Neural Networks and Learning Systems
TBMI26 / 732A55
2023

Lecture 6
Ensemble Learning

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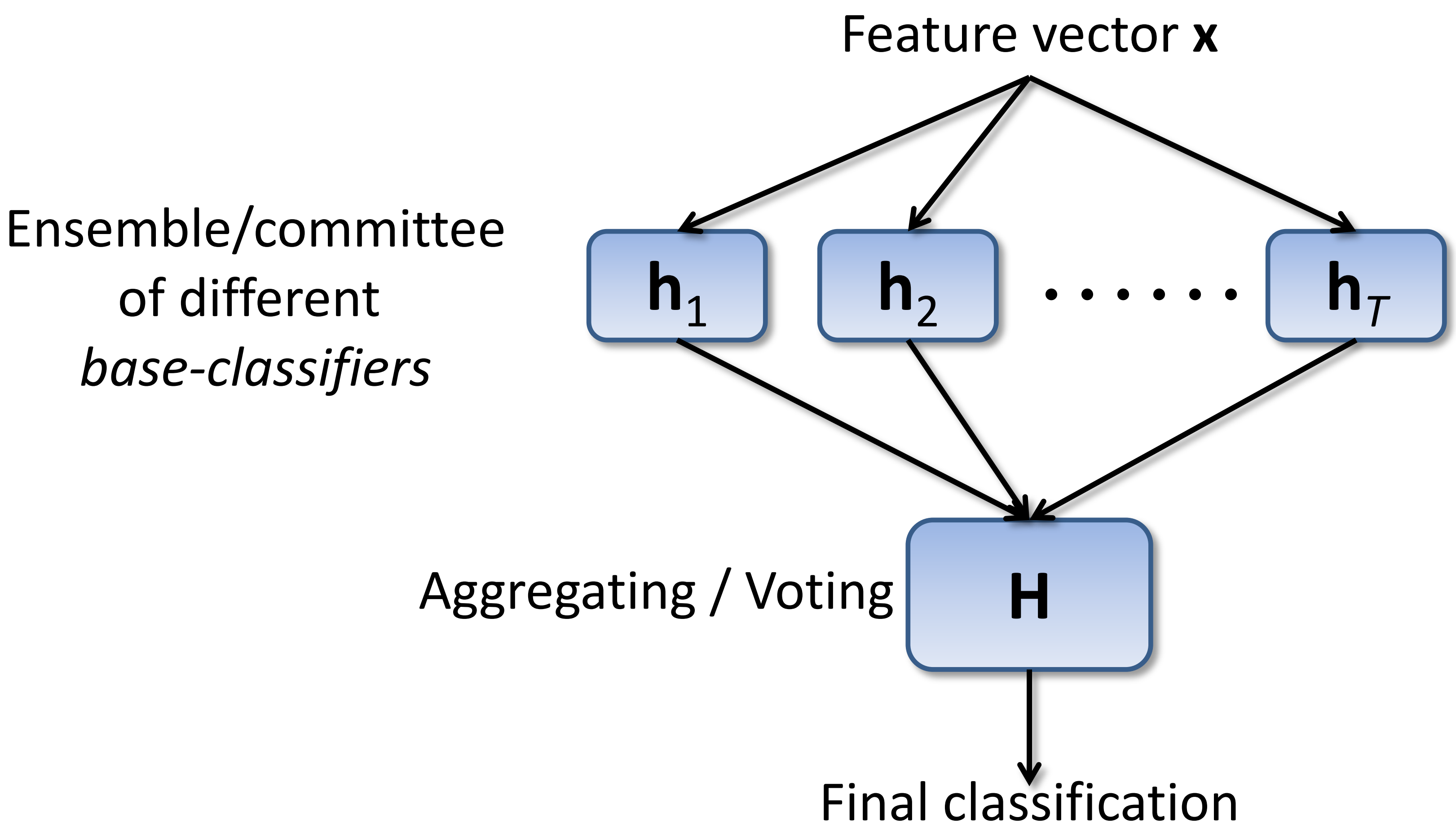
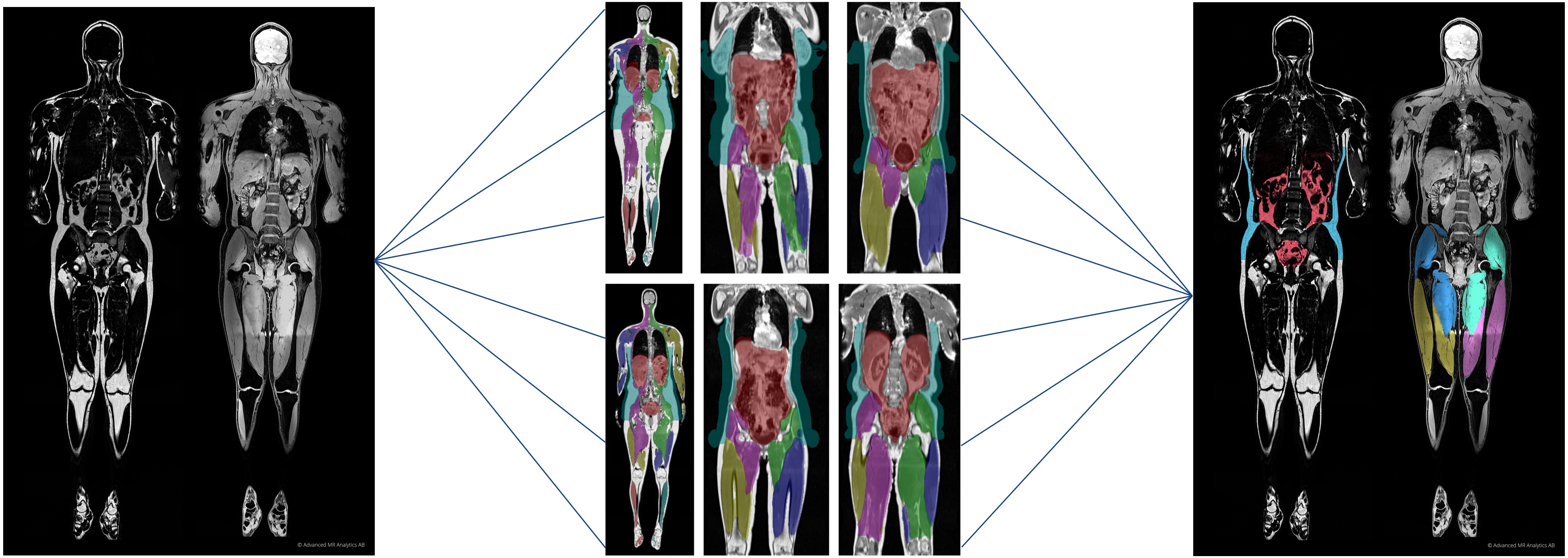
Classifier ensemble



Example – Atlas-based image segmentation

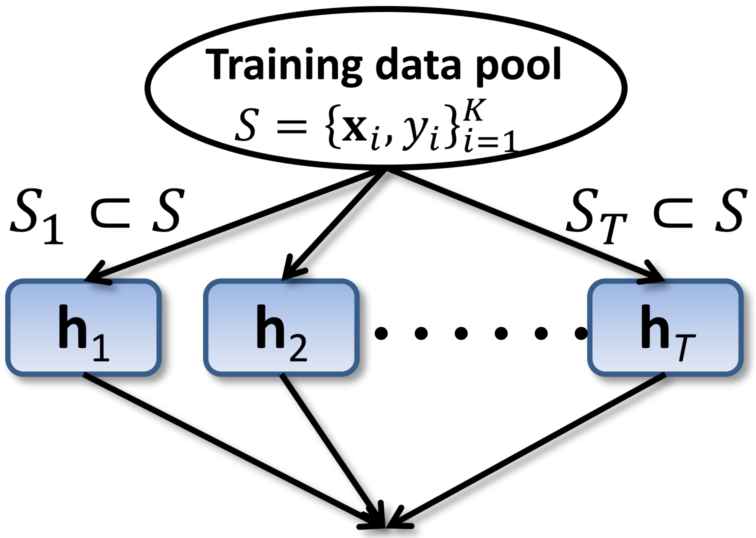


Images from AMRA® Medical AB

Bootstrap **Agg**regat**ing** (Bagging)

