Have you ever looked closely at a snowflake that landed on your palm or the opening of a Nautilus shell? As examples of fractals in nature, they both have the feature of repeating patterns. Fractals are often viewed as intriguing figures that hold the beauty of infinite, repetitive patterns. In this essay, we will examine how fractals appear in nature, and why they are useful in mathematics as the graphical visualization of some equations share similar properties as a fractal.

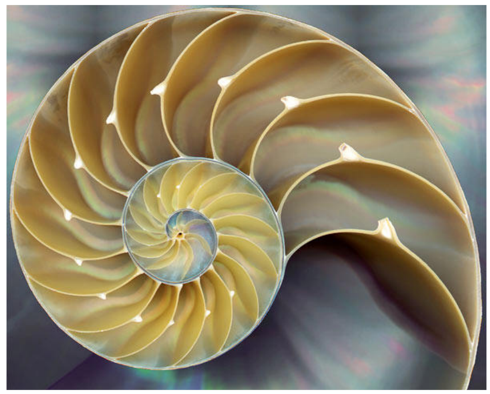

Figure 1: Cross-section of a Nautilus shell that demonstrates the logarithmic spiral fractal[1]

There was a long process to settle on the formal definition of a fractal. Benoit Mandelbrot, the mathematician who first used the term "fractal" to describe this kind of structures in 1975, simplified his original definition to be that "a fractal is a shape made of parts similar to the whole in some way".[2] Fractals are *self-similar* objects that has the property of being approximately the same to a part of itself. The definition might not tell us so much about fractals, so let's go back to the snowflake example!

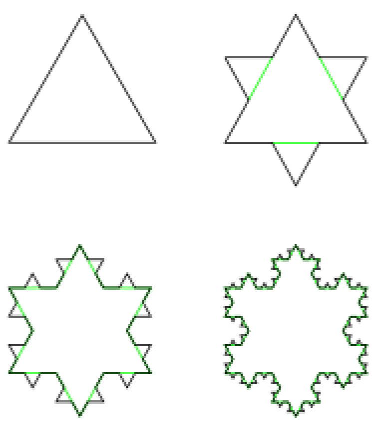

Figure 2: The first four iterations of the Koch snowflake[3]

[1] Picture source

[2] "Fractal." Wikipedia, Wikimedia Foundation, 28 Jan. 2020, https://en.wikipedia.org/wiki/Fractal.

The picture above is how the *Koch snowflake* looks like. Beginning with an equilateral triangle, we paste a smaller equilateral triangle onto the middle part of each side. If one repeats the pasting business for infinite times, a fractal of the Koch snowflake is defined. This probably sounds very bizarre as it is difficult to imagine repeating a process by infinite times. The idea behind it is simple; if we keep zooming into any segment of the Koch snowflake, the visual pattern seems to be repeating itself, but in fact each edge and spike of the snowflake is smaller compared to the initial ones.

You might have noticed that the perimeter of the Koch snowflake is infinite as there are infinitely many repeating spikes. On the other hand, the area of the Koch snowflake is finite. Clearly, we can inscribe the Koch snowflake in a circle, and a circle has a finite area. This is a very counter-intuitive property of fractals.

