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## **Module 12 Lecture - Analysis of Covariance (ANCOVA)**

Analysis of Variance

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# 1 Overview and Introduction

## 1.1 Textbook Learning Objectives

- Discuss basic ideas of linear regression and correlation.
- Create and interpret a line of best fit.
- Calculate and interpret the correlation coefficient.

## 1.2 Instructor Learning Objectives

- Understand the concept of prediction and how it ties into linear regression
- Begin to see the broader applications of regression outside the simple applications

## 1.3 Introduction

- In this unit, we will be describing both \_\_\_\_\_ and \_\_\_\_\_ linear regression
  - At a basic level, these statistics are appropriate to \_\_\_\_\_ numeric variables with a ratio or interval scale of measurement
  - There are more advanced \_\_\_\_\_ that can work with ordinal and categorical data, but they are out of scope for this class

 Discuss: As review, describe what it means for a variable to be continuous and what the difference between ratio and interval scale data are?

- Furthermore, we'll be primarily focusing on the **bivariate** scenario, where there are \_\_\_\_\_ variables we are comparing
  - This is usually set up as one **predictor** variable and one **criterion** variable
  - There are important (and commonly) used extensions for **multivariate** scenarios, but they involve more complex interpretation and assumptions
- Your book uses the terms "independent" and "dependent" to describe predictor and criterion variables, but I stay away from those terms unless the \_\_\_\_\_ of the study was experimental in nature

**!** Important

Linear regression often shows up in cross-sectional or longitudinal research where it is difficult to say it can fully support a causal claim. Be careful in describing what exactly a regression model shows about the relationship between two or more variables

## 2 Basic Review of Linear Equations

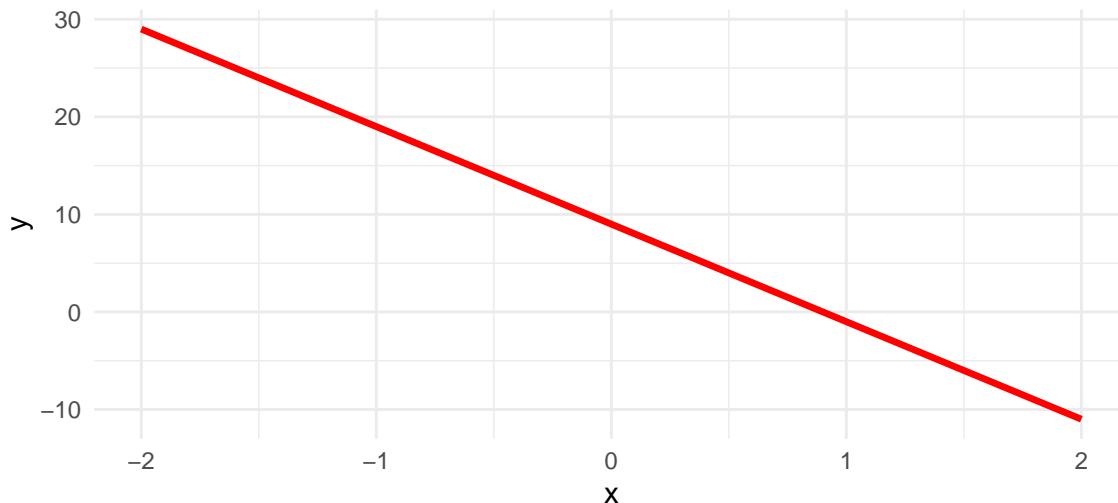
### 2.1 Introduction

- Many folks are introduced to the basic \_\_\_\_\_ equation in algebra and/or geometry:  $y = a + bx$ 
  - Where:
    - \*  $y$  = \_\_\_\_\_ variable
    - \*  $a$  = \_\_\_\_\_ or the value of  $y$  when  $x = 0$
    - \*  $b$  = \_\_\_\_\_
    - \*  $x$  = \_\_\_\_\_ variable
  - \_\_\_\_\_ forms include:  $y = b + mx$  or  $y = mx + b$ 
    - \* All of these formulas mean the \_\_\_\_\_ thing
  - Signs of the slope and y-intercept can be \_\_\_\_\_
- Examples:
  - $y = 8 + 4x$
  - $y = 12x - 3$
  - $y = -10x + 9$  - shown in the graph below

