

COMPUTER ASSIGNMENT

Write an individual report (pdf file) and submit it via LISAM. **Deadline is Monday 6th November at 23:30.** The grade for the computer assignment is PASS/FAIL.

1. Monte Carlo Methods

Monte Carlo methods prove highly effective for estimating probabilities, expectations, and various distribution characteristics, especially in complex scenarios where manual calculations become challenging. According to the Monte Carlo approach, we generate an extensive sequence of random variables following the distribution of interest. Subsequently, we estimate probabilities through long-term proportions and expectations through long-term averages. To gain a comprehensive understanding of Monte Carlo Simulation, I recommend studying Chapter 5 of the second course book (Probability and Statistics for Computer Scientists) or exploring additional references on the subject.

Now, let's apply Monte Carlo methods to solve the following exercises:

1-Derive a formula and explain how to generate a random variable with the density

$$f(x) = (1.5)\sqrt{x} \quad \text{for } 0 < x < 1$$

if your random number generator produces a Standard Uniform random variable U . Use the inverse transform method. Compute this variable if $U = 0.001$.

2-Twenty computers are connected in a network. One computer becomes infected with a virus. Every day, this virus spreads from any infected computer to any uninfected computer with probability 0.1. Also, every day, a computer technician takes 5 infected computers at random (or all infected computers, if their number is less than 5) and removes the virus from them. Estimate:

- (a) the expected time it takes to remove the virus from the whole network;
- (b) the probability that each computer gets infected at least once;
- (c) the expected number of computers that get infected.

2. Computer Exercises from Course's book (Mathematical Statistics with Applications)

Do the following applet exercises from the book using R (see Appendix below for useful functions) and write your comments on the results. Explain what you have learned from each exercise.

Exercises: 4.84, 4.117, 4.118, 10.19, 10.21, 11.31, 11.69

Good luck! Lycka till!

APPENDIX: Functions for random variables in R (d,p,q,r)

The functions `d`, `p`, `q`, `r` allow for a quick calculation of densities, probabilities, quantiles and samples from random variables in R. Below I show examples for the normal distribution, but you have equivalent functions for gamma, beta, T, etc. For a comprehensive list of the available distributions visit the CRAN Task View on probability distributions:

<https://cran.r-project.org/web/views/Distributions.html>

- `dnorm`: Calculates densities for a normal distribution, i.e. $f(y)$

```
> dnorm(0,mean=0,sd=1)
[1] 0.3989423
```
- `pnorm`: Calculates probabilities for a normal distribution, i.e. $F(y)$

```
> pnorm(0,mean=0,sd=1)
[1] 0.5
```
- `qnorm`: Obtains the quantile q from a normal probability $0 < p < 1$, i.e. $F(q) = p$

```
> qnorm(0.5,mean=0,sd=1)
[1] 0
```
- `rnorm`: Obtains a random sample from a normal distribution

```
> v = rnorm(10000,mean=0,sd=1)
> hist(v,breaks=50)
```

