```
# Cell 1: Imports and global style
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from matplotlib.lines import Line2D
import json
import networkx as nx
from networkx.readwrite import json_graph

# Global seaborn style
sns.set_theme(style="white", context="talk")
```

```
# Cell 2: Load Harry Potter networks (HP2, HP3)
with open("HP2.json", "r", encoding="utf-8") as f:
    hp2_data = json.load(f)
with open("HP3.json", "r", encoding="utf-8") as f:
    hp3_data = json.load(f)

HP2 = json_graph.node_link_graph(
    hp2_data,
    directed=hp2_data.get("directed", False),
    multigraph=hp2_data.get("multigraph", False),
    edges="links",
)

HP3 = json_graph.node_link_graph(
    hp3_data,
    directed=hp3_data.get("directed", False),
    multigraph=hp3_data.get("multigraph", False),
    edges="links",
)

print("HP2:", HP2.number_of_nodes(), "nodes,", HP2.number_of_edges(), "edges")
print("HP3:", HP3.number_of_nodes(), "nodes,", HP3.number_of_edges(), "edges")
```

```
HP2: 64 nodes, 91 edges
HP3: 64 nodes, 87 edges
```

```
# Cell 3: Quick basic stats function for any directed graph
def basic_stats(G, name="G"):
    n = G.number_of_nodes()
    m = G.number_of_edges()
    density = nx.density(G)
    avg_out_degree = m / n if n > 0 else np.nan

    print(f"=== {name} ===")
    print(f"Nodes: {n}")
    print(f"Edges: {m}")
    print(f"Density: {density:.4f}")
    print(f"Average out-degree: {avg_out_degree:.2f}")

    if isinstance(G, nx.DiGraph):
        rec = nx.reciprocity(G)
        print(f"Reciprocity: {rec:.4f}")
```

```
    largest_wcc = max(nx.weakly_connected_components(G), key=len)
    print(f"Largest weakly connected component size: {len(largest_wcc)}")
    print()

basic_stats(HP2, "HP2")
basic_stats(HP3, "HP3")
```

```
=== HP2 ===
Nodes: 64
Edges: 91
Density: 0.0226
Average out-degree: 1.42
Reciprocity: 0.7912
Largest weakly connected component size: 18

=== HP3 ===
Nodes: 64
Edges: 87
Density: 0.0216
Average out-degree: 1.36
Reciprocity: 0.8046
Largest weakly connected component size: 17
```

```
# Cell 4: Load Czech data and build classroom networks
rel_path  = "relational data.xlsx"
attr_path = "student attributes.xlsx"

# Student attributes: two-row header -> use the second row as variable names
attr_raw = pd.read_excel(attr_path, header=[0, 1])
attr_raw.columns = [c[1] if isinstance(c, tuple) else c for c in attr_raw.colu
attributes = attr_raw

# Node attribute fields to attach to the graphs
attr_fields = [
    "gender",
    "HISEI",
    "whole_classroom_talk_start",
    "whole_classroom_talk_end",
    "literacy_start",
    "literacy_end",
    "need_of_success",
    "avoidance_of_failure",
]

rel_xls = pd.ExcelFile(rel_path)
print("Sheets in relational data:", rel_xls.sheet_names)

graphs_cz = {}

for sheet in rel_xls.sheet_names:
    print(f"\nProcessing sheet: {sheet}")
    df = pd.read_excel(rel_path, sheet_name=sheet, index_col=0)
    df = df.fillna(0)

    # Row labels are 101,102,...; columns are "0101","0102",... etc.
    # Convert both to integer student IDs and keep the common intersection.
    df.index = pd.to_numeric(df.index, errors="coerce")
    df.columns = pd.to_numeric(df.columns, errors="coerce")
```

```
    df = df.loc[df.index.notna(), df.columns[df.columns.notna()]]
    df.index = df.index.astype(int)
    df.columns = df.columns.astype(int)

    common_ids = sorted(set(df.index) & set(df.columns))
    df = df.loc[common_ids, common_ids]

    # Binarize nominations: >0 means a directed tie
    df_bin = (df > 0).astype(int)

    G = nx.from_pandas_adjacency(df_bin, create_using=nx.DiGraph)

    # Attach node attributes
    for node in list(G.nodes()):
        sid = int(node)
        G.nodes[node]["studentID"] = sid

        matched = attributes.loc[attributes["studentID"] == sid]
        if len(matched) >= 1:
            row = matched.iloc[0]
            for col in attr_fields:
                if col in row.index:
                    G.nodes[node][col] = row[col]

    graphs_cz[sheet] = G
    print(f"  nodes: {G.number_of_nodes()}, edges: {G.number_of_edges()}")

print("\nAll Czech classroom graphs created.")
print("Available graphs:", list(graphs_cz.keys()))

# Use 01start and 01end as the main Czech networks for comparison
G_cz_start = graphs_cz["01start"]
G_cz_end   = graphs_cz["01end"]

print(f"\nCzech 01start: {G_cz_start.number_of_nodes()} nodes, {G_cz_start.num
print(f"Czech 01end:   {G_cz_end.number_of_nodes()} nodes, {G_cz_end.number_of
```