Breiman,1994

Train each base-classifier using a subset of the training data



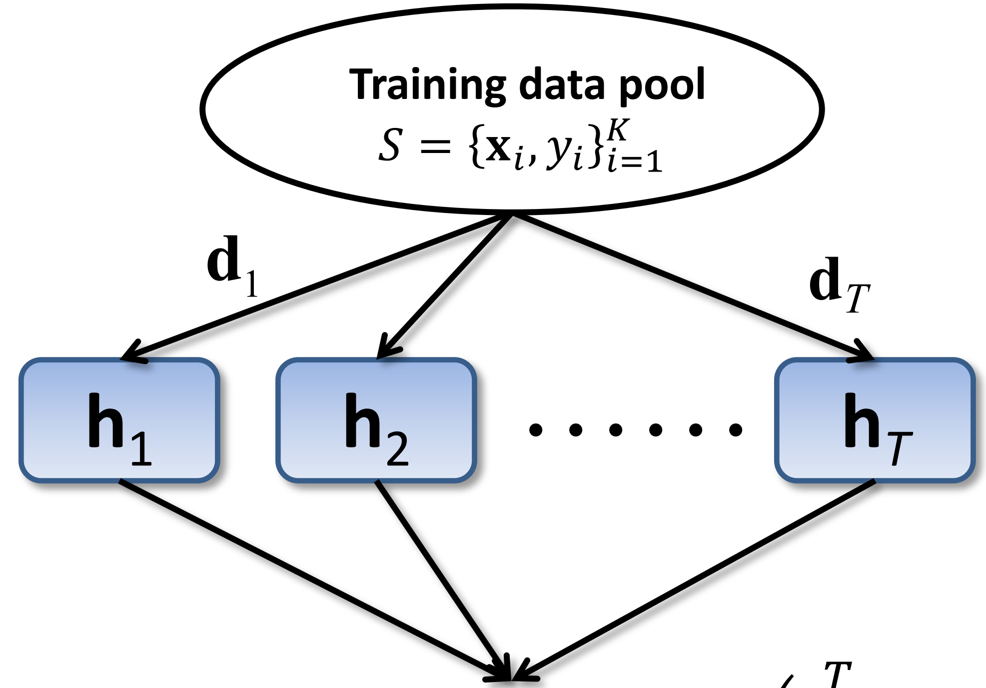
Reduced risk of overfitting

Boosting

Schapire and others, (1989-1990)

Train each base-classifier using all training data but with weights indicating how important each training sample is.

$$\mathbf{d} = \{d_i\}_{i=1}^{K}$$



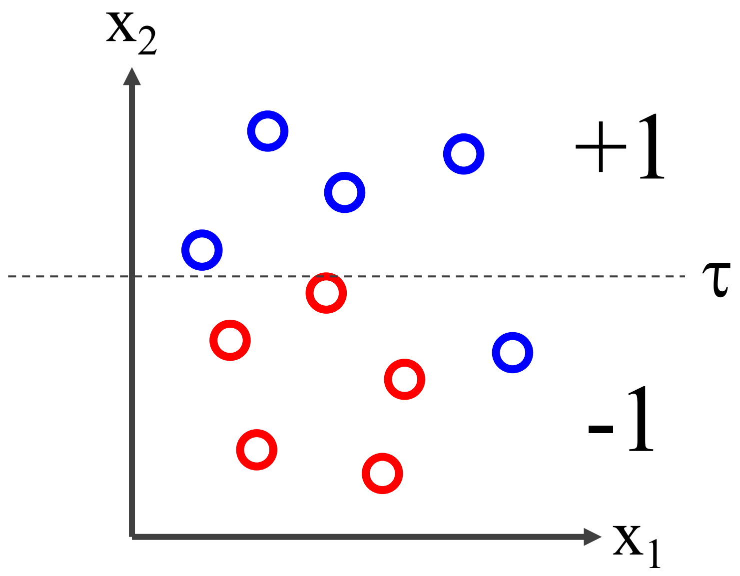
$$H(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(\mathbf{x})\right)$$

Simple/Weak classifiers

$$\mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}$$

Example: Threshold ***one*** feature

$$h(x_2) = \begin{cases} +1 & x_2 \geq \tau \\ -1 & x_2 < \tau \end{cases}$$

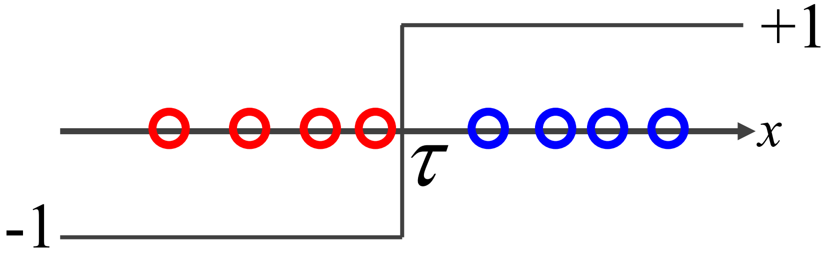


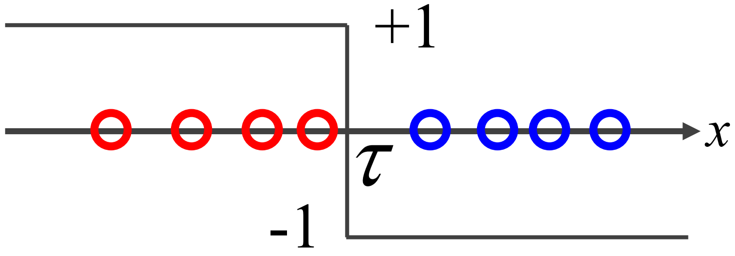
Decision stump

Polarity $p = \{-1, 1\}$

$p = 1$ $\qquad\qquad$ $p = -1$

$$h(x) = \begin{cases} +1 & x \geq \tau \\ -1 & x < \tau \end{cases} \qquad\qquad h(x) = \begin{cases} +1 & x \leq \tau \\ -1 & x > \tau \end{cases}$$

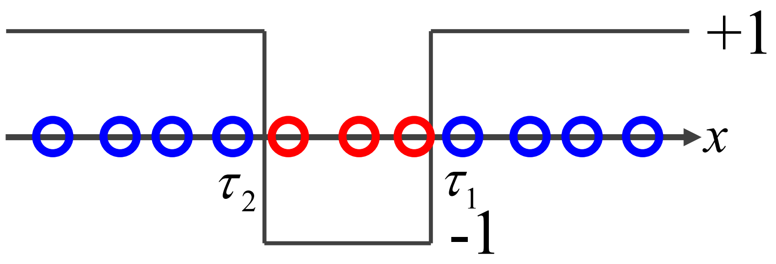




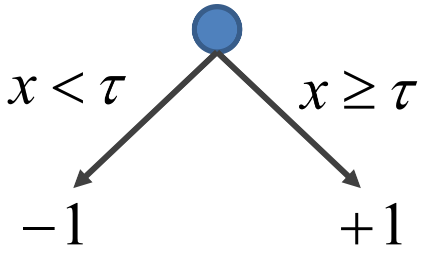
Classification and Regression Trees (CART)