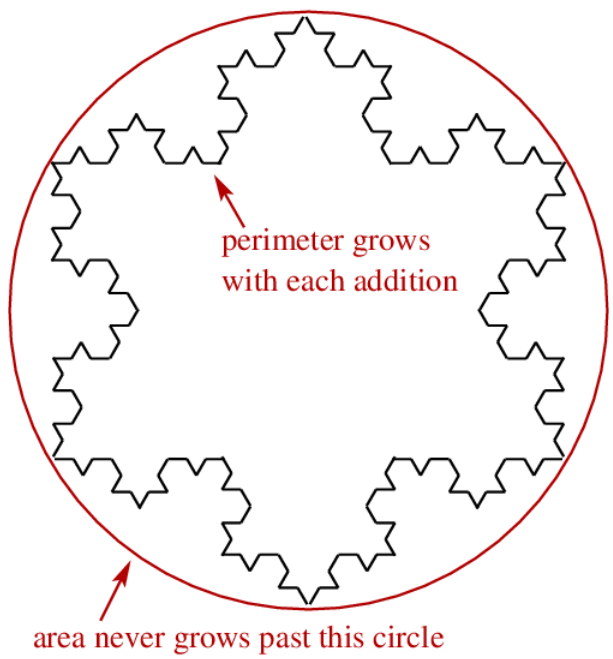

Figure 3: How Koch snowflake has a finite area but infinite perimeter[4]

The reason why fractals are defined in mathematics is not only because its appearance in nature, but the fact that the graphical visualization of mathematical models for the nature have similar characteristics as fractals. A famous example is the *Lorenz attractor*. If we plot out the the equations that Lorenz used for the weather simulation along with a wide range of inputs, we can get the following beautiful, butterfly-like graph. Notice that none of the paths cross each other or form a loop. This figure shares the fractal dimension property of the Koch Snowflake; it also has an infinite perimeter and finite area inscribed in a butterfly shape figure. It is an infinite complex of surfaces.

[3] "Koch Snowflake." Wikipedia, Wikimedia Foundation, 15 Jan. 2020, https://en.wikipedia.org/wiki/Koch_snowflake.

[4] Francis, Matthew. "Fractals for Fun." Galileo's Pendulum, 31 Jan. 2012, www.galileospendulum.org/2012/01/31/fractals-for-fun/.

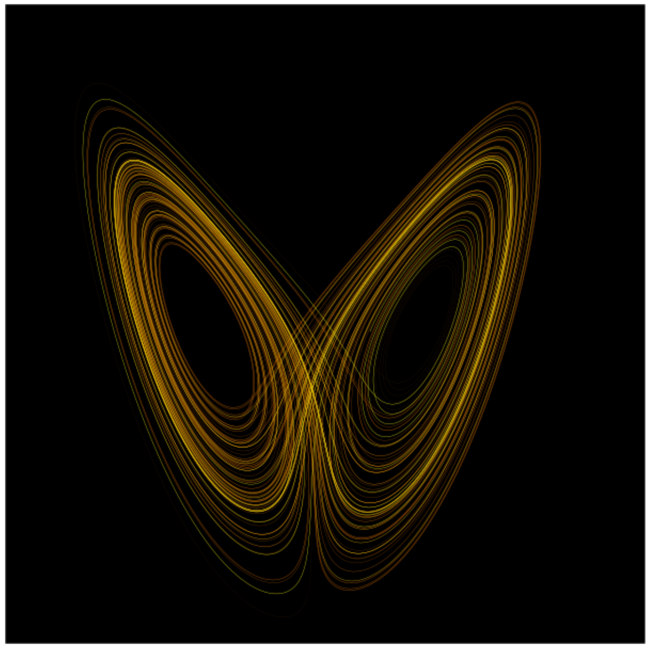

Figure 4: The visualization of Lorenz Attractor[5]

Fractals interact closely with dynamical systems and provide geometric interpretations of chaos, for example, the *Lorenz attractor*, so that people can study the mathematical model better. In practices, modern cell phones use a fractal-looking antenna that has different shapes nested within one another. Each shape corresponds to a radio signal so that the antenna can work at different frequencies at the same time with a higher efficiency than regularly shaped antennas.[6] Using Mandelbrot's words to summarize: "beautiful, damn hard, increasingly useful. That's fractals".[2]

[5] "Butterfly Effect." Wikipedia, Wikimedia Foundation, 16 Mar. 2020, https://en.wikipedia.org/wiki/Butterfly_effect.

[6] Hein, Simeon. "What Are Fractals? The Complex History of Fractals." What Are Fractals?, Gaia, 12 Dec. 2016, www.gaia.com/article/what-are-fractals.