Time Series and Sequence Learning

Discussion seminar for Lecture 4

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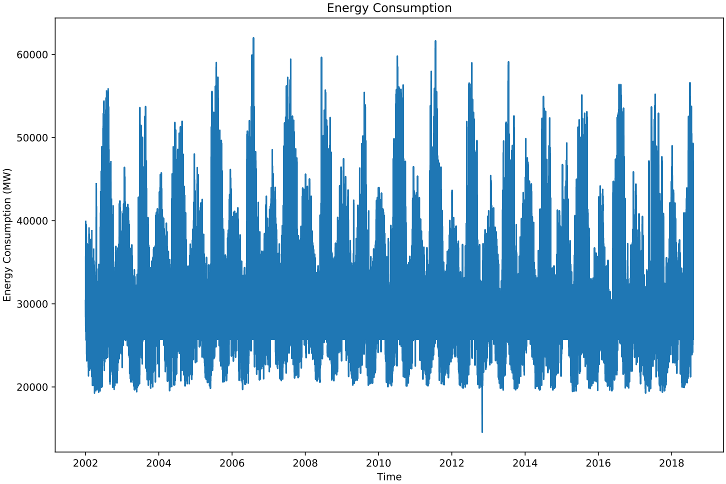
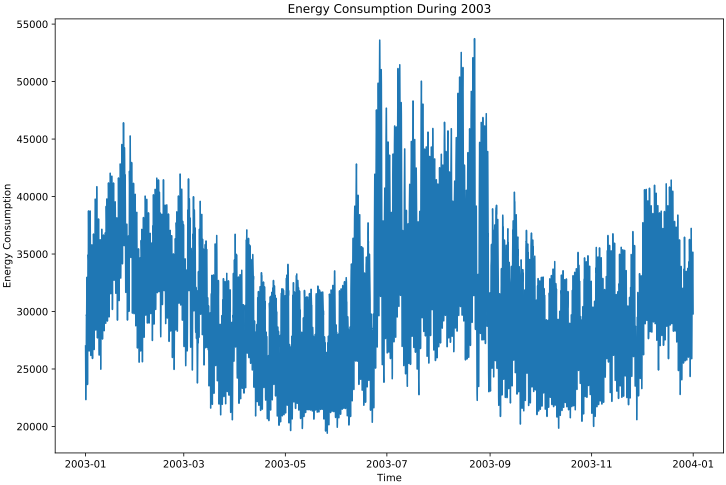
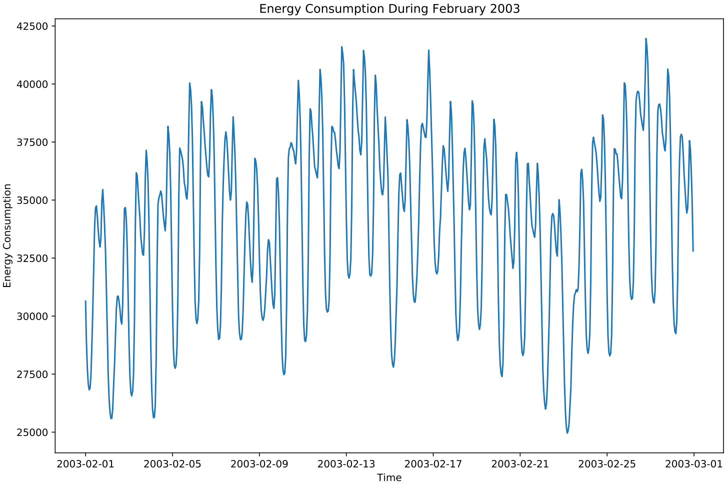
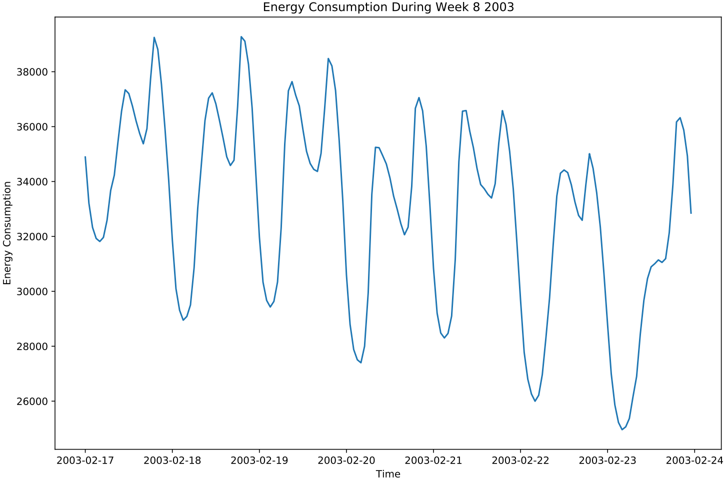

Energy Consumption
Energy Consumption (MW)
60000
50000
40000
30000
20000
2002
2004
2006
2008
2010
2012
2014
2016
2018
Time