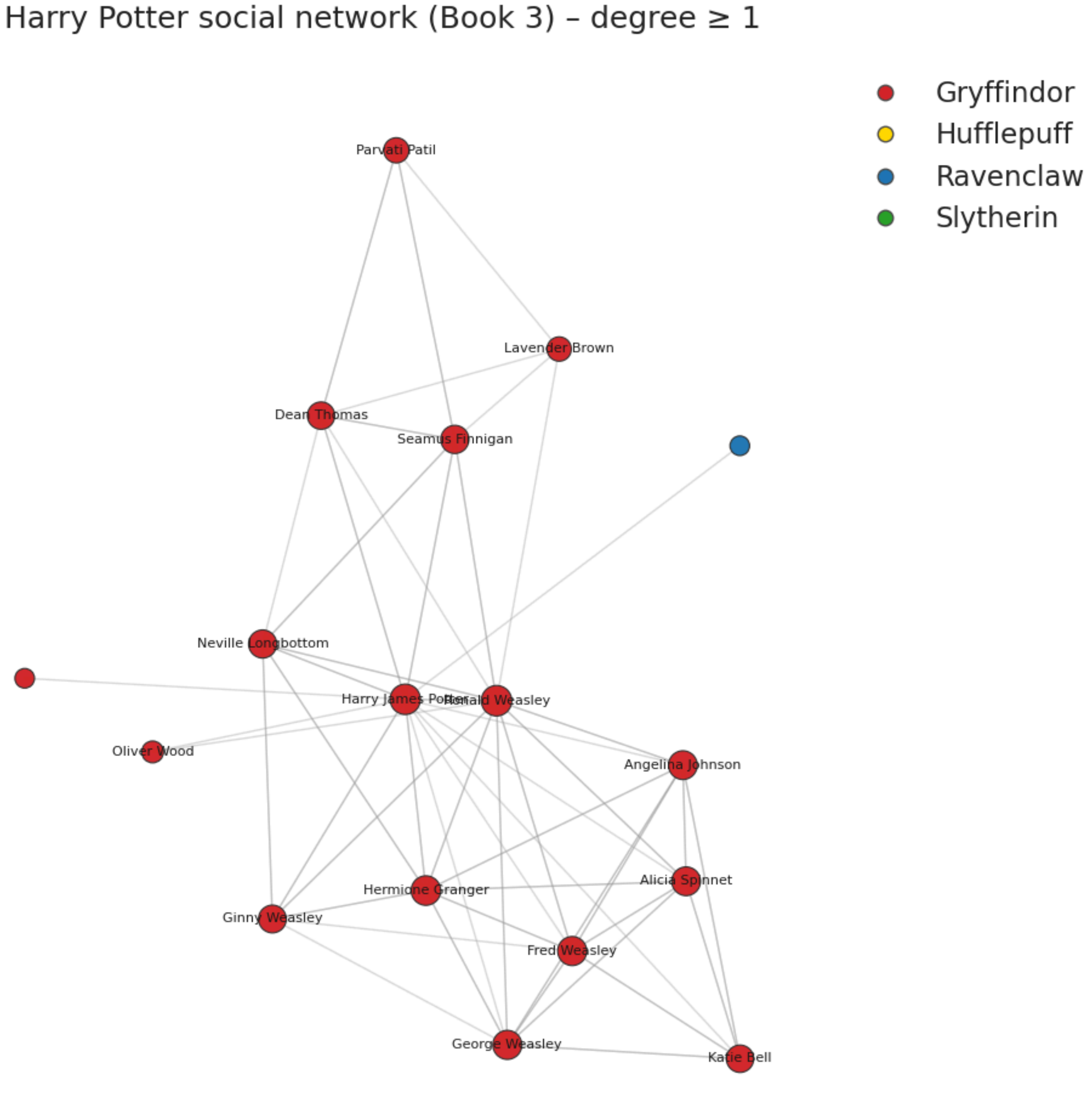
```
Processing sheet: 04end
  nodes: 24, edges: 240
```

```
Processing sheet: 08end
  nodes: 22, edges: 95

Processing sheet: 09start
  nodes: 20, edges: 146

Processing sheet: 09end
  nodes: 20, edges: 129

Processing sheet: 10start
  nodes: 22, edges: 134

Processing sheet: 10end
  nodes: 22, edges: 250

Processing sheet: 11start
  nodes: 23, edges: 122

Processing sheet: 11end
  nodes: 23, edges: 139

Processing sheet: 12start
  nodes: 22, edges: 238

Processing sheet: 12end
  nodes: 22, edges: 275

All Czech classroom graphs created.
Available graphs: ['01start', '01end', '02start', '02end', '03start', '03end',

Czech 01start: 24 nodes, 208 edges
Czech 01end:   24 nodes, 237 edges
```

```
# Cell 5: Visualize HP3 core network (degree >= 1)
deg_all_hp3 = dict(HP3.degree())
core_nodes_hp3 = [v for v, d in deg_all_hp3.items() if d >= 1]

HP3_core = HP3.subgraph(core_nodes_hp3).copy()
print(f"HP3 core network: {HP3_core.number_of_nodes()} nodes, {HP3_core.number

plt.figure(figsize=(10, 10))

# House color map
color_map_scheme = {
    "1": "tab:red",   # Gryffindor
    "2": "gold",      # Hufflepuff
    "3": "tab:blue",  # Ravenclaw
    "4": "tab:green", # Slytherin
}

houses_core = nx.get_node_attributes(HP3_core, "house")
deg_core = dict(HP3_core.degree())

scale_core = 80
node_colors_core = [
    color_map_scheme.get(houses_core.get(v), "lightgrey")
    for v in HP3_core.nodes()
]
node_sizes_core = [
    80 + scale_core * np.log1p(deg_core[v])
    for v in HP3_core.nodes()
]
```

```
pos_core = nx.kamada_kawai_layout(HP3_core)

nx.draw_networkx_edges(
    HP3_core, pos_core,
    alpha=0.35,
    width=1.0,
    edge_color="0.6",
    arrows=False,
)

nx.draw_networkx_nodes(
    HP3_core, pos_core,
    node_color=node_colors_core,
    node_size=node_sizes_core,
    linewidths=0.8,
    edgecolors="0.2",
    alpha=0.98,
)

labels_core = {
    v: HP3_core.nodes[v].get("name", str(v))
    for v, d in deg_core.items() if d >= 2
}
nx.draw_networkx_labels(
    HP3_core, pos_core,
    labels=labels_core,
    font_size=8,
)

legend_elements = [
    Line2D([0], [0], marker="o", linestyle="",
           markerfacecolor="tab:red", markeredgecolor="0.3",
           label="Gryffindor"),
    Line2D([0], [0], marker="o", linestyle="",
           markerfacecolor="gold", markeredgecolor="0.3",
           label="Hufflepuff"),
    Line2D([0], [0], marker="o", linestyle="",
           markerfacecolor="tab:blue", markeredgecolor="0.3",
           label="Ravenclaw"),
    Line2D([0], [0], marker="o", linestyle="",
           markerfacecolor="tab:green", markeredgecolor="0.3",
           label="Slytherin"),
]

plt.legend(
    handles=legend_elements,
    frameon=False,
    loc="upper left",
    bbox_to_anchor=(1.02, 1.0),
)

plt.title("Harry Potter social network (Book 3) — degree ≥ 1", pad=16)
plt.axis("off")
plt.tight_layout()
plt.show()
```

```
HP3 core network: 17 nodes, 87 edges
```




```
# Cell 6: Global structural statistics and change over time
def graph_summary(G, name, directed=True):
    n = G.number_of_nodes()
    m = G.number_of_edges()

    if directed and isinstance(G, nx.DiGraph):
        density = nx.density(G)
        reciprocity = nx.reciprocity(G)
        G_und = G.to_undirected()
    else:
        density = nx.density(G)
        reciprocity = np.nan
        G_und = G

    transitivity = nx.transitivity(G_und)
    avg_clustering = nx.average_clustering(G_und)

    if nx.is_connected(G_und):
        avg_path_len = nx.average_shortest_path_length(G_und)
    else:
```

```
        gcc_nodes = max(nx.connected_components(G_und), key=len)
        avg_path_len = nx.average_shortest_path_length(G_und.subgraph(gcc_node

    return pd.DataFrame([{
        "name": name,
        "n_nodes": n,
        "n_edges": m,
        "density": density,
        "reciprocity": reciprocity,
        "transitivity": transitivity,
        "avg_clustering": avg_clustering,
        "avg_path_len_gcc": avg_path_len,
    }])

df_stats = pd.concat(
    [
        graph_summary(HP2, "HP2", directed=True),
        graph_summary(HP3, "HP3", directed=True),
        graph_summary(G_cz_start, "Czech_01start", directed=True),
        graph_summary(G_cz_end,   "Czech_01end",   directed=True),
    ],
    ignore_index=True,
)
df_stats
```

