

# Time Series and Sequence Learning

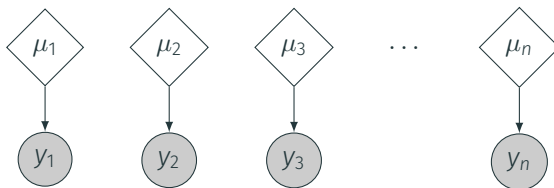
Modeling time series with stochastic processes

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# A graphical representation

Classical regression applied to time series data.



$$\mu_t = \theta_0 + \theta_1 u_t, \quad u_t = 1993 \frac{4}{365}, 1993 \frac{14}{365}, \dots, 2020 \frac{20}{365}$$

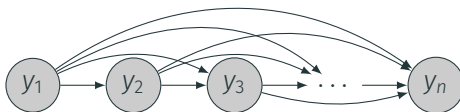
A fundamental approach to time series analysis is to model the data as a **stochastic process**,

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Probabilistic graphical model:



# A model for time series data

How can we model the stochastic process?

A complete **probabilistic description** is given by the *joint probability density function*

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**Cumbersome to work with directly:**

1. Dimension grows with time horizon  $n$
2. High-dimensional for large  $n$
3. Can not be used to *forecast* values  $y_t$  for  $t > n$

## A model for time series data

$$p(y_{1:n}) = p(y_n | y_{1:n-1}) p(y_{1:n-1})$$

$$= p(y_n | y_{1:n-1}) p(y_{n-1} | y_{1:n-2}) p(y_{1:n-2})$$

$$= p(y_n | y_{1:n-1}) p(y_{n-1} | y_{1:n-2}) \dots p(y_2 | y_1) p(y_1)$$

# A model for time series data

Factorize joint pdf as

$$p(y_{1:n}) = \prod_{t=1}^n p(y_t | y_{1:t-1})$$

It is enough to model the **one-step predictive pdf**  $p(y_t | y_{1:t-1})$ .

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*[For notational brevity we define  $p(y_1 | y_{1:0}) := p(y_1)$ ]*



# First-order auto-regressive model

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ex) If we believe that  $y_t$  is **similar to**  $y_{t-1}$ , one possibility is to assume

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For a first-order AR model we have,

$$p(y_t | y_{1:t-1}) = p(y_t | y_{t-1}) = \mathcal{N}(y_t | a y_{t-1}, \sigma_\varepsilon^2).$$

# First-order auto-regressive model

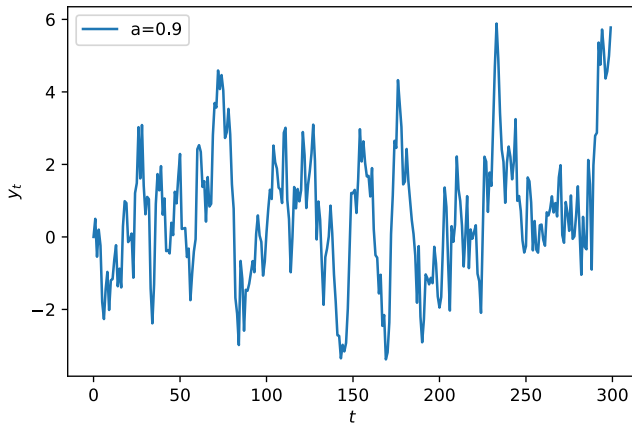
The dependency structure of the predictive pdf,

$$p(y_t | y_{1:t-1}) = p(y_t | y_{t-1}) = \mathcal{N}(y_t | \textcolor{red}{a}y_{t-1}, \sigma_\epsilon^2).$$

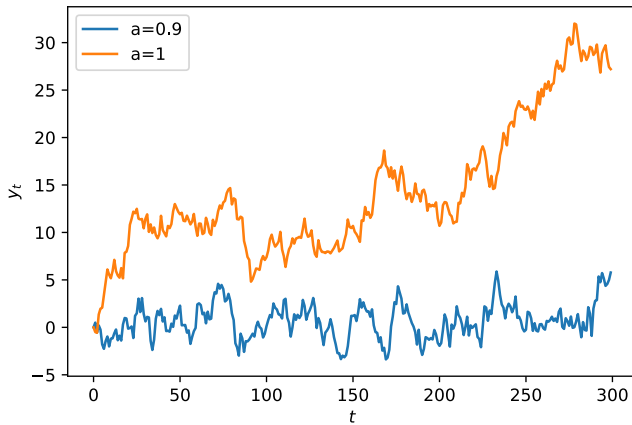
can be illustrated graphically as



# Simulation of AR(1)



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