LINKÖPING UNIVERSITY

Time Series and Sequence Learning

Discussion seminar for Lecture 3

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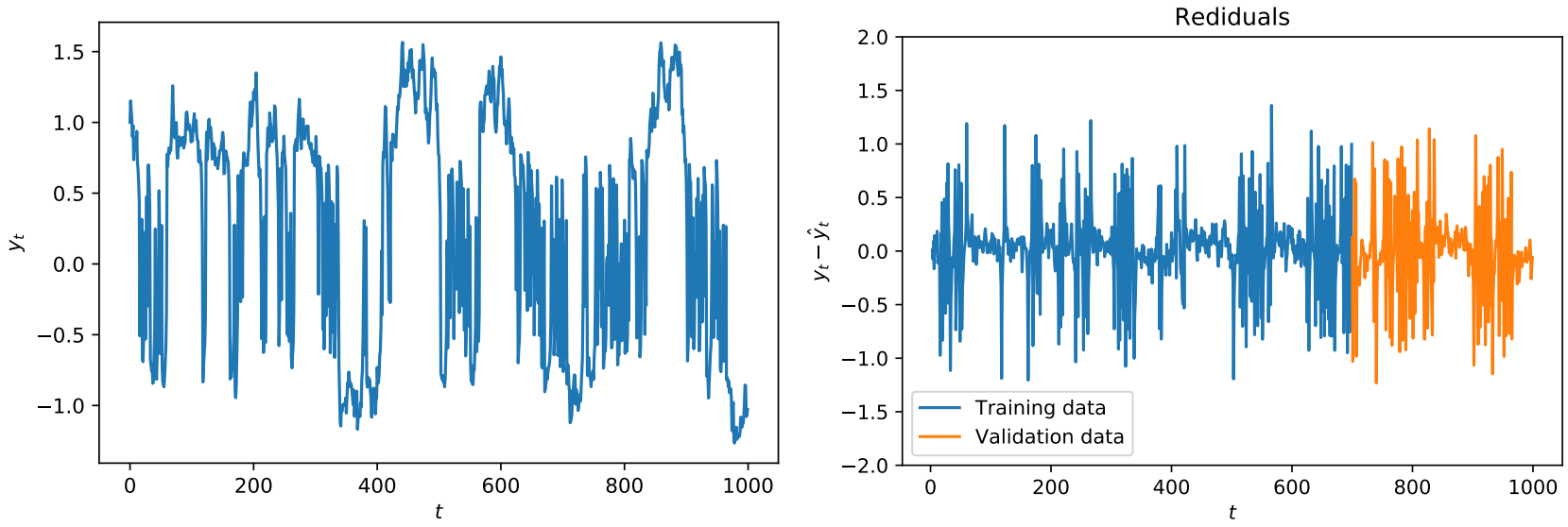
lecture3a – Detecting nonlinearity



Left: Data. *Right:* Residuals for a fitted AR(3) model.

Discussion questions:

1. How can we see from the residuals that the data is not well explained by the model?
2. Is it possible to see that that data is not well explained by a *linear process* by just looking at the data? How?

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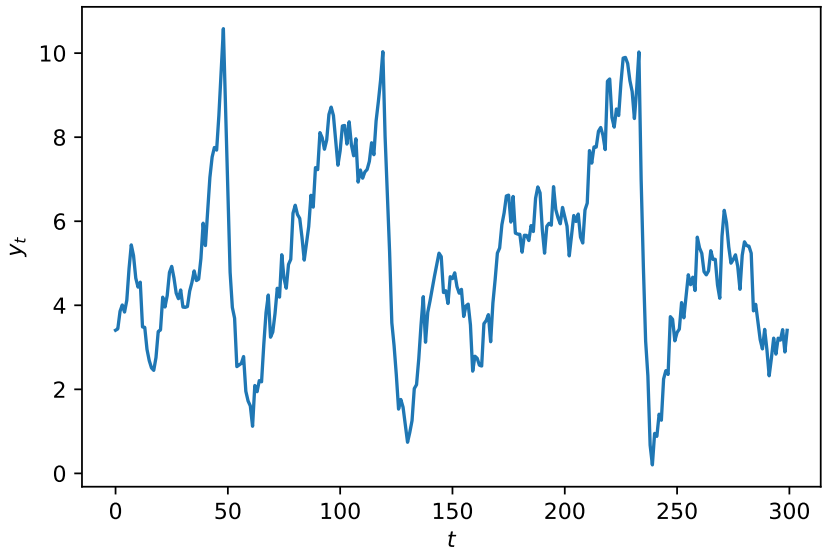


