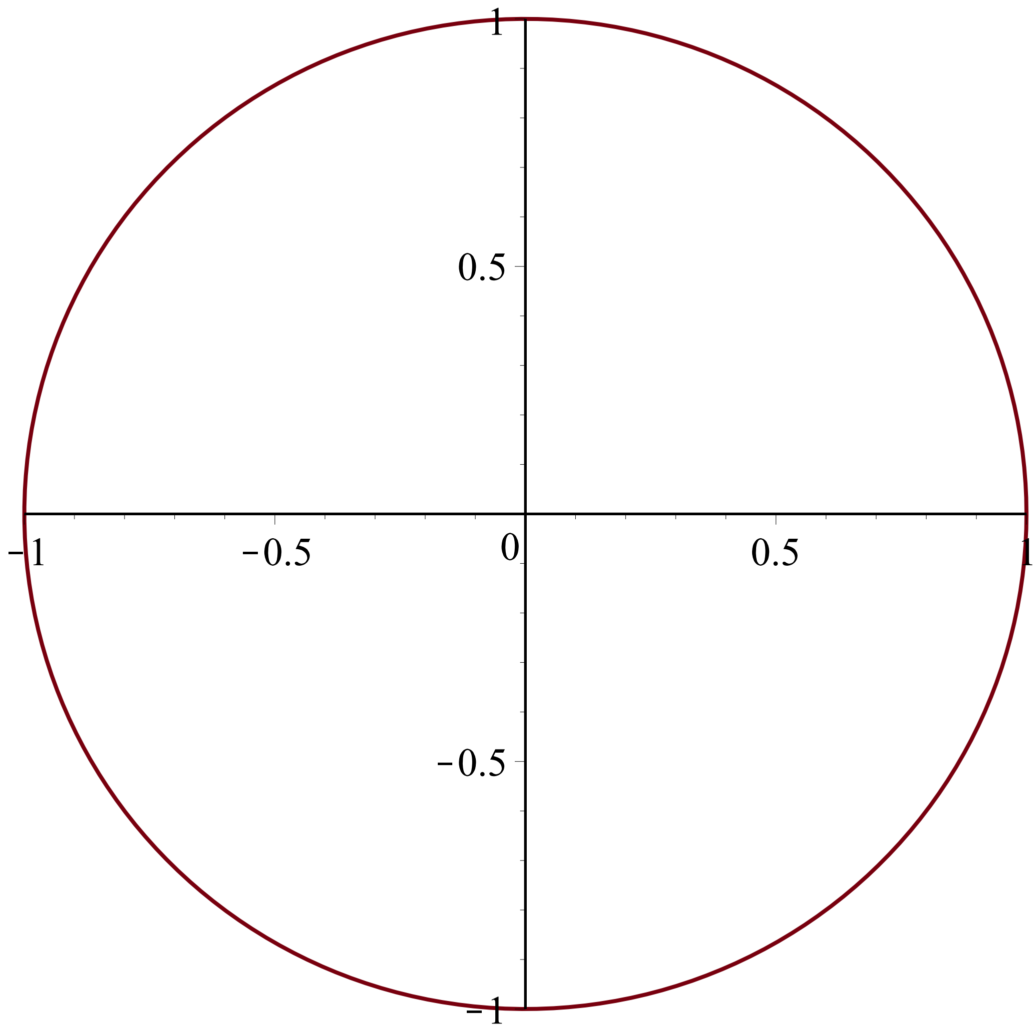

```
>
```

14. First plot in your notebook the planar curve of parametric equations $x = \cos(t)$, $y = \sin(t)$ in each of the intervals: $\left[0, \frac{\pi}{6}\right]$, $\left[0, \frac{\pi}{3}\right]$, $\left[0, \frac{\pi}{2}\right]$, $[0, \pi]$, $\left[0, \frac{3 \cdot \pi}{2}\right]$, $[0, 2 \cdot \pi]$, $[0, 4 \cdot \pi]$.
Then do the same using Maple. Write your comments in your notebook.

```
> plot([cos(t),sin(t),t=0..Pi]);
```




```
> plot([cos(t),sin(t),t=0..2*Pi]);
```