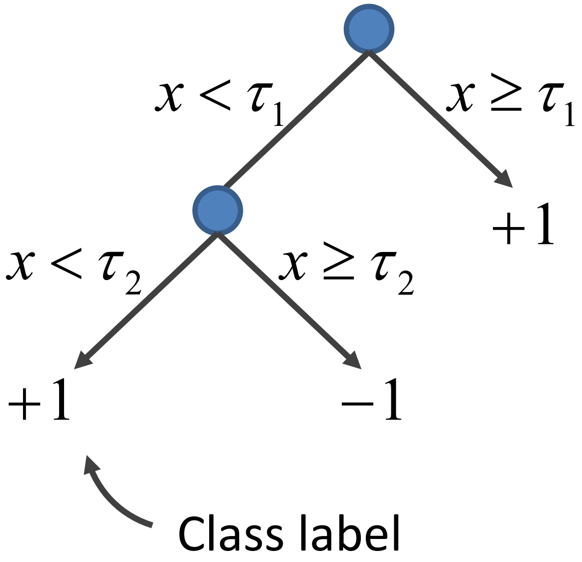
$$h(x) = \begin{cases} +1 & x \geq \tau \\ -1 & x < \tau \end{cases}$$

$$h(x) = \begin{cases} +1 & x \geq \tau_1 \\ -1 & x < \tau_1 \text{ and } x \geq \tau_2 \\ +1 & x < \tau_2 \end{cases}$$

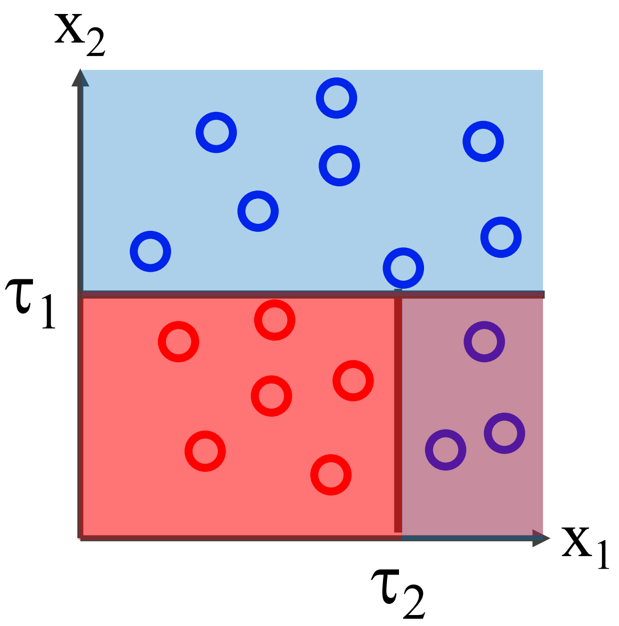


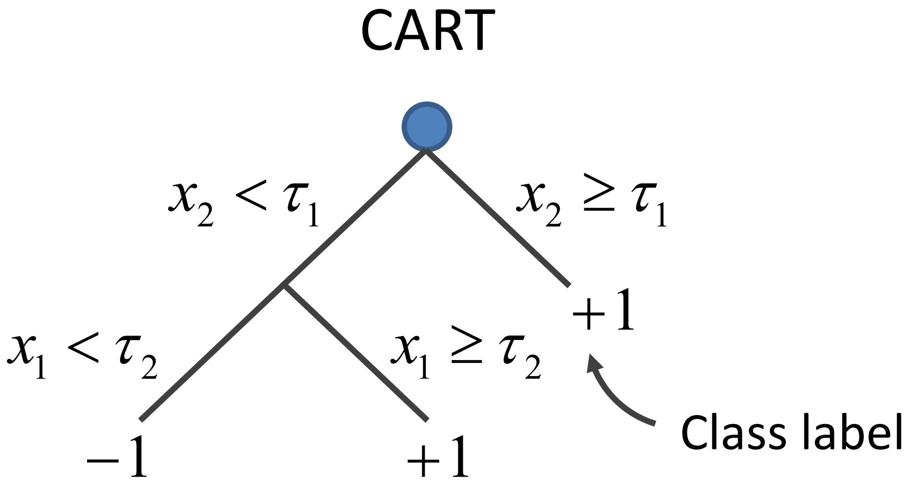
Decision stump





CART – 2D example





Piecewise flat classification function

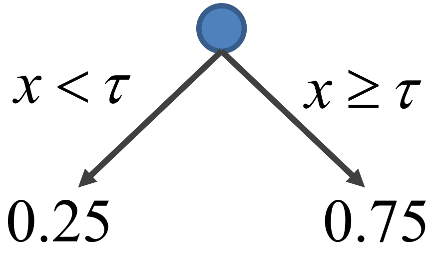
$$f(\mathbf{x}; \underbrace{w_1, \dots, w_N}_{\text{Feature indices, thresholds and polarities}}) \rightarrow \Omega$$

Regression Tree

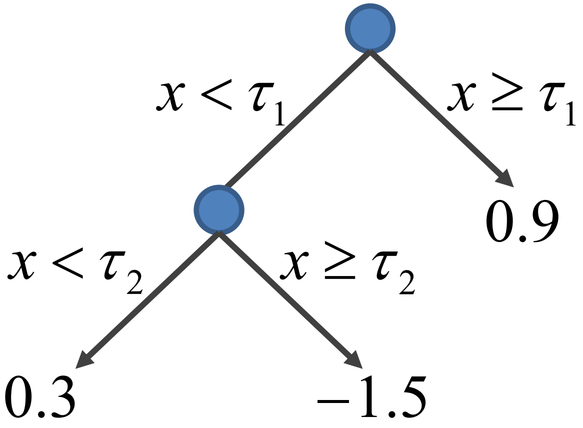
Same, but with real-valued output!

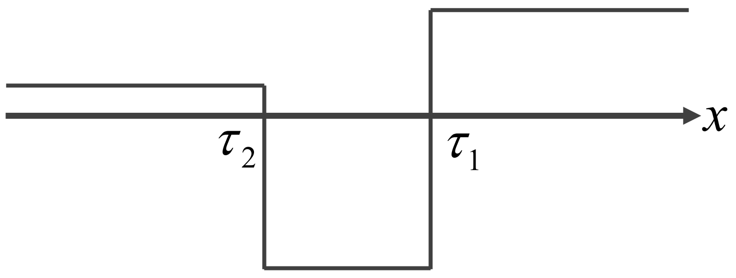
$$h(x) = \begin{cases} 0.75 & x \geq \tau \\ 0.25 & x < \tau \end{cases}$$

Decision stump

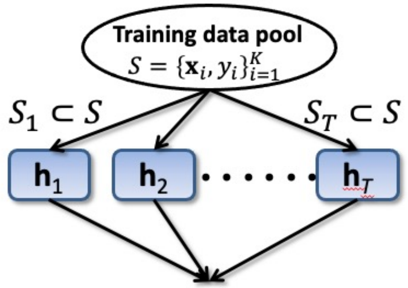


$$h(x) = \begin{cases} 0.9 & x \geq \tau_1 \\ -1.5 & x < \tau_1 \text{ and } x \geq \tau_2 \\ 0.3 & x < \tau_2 \end{cases}$$

