Module I Homework 1

qquantt

Prep24AutumnM1

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Problem 1a.

Intuition: $P(A \cup B)$

Solution. □

$$\begin{aligned} \frac{1}{3} + \frac{1}{3} - \left(\frac{1}{3}\right)^2 &= \frac{5}{9} \\ &\approx \boxed{0.5556} \end{aligned}$$

Problem 1b.

Intuition: $P(A \cap B \mid C)$

Solution. □

$$\begin{aligned} \frac{\left(\frac{1}{3}\right)^2}{\frac{5}{9}} &= \frac{1}{5} \\ &= \boxed{0.2000} \end{aligned}$$

Problem 2.

Intuition: Find the count of favorable arrangements (*i.e.*, that satisfy that no women sit at table A) in consideration of the count of total possible arrangements

Solution. □

$$\begin{aligned} p &= \frac{\binom{17}{5}\binom{15}{5}\binom{10}{5}\binom{15}{5}}{\binom{20}{5}\binom{15}{5}\binom{10}{5}\binom{15}{5}} \\ &= \frac{\binom{17}{5}}{\binom{20}{5}} \\ &= \frac{6188}{15504} \\ &\approx \boxed{0.3991} \end{aligned}$$

Problem 3.

Intuition: Independent events with unconditional probability

Solution. □

$$\begin{aligned} P(Party) &= 1 - P(NotSunnySaturday \cap NotSunnySunday) \\ &= 1 - (0.20)(0.60) \\ &= 1 - 0.12 \\ &= \boxed{0.8800} \end{aligned}$$

Problem 4.

Intuition: Independent events with conditional probability

Solution. □

$$\begin{aligned} &\frac{P(SunnyOnSaturday \cap NotSunnyOnSunday) + P(SunnyOnSunday \cap NotSunnyOnSaturday)}{P(Party)} \\ &= \frac{0.8 \times 0.6 + 0.2 \times 0.4}{0.88} \\ &= \frac{0.56}{0.88} \\ &\approx \boxed{0.6364} \end{aligned}$$

Problem 5.

Intuition: There are various ways to apply the multiplication rule based on interpretations of the problem

Solution. □

$$\begin{aligned} &\frac{52 \times 12}{52 \times 51} \\ &= \frac{4 \times 13 \times 12}{52 \times 51} \\ &= \frac{4 \times \binom{13}{2}}{\binom{52}{2}} \\ &\approx \boxed{0.2353} \end{aligned}$$