# Prep Course

# Module I Homework 6

Due 23:30, Saturday, August 17th, 2024 (Chicago time)

For this Homework, you should submit a single PDF named: **M1HW6_LASTNAME_Firstname.pdf**

## Problem 1

Let $(X, Y)$ have joint density

$$f(x, y) = 2x \quad \text{for } 0 \leq x \leq 1, 0 \leq y \leq 1$$

and 0 elsewhere.

(a) Find $\mathbb{P}(XY > z)$ for $0 \leq z \leq 1$. Your final answer should be a function of $z$. (Hint: if you pick up a particular $z$, say, $\frac{1}{2}$, what is the area within the unit square of $0 \leq x \leq 1$ and $0 \leq y \leq 1$ such that $xy > z$? P1.68 shows what you need to do, i.e., a double integral.)

(b) Find the cumulative distribution function of the random variable $Z \coloneqq XY$. Your final answer should be a function of $z$. (Hint: what's the definition of CDF? Can you leverage the result of (a) to figure out the CDF?)

(c) Find $\mathbb{E}Z$. Your final answer should be a number. (Hint: now that you have the CDF, figure out the PDF, and then follow P2.7.)

Each of (a), (b), and (c) is worth 10/10 points.

## Problem 2

Find all the maxima and inflection points of the following function:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

## Problem 3

<u>No justification or proof is required for this problem.</u>

The percentage of time $X$ that a lathe is in use during a typical 40-hour workweek is a random variable whose probability density function is given by

$$f(x) = \begin{cases} 3x^2 & for\ 0 \leq x \leq 1 \\ 0 & elsewhere \end{cases}.$$

Then the actual number of hours, out of a 40-hour week, that the lathe is not in use is

$$U = 40(1 - X).$$

Which one of the following options is the CDF for $U$?

(a) $1 - \left(1 - \frac{u}{40}\right)^3$

(b) $\left(\frac{u}{40}\right)^3$

(c) $\left(\frac{1}{2} - \frac{u}{80}\right)^3$

(d) $\left(\frac{u}{40} - \frac{1}{3}\right)^2$

# Problem 4

No justification or proof is required for this problem.

Following Problem 3, which one of the following options is the probability density function for $U$?

(a) $\frac{3}{40}\left(1 - \frac{u}{40}\right)^2$

(b) $\frac{3u^2}{64000}$

(c) $-\frac{3(u-40)^2}{512000}$

(d) $\frac{u}{800} - \frac{1}{60}$