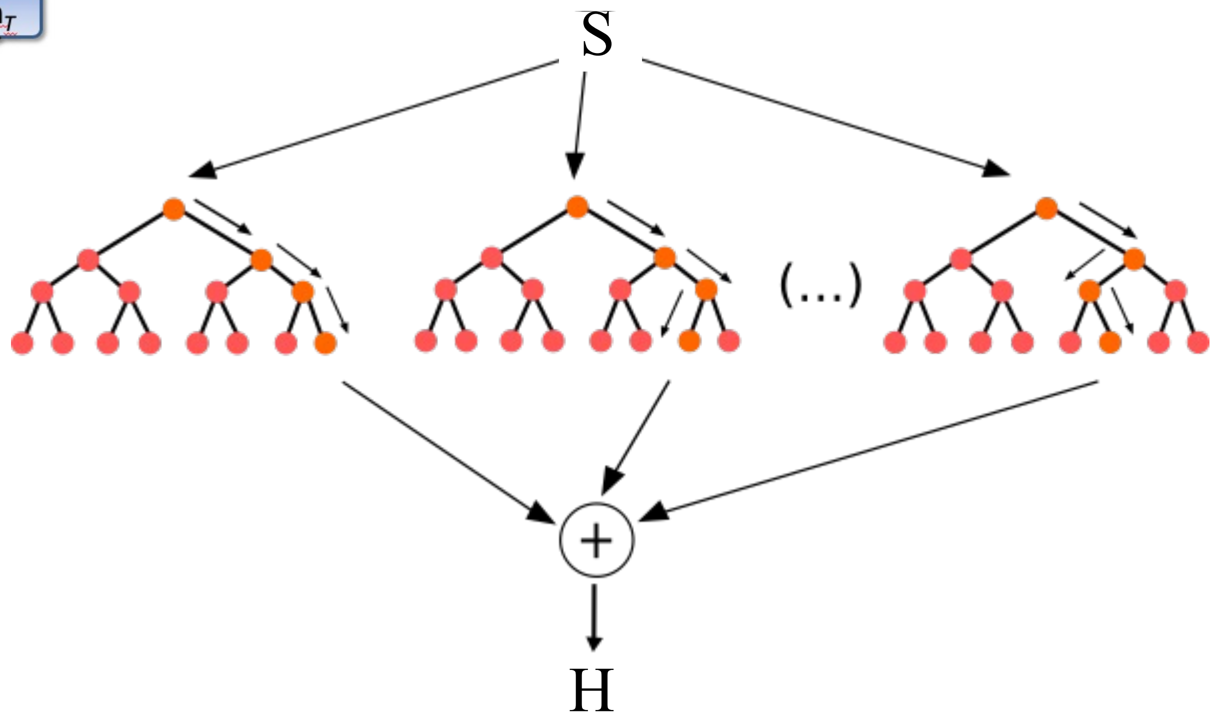




Random Forest



Bagging + Decision Trees



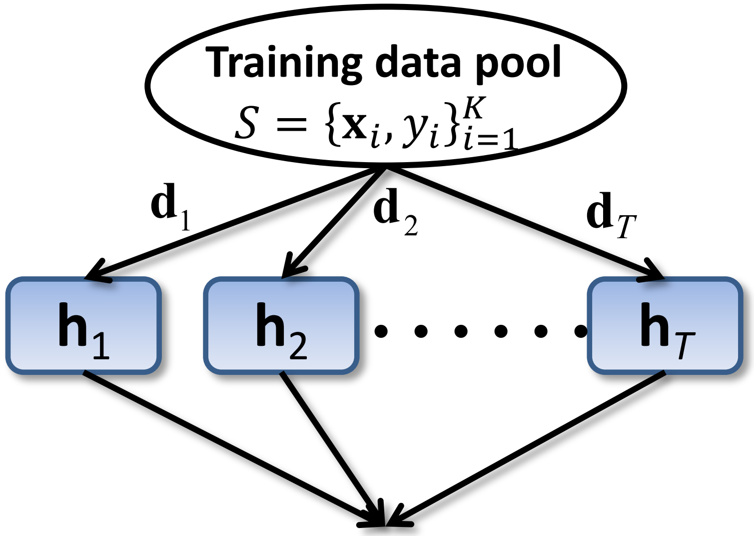
1. For each tree, select a random subset of the training samples
2. For each split, select a random subset of the features

} Make the trees "independent"

Image from https://towardsdatascience.com/random-forest-learning-essential-understanding-1ca856a963cb

General boosting algorithm

Train weak classifiers sequentially!



$$H(\mathbf{x}) = \operatorname{sign}\left(\sum_{t=1}^{T} \alpha_t \mathrm{h}_t(\mathbf{x})\right)$$

1. Set weights $d_1 = 1/K$
2. Train weak classifier $h_1(\mathbf{x})$ using weights $\mathbf{d}_1$
3. Increase and decrease weight for wrongly and correctly classified training examples respectively -> $\mathbf{d}_2$
4. Train weak classifier $h_2(\mathbf{x})$ using weights $\mathbf{d}_2$
5. Repeat until $h_T(\mathbf{x})$

Training a decision stump

Find best split threshold τ!

Class label {-1,+1}

Training input: $\{x_i, y_i, d_i\}_{i=1}^{K}$ Normalized weights: $\sum_{i=1}^{K} d_i = 1$

Consider only one feature

Weight

Threshold function: $h(x;\tau,p) = \begin{cases} +1 & p\,x \geq p\,\tau \\ -1 & p\,x < p\,\tau \end{cases}$

Polarity {-1,1}

weighted 0-1 loss function: $\min_{\tau,p} \varepsilon(\tau,p) = \sum_{i=1}^{K} d_t(i) I(y_i \neq h_t(\mathbf{x}_i))$

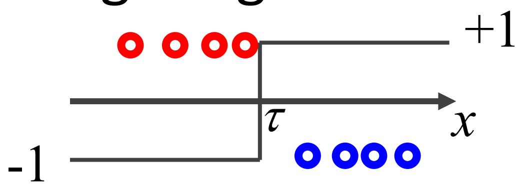
1 for false classifications

Training a decision stump, cont.

$$\min_{\tau,p} \varepsilon(\tau,p) = \sum_{i=1}^{K} d_i\, I\big(y_i \neq h(x_i;\tau,p)\big) \text{is always} \leq 0.5!$$

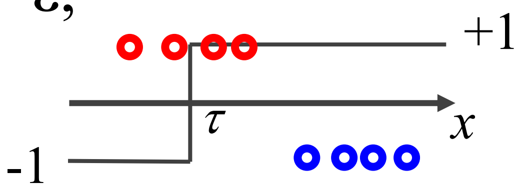
Why? If we classify <u>all</u> training samples wrong we get:

$$\varepsilon(\tau,p) = \sum_{i=1}^{K} d_i = 1$$



But we can then just change polarity/sign and get all samples correct, i.e., $\varepsilon = 0$!

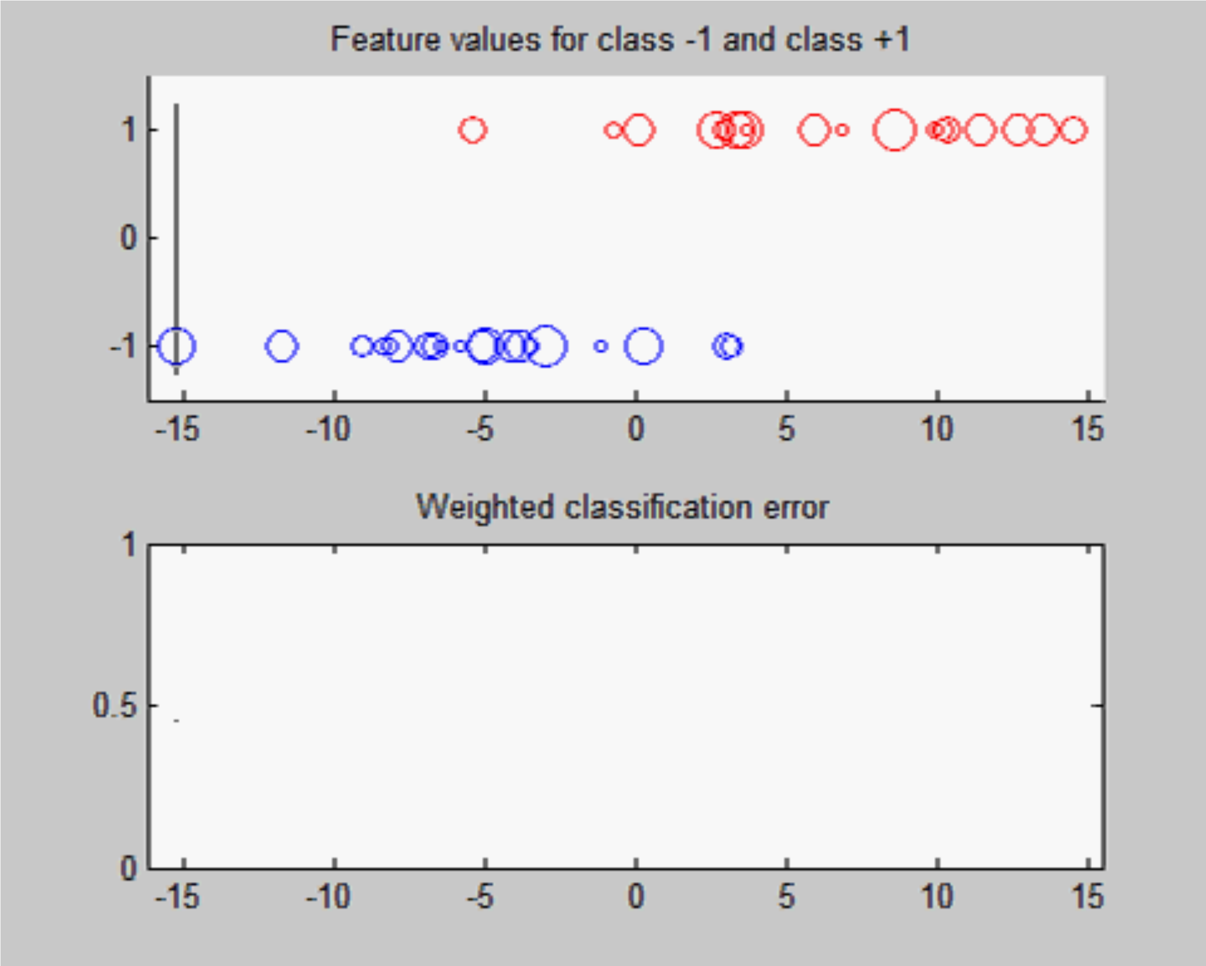
In general, if we obtain a loss ε between 0.5 and 1.0, we can switch polarity and get the loss $1.0 - \varepsilon$, which is smaller than 0.5.