>

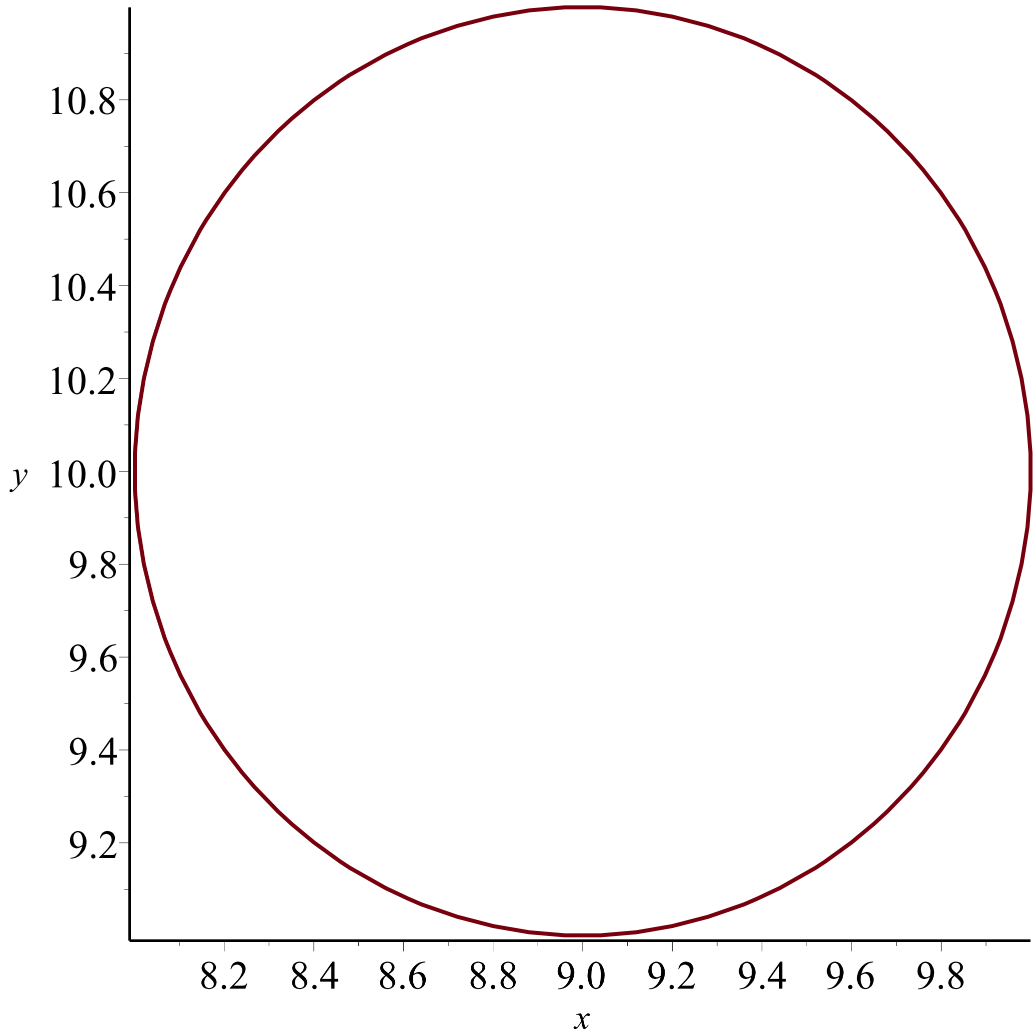
15. Plot the planar curves of parametric equations a) $x = 2 \cdot \cos\left(\frac{t}{3}\right)$, $y = 2 \cdot \sin\left(\frac{t}{3}\right)$

b) $x = \cos(4 \cdot t)$, $y = \sin(4 \cdot t)$ in different intervals at your choice. Write your comments in your notebook.