**Listing 1** Line Plot of  $y = -10x + 9$ 

```
x_vals <- seq(-2, 2, by = 0.1)
y_vals <- -10 * x_vals + 9 # NOTE: See equation here!
data <- data.frame(x = x_vals, y = y_vals)
```

```
ggplot(data, aes(x = x, y = y)) +
  geom_line(color = "red", size = 1.2) +
  labs(
    title = "",
    x = "x",
    y = "y"
  ) +
  theme_minimal()
```

**!** Important

Pay attention to where the 0 on the x-axis is! It may not always be on the far left

- If put on a \_\_\_\_\_ plot, this equation result in a straight line
  - If  $b > 0 \rightarrow$  upward to right
  - If  $b < 0 \rightarrow$  downward to right
  - If  $b = 0 \rightarrow$  straight horizontal across
  - A \_\_\_\_\_  $a / y$ -intercept will result in the line crossing the y-axis ( $x = 0$ ) at a higher point



? Looking at the 3 graphs above, which one has the lowest y-intercept

- A) Left
- B) Center
- C) Right
- D) All have the same y-intercept

Explanation:

## 3 Scatterplots & Graphical Methods

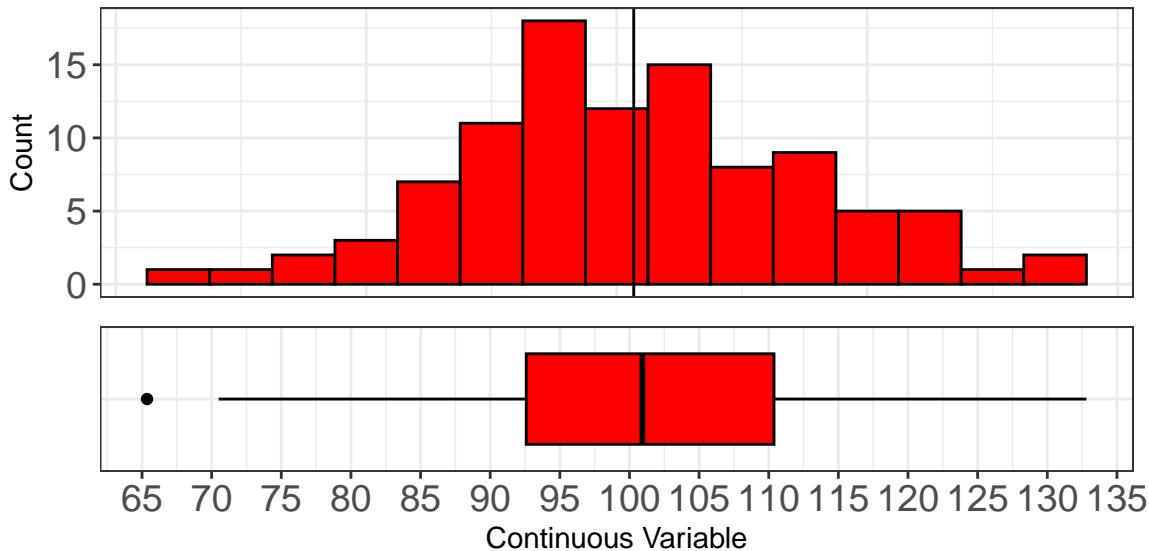
### 3.1 Introduction

- Early in the semester, we learned about graphical methods for showing information about individual, continuous variables, i.e., boxplots, histograms, stem-and-leaf plots

► Discuss: Review: Take a look at the histogram and boxplot below, what can you say to describe this variable based on what they plots tell you?

**Listing 2** Histogram and boxplot of a hypothetical continuous variable

```
histbox(normal_data, value, "Continuous Variable")
```