	name	n_nodes	n_edges	density	reciprocity	transitivity	avg_cluster
0	HP2	64	91	0.022569	0.791209	0.640288	0.199
1	HP3	64	87	0.021577	0.804598	0.636364	0.184
2	Czech_01start	24	208	0.376812	0.673077	0.661432	0.725
3	Czech_01end	24	237	0.429348	0.767932	0.711697	0.791

Next steps: Generate code with df_stats New interactive sheet

```
# Cell 7: Structural change between t1 and t2 (HP vs Czech)
def structural_change(G_start, G_end, label):
    s1 = graph_summary(G_start, label + "_start", directed=True).iloc[0]
    s2 = graph_summary(G_end,   label + "_end",   directed=True).iloc[0]
    return pd.DataFrame([{
        "pair": label,
        "delta_density":         s2["density"]        - s1["density"],
        "delta_reciprocity":     s2["reciprocity"]    - s1["reciprocity"],
        "delta_transitivity":    s2["transitivity"]   - s1["transitivity"],
        "delta_avg_clustering":  s2["avg_clustering"] - s1["avg_clustering"],
    }])

df_struct_change = pd.concat(
    [
        structural_change(HP2, HP3, "HP"),
        structural_change(G_cz_start, G_cz_end, "Czech_01"),
    ],
    ignore_index=True,
)
df_struct_change
```

	pair	delta_density	delta_reciprocity	delta_transitivity	delta_avg_clus
0	HP	-0.000992	0.013389	-0.003924	-(
1	Czech_01	0.052536	0.094856	0.050265	(

Next steps: Generate code with df_struct_change New interactive sheet

```
# Cell 8: Node-level measures for all four networks
def node_level_measures(G, name, directed=True):
    if directed and isinstance(G, nx.DiGraph):
        indeg = dict(G.in_degree())
        outdeg = dict(G.out_degree())
        deg = dict(G.degree())
        G_und = G.to_undirected()
    else:
        deg = dict(G.degree())
        indeg = {v: np.nan for v in G.nodes()}
        outdeg = {v: np.nan for v in G.nodes()}
        G_und = G

    # Use the largest connected component for betweenness and eigenvector
    gcc_nodes = max(nx.connected_components(G_und), key=len)
    G_gcc = G_und.subgraph(gcc_nodes)

    betw = nx.betweenness_centrality(G_gcc, normalized=True)
    eig = nx.eigenvector_centrality_numpy(G_gcc)

    rows = []
    for v in G.nodes():
        attrs = G.nodes[v]
        row = {
            "graph": name,
            "node": v,
            "name": attrs.get("name", v),
            "degree": deg[v],
            "indegree": indeg[v],
            "outdegree": outdeg[v],
            "betweenness": betw.get(v, 0.0),
            "eigenvector": eig.get(v, 0.0),
            "house": attrs.get("house", None),
            "gender": attrs.get("gender", None),
            "schoolyear": attrs.get("schoolyear", None),
        }
        # Czech-specific attributes if present
        for extra in [
            "HISEI",
            "whole_classroom_talk_end",
            "literacy_end",
            "need_of_success",
            "avoidance_of_failure",
        ]:
            if extra in attrs:
                row[extra] = attrs[extra]
        rows.append(row)
    return pd.DataFrame(rows)
```

```
df_nodes_hp2 = node_level_measures(HP2, "HP2", directed=True)
df_nodes_hp3 = node_level_measures(HP3, "HP3", directed=True)
df_nodes_cz_start = node_level_measures(G_cz_start, "Czech_01start", directed=
df_nodes_cz_end   = node_level_measures(G_cz_end,   "Czech_01end",   directed=

df_nodes_all = pd.concat(
    [df_nodes_hp2, df_nodes_hp3, df_nodes_cz_start, df_nodes_cz_end],
    ignore_index=True,
)
df_nodes_all.head()
```

	graph	node	name	degree	indegree	outdegree	betweenness	eigenvector	h
0	HP2	1	Adrian Pucey	0	0	0	0.000000	0.000000	
1	HP2	2	Alicia Spinnet	15	7	8	0.002451	0.309742	
2	HP2	3	Angelina Johnson	13	7	6	0.002451	0.309742	
3	HP2	4	Anthony Goldstein	0	0	0	0.000000	0.000000	
4	HP2	5	Blaise Zabini	0	0	0	0.000000	0.000000	

