Portfolio Optimization Using Sharpe Ratio and Volatility Minimization

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***Abstract*—This study explores portfolio optimization strategies with a focus on maximizing the Sharpe ratio and minimizing volatility. Using monthly returns of three assets (AAPL, GOOGL, MSFT), we evaluate portfolio performance through metrics such as return, variance, and Sharpe ratio. The results highlight optimal asset allocation strategies for achieving desired risk-return trade-offs.**

I. Introduction

Portfolio optimization is a cornerstone of modern finance, aiming to allocate assets efficiently to maximize returns while minimizing risk. This project analyzes a three-asset portfolio using historical data for AAPL, GOOGL, and MSFT. Key objectives include:

- Calculating portfolio returns, variance, and covariance matrix.
- Identifying portfolios with minimum volatility and maximum Sharpe ratio.
- Providing actionable insights for risk-adjusted investment strategies.

II. Methodology

A. Data Description

The analysis uses monthly return data for three assets:

- **AAPL**: Mean monthly return: 2.46%
- **GOOGL**: Mean monthly return: 2.85%
- **MSFT**: Mean monthly return: 1.26%

B. Metrics

- **Portfolio Return**: Weighted average of individual asset returns.
- **Portfolio Variance**: $\sigma_p^2 = \mathbf{w}^T \Sigma \mathbf{w}$, where Σ is the covariance matrix.
- **Sharpe Ratio**: $S = \frac{R_p - R_f}{\sigma_p}$, where R_p is portfolio return, R_f is the risk-free rate, and σ_p is portfolio volatility.

C. Optimization Approach

Portfolios are evaluated for:

- **Minimum Volatility**: Minimizing σ_p.
- **Maximum Sharpe Ratio**: Maximizing S.

III. Results

A. Key Metrics

- **Monthly Portfolio Return**: 2.19%
- **Monthly Portfolio Variance**: 0.12%

B. Covariance Matrix

The covariance matrix of asset returns is shown in Table I.

TABLE I
Covariance Matrix of Asset Returns

Ticker	AAPL	GOOGL	MSFT
AAPL	0.002777	0.000007	0.001083
GOOGL	0.000007	0.002817	0.000257
MSFT	0.001083	0.000257	0.002725

C. Optimization Results

1) Portfolio with Minimum Volatility:

- **Annual Sharpe Ratio**: 1.24
- **Annual Return**: 22.31%
- **Annual Volatility**: 3.26%
- **Asset Allocation**:
  - AAPL: 38.16%
  - MSFT: 8.10%
  - GOOGL: 53.74%

2) Portfolio with Maximum Sharpe Ratio:

- **Annual Sharpe Ratio**: 1.52
- **Annual Return**: 30.37%
- **Annual Volatility**: 3.99%
- **Asset Allocation**:
  - AAPL: 35.0%
  - MSFT: 15.0%
  - GOOGL: 50.0%

IV. Discussion

The results demonstrate that portfolios optimized for Sharpe ratio prioritize higher returns at the expense of marginally increased volatility. The minimum volatility portfolio offers a more conservative allocation suitable for risk-averse investors.

V. Conclusion

This analysis highlights the trade-offs between risk and return in portfolio optimization. Future work could incorporate additional assets, risk-free rates, and macroeconomic factors to refine the model.

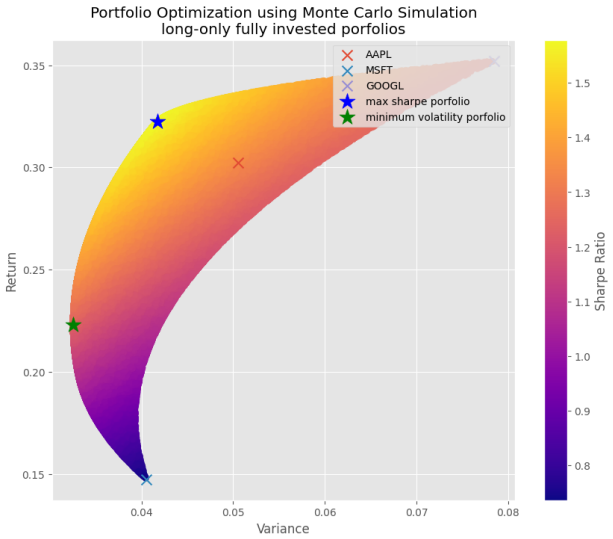

Fig. 1. Portfolio Optimization using Monte Carlo Simulation: Long-only Fully Invested Portfolios

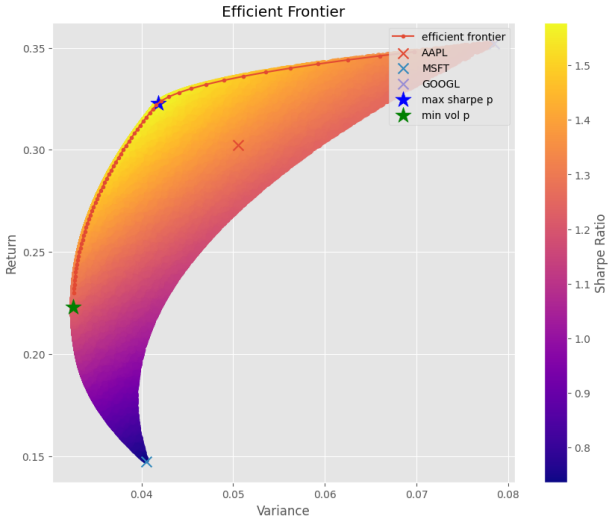

Fig. 2. Efficient Frontier of Portfolio Optimization

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