We encounter dynamical systems everywhere in daily life. Imagine how moon orbits in the outer space. Its position changes over the course of time. This is a simple example of a continuous dynamical system. A dynamical system is all about evolution of something over time. The evolution can occur smoothly, or in discrete time steps.[1] We can also change the orbit of the moon to a discrete dynamical system by taking a snapshot of its position once per day instead of recording in a continuous manner.[2] Its daily phases in a month are used to determine the Lunar Calendar. On the other hand, the dynamics of weather is very complex. This is probably why weather is hard to predict and the forecast is often inaccurate. Dynamical systems, modeled by equations, are very useful to stimulate natural phenomena and have many interesting applications in modern technology.

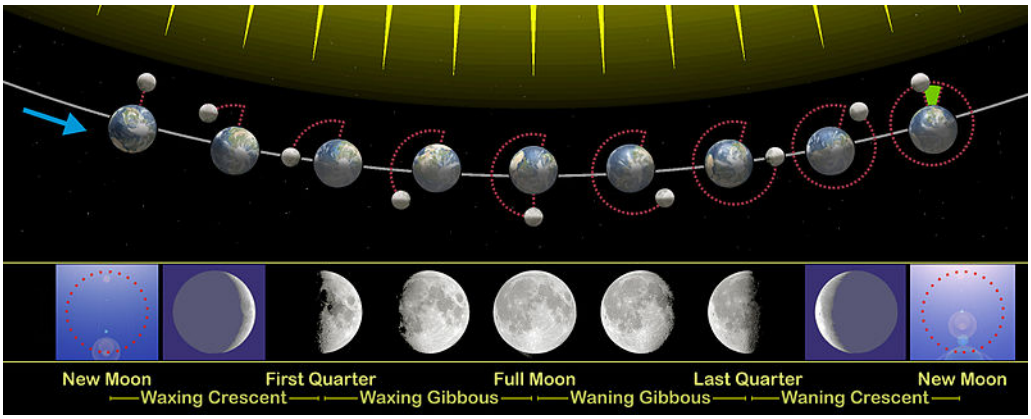



Figure 1: The phases of the Moon based on its orbit position around the Earth[3]

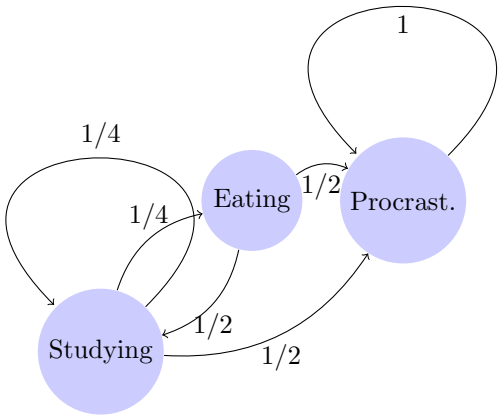
We will focus on the applications of *Markov chains* that are modeled by discrete dynamical systems. