HPS391 Term Paper
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Topic: History of the "new mathematics" education movement in the United States

Education in mathematics is always a controversial subject filled with obstacles throughout history, where new ideas and methods to deliver teaching are introduced continuously. One might wonder what some revolutionary movements are, and how they result in the development of the current mathematics education system, whether it is the practical or philosophical focus. The differences and similarities of these two revolutionary mathematics education reforms that happened in the United States, and as well as their impact will be examined in this paper. These experimental, revolutionary reforms which led to the current system are not sufficient and only mark the start of developing an adequate mathematics education. A mathematics education reform that is able to maximize the effectiveness of teaching and focuses on problem-solving skills is required to satisfy the current demand of mathematical skills that are largely used in computer science and other subjects of interests, and one of the possible reforms in the education system of mathematics currently taking place is the implementation of the inverted style of teaching.

The first reform mathematics education that was primarily focused on abstraction and rigorous languages occurred from approximately 1950 to 1970 and struggled to make a positive impact because of its difficult level of abstract and lack of preparation of teachers. It was partly inspired by Bourbaki, one of the most influential mathematicians in the 20th century. Another reason behind the reform was the technology competition between America and Russia after World War II and after the launch of Sputnik by the Soviet Union. America had a good understanding of the importance of strong mathematics in creating new technology – to train more technical professionals and maintain a military force lead (Herrera & Owen, 2001). In

1955, the College Entrance Examination Board (CEEB) became directly involved in this by appointing a Commission on Mathematics to consider how their examinations should reflect the changes in the field of mathematics that had taken place in the previous 50 years. The commission's nine-point program, finalized in 1959, "called for preparation in concepts and skills to prepare for calculus and analytic geometry at college entry" (p. 85, Herrera & Owen, 2001), paying large attention to mathematical structure in properties of natural, rational, real, and complex numbers, and recommending the use of unifying ideas such as sets, variables, functions, and relations (CEEB, 1959).

Many programs were introduced to serve the purpose of enforcing the initiatives of the nine-point program for Junior and High School adaption. The UICSM program, for example, focuses on the significant changes in the curriculum to identify unifying mathematical themes and use precise language and symbols (Herrera & Owen, 2001). They are mainly required and constantly tested in college-level mathematics courses even now. Moreover, the School Mathematics Study Group (SMSG), funded by the National Science Foundation (NSF), was operated by a process of classroom trials, summer writing sessions, rewriting, and publishing for national distribution in order to train the teachers (Herrera & Owen, 2001). The Madison Project is characterized by the intention to prepare creative supplementary materials to interest students in Mathematics at a younger age.

On the other hand, elementary school implementation is slower and more difficult, since with limited training available, few elementary school teachers have the knowledge that is required for the new curriculum. "Parents reacted to the new mathematics curriculum with consternation due to their inability to help their children with mathematics" (p. 87), while the teachers showed enthusiasm for the advantage of NSF funded summer-long institutes and training offered by the innovative programs. A piece of indirect evidence from the National Assessment of Educational Progress NAEP that accesses 13 years old students indicates a not dramatic upheaval in student's performance after the implementation of these programs (Herrera & Owen, 2001).

It is possible to claim that the reform was a reasonable success. However, as a strong critic cited the College Board's report of a 10-year decline in Scholastic Aptitude Test scores (Usiskin, 1985), the idea that the new math was a failure and people should return to basics was widespread. Consequently, the era after 1970 is called "back to basics", which is the foundation of what mathematics education currently has. People even linked education to a national crisis through the publication of A Nation at Risk (1983), commissioned by the National Commission for Excellence in Education (NCEE). More and more concerns were heard from papers and conferences. As a summary, problem-solving and application are seen to be the curricular focus, while the public "views math as a set of arithmetic skills" (p. 88). People believe that more measurable mathematics contents and observable behaviours such as computation and algebraic manipulation should be included in the learning objectives, rather than focusing on "the precise, structuralist language of sets, logic, and algebraic structures" (p. 87, Herrera & Owen, 2001).

The second reform marked the start of the "back to basics" era, which focuses more on application rather than abstraction or rigorousness of mathematics. In 1989, the National Council of Teachers of Mathematics (NCTM) started to reform mathematics education by changes in content and pedagogy based on the NCTM *Standards*. The key difference compared to the first reform is that NCTM's emphasis is creating mathematics content and instruction suitable to all students, not only designed to prepare college-level studies of mathematics. Its focus shifts to giving practices to students as they are required to use more application and problem-solving knowledge with arithmetic skills. The use of technologies such as calculators or computers is restricted to computational skills or statistics. Instead of listing out specific mathematical topics to be covered by the end of each grade, the *Standards* outline and provide examples to clarify the contents by the followings. The inclusion of discrete mathematics, statistics, and mathematical modelling in secondary studies, with increased attention to applications, the stress on connections of mathematics to the real world, and the emphasis on higher-order thinking through problem-solving, communication, connections and reasoning, are some of the highlights of the curriculum content (Herrera & Owen, 2001). In contrast, some highlights for the pedagogy part are activations of student involvement in discovering and constructing mathematical relationships, rather than merely memorizing procedures and following them. Students can use representations such as computer graphics, tables, pictures, or others as a means to help themselves grasp abstract concepts, with group work to practice mathematical communication with peers and train themselves to justify their ideas properly.

The public views on this second reform are diversified. One positive outcome is the cooperation among the National Science Foundation to produce a curriculum series aligned with the *Standards*. Also, schools have available instructional materials that incorporate the goals envisioned in the reform in hand. However, the change requires teachers, in practice, alter their role from a transmitter of knowledge to a facilitator. There are more responsibilities and new skills needed. The teachers have to engage the class in mathematical discovery and encourage students to discuss mathematics with classmates. This forces the teachers to step out of their accepted position, their comfort zone, to an uncomfortable, new position (Dossey & Halvorsen, 2016). The *Standards* are also vague about the importance of basic computational skills. Students might think there is no need to know the multiplication rule as they can always use a calculator in class, but fundamental arithmetic skills are necessary and frequently used even in daily life. The issue of accountability is another negative consequence that one needs to consider. Not only parents and teachers but also students are increasingly worried and frustrated about their performance on standardized testing for graduation or university entrance exams, as the *Standards* does not align with these high-stakes testing (Herrera & Owen, 2001). Many graduates from secondary school who continues their studies in mathematics at postsecondary institutions or universities experience a huge increase of the level of difficulties on contents of mathematics courses – writing rigorous math proofs and solving challenging assignment problems, as the mathematics curriculum in secondary school was way easier and less critical thinking needed to complete homework.

Both of the reforms have positive and negative aspects that result in the formation of current mathematics education, while the negative effects were more impactful. The negative aspects of the first and second reform, in simple words, were caused by the inclusion of too abstract and challenging contents of mathematics, and the inclusion of not abstract enough contents of the mathematics curriculum in elementary and secondary schools respectively. None of them was sufficient to claim itself as a perfect approach to deliver mathematics teaching. As technology advances, educators need to update the mathematics education content and pedagogy to match with the demand. One of the many new methods proposed and implemented in the 21st century to counteract these negative effects is the inverted/flipped learning approach for math courses, and this approach might be the candidate of the most significant revolutionary reforms to teaching in mathematics education in the long-term span.

In recent years, in order to recover the negative impacts and maintain the positive impacts from the past two reforms, student-centred learning modes have been widely used and have produced many positive outcomes on the effectiveness of learning, which leads to the implementation of the inverted learning approach in different educational institutions and subjects of study, including U of T. There are many forms of the inverted learning approach such as providing interactable e-books. For instance, the *Active Calculus* textbook series by Matthew Boelkins and Steven Schlicker is "a free, open-source [online] calculus text that is designed to support an active learning approach in the standard first two semesters of calculus" (p. 1, Matthew & Steven) in the university level. It has more than 250 interactive exercises in the HTML version, which is also accessible through smartphones, while some other "e-book system … provides an additional function for teachers to realize the learning status of the students, so that proper in-class remedial instruction or activities can be provided accordingly" (p. 189, Hwang, G.-J., & Lai, C.-L., 2017). Through these intense practices, students are actively exposed to abstract mathematics that is challenging to understand compares to the practical mathematics problems that they are used to solving by arithmetic skills in secondary schools, due to the second reform.