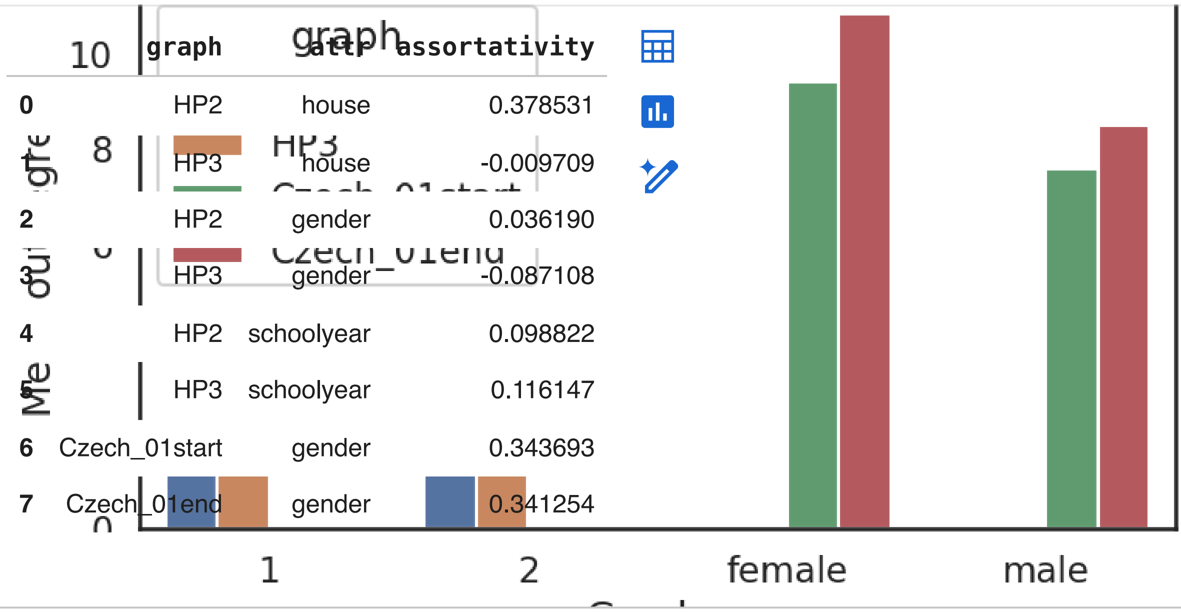
Next steps: Generate code with df_nodes_all New interactive sheet

```
# Cell 9: Outdegree distributions and gender differences
# 9a. Outdegree distribution by graph (boxplot)
plt.figure(figsize=(8, 5))
sns.boxplot(
    data=df_nodes_all,
    x="graph",
    y="outdegree",
)
plt.xlabel("")
plt.ylabel("Outdegree")
plt.title("Outdegree distribution by graph")
plt.tight_layout()
plt.show()

# 9b. Outdegree ECDF by graph
plt.figure(figsize=(8, 5))
sns.ecdfplot(
    data=df_nodes_all,
    x="outdegree",
    hue="graph",
)
plt.xlabel("Outdegree")
plt.ylabel("Empirical CDF")
plt.title("Outdegree ECDF: HP2 / HP3 / Czech_01start / Czech_01end")
plt.tight_layout()
plt.show()

# 9c. Mean outdegree by gender and graph
def mean_outdeg_by_gender(df_nodes, graph_name):
```

```
    sub = df_nodes[df_nodes["graph"] == graph_name]
    return (
        sub.groupby("gender")["outdegree"]
        .agg(["count", "mean", "std"])
        .reset_index()
        .assign(graph=graph_name)
    )

df_gender_hp2 = mean_outdeg_by_gender(df_nodes_all, "HP2")
df_gender_hp3 = mean_outdeg_by_gender(df_nodes_all, "HP3")
df_gender_cz_start = mean_outdeg_by_gender(df_nodes_all, "Czech_01start")
df_gender_cz_end   = mean_outdeg_by_gender(df_nodes_all, "Czech_01end")

df_gender = pd.concat(
    [df_gender_hp2, df_gender_hp3, df_gender_cz_start, df_gender_cz_end],
    ignore_index=True,
)

df_gender_plot = df_gender.dropna(subset=["gender"]).copy()

plt.figure(figsize=(8, 5))
sns.barplot(
    data=df_gender_plot,
    x="gender",
    y="mean",
    hue="graph",
)
plt.ylabel("Mean outdegree")
plt.xlabel("Gender")
plt.title("Mean outdegree by gender and graph")
plt.tight_layout()
plt.show()
```


```
# === Cell 10: Assortativity (homophily) by attribute ===
def assortativity_by_attr(G, attr, directed=True):
    if directed and isinstance(G, nx.DiGraph):
        H = G.to_undirected()
    else:
        H = G
    nodes_with_attr = [v for v in H.nodes() if attr in H.nodes[v]]
    H = H.subgraph(nodes_with_attr).copy()
    if H.number_of_edges() == 0:
        return np.nan
    return nx.attribute_assortativity_coefficient(H, attr)

assort_hp2_house = assortativity_by_attr(HP2, "house", directed=True)
assort_hp3_house = assortativity_by_attr(HP3, "house", directed=True)
assort_hp2_gender = assortativity_by_attr(HP2, "gender", directed=True)
assort_hp3_gender = assortativity_by_attr(HP3, "gender", directed=True)
assort_hp2_year   = assortativity_by_attr(HP2, "schoolyear", directed=True)
assort_hp3_year   = assortativity_by_attr(HP3, "schoolyear", directed=True)

assort_cz_start_gender = assortativity_by_attr(G_cz_start, "gender", directed=
assort_cz_end_gender   = assortativity_by_attr(G_cz_end,   "gender", directed=

df_assort = pd.DataFrame(
    [
        {"graph": "HP2",          "attr": "house",      "assortativity": assor
        {"graph": "HP3",          "attr": "house",      "assortativity": assor
        {"graph": "HP2",          "attr": "gender",     "assortativity": assor
        {"graph": "HP3",          "attr": "gender",     "assortativity": assor
        {"graph": "HP2",          "attr": "schoolyear", "assortativity": assor
        {"graph": "HP3",          "attr": "schoolyear", "assortativity": assor
        {"graph": "Czech_01start","attr": "gender",     "assortativity": assor
        {"graph": "Czech_01end",  "attr": "gender",     "assortativity": assor
    ]
)
df_assort
```

	graph	attr	assortativity
0	HP2	house	0.378531
1	HP3	house	-0.009709
2	HP2	gender	0.036190
3	HP3	gender	-0.087108
4	HP2	schoolyear	0.098822
5	HP3	schoolyear	0.116147
6	Czech_01start	gender	0.343693
7	Czech_01end	gender	0.341254