Discussion questions:

1. Does the data above appear to be generated by a linear AR(p) model?

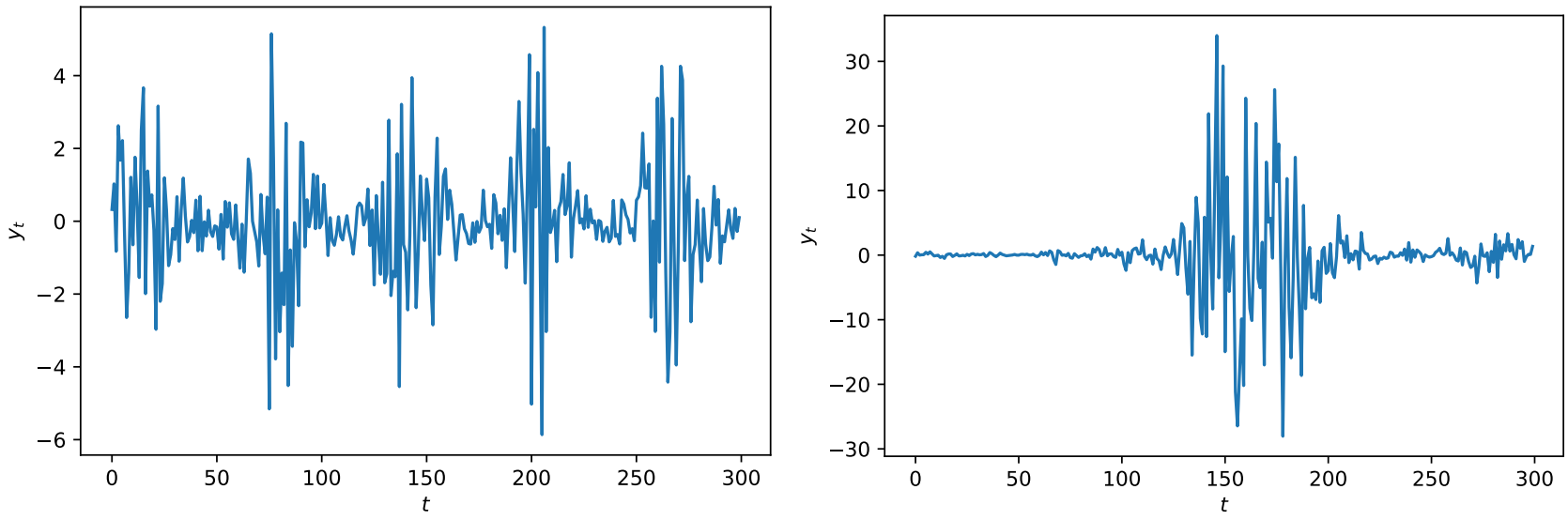
   It is not a stationary AR, and it does not have the same pattern along the time

2. If $\{y_t\}_{t\geq 1}$ is a stationary process generated by an AR(p) model with coefficients $(a_1, \ldots, a_p)$, then what can be said about the negated process $\{-y_t\}_{t\geq 1}$

   Yes, it is a mirror, it also means it change the ar's sign, so it is still linear stationary

AR model can have pattern, but only a same pattern and constantly pattern, not wide and narrow wave at the same time

lecture3a – Detecting nonlinearity



Left: Non-stationary (cyclic variance).

Right: Stationary but nonlinear (stochastic variance).

When we are asked the question *"Is this time series stationary?"*, the meaning is often *"Could this time series have been generated by a stationary linear process?"*

In practice it is hard to distinguish between non-stationarity and nonlinearity!

Nonlinear auto-regressive model, NAR(p):

$$y_t = f_\theta(y_{t-1}, y_{t-2}, \ldots, y_{t-p}) + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2),$$

Using an 2-layer NN to model f_θ:

$$\begin{aligned} \mathbf{h}_t &= \sigma(W^{(1)} H_{t-1} + b^{(1)}) \\ y_t &= W^{(2)} \mathbf{h}_t + b^{(2)} + \varepsilon_t. \end{aligned}$$

Discussion questions:

1. Contrary to a linear AR(p) model it is customary to include "bias terms" $b^{(j)}$ in the MLP-NAR. Why? Why don't we include a bias/intercept in the linear AR model?