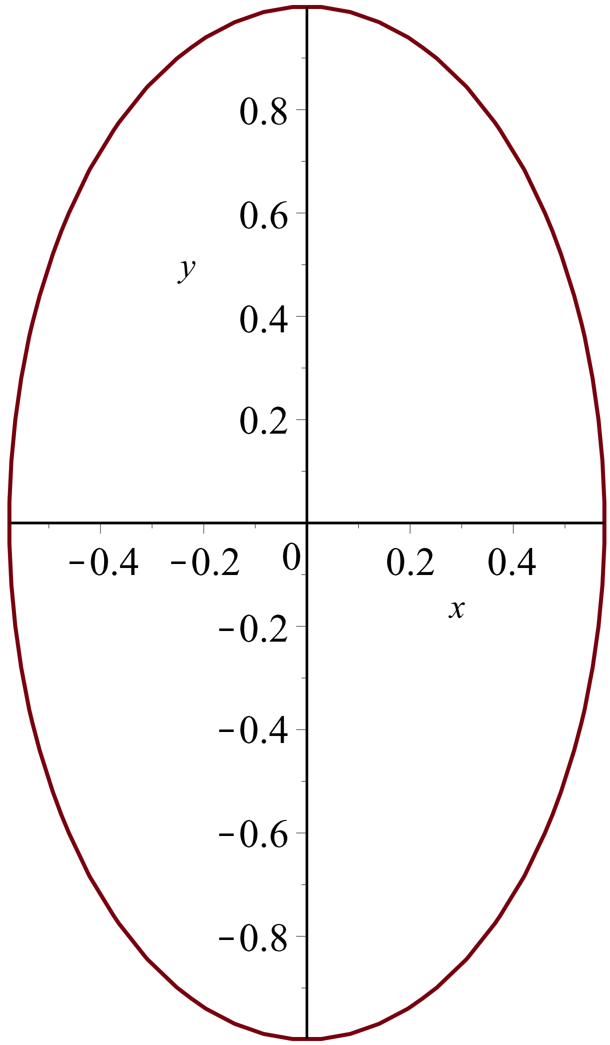
16. Write in your notebook the implicit equations of a circle centered in (9,10), an ellipse and a parabola. Then plot at leats a circle, an ellipse and a parabola.

```
> with(plots); implicitplot((x-9)^2+(y-10)^2=1,x=8..10,y=9..11);
```

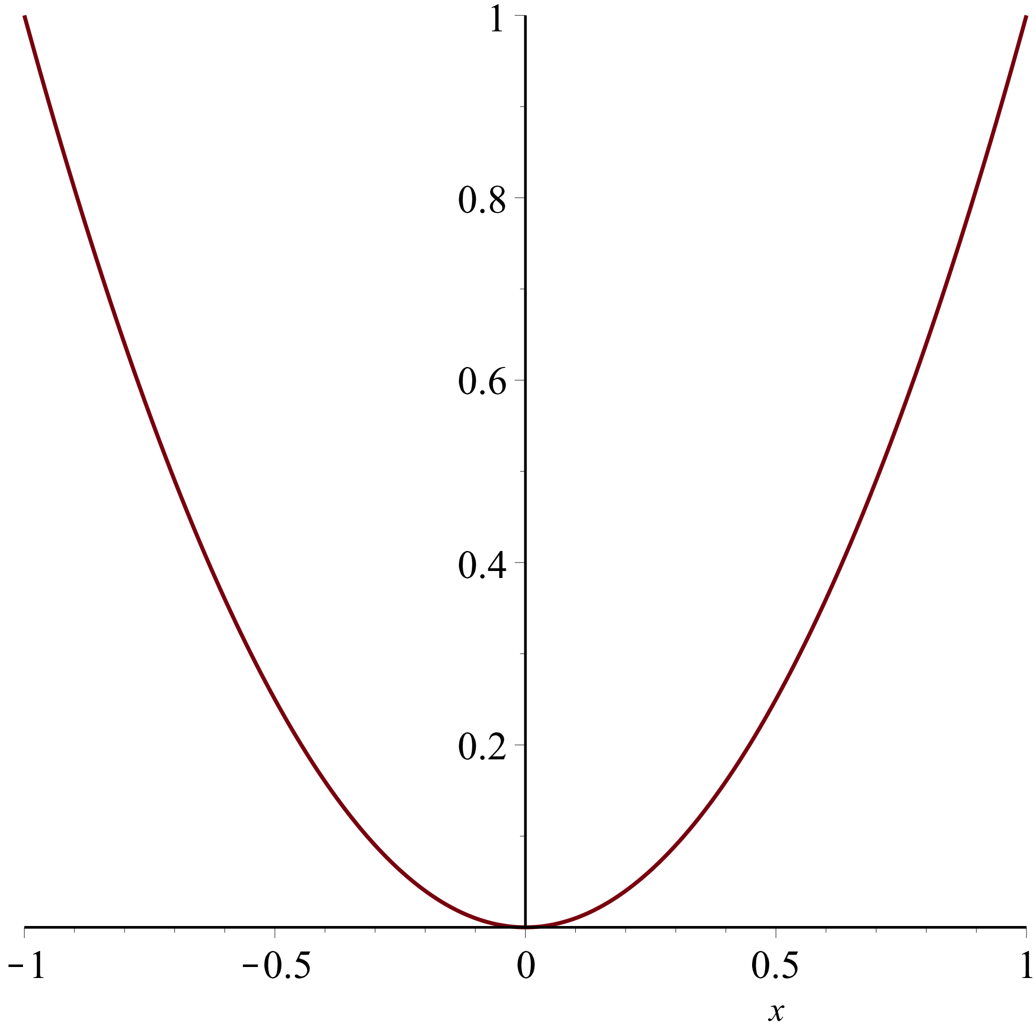
[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *shadebetween*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*]