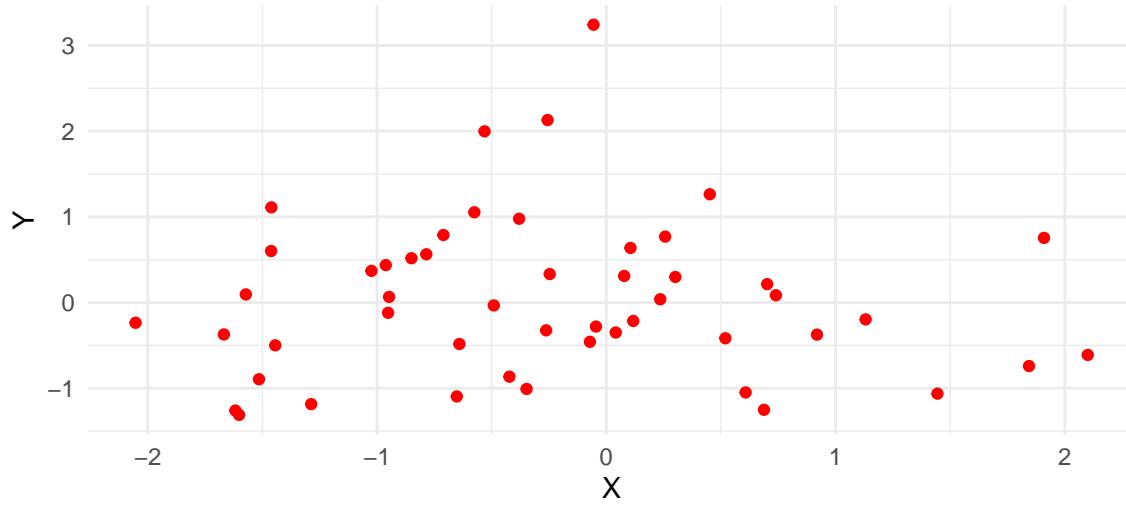


- In addition to ways we can graphically represent a single variable, we can also represent the \_\_\_\_\_ scenario with a **scatterplot**
  - A scatterplot will be set up so that one of our \_\_\_\_\_ is on the x-axis, and one of our variables is on the \_\_\_\_\_
  - It's convention to put the \_\_\_\_\_ variable on the x-axis, and the \_\_\_\_\_ variable on the y-axis, but it's not necessarily "wrong" to do the other way

**Listing 3** Scatterplot Example

```
# Create sample data
df <- data.frame(x = rnorm(50), y = rnorm(50))

# Create scatterplot with red points
ggplot(df, aes(x = x, y = y)) +
  geom_point(color = "red") +
  labs(title = "", x = "X", y = "Y") +
  theme_minimal()
```

**!** Important

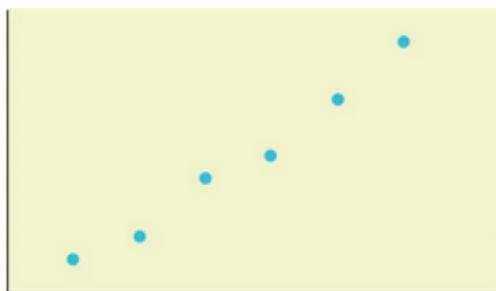
Remember, various graphs and plots are a great way for us to best understand our variables and relationships - we can and should use them often alongside our numeric results.

**3.2 Strength and Direction**

- In a scatterplot, we can \_\_\_\_\_ inspect the **strength** of the correlation, as well as the **direction** of the relationship
- The strength of a relationship indicates how closely \_\_\_\_\_ two variables are to one another
  - A \_\_\_\_\_ strength indicates close/strong relationship, and is visually shown by points on a scatterplot sitting along a clear linear trend

*"I don't mind not knowing. It doesn't scare me." — Richard P. Feynman*

- Conversely, a \_\_\_\_\_ strength would represent two variables not very related to one another, and be shown by a scatterplot where points do not seem to follow a line
- The direction of a relationship indicates how one variable \_\_\_\_\_ to change in the other
  - A **negative relationship** indicates that as one variable increases, the other \_\_\_\_\_
  - A **positive relationship** indicated that as one variable \_\_\_\_\_, so does the other



(a) Positive linear pattern (strong)

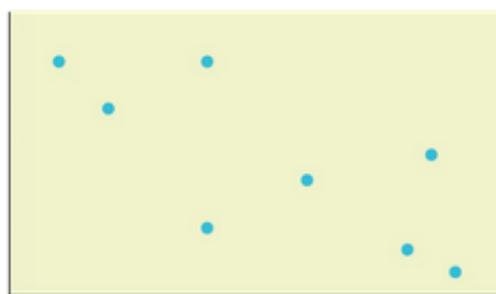


(b) Linear pattern w/ one deviation

Figure 12.6



(a) Negative linear pattern (strong)

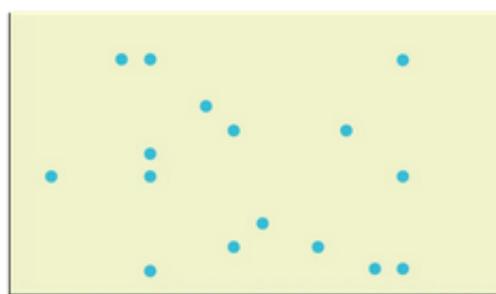


(b) Negative linear pattern (weak)

