# The BPF likelihood estimator

a) We have $p(y_{1:n}) = \prod_{t=1}^{n} p(y_t|y_{1:t-1})$ Using $p(y_1|y_{1:0}) = p(y_1)$

giving that $l(y_{1:n}) = \log p(y_{1:n}) = \sum_{t=1}^{n} \log p(y_t|y_{1:t-1})$

b) $p(y_t|y_{1:t-1}) = \int p(y_t, \alpha_t|y_{1:t-1})\, d\alpha_t$

$$= \int g(y_t|\alpha_t) \cdot p(\alpha_t|y_{1:t-1})\, d\alpha_t \quad (*)$$

c) $\left\{\alpha_t^i, \frac{1}{N}\right\}_{i=1}^{N}$ is a sample from $p(\alpha_t|y_{1:t-1})$ into

gives

$$(*) = \int g(y_t|\alpha_t) \cdot \frac{1}{N} \sum_{i=1}^{N} \delta_{\alpha_t^i}(\alpha_t)\, d\alpha_t = \sum_{i=1}^{N} \frac{1}{N} g(y_t|\alpha_t^i)$$

$$= \sum_{i=1}^{N} \frac{1}{N} w_t^i$$

d) $l(y_{1:n}) \approx \sum_{t=1}^{n} \log\left(\frac{1}{N} \sum_{i=1}^{N} w_t^i\right)$

# Log Weights:

Let $\log \tilde{w}_t^i = \log w_t - c_t$, $c_t = \max\{\log w_t^i\}_{i=1}^N$

a) $$\frac{\tilde{w}_t^i}{\tilde{Z}_t} = \frac{w_t^i \cdot e^{-c_t}}{\sum_{i=1}^N w_t^i e^{-c_t}} = \frac{w_t^i}{\sum_{i=1}^N w_t^i} = \frac{w_t^i}{Z_t}$$

b) $$\max \log \tilde{w}_t^i = \max \log w_t^i - c_t = c_t - c_t = 0$$

$$\Rightarrow \max \tilde{w}_t^i = 1.$$

c) $$\log\left(\frac{1}{N}\sum_{i=1}^N w_t^i\right) = \log\left(\frac{1}{N}\sum_{i=1}^N \tilde{w}_t^i \cdot e^{c_t}\right)$$

$$= \log\left(\frac{1}{N}\sum_{i=1}^N \tilde{w}_t^i\right) + c_t.$$