The Loss Function - Example

$$\min_{\tau,p} \varepsilon(\tau,p) = \sum_{i=1}^{K} d_i \, I(y_i \neq h(x_i;\tau,p))$$

Movie!

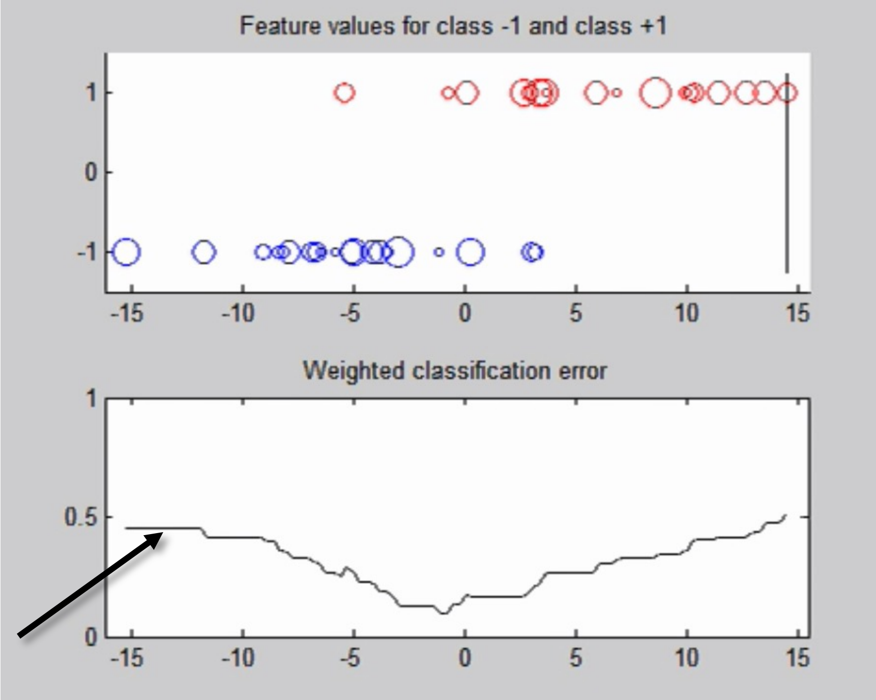


Brute force optimization

$$\min_{\tau,p} \varepsilon(\tau,p) = \sum_{i=1}^{K} d_i \, I(y_i \neq h(x_i;\tau,p))$$

Loss-function jumps at the x_i:s. Enough to test all thresholds in the set $\tau \in \{x_i\}_{i=1}^{K}$ and see which one gives the smallest error.

Loss function is piece-wise constant!



Brute force optimization

$$\text{Training samples } \mathbf{x}_i = \begin{pmatrix} x_{1,i} \\ \vdots \\ x_{N,i} \end{pmatrix}, i = 1 \cdots K$$

training samples

feature components

Pseudo code:

$\varepsilon_{\min} = \inf$

for all feature components k = 1:N

  for all thresholds $\tau \in \{x_{k,i}\}_{i=1}^{M}$

$$\varepsilon(\tau, p = 1) = \sum_{i=1}^{K} d_i I(y_i \neq h(x_{k,i}; \tau, p = 1))$$

    if $\varepsilon > 0.5$

      $p = -1$

      $\varepsilon = 1 - \varepsilon$

    end

   if $\varepsilon < \varepsilon_{\min}$ ...... **end**

  end

end

Discrete AdaBoost

Freund & Schapire, 1995

Training data: $\{\mathbf{x}_i, y_i\}_{i=1}^{K}, y_i \in \{-1, +1\}$

Initialization: $d_1 = \frac{1}{K},\ T =$ # base classifiers

for t = 1 to *T*

Find weak classifier $h_t(\mathbf{x}) = \{-1,+1\}$ that minimizes the weighted classification error: $\varepsilon_t = \sum_{i=1}^{K} d_t(i) I(y_i \neq h_t(\mathbf{x}_i))$

Update weights:

$$d_{t+1}(i) = d_t(i) e^{-\alpha_t y_i h_t(x_i)}, \text{ where } \alpha_t = \frac{1}{2} \ln \frac{1-\varepsilon_t}{\varepsilon_t}$$

and renormalize so that $\sum_{i=1}^{K} d_{t+1}(i) = 1$

end

Final strong classifier: $H(\mathbf{x}) = \operatorname{sign}\left(\sum_{t=1}^{T} \alpha_t \mathrm{h}_t(\mathbf{x})\right)$

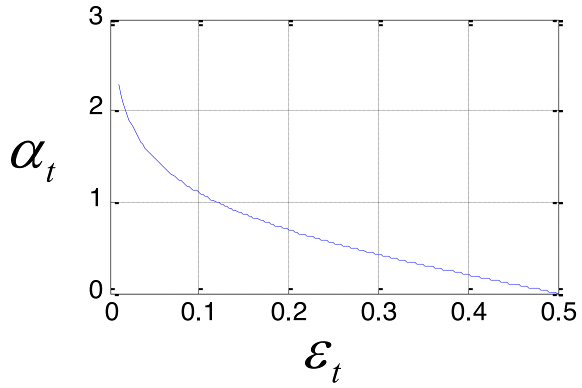
Discrete AdaBoost

- Discrete output from the weak classifier
  $h_t(\mathbf{x}) = \{-1,+1\}$
- Weight update

$$d_{t+1}(i) = d_t(i)e^{-\alpha_t y_i h_t(\mathbf{x}_i)} = \begin{cases} d_t(i)e^{-\alpha_t} & \text{If } \mathbf{x}_i \text{ correctly classified} \\ d_t(i)e^{\alpha_t} & \text{If } \mathbf{x}_i \text{ wrongly classified} \end{cases}$$

- Performance of weak classifier:

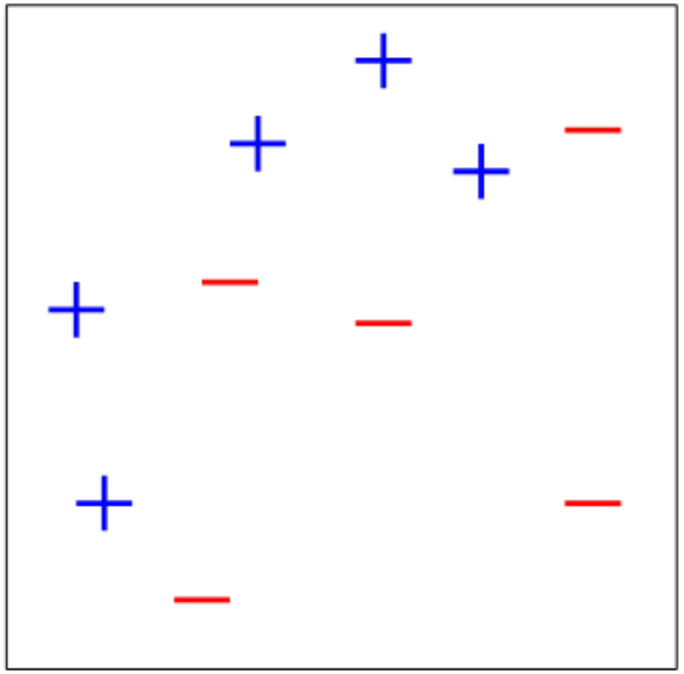
$$\alpha_t = \frac{1}{2}\ln\frac{1-\varepsilon_t}{\varepsilon_t}$$



Final strong classifier: $H(\mathbf{x}) = \mathrm{sign}\left(\sum_{t=1}^{T} \alpha_t \mathrm{h}_t(\mathbf{x})\right)$

Discrete AdaBoost – Toy example

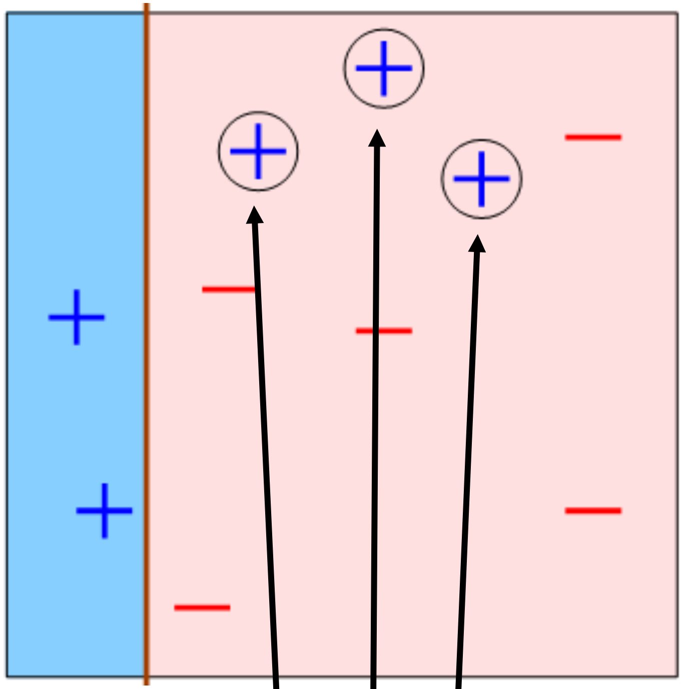
Initial weights $d_1(i) = \frac{1}{10}$, $T = 3$

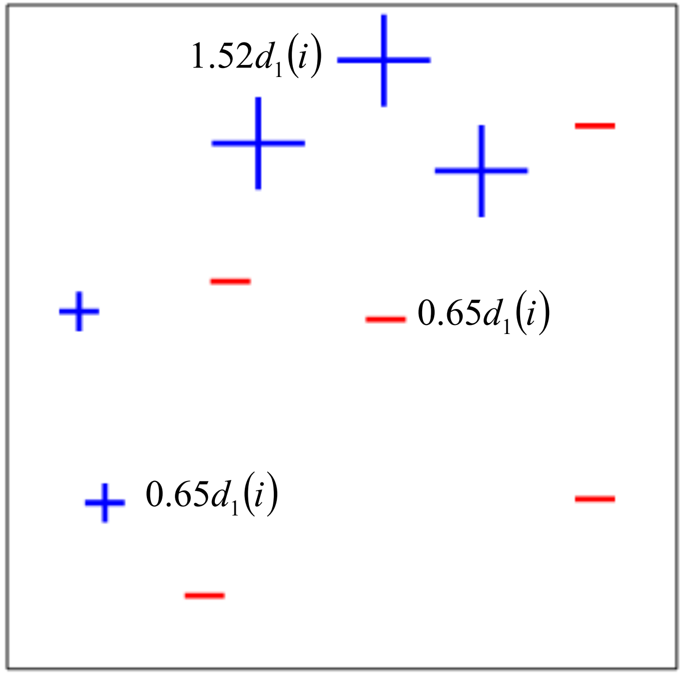


10 training samples $\mathbf{x}_i$, $i = 1 \cdots 10$

Images from R. Schapire, ”A Boosting Tutorial”

Discrete AdaBoost – Toy example





$$d_1(i) = \frac{1}{10}$$

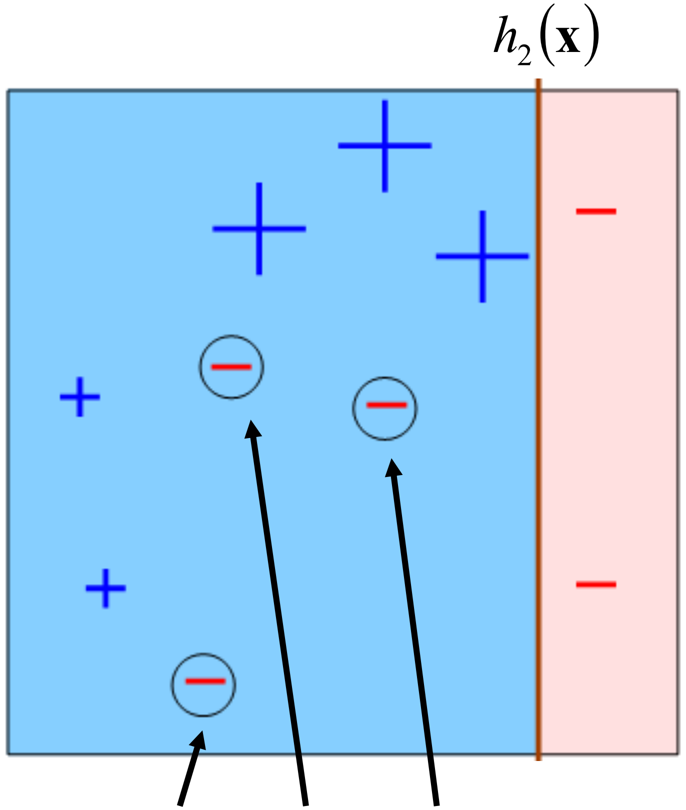
$$\varepsilon_1 = \left(\frac{1}{10} + \frac{1}{10} + \frac{1}{10}\right) = 0.3$$