Gender

Next steps: Generate code with df_assort New interactive sheet

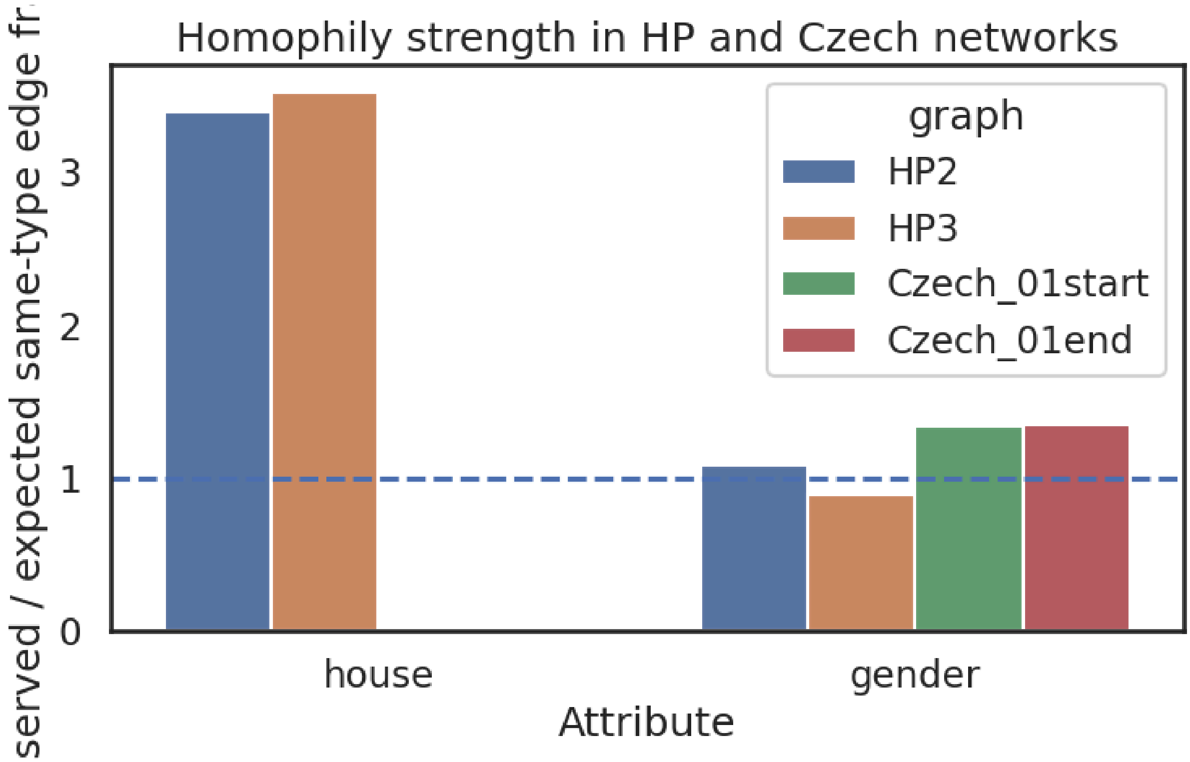
```
# Cell 11: Mixing summary (observed vs expected homophily)
def mixing_summary(G, attr, directed=True):
    """
    Return mixing matrix, observed same-type edge fraction, expected same-type
    fraction under random mixing with same attribute distribution, and their r
    """
    if directed and isinstance(G, nx.DiGraph):
        H = G.to_undirected()
    else:
        H = G

    keep = [v for v in H.nodes() if attr in H.nodes[v]]
    H = H.subgraph(keep).copy()
    if H.number_of_edges() == 0:
        return None

    mix = nx.attribute_mixing_matrix(H, attr, normalized=True)
    same_frac = np.trace(mix)

    values = [H.nodes[v][attr] for v in H.nodes()]
    vc = pd.Series(values).value_counts(normalize=True)
    expected_same_frac = (vc ** 2).sum()

    ratio = same_frac / expected_same_frac if expected_same_frac > 0 else np.n
    return mix, same_frac, expected_same_frac, ratio

# HP: house / gender / schoolyear
mix_hp2_house = mixing_summary(HP2, "house", directed=True)
mix_hp3_house = mixing_summary(HP3, "house", directed=True)
mix_hp2_gender = mixing_summary(HP2, "gender", directed=True)
mix_hp3_gender = mixing_summary(HP3, "gender", directed=True)

# Czech: gender
mix_cz_start_gender = mixing_summary(G_cz_start, "gender", directed=True)
mix_cz_end_gender   = mixing_summary(G_cz_end,   "gender", directed=True)

def summarize_mixing(label, attr, result):
    if result is None:
        return None
    mix, same_frac, exp_same, ratio = result
    return {
        "graph": label,
        "attr": attr,
        "same_fraction": same_frac,
        "expected_same_fraction": exp_same,
        "ratio_obs_over_expected": ratio,
    }

rows = []
rows.append(summarize_mixing("HP2", "house", mix_hp2_house))
rows.append(summarize_mixing("HP3", "house", mix_hp3_house))
rows.append(summarize_mixing("HP2", "gender", mix_hp2_gender))
rows.append(summarize_mixing("HP3", "gender", mix_hp3_gender))
rows.append(summarize_mixing("Czech_01start", "gender", mix_cz_start_gender))
rows.append(summarize_mixing("Czech_01end",   "gender", mix_cz_end_gender))

df_mixing_summary = pd.DataFrame([r for r in rows if r is not None])
```

```
df_mixing_summary
```

