Time Series and Sequence Learning

Modeling time series with stochastic processes

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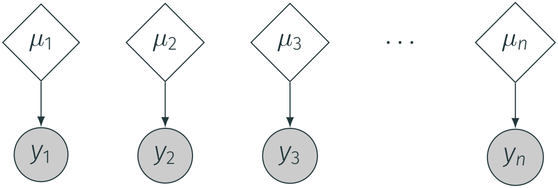
A graphical representation

Classical regression applied to time series data.



$$\mu_t = \theta_0 + \theta_1 u_t, \qquad u_t = 1993\tfrac{4}{365},\ 1993\tfrac{14}{365},\ \ldots,\ 2020\tfrac{20}{365}$$

Stochastic process

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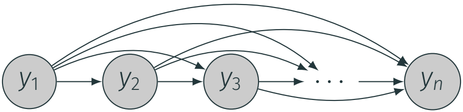
Probabilistic graphical model:



A model for time series data

How can we model the stochastic process?

A complete probabilistic description is given by the *joint probability density function*

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Cumbersome to work with directly:

1. Dimension grows with time horizon n
2. High-dimensional for large n
3. Can not be used to *forecast* values y_t for $t > n$

A model for time series data

$$p(y_{1:n}) = p(y_n | y_{1:n-1})\, p(y_{1:n-1})$$

$$= p(y_n | y_{1:n-1})\, p(y_{n-1} | y_{1:n-2})\, p(y_{1:n-2})$$

$$= p(y_n | y_{1:n-1})\, p(y_{n-1} | y_{1:n-2}) \cdots p(y_2 | y_1)\, p(y_1)$$

A model for time series data

Factorize joint pdf as

$$p(y_{1:n}) = \prod_{t=1}^{n} p(y_t \mid y_{1:t-1})$$

It is enough to model the one-step predictive pdf $p(y_t \mid y_{1:t-1})$.

[For notational brevity we define $p(y_1 \mid y_{1:0}) := p(y_1)$]

y_1:0 is an empty set

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ex) If we believe that y_t is **similar to** y_{t-1}, one possibility is to assume

$$y_t = a y_{t-1} + \varepsilon_t, \qquad \varepsilon_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2),$$

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For a first-order AR model we have,

$$p(y_t \mid y_{1:t-1}) = \overset{\text{we only focus on 1 past value here}}{p(y_t \mid y_{t-1})} = \mathcal{N}\left(y_t \mid a y_{t-1}, \sigma_\varepsilon^2\right).$$

First-order auto-regressive model

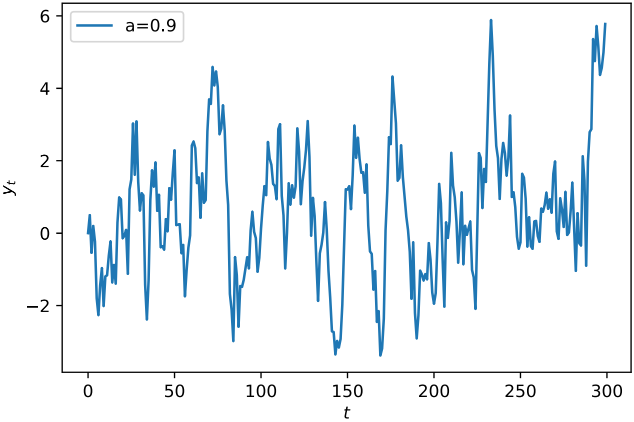
The dependency structure of the predictive pdf,

$$p(y_t \mid y_{1:t-1}) = p(y_t \mid y_{t-1}) = \mathcal{N}\left(y_t \mid a y_{t-1}, \sigma_\varepsilon^2\right).$$

can be illustrated graphically as

$$y_1 \to y_2 \to y_3 \to \cdots \to y_n$$

Simulation of AR(1)



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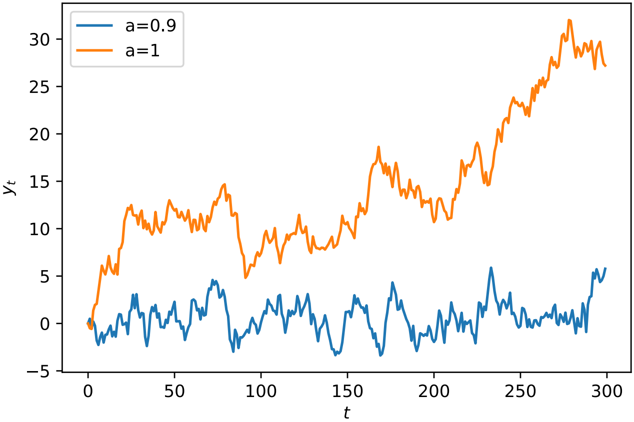

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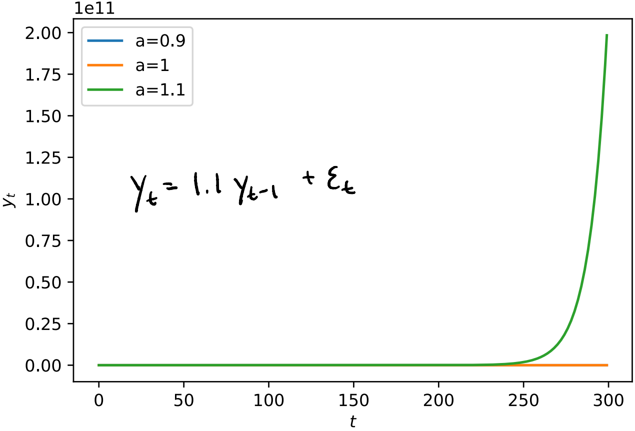

$y_t = 1.1\, y_{t-1} + \varepsilon_t$