Figure 12.7



(a) Exponential growth pattern



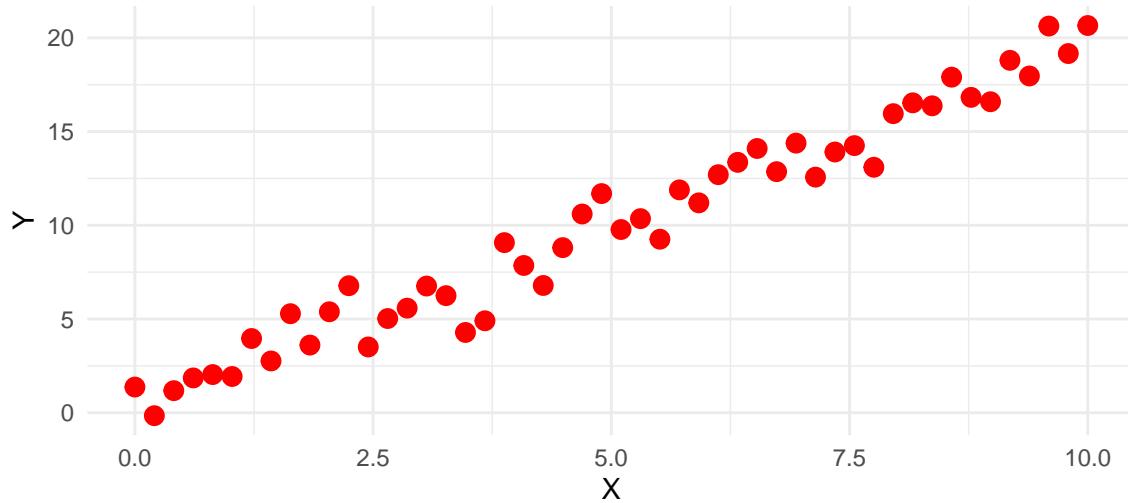
(b) No pattern

Discuss: Using the scatterplot below, describe the direction and strength of the relationship between the two variables x and y. Think about how an imaginary line between these points would look

#### Listing 4 Hypothetical Scatterplot

```
set.seed(42)
x <- seq(0, 10, length.out = 50)
y <- 2 * x + rnorm(50, mean = 0, sd = 1)
df <- data.frame(x = x, y = y)

# Create scatterplot
ggplot(df, aes(x = x, y = y)) +
  geom_point(color = "red", size = 3) +
  labs(title = "", x = "X", y = "Y") +
  theme_minimal()
```



## 4 Correlation Coefficient r

### 4.1 Introduction

- Pearson's Product-moment Correlation Coefficient or  $r$  for short, is the most commonly used statistic to numerically describe a \_\_\_\_\_, linear relationship between two variables
  - In this case, saying "non-directional" just means that it doesn't necessarily tell us about one predicting or \_\_\_\_\_ another

#### ! Important

It is incredibly important to remember that  $r$  can only be applied when we think there is a \*linear\* relationship between two variables. A scatterplot can help identify if this does appear to be a linear relationship

### 4.2 Calculation

$$r = \frac{n \sum (xy) - (\sum x)(\sum y)}{\sqrt{[n \sum x^2 - (\sum x)^2][n \sum y^2 - (\sum y)^2]}}$$

- Where:
  - $n$ : the number of \_\_\_\_\_ data points
  - $x$ : individual data points in variable  $X$
  - $y$ : individual data points in variable  $Y$

#### ! Important

Like with everything in statistics, there are several ways to write this same formula - but this is the most simplified version.

- We'll come back to work though this equation in the [Full Worked Example](#)

### 4.3 Interpretation

- Value:
  - Always true:  $-1 \leq r \leq 1$
  - $r = 0$  suggests \_\_\_\_\_ relationship

- $r = 1$  or  $r = -1$  suggest a \_\_\_\_\_ relationship, i.e., a straight line on a scatterplot
- $r$  closer to -1 or 1 (away from 0) suggests a \_\_\_\_\_ linear relationship
- Sign:
  - A \_\_\_\_\_ sign for  $r$  suggests a negative relationship
  - A \_\_\_\_\_ sign for  $r$  suggests a positive relationship
  - In relation to regression (which we'll discuss in a moment),  $r$  will have the same sign as  $b$ , or the slope of our line-of-best-fit
- Example:
  - I find that there is a strong positive correlation of GPA and SAT Score of  $r = 0.76$
  - I find that there is weak negative correlation between salary and happiness (assume continuous) of  $r = -0.21$
  - I find no relationship between mileage on cars and cost of cars of  $r = 0.02$

 Discuss: Describe the relationship between two variables with  $r = -0.80$

## 4.4 Significance

- $r$  is a \_\_\_\_\_ statistic, and its corresponding \_\_\_\_\_ in the population is represented as  $\rho$  (greek letter "rho")
  - We can \_\_\_\_\_ test to see if  $\rho$  is significantly different than 0

### ! Important

Confusingly, there is a similar correlation coefficient for ordinal data called Spearman's rho, but that is not the same as what we are talking about here.