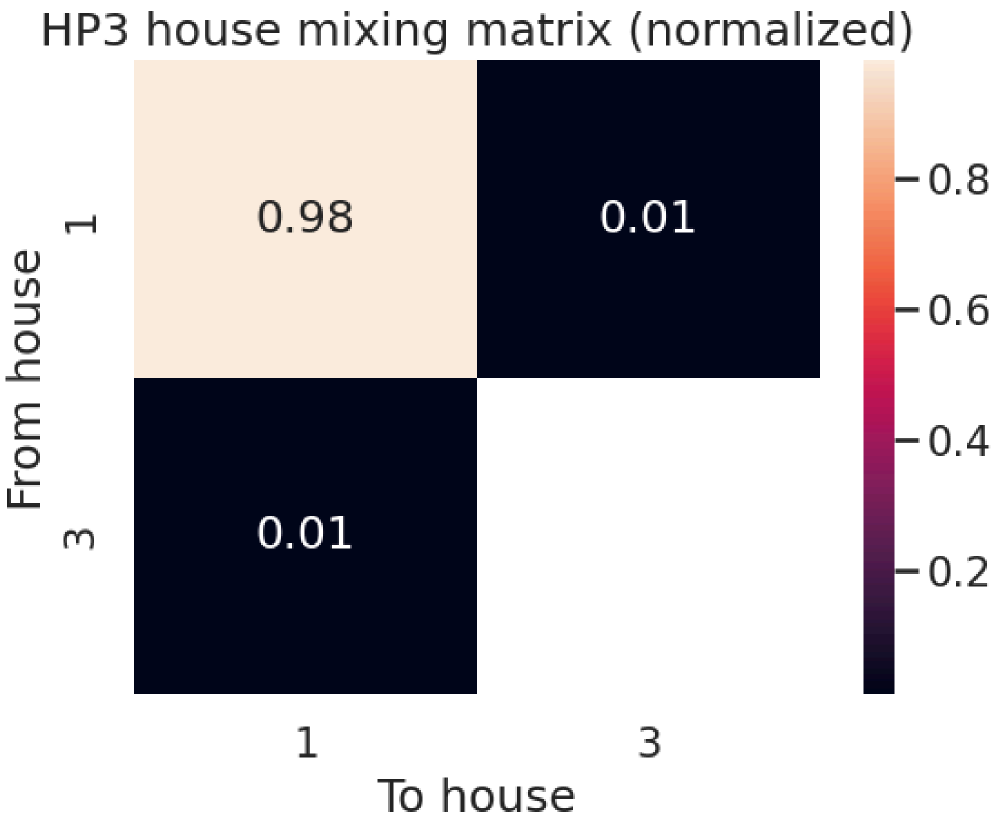
	graph	attr	same_fraction	expected_same_fraction	ratio_obs_over_exp
0	HP2	house	0.945455	0.278320	3.3
1	HP3	house	0.980769	0.278320	3.5
2	HP2	gender	0.581818	0.539551	1.0
3	HP3	gender	0.480769	0.539551	0.8
4	Czech_01start	gender	0.688406	0.513889	1.3
5	Czech_01end	gender	0.691781	0.513889	1.3