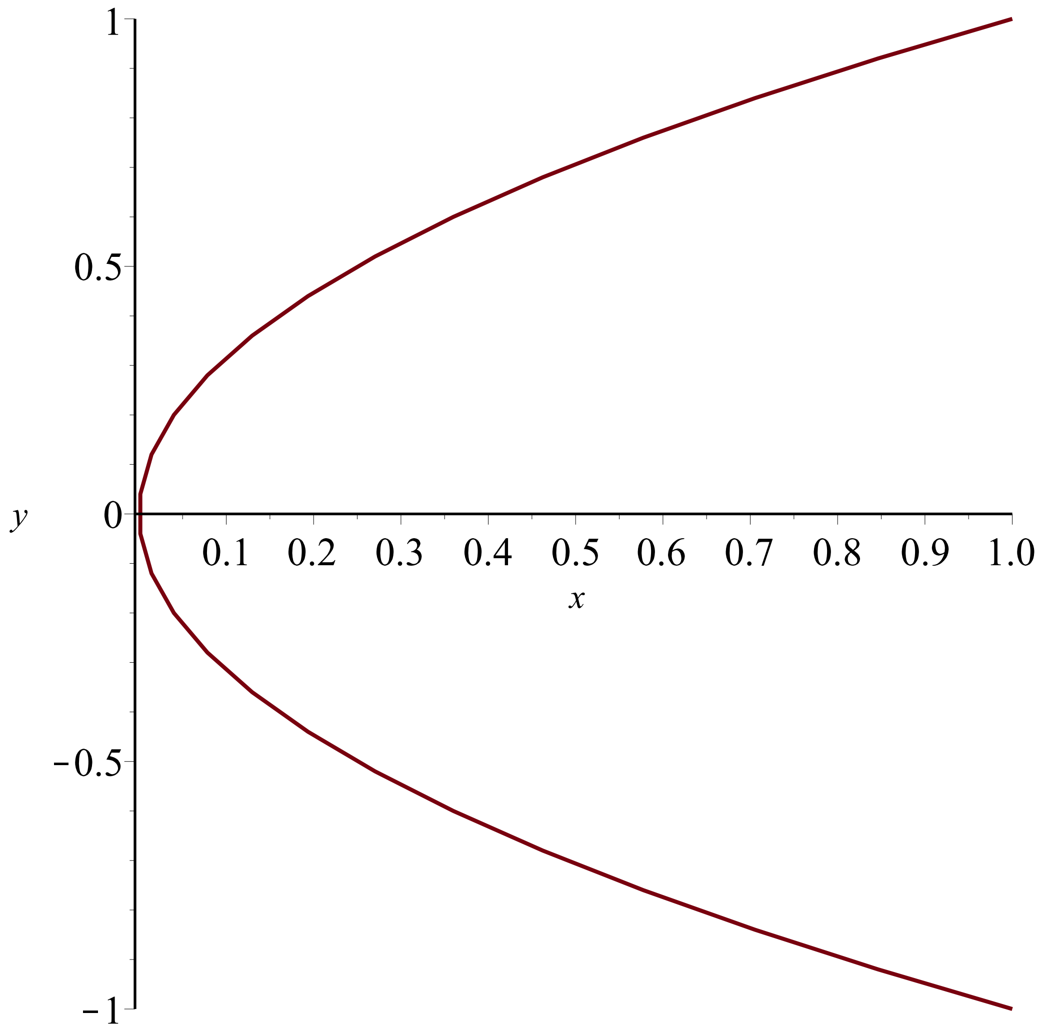

```
> implicitplot(3*x^2+y^2=1,x=-0.7..0.7,y=-1..1);
```




```
> plot(x^2,x=-1..1);
```