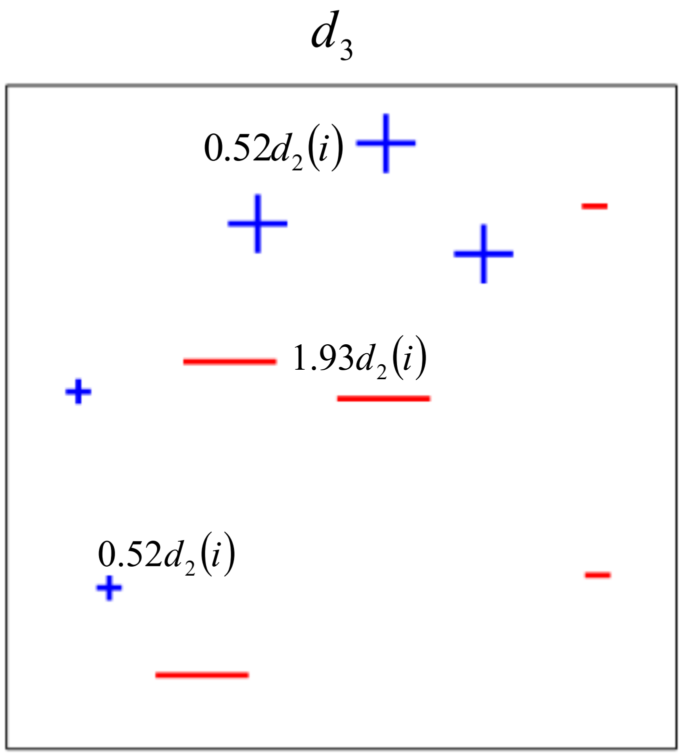
$$\alpha_1 = \frac{1}{2}\ln\left(\frac{1-0.3}{0.3}\right) = 0.42$$

$$d_2(i) = \begin{cases} d_1(i)e^{-0.42} = 0.65d_1(i) & \text{If } \mathbf{x}_i \text{ correctly classified} \\ d_1(i)e^{0.42} = 1.52d_1(i) & \text{If } \mathbf{x}_i \text{ wrongly classified} \end{cases}$$

Normalize d_2!

Discrete AdaBoost – Toy example





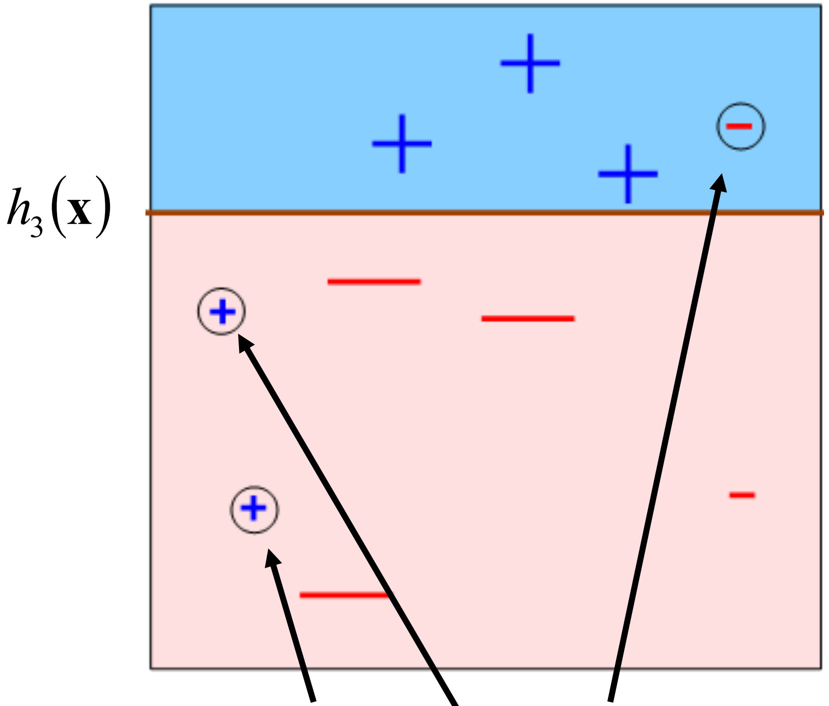
$$\varepsilon_2 = \left(d_2(j) + d_2(k) + d_2(l)\right) = 0.21$$

$$\alpha_2 = \frac{1}{2}\ln\left(\frac{1-0.21}{0.21}\right) = 0.66$$

$$d_3(i) = \begin{cases} d_2(i)e^{-0.66} = 0.52d_2(i) & \text{If } \mathbf{x}_i \text{ correctly classified} \\ d_2(i)e^{0.66} = 1.93d_2(i) & \text{If } \mathbf{x}_i \text{ wrongly classified} \end{cases}$$

Normalize d_3!

Discrete AdaBoost – Toy example

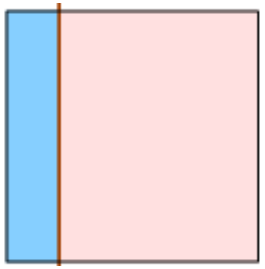
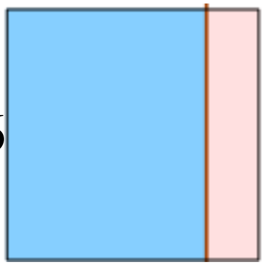
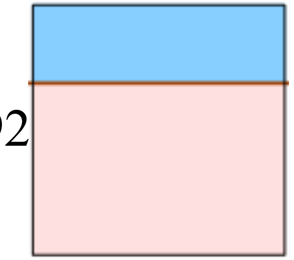


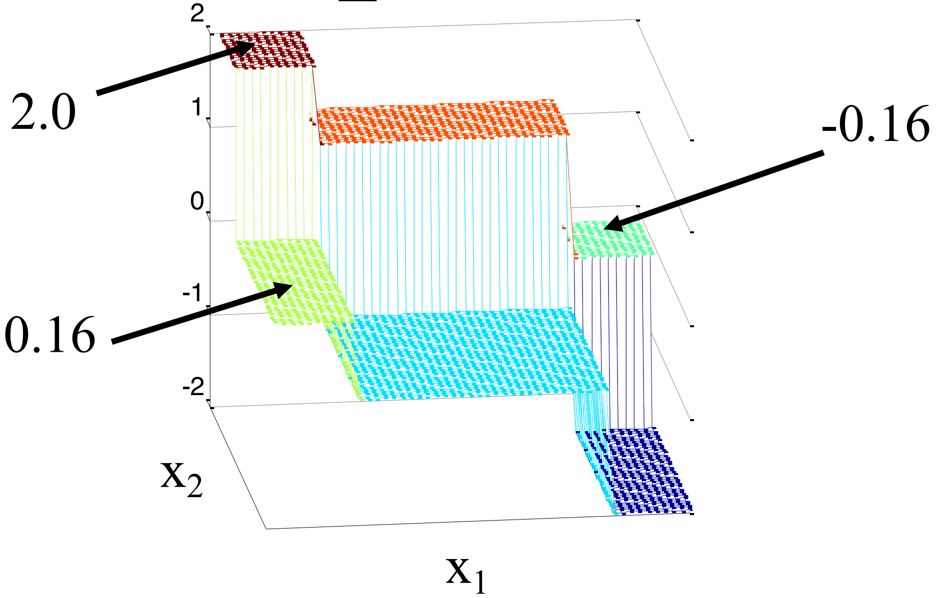
$$\varepsilon_3 = \left(d_3(p) + d_3(q) + d_3(r)\right) = 0.14$$

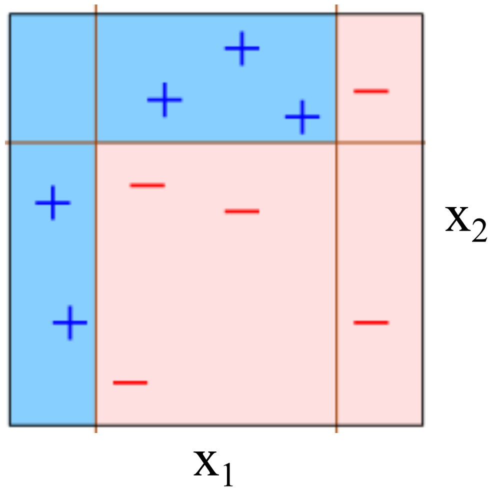
$$\alpha_3 = \frac{1}{2}\ln\left(\frac{1-0.14}{0.14}\right) = 0.92$$