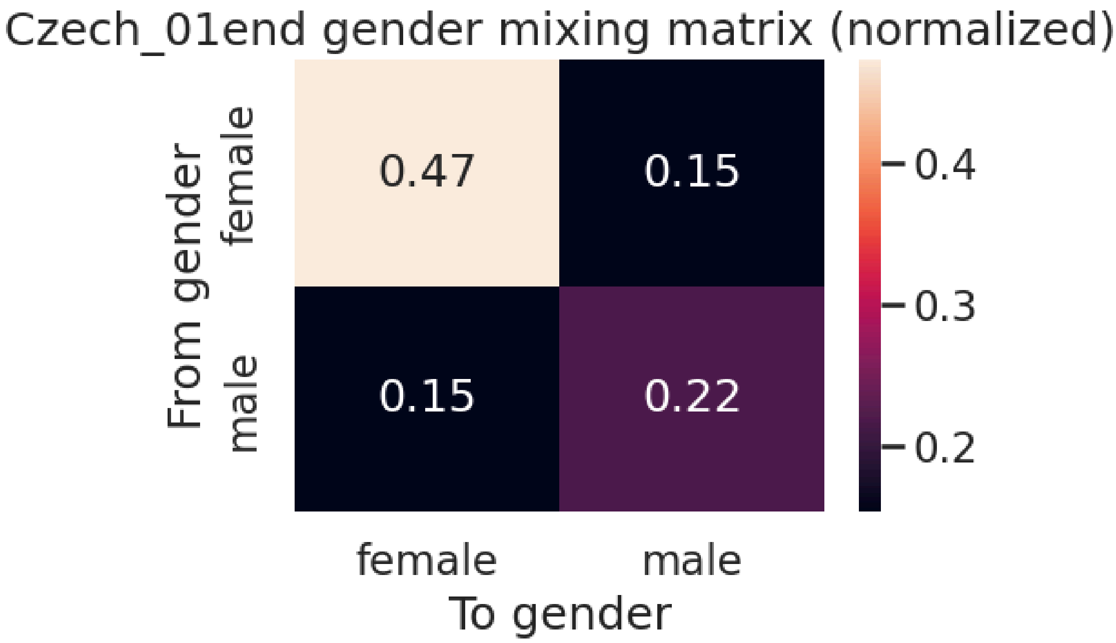
Next steps: Generate code with df_mixing_summary New interactive sheet

```
# Cell 12: Plot homophily strength (observed / expected)
plt.figure(figsize=(8, 5))
sns.barplot(
    data=df_mixing_summary,
    x="attr",
    y="ratio_obs_over_expected",
    hue="graph",
)
plt.axhline(1.0, linestyle="--")
plt.ylabel("Observed / expected same-type edge fraction")
plt.xlabel("Attribute")
plt.title("Homophily strength in HP and Czech networks")
plt.tight_layout()
plt.show()
```



```
# === Cell 13: Mixing matrix heatmaps (HP3 house, Czech_01end gender) ===
# HP3: house mixing
H_hp3 = HP3.to_undirected()
keep_hp3 = [v for v in H_hp3.nodes() if "house" in H_hp3.nodes[v]]
H_hp3 = H_hp3.subgraph(keep_hp3).copy()

mix_dict_hp3 = nx.attribute_mixing_dict(H_hp3, "house", normalized=True)
labels_house = sorted(mix_dict_hp3.keys())
mix_mat_hp3 = pd.DataFrame(mix_dict_hp3).reindex(
    index=labels_house, columns=labels_house
)

plt.figure(figsize=(6, 5))
sns.heatmap(
    mix_mat_hp3,
    annot=True,
    fmt=".2f",
)
plt.xlabel("To house")
plt.ylabel("From house")
plt.title("HP3 house mixing matrix (normalized)")
plt.tight_layout()
plt.show()

# Czech_01end: gender mixing
H_cz = G_cz_end.to_undirected()
keep_cz = [v for v in H_cz.nodes() if "gender" in H_cz.nodes[v]]
H_cz = H_cz.subgraph(keep_cz).copy()

mix_dict_cz = nx.attribute_mixing_dict(H_cz, "gender", normalized=True)
labels_gender = sorted(mix_dict_cz.keys())
mix_mat_cz = pd.DataFrame(mix_dict_cz).reindex(
    index=labels_gender, columns=labels_gender
)

plt.figure(figsize=(5, 4))
sns.heatmap(
    mix_mat_cz,
    annot=True,
    fmt=".2f",
)
plt.xlabel("To gender")
plt.ylabel("From gender")
plt.title("Czech_01end gender mixing matrix (normalized)")
plt.tight_layout()
plt.show()
```






```
# Cell 14: Structural holes and bridging centrality
def add_structural_hole_measures(G, df_nodes, graph_name, directed=True):
    """
    For a given graph, compute:
      - k-core index
      - bridging coefficient
      - bridging centrality (bridging coefficient * betweenness)
    and add them to df_nodes for rows with graph == graph_name.
    """
    if directed and isinstance(G, nx.DiGraph):
        H = G.to_undirected()
    else:
        H = G
```

```
    core_num = nx.core_number(H)

    sub = df_nodes[df_nodes["graph"] == graph_name]
    if "betweenness" in sub.columns and sub["betweenness"].notna().any():
        betw_dict = dict(zip(sub["node"], sub["betweenness"]))
    else:
        betw_dict = nx.betweenness_centrality(H, normalized=True)

    deg = dict(H.degree())
    neighbors = {v: set(H.neighbors(v)) for v in H.nodes()}

    bridging_coeff = {}
    bridging_centrality = {}

    for v in H.nodes():
        k_v = deg.get(v, 0)
        if k_v == 0:
            bridging_coeff[v] = 0.0
            bridging_centrality[v] = 0.0
            continue
        neigh = neighbors[v]
        if not neigh:
            bridging_coeff[v] = 0.0
            bridging_centrality[v] = 0.0
            continue
        denom = sum(1.0 / deg.get(u, 1) for u in neigh)
        bc = (1.0 / k_v) / denom if denom > 0 else 0.0
        bridging_coeff[v] = bc
        bridging_centrality[v] = bc * betw_dict.get(v, 0.0)

    df_nodes2 = df_nodes.copy()
    mask = df_nodes2["graph"] == graph_name
    df_nodes2.loc[mask, "kcore"] = df_nodes2.loc[mask, "node"].map(core_num)
    df_nodes2.loc[mask, "bridging_coeff"] = df_nodes2.loc[mask, "node"].map(br
    df_nodes2.loc[mask, "bridging_centrality"] = df_nodes2.loc[mask, "node"].m

    return df_nodes2

df_nodes_all = add_structural_hole_measures(HP2,        df_nodes_all, "HP2",
df_nodes_all = add_structural_hole_measures(HP3,        df_nodes_all, "HP3",
df_nodes_all = add_structural_hole_measures(G_cz_start, df_nodes_all, "Czech_0
df_nodes_all = add_structural_hole_measures(G_cz_end,   df_nodes_all, "Czech_0

df_nodes_all.head()
```

	graph	node	name	degree	indegree	outdegree	betweenness	eigenvector	h
0	HP2	1	Adrian Pucey	0	0	0	0.000000	0.000000	
1	HP2	2	Alicia Spinnet	15	7	8	0.002451	0.309742	
2	HP2	3	Angelina Johnson	13	7	6	0.002451	0.309742	
3	HP2	4	Anthony Goldstein	0	0	0	0.000000	0.000000	
4	HP2	5	Blaise Zabini	0	0	0	0.000000	0.000000	

Next steps: Generate code with df_nodes_all New interactive sheet

```
# Cell 15: Top bridging nodes (structural holes)
def top_bridgers(df_nodes, graph_name, k=10):
    sub = df_nodes[df_nodes["graph"] == graph_name].copy()
    return sub.sort_values("bridging_centrality", ascending=False)[
        [
            "name",
            "house",
            "gender",
            "schoolyear",
            "degree",
            "betweenness",
            "kcore",
            "bridging_coeff",
            "bridging_centrality",
        ]
    ].head(k)

print("HP2 top bridging nodes:")
display(top_bridgers(df_nodes_all, "HP2", k=10))

print("HP3 top bridging nodes:")
display(top_bridgers(df_nodes_all, "HP3", k=10))

print("Czech_01start top bridging students:")
display(top_bridgers(df_nodes_all, "Czech_01start", k=10))

print("Czech_01end top bridging students:")
display(top_bridgers(df_nodes_all, "Czech_01end", k=10))
```

