Prep Course

Module I Homework 7

Due 23:30, Saturday, August 24th, 2024 (Chicago time)

For this Homework, you should submit a single PDF named: **M1HW7_LASTNAME_Firstname.pdf**

Problem 1

Do P2.14.

Hint: for each slot in the queue, define an indicator random variable. Apply the rule of total conditional expectation to evaluate the expectation of each indicator. Note that different indicators might need to be treated differently based on where their slot is in the queue. If possible, run a Monte Carlo simulation to verify your answer.

This problem is worth 15/10.

Problem 2

Do P2.18 but with dice rolls instead of coin flips: What is the expectation of the number of times you need to roll a fair die until you get two 6's in a row?

Problem 3

Do P2.19, but with a square instead of a cube: A bug is at one corner of a square. What's the expectation of the number of steps it takes, to reach the opposite corner? Each step takes it to an adjacent corner, with either corner equally likely.

Problem 4

Do P2.10.

Apply the bottom formula on P2.8. If we measure time in hours starting from noon, then each arrival time is uniformly distributed in [0,1], so the joint density of the two arrival times (X, Y) is $f(x, y) = 1$ for $0 \leq x \leq 1$, $0 \leq y \leq 1$. How to express the waiting time $g(x, y)$ in terms of x and y?