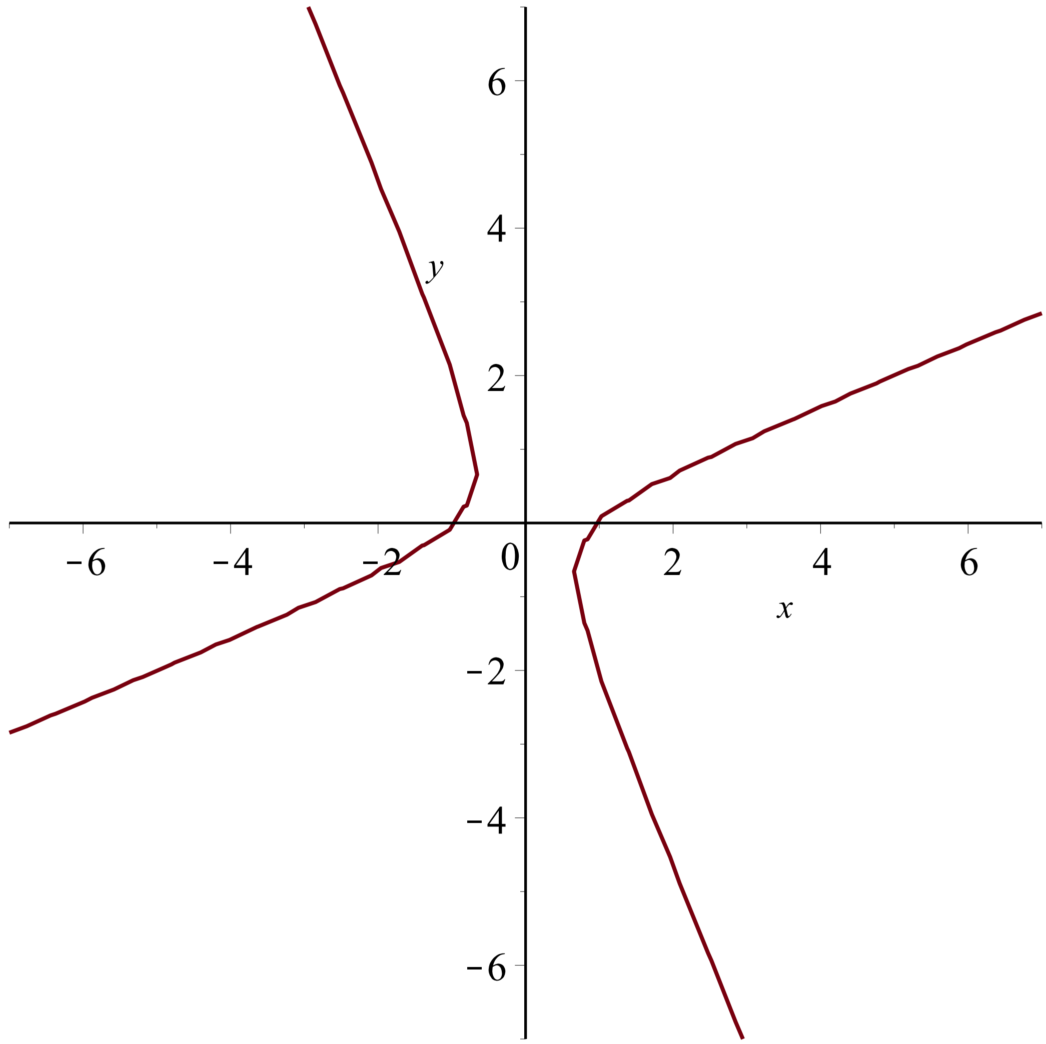

```
> implicitplot(x=y^2,x=0..1,y=-1..1);
```



