Prep Course

Module I Homework 2

Due 23:30, Saturday, July 20th, 2024 (Chicago time)

For this Homework, you should submit a single PDF named: **M1HW2_LASTNAME_Firstname.pdf**

Problem 1

Do P1.38.

Problem 2

Let's assume that 20% of the population in a country do not have any health insurance. Let X equal the number with no health insurance in a random sample of $n = 15$ people. Find $\mathbb{P}(X \geq 2)$. Please round your answer to four decimal places.

Problem 3

Suppose that an airplane engine will fail, when in flight, with probability $1 - p$ independently from engine to engine; suppose that the airplane will make a successful flight if at least 50 percent of its engines remain operative. For what values of p is a four-engine plane preferable to a two-engine plane?

Problem 4

Consider a deck of 52 cards, ordered such that $A > K > Q > \cdots > 3 > 2$. Player A randomly picks one first, then player B randomly picks one out of the rest. What is the probability that player A's card is greater than player B's? Please round your answer to four decimal places.

Problem 5

The article

https://www.cnn.com/2020/05/26/health/antibody-tests-cdc-coronavirus-wrong/index.html

quotes the CDC's statement that

"in a population where the prevalence is 5%, a test with 90% sensitivity and 95% specificity will yield a positive predictive value of 49%. In other words, less than half of those testing positive will truly have [COVID-induced] antibodies. . . Alternatively, the same test in a population with an antibody prevalence exceeding 52% will yield a positive predictive greater than 95%, meaning that less than one in 20 people testing positive will have a false positive test result."

How did the CDC obtain the numbers 49% and 95% that are marked in grey? Provide calculations for each of these two numbers.

Definitions:

- "Positive predictive value" is the probability of COVID, given a positive test result.
- "Prevalence" is the percentage of the population that had COVID.
- "Sensitivity" is the probability of a positive test, given a test-taker who had COVID.
- "Specificity" is the probability of a negative test, given a test-taker who did not have COVID.

Hints:

Start by writing the CDC's assumptions using the notation of probabilities and conditional probabilities. For example, positive predictive value is $\mathbb{P}(\text{COVID}|\text{positive})$. Prevalence is $\mathbb{P}(\text{COVID})$, which is the prior probability, before considering the evidence from the test. Then use Bayes' rule. (Note that this article was from the pre-vaccine era, when the presence of those antibodies was highly indicative of COVID history, as those antibodies could not have been attributable to vaccinations which were not available yet.)