Energy Consumption During 2003
55000
50000
45000
40000
35000
30000
25000
20000
Energy Consumption
2003-01
2003-03
2003-05
2003-07
2003-09
2003-11
2004-01
Time

lecture4a - Classical Decomposition




Energy Consumption During Week 8 2003
Energy Consumption
Time
38000
36000
34000
32000
30000
28000
26000
2003-02-17
2003-02-18
2003-02-19
2003-02-20
2003-02-21
2003-02-22
2003-02-23
2003-02-24

Discussion questions:

1. What components can you find in the data? Can you explain them?
2. Would this be an additive or multiplicative model?

this is an additive model

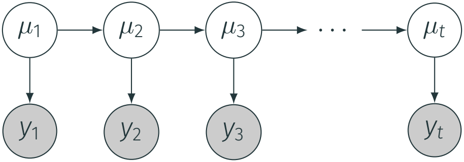
3. How do we calculate the seasonality?

1. check the data, plots several plots and compare
2. choose the whole time period, like year, month and week

lecture4b – State-Space Model

The Local Level Model Two stochastic processes $y_1, y_2, y_3, \ldots$ and $\mu_1, \mu_2, \mu_3, \ldots$

$$
\begin{aligned}
y_t &= \mu_t + \varepsilon_t, & \varepsilon_t &\overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2) \\
\mu_{t+1} &= \mu_t + \eta_t, & \eta_t &\overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_\eta^2) \\
& & \mu_1 &\sim \mathcal{N}(a_1, P_1)
\end{aligned}
$$



Discussion questions:

1. What can we say about the joint process $\{(\mu_t, y_t)\}_{t\geq 1}$? Is this a linear model? YES, it's linear
2. Which of the following problems relate to filtering, prediction and smoothing? Also think about what could the state and observations be.

state is Y value which is unknown, real dist

Real time:prediction
Collect data:smoothing

a) Find the trajectory an airplane based on noisy observations. (Does it change of we do it realtime or if we collect all the data first?)

Prediction

b) Make a statement about the weather tomorrow.

c) Calculate the current position of a robot based on sensors.

Filtering
if change to next pos, then this will be prediction

Find \mu_{t} l y:{1:n}

n < t : This is known as the forecasting problem.
n = t : This is known as the filtering problem.
n > t : This is known as the smoothing problem.

1 and 2 use kalman filter
3 use kalman smoother

Kalman Filter for local-level model

For each iteration $t = 1, 2, 3, \ldots$ repeat the following steps:

- Measurement updates
  1. Forecasting error: $v_t = y_t - \hat{\mu}_{t\,|\,t-1}$
  2. Forecasting variance: $F_t = P_{t\,|\,t-1} + \sigma_\varepsilon^2$
  3. Kalman gain: $K_t = P_{t\,|\,t-1}/F_t$
  4. Filter mean: $\hat{\mu}_{t\,|\,t} = \hat{\mu}_{t\,|\,t-1} + K_t v_t$
  5. Filter variance: $P_{t\,|\,t} = P_{t\,|\,t-1}(1 - K_t)$
- Time updates
  6. Predictor mean: $\hat{\mu}_{t+1\,|\,t} = \hat{\mu}_{t\,|\,t}$ because μt+1 =μt +ηt, so expected value -> what we see this line
  7. Predictor variance: $P_{t+1\,|\,t} = P_{t\,|\,t} + \sigma_\eta^2$

Initialized using $\hat{\mu}_{1\,|\,0} = a_1$ and $P_{1\,|\,0} = P_1$.

based on different a, we will change the mean.

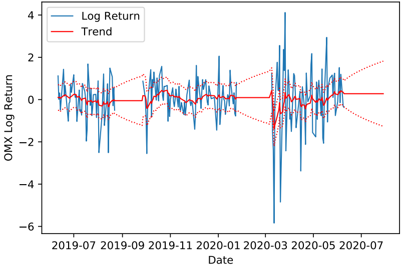
Discussion questions:

1. If we change the state equation to $\mu_t = a \cdot \mu_{t-1} + \eta_t$, how would that change the Kalman filter?
2. We are able to calculate the log-likelihood as

$$\ell(\theta) = -\frac{n}{2} \log 2\pi - \frac{1}{2} \sum_{t=1}^{n} \left(\log F_t(\theta) + \frac{(y_t - \hat{\mu}_{t\,|\,t-1}(\theta))^2}{F_t(\theta)} \right),$$

how would you try to maximize this? Is direct calculations of derivatives feasible?

lecture4f – Forecasting and Missing Data



Discussion questions:

from step 6,7, we know the mean is same, and var is growing

1. What can we say about the mean and variance estimates for periods of missing data and forecasting?
2. What would happen to mean and variance estimates if we changed the model to $\mu_t = a \cdot \mu_{t-1} + \eta_t$?