? If we are testing our continuous, numeric coefficient against the arbitrary standard of 0, what test does that sound most similar to, of the ones we have covered so far?

- A) Welch's t-test
- B) One-sample t-test
- C) Chi-square goodness-of-fit
- D) Chi-square test-of-independence

Explanation:

- Our hypothesis pair:
  - $H_0 : \rho = 0$
  - $H_A : \rho \neq 0$
  - We decided whether we can \_\_\_\_\_ the null hypothesis that  $\rho = 0$
- There are several ways to calculate the p-value (which we won't do by hand), but one of the common ways is via the \_\_\_\_\_!

## 4.5 Coefficient of Determination (r-squared)

- We can also get  $r^2$ , called r-squared or the **coefficient of determination**
  - This is a way of representing how much variance in one variable is or accounted for by change in the other
- Example:
  - GRE scores correlate with later happiness,  $r = 0.50 \rightarrow r^2 = 0.25$ , thus 25% of the variance in happiness is explained by GRE score

Discuss: If I say that annual household spending and child GPA are correlated  $r = 0.12$ , write out how much variance is explained in the same manner as the above example.

## 4.6 Full Worked Example

- Consider the following small sets of data:
  - $X : 1, 2, 3, 4 \rightarrow \bar{x} = 2.5, s_x = 1.29$
  - $Y : 2, 3, 6, 3 \rightarrow \bar{y} = 3.5, s_y = 1.73$
  - $n = 4$  sets of data points

$$r = \frac{n \sum (xy) - (\sum x)(\sum y)}{\sqrt{[n \sum x^2 - (\sum x)^2][n \sum y^2 - (\sum y)^2]}}$$

i	X	Y	X·Y	$X^2$	$Y^2$
1	1	2	2	1	4
2	2	3	6	4	9
3	3	6	18	9	36
4	4	3	12	16	9
$\Sigma$	<b>10</b>	<b>14</b>	<b>38</b>	<b>30</b>	<b>58</b>

$$r = \frac{4(38) - (10)(14)}{\sqrt{[4(30) - 10^2] * [4(58) - 14^2]}} = 0.4472$$

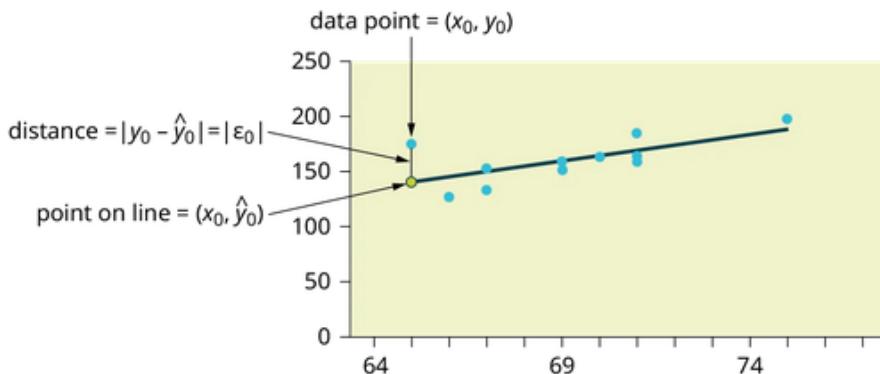
- $r = 0.4472$  is
  - A moderately strong, positive correlation coefficient
  - $r = 0.4472 \rightarrow r^2 = 0.20 \rightarrow 20\%$  of variance in Y is accounted for by X

## 5 Regression Equation

### 5.1 Introduction

- While graphical methods are useful, we often want \_\_\_\_\_ to back up our conclusions
- The purpose of simple linear \_\_\_\_\_ analysis is to “fit” or “draw” a linear, straight line that goes through the center of all of our points, sometimes called the **line-of-best-fit**
  - There are \_\_\_\_\_ methods for the exact equation is used to minimize the distance to each point from our line-of-best-fit or **error**
  - For our simple use, the most common method is the **ordinary least-squares**