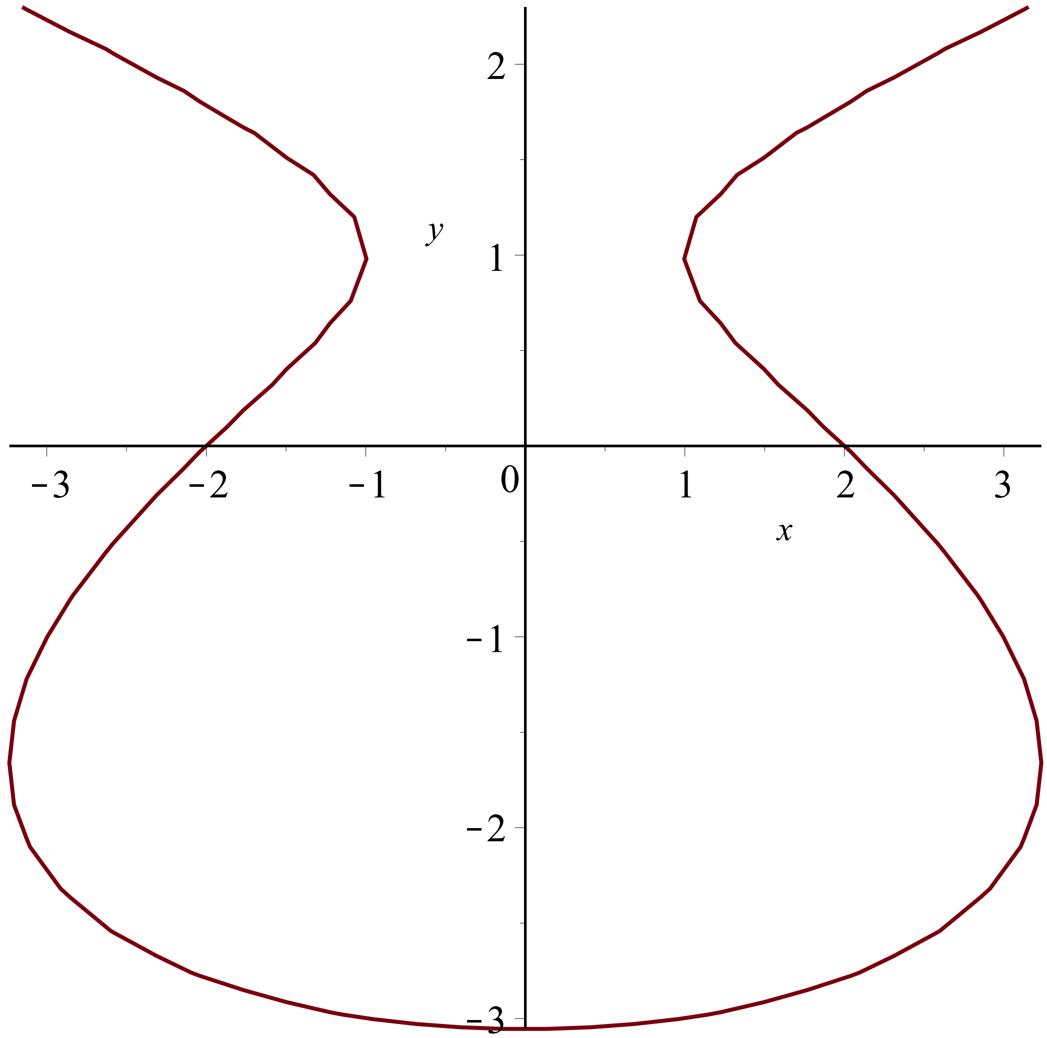
>

17. Plot the planar curves of implicit equations a) $x^2 - 2 \cdot x \cdot y - y^2 = 1$ b) $y^3 + y^2 - 5 \cdot y - x^2 = -4$.
You have to choose properly a rectangle where to see the curve.

```
> implicitplot(x^2-2*x*y-y^2=1,x=-7..7,y=-7..7);
```



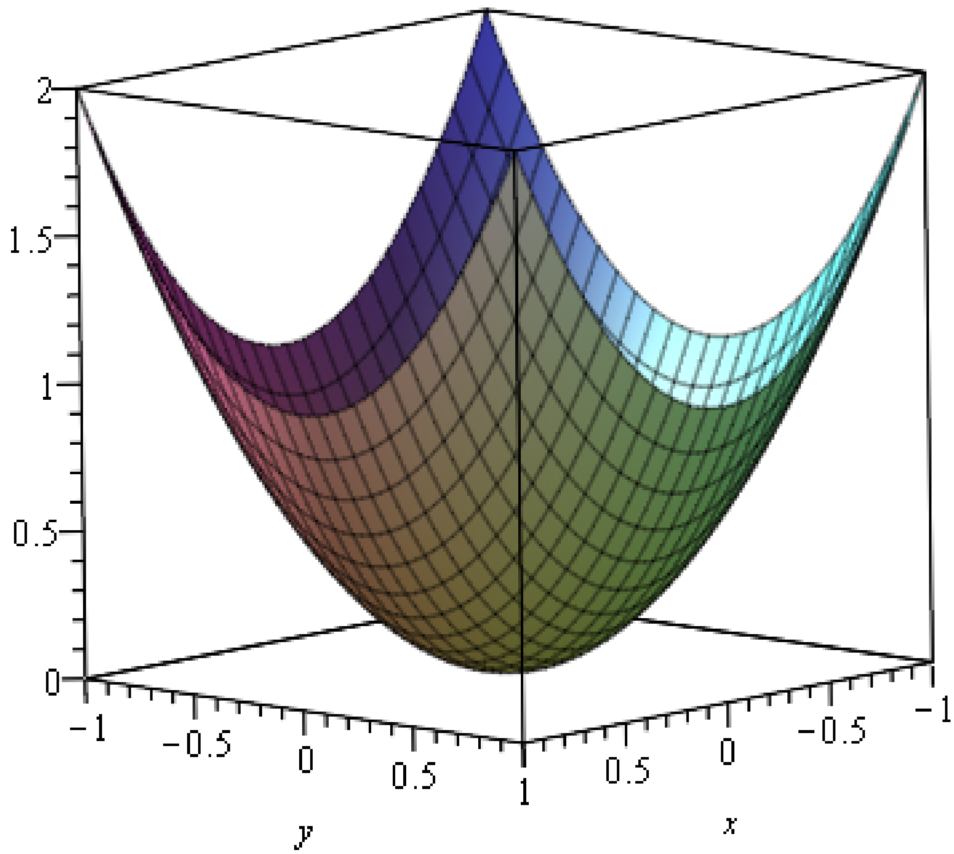

```
> implicitplot(y^3+y^2-5*y-x^2=-4,x=-3.4..3.4,y=-3.2..2.3);
```




```
>
```

18. First draw in your notebook the graph of the function $H(x, y) = x^2 + y^2$. Then plot it in 3d using Maple. For the variable (x,y) choose a rectangle centered at (0,0). What remarkable planar curves are the level curves of H? Write your comments in your notebook.

```
> plot3d(x^2+y^2,x=-1..1,y=-1..1);
```



2
1.5
1
0.5
0
−1
−0.5
0
0.5
1
y
1
0.5
0
−0.5
−1
x

>