In addition, the implementation specifically in mathematics at the University of Toronto uses another form of inverted learning. Many first-year math courses such as calculus and linear algebra, which are required for students who are interested in pursuing a program in mathematics at U of T, have been reconstructed. Students now have access to the course materials beforehand, as the course contents are posted priory online for previewing and for later reviewing. In lectures, students are more concentrated on discussing math problems from a carefully designed problem package with peers, with assistance from instructors and going over answers together as a class afterwards.

The transition from secondary to postsecondary is harsh due to the rigorous requirement of solving mathematics, which is caused by the negative impact of the last reform as it focused more on the application than the abstraction of mathematics in the secondary curriculum. However, with this learning environment, rather than passively copying down notes instructors presented on board without thinking, students will always find themselves being intellectually challenged by attending these engaging lectures fulfilled with thought-provoking questions. Students are provided with the opportunities to gain more experiences on how to interpret precise math definitions and write mathematical proofs with concise language dealing with the abstract aspects of mathematics. These more hands-on practices are not only done outside of class times through assignments that are submitted for marks which gives pressure to students, but also done inside of class times, as instructors can support students thoroughly. Students feel more confident and comfortable with the materials through in-class exercises with peers, which leads to better performance in assignments and exams. This learning approach and environment substantially counteracts the negative influences on students from the last reform and helps students to feel more enjoyable with the course materials. Another famous example of videos as a form of inverted learning is Khan Academy, a free educational website with thousands of learning videos correlates with k-12 studies founded by Khan. Because of the increasing usage of internet, computers, electronic devices, and smartphones in people's daily life, this kind of medium to deliver knowledge becomes very popular and useful because of its easy accessibility (Zengin, 2017).

Some might question if a single learning method is enough to be considered as an influential reform to mathematics education. However, the potential of inverted learning should not be underestimated. By carefully designing how inverted learning is implemented in mathematics courses of different level, ranging from elementary to university, the negative outcomes from the first and second reform can be avoided while achieving the same learning objectives, especially the part of concluding more abstract mathematics into the secondary school curriculum. In order to achieve that, one need to "specifically define the learning objectives for the course [by stating] the competences that students should acquire in the course", "ensure that the students are going through activities in the course that enables them to meet the learning objectives", such as formerly watching course material videos or completing E-book exercises, and "choose an assessment form that shows whether students have achieved the stated learning objectives" (p. 1, Bennedsen & Birkkaer, 2014). By watching course material before the class, students who get to expose to the abstract contents earlier have more times to absorb the materials, along with more practices in class with the guidance from expertise – the teacher, teaching assistant, or instructor to explain any confusions or questions that they might have. Choosing the proper assessment solves the issue from the first reform of covering more abstract mathematics in elementary or secondary school, which is the public concern of "course materials need to have more measurable mathematics contents", as the assessment is a good measurement of students' understanding. It also solves the accountability issue of the second reform, which causes the hard transition for students from secondary to post-secondary, especially for those who want to study pure and rigor mathematics, since more abstract contents that universities might consider to be "prerequisite mathematics" from secondary are actually covered, well explored, and deeply entrenched into student's mind.

Learning from the past two somewhat unsuccessful reforms carried out by CEEB and the *Standards* in mathematics education, the inverted learning style of teaching, which is supported by researches on its effectiveness mostly done at the college level (p. 5, Szparagowski, 2014), has its capability to provide a better learning environment at middle school and high school. The prompt improvement to the current mathematics education needs to be carried to satisfy the technology advancing demand. It is certainly true that the inverted learning classroom might not be the perfect approach to improve mathematics education and takes similar risks as the implementations of the last two reforms, but it definitely provides a direction to what future education, not limited to mathematics, might be like.

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