Prep Course

Module I Homework 8

Due 23:30, Saturday, August 31st, 2024 (Chicago time)

For this Homework, you should submit a single PDF named: **M1HW8_LASTNAME_Firstname.pdf**

Please note that in this homework every time before you apply L'Hospital's rule, you must show that either the $\frac{0}{0}$ or $\frac{\infty}{\infty}$ indeterminate form is satisfied; otherwise, you may receive 0 credits for the problem, even if your final answer is correct.

Problem 1

Evaluate $\lim_{x\to\infty}\left(\frac{x+a}{x-a}\right)^x$, where $a \in \mathbb{R}$

Problem 2

No justification or proof is required for this problem.

Using the setup on slide 4 of Calculus Lecture 1 and assuming $S_0, K,\ T,\ \sigma$ are all positive and $r = 0$, let's define the following two numbers:

$$m \coloneqq \mathbb{E}[(S_T - K)^+] \text{ and } n \coloneqq [(\mathbb{E}S_T) - K]^+.$$

Which of the following is true?

(a) $m > n$;
(b) $m = n$;
(c) $m < n$;
(d) any of the above three options is possible.

Problem 3

The random variable E follows an Exponential distribution with parameter $\lambda > 0$. The probability density function of an Exponential distribution with parameter λ is as follows:

$$f_E(x) = \lambda e^{-\lambda x}, where\ x \geq 0$$

Given $t < \lambda$, please find its moment generating function, i.e., please evaluate the following integral:

$$M_E(t) = \mathbb{E}[e^{tE}] = \int_0^\infty e^{tx} f_E(x)dx.$$

Problem 4

No justification or proof is required for this problem.

A wise man presents you with an urn containing an equal number of red and black marbles. Now you reach in and draw two marbles at random without replacement. Let p denote the probability that the two marbles are of matching color. Which of the following is true?

a) $p < \frac{1}{2}$
b) $p = \frac{1}{2}$
c) $p > \frac{1}{2}$

Problem 5

Evaluate

$$\lim_{x \to 0^+} x^x$$

Problem 6

Evaluate

$$\lim_{x \to 1} \frac{x^{\frac{3}{2}} - \frac{3}{2}x + \frac{1}{2}}{(x-1)^2}$$

Problem 7

(a) Find the Taylor expansion (without remainder) of $\sin x$ at $x = 0$.
(b) Find the Taylor expansion (without remainder) of $\log x$ at $x = 1$.

Each part is worth 10 points.