

Computational Statistics Computer Lab 6 (Group 7)

Qinyuan Qi(qinqi464)

Satya Sai Naga Jaya Koushik Pilla (satpi345)

2023-12-22

Question 1: Genetic algorithm (Solved by Qinyuan Qi)

Answer:

(1):

(2):

(3):

(4):

(5):

(6):

(7):

(8):

(9):

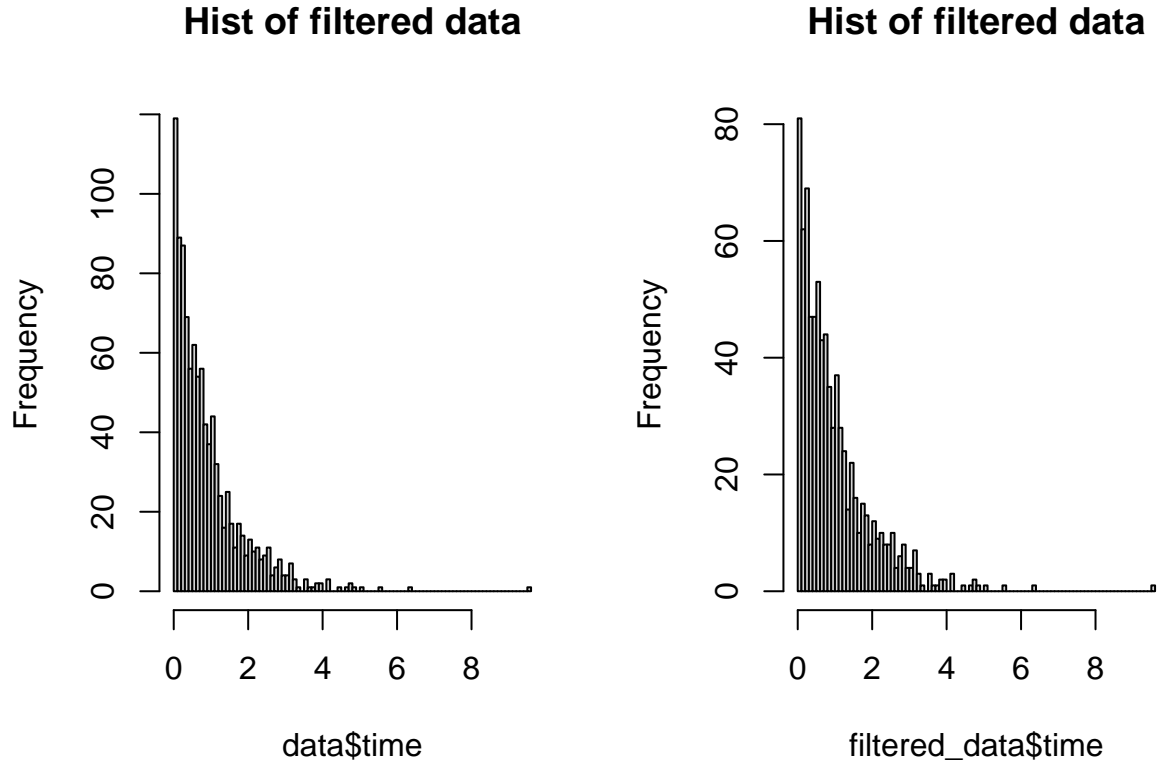
Question 2: EM algorithm (Solved by Satya Sai Naga Jaya Koushik Pilla)

Answer:

(1) Plot 2 histograms:

According to the plots generated, we found that the plot seems follow exponential distribution.

```
##### (2.1) #####
# Load data
data <- read.csv("censoredproc.csv",
                 sep = ";", header = TRUE)
# We will filter out the left-censored data which cens=2
filtered_data <- data[data$cens ==1,]
layout(matrix(c(1:2), 1, 2))
# plot the data
hist(data$time, breaks = 100, main="Hist of filtered data")
# plot the filtered data
hist(filtered_data$time, breaks = 100, main="Hist of filtered data")
```



(2):

The general CDF form of an exponential distribution is:

$$F(x, \lambda) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

So PDF of an exponential distribution is derivative of F on x:

$$f(x, \lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Likelihood function for the exponential distribution is as follows.

$$L(\lambda; x_1, x_2, \dots, x_n) = \prod f(x, \lambda) = \lambda^n \exp(-\lambda \sum_{j=1}^n x_j)$$

PDF for the truncated exponential distribution is derived as follows.

$$P(X \leq x | X \leq c) = \frac{P(X \leq x, X \leq c)}{P(X \leq c)} = \frac{P(X \leq \lambda)}{P(X \leq c)} = \frac{\lambda e^{-\lambda x}}{c e^{-\lambda c}} = \frac{\lambda}{c} e^{-\lambda(x-c)}$$

So likelihood function for the truncated exponential distribution is as follows.

$$L(\lambda|X \leq c; x_1, x_2 \dots x_n) = \prod P(X \leq x|X \leq c) = (\frac{\lambda^n}{c^n}) \exp(-\lambda \sum_{j=1}^n x_j)$$

(3):

Since it's relative straight forward , we will use the likelihood function directly.

So we will derive the EM function using the likelihood function in 2.2.

E-Step: Let's compute the expectation of likelihood as follows.

$$Q(\lambda, \lambda^t) = E(L(\lambda|X \leq c; x_1, x_2 \dots x_n))$$

(4):

(5):

(6):