Discrete AdaBoost – Toy example

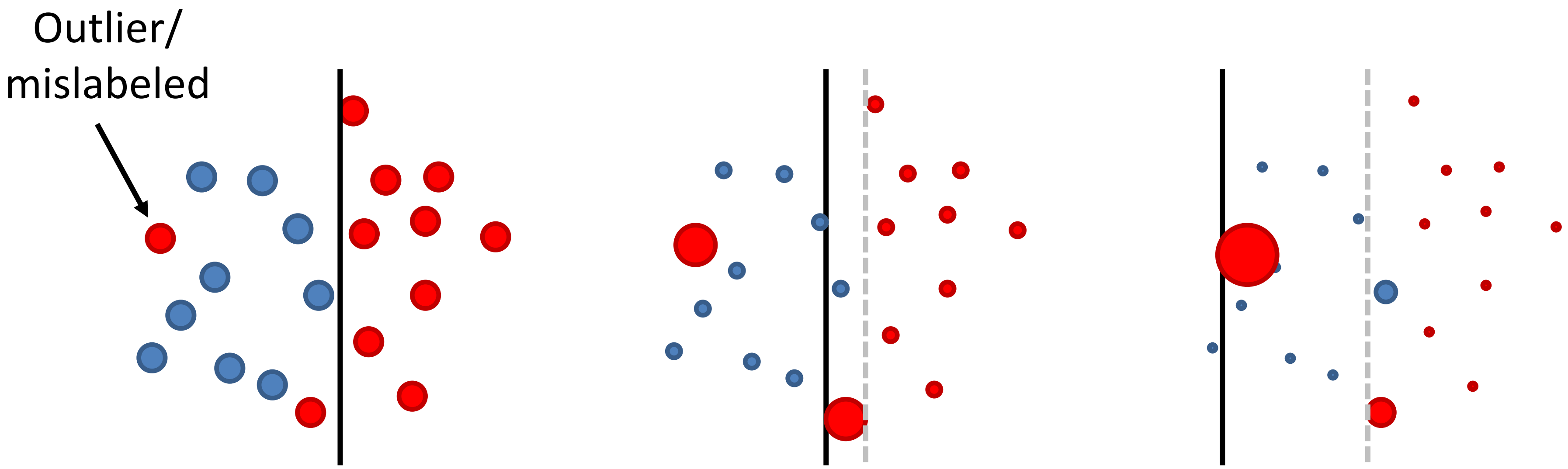
Final strong classifier: $H(\mathbf{x}) = \operatorname{sign}\left(\sum_{t=1}^{T} \alpha_t \mathrm{h}_t(\mathbf{x}) \right)$

$\mathrm{H}(\mathbf{x}) = \operatorname{sign}\Big[\, 0.42$  $+\ 0.66$  $+\ 0.92$  $\Big]$



$\mathrm{H}(\mathbf{x}) =$ 

Problem - Outliers



- Outliers gain weight exponentially
- Will eventually result in bad weak classifiers
- May ruin the final ensemble classifier

Outlier strategies

- Keep an eye on the weights (plot them!)
- Weight trimming
  - Don’t allow weights larger than a certain threshold
  - Disregard training samples with too large weights
- Use alternative weight update schemes with less aggressive increases for misclassified training data
  - LogitBoost
  - GentleBoost

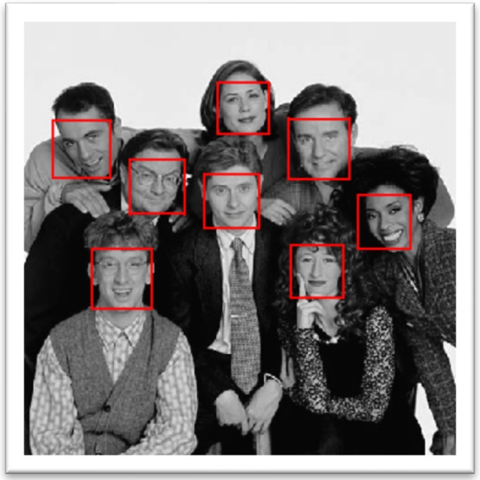
Summary Ensemble Learning

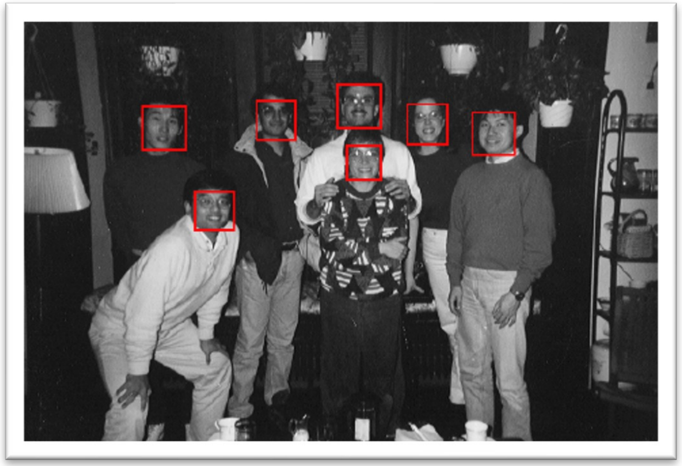
(using decision trees)

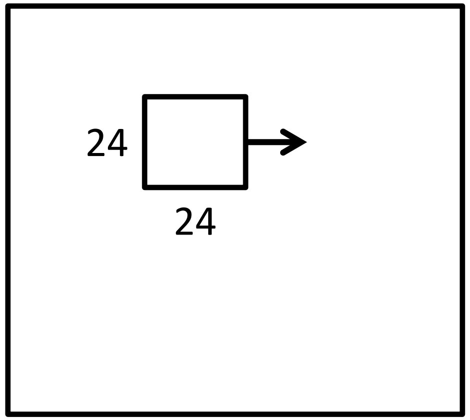
- Nonlinear classifiers that are easy to implement
- Easy to use - just one or a few parameters (T)
- Inherent feature selection
- Slow to train – fast to classify (real-time)
- Look out for outlier problems (AdaBoost)

Real-time object detection

P. Viola and M. Jones "*Rapid Object Detection using a Boosted Cascade of Simple Features*", 2001



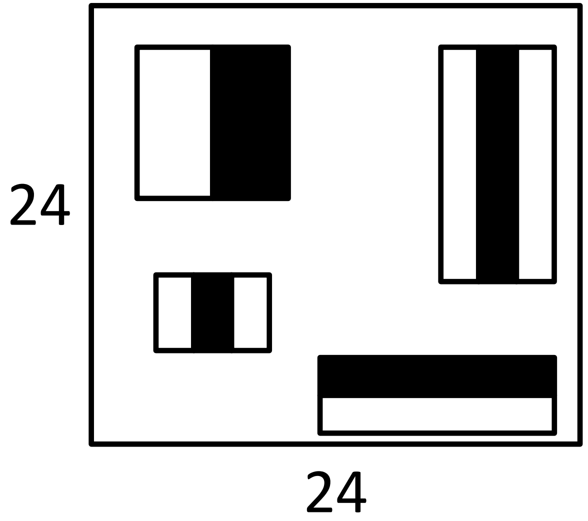




Sweep a sub-window over the image. for each position, determine if the sub-window contains a face or not.

Haar-features

Rectangle filters



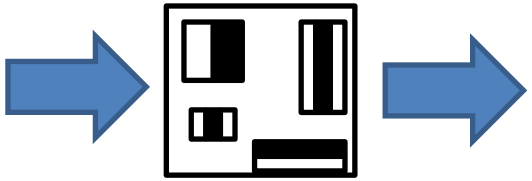
From the sub-window, calculate contrast features (Haar features) at different locations and scales, and in different orientations. LOTS of different combinations, can be several 100,000 features!

Train detector with AdaBoost

As described previously!

24 x 24 pixels face images

Apply Haar filters
to each image



$$\mathbf{x}_i = \begin{pmatrix} x_{1,i} \\ \vdots \\ x_{N,i} \end{pmatrix}, i = 1 \ldots K$$

Similar with non-face images to obtain negative examples.