HP2 top bridging nodes:

	name	house	gender	schoolyear	degree	betweenness	kcore	bridging_co
24	Harry James Potter	1	1	1991	24	0.586397	7.0	0.01(
55	Ronald Weasley	1	1	1991	17	0.104779	6.0	0.04ŧ
20	Ginny Weasley	1	2	1992	4	0.003676	2.0	0.51:
19	George Weasley	1	1	1989	17	0.012868	7.0	0.09⁄
18	Fred Weasley	1	1	1989	17	0.012868	7.0	0.09⁄
1	Alicia Spinnet	1	2	1989	15	0.002451	7.0	0.13ī
25	Hermione Granger	1	2	1991	13	0.002451	7.0	0.13ī
2	Angelina Johnson	1	2	1989	13	0.002451	7.0	0.13ī
8	Colin Creevey	1	1	1992	0	0.000000	0.0	0.00(
9	Cormac McLaggen	1	1	1990	0	0.000000	0.0	0.00(

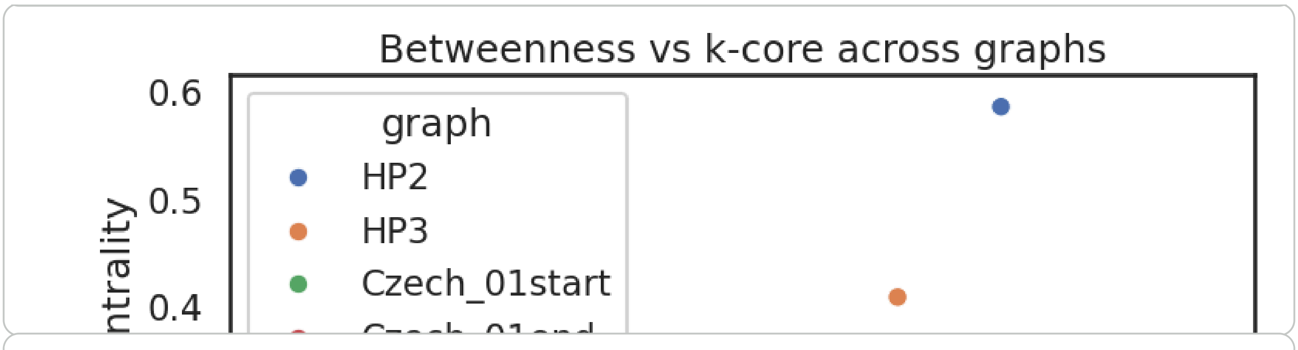
HP3 top bridging nodes:

	name	house	gender	schoolyear	degree	betweenness	kcore	bridging_
74	Dean Thomas	1	1	1991	9	0.059722	4.0	0.·
121	Seamus Finnigan	1	1	1991	11	0.059722	4.0	0.·

```
# Cell 16: Plots for structural holes
# 16a. Betweenness vs k-core by graph
plt.figure(figsize=(8, 6))
sns.scatterplot(
    data=df_nodes_all,
    x="kcore",
    y="betweenness",
    hue="graph",
)
plt.ylabel("Betweenness centrality")
plt.xlabel("k-core index")
plt.title("Betweenness vs k-core across graphs")
plt.tight_layout()
plt.show()

# 16b. Bridging centrality distribution by graph
plt.figure(figsize=(8, 5))
sns.boxplot(
    data=df_nodes_all,
    x="graph",
    y="bridging_centrality",
)
```

```
plt.ylabel("Bridging centrality")
plt.xlabel("")
plt.title("Bridging centrality distribution")
plt.tight_layout()
plt.show()

# 16c. Bridging centrality by house (HP only)
df_hp_bridge = df_nodes_all[
    df_nodes_all["graph"].isin(["HP2", "HP3"])
].dropna(subset=["house"])

plt.figure(figsize=(9, 5))
sns.boxplot(
    data=df_hp_bridge,
    x="house",
    y="bridging_centrality",
    hue="graph",
)
plt.ylabel("Bridging centrality")
plt.xlabel("House")
plt.title("Bridging roles by house (HP2 vs HP3)")
plt.tight_layout()
plt.show()
```

165	114	None	female	None	22	0.052903	9.0	0.04631[illegible]
152	101	None	male	None	20	0.024827	8.0	0.068421
171	120	None	female	None	21	0.014879	9.0	0.07917[illegible]
153	102	None	female	None	14	0.005703	8.0	0.17596[illegible]
158	107	None	female	None	26	0.019005	9.0	0.04854[illegible]
160	109	None	female	None	30	0.019005	9.0	0.04854[illegible]
175	124	None	female	None	27	0.019005	9.0	0.04854[illegible]




```
# Cell 17: New edge diagnostics (reciprocity and triadic closure)
def edge_set(G):
    return set(G.edges())

def new_edge_diagnostics(G_start, G_end, label):
    """
    For edges that appear in G_end but not in G_start, compute:
      - fraction that are reciprocal with respect to G_start
      - fraction that close at least one triangle using G_start's undirected st
    """
    E1 = edge_set(G_start)
    E2 = edge_set(G_end)
    new_edges = list(E2 - E1)
    if not new_edges:
        return pd.DataFrame([{
            "pair": label,
            "n_new_edges": 0,
            "frac_new_reciprocal": np.nan,
            "frac_new_closing_triangles": np.nan,
        }])

    G_und_start = G_start.to_undirected()
    neighbors_start = {v: set(G_und_start.neighbors(v)) for v in G_und_start.no

    reciprocal_flags = []
    triad_flags = []

    for i, j in new_edges:
        reciprocal_flags.append((j, i) in E1)

        neigh_i = neighbors_start.get(i, set())
        neigh_j = neighbors_start.get(j, set())
        closing = len(neigh_i & neigh_j) > 0
        triad_flags.append(closing)

    return pd.DataFrame([{
        "pair": label,
        "n_new_edges": len(new_edges),
        "frac_new_reciprocal": np.mean(reciprocal_flags),
        "frac_new_closing_triangles": np.mean(triad_flags),
    }])

diag_hp = new_edge_diagnostics(HP2, HP3, "HP2→HP3")
diag_cz = new_edge_diagnostics(G_cz_start, G_cz_end, "Czech 01start→01end")

df_new_edge_diag = pd.concat([diag_hp, diag_cz], ignore_index=True)
df_new_edge_diag
```