   in ar model,we will sum yt and it wil sum to inf

2. How is the squared loss function

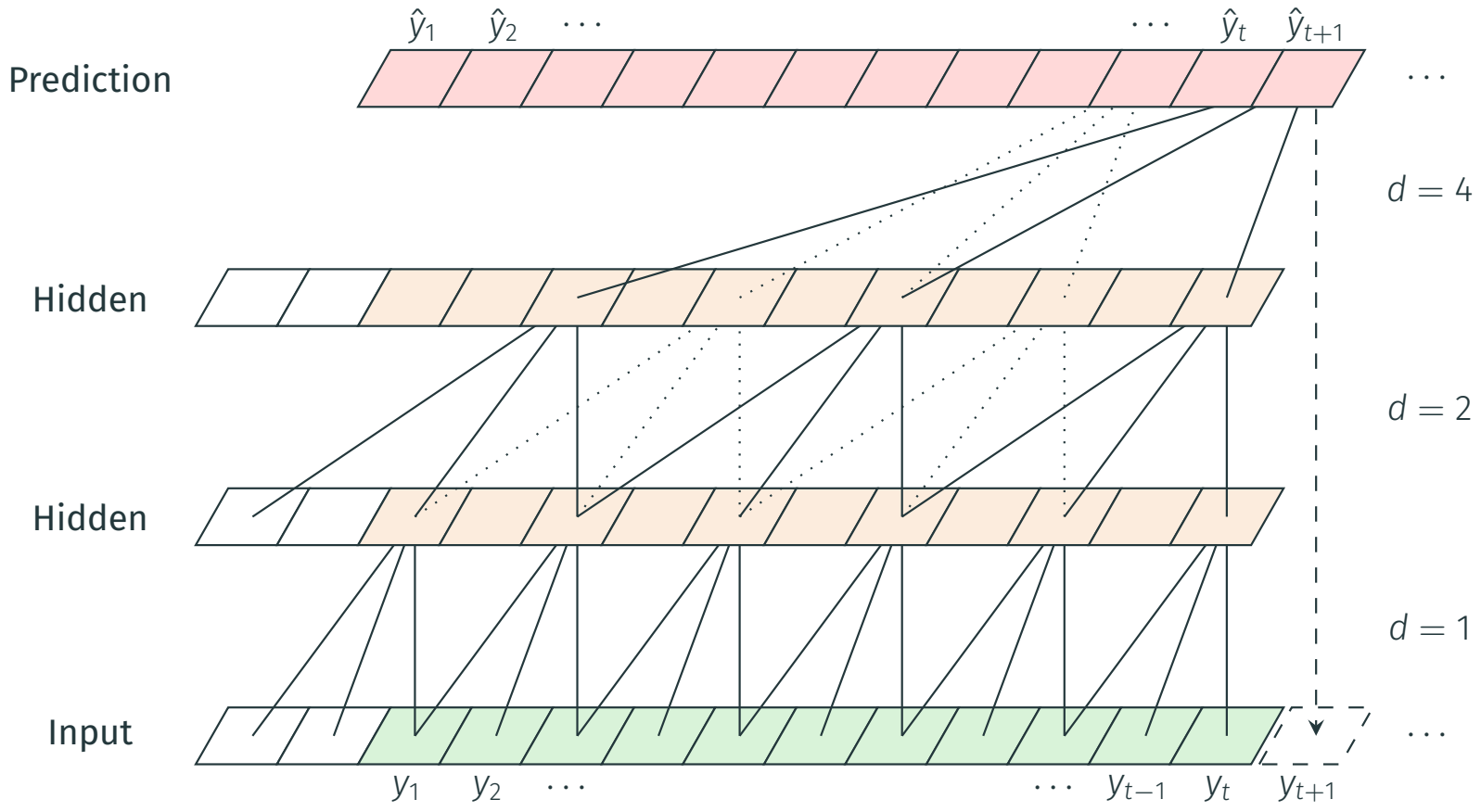
   activate function will help to not over the limit

$$L(\theta) = \frac{1}{n-p} \sum_{t=p+1}^{n} \left(y_t - f_\theta(y_{t-1}, y_{t-2}, \ldots, y_{t-p})\right)^2$$

   we can not predict the first p value, we just skip first p values in NN network in the next slide using d, several value will not be used or predicted.

   related to the data likelihood?

lecture3{b,c} - TCN



Discussion questions:

1. Which statements are correct?
   a) NAR is a special case of TCN.
   b) TCN is a special case of NAR.
   c) TCN and NAR are equivalent model classes.
2. How is the receptive field of a TCN related to the order p of a NAR(p) model? P is same as receptive value
3. Will the receptive field always increase exponentially with depth for a TCN with dilated convolutions?