## 5.2 Minimizing Error



- Our line-of-best-fit is made up of \_\_\_\_\_ many predicted or expected points with coordinates:  $(\hat{x}, \hat{y})$  and passes through our observed data with coordinates:  $(x, y)$ 
  - Thus, our error is the distance between the observed and the predicted points:  $|y - \hat{y}| = \epsilon$ , put another way, the \_\_\_\_\_ on the y-axis that the predicted line is away from the observed point
  - We get an  $\epsilon$  or error for each observed point:

$$(\epsilon_1)^2 + (\epsilon_2)^2 + \dots + (\epsilon_n)^2 = \sum_{i=1}^n \epsilon^2 = SSE$$

- The above equation gives us the **sum of squared errors (SSE)**
  - In least-squares regression, our goal is to \_\_\_\_\_ a line that \_\_\_\_\_ the SSE

## 5.3 The Regression Equation Calculation

- The line-of-best-fit that is fit to \_\_\_\_\_ SSE can be found in the sample, using this formula:

$$\hat{y} = a + bx$$

- Where:
  - $\hat{y}$ : our \_\_\_\_\_ y-value at any given point
  - $a = \bar{y} - b\bar{x}$ :
  - $b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum x-\bar{x}^2}$  or  $b = r\left(\frac{s_y}{s_x}\right)$ : line \_\_\_\_\_
  - $\bar{x}$ : Sample \_\_\_\_\_ of variable  $X$
  - $\bar{y}$ : Sample mean of variable  $Y$
  - $s_x$ : Sample standard \_\_\_\_\_ of variable  $X$

- $s_y$ : Sample standard deviation of variable  $Y$
- $r$ : Pearson's \_\_\_\_\_ coefficient
- As you can see, there are multiple steps and \_\_\_\_\_, but our hope is to get to a point in which we have values for both  $a$  and  $b$

## 5.4 Worked Example

- Consider the following small sets of data:
  - $X : 1, 2, 3, 4 \rightarrow \bar{x} = 2.5, s_x = 1.29$
  - $Y : 2, 3, 6, 3 \rightarrow \bar{y} = 3.5, s_y = 1.73$
  - $n = 4$  sets of data points
  - $r = 0.4472$
- Regression math
  - $a = 3.5 - 0.60(2.5) = 2$
  - $b = 0.4472(\frac{1.73}{1.29}) = 0.60\text{-ish}$
  - Final line equation:  $\hat{Y} = 2.0 + 0.60x$
  - Practical reading: For every one unit increase in X, there is a 0.60 unit increase in Y; at  $x = 0, y = 2.0$
- Predicting Y off specific value of X
  - If  $x = 2$  then...
  - $\hat{Y} = 2.0 + 0.60(2) = 3.2$

## 6 Conclusion

### 6.1 Recap

- Correlation and regression both give us some helpful ways of calculating information about the relationships between two continuous variables (bivariate)
- One can determine both the direction and relative strength of the relationships between the variables
- One can also make a determination on predicting one variable based upon another, with the caveat that there can always be the chance of error in that prediction

## Key Terms

### B

**bivariate** A description of data that is two variables 2

### C

**coefficient of determination** r-squared, the percent of variable y variance explained by variable x 13

**criterion** A description of a variable that is treated as an outcome in a regression model 2

### D

**direction** In correlation, the trend of how one variable reacts to change in another 7

### E

**error** The distance between the predicted value and the observed value 14

### L

**line-of-best-fit** In regression, the imaginary line drawn that minimize errors across the points 14

### M

**multivariate** A description of data that is more than two variables 2

### N

**negative relationship** In correlation, a trend where, as one variable increases, the other decreases 8

### O

**ordinary least-squares** The mostly commonly used method to calculate error from each point from a regression line 14

### P

**positive relationship** In correlation, a trend where, as one variable increases, so does the other 8

**predictor** A description of a variable that suggests that it is used as one of the data points to predict an outcome in a regression model 2

### S

**scatterplot** A graphical way to represent two numeric variable with one variable on the x-axis and one variable on the y-axis 6

**strength** In correlation, how strongly related two variables are **7**

**sum of squared errors (SSE)** In regression, the total sum of the of all of squared errors resulting from differences in the predicted and observed values **15**

*The instructor-provided glossary may not include all terms worth memorizing, make sure you consider using the vocabulary list in your book and your own judgment to make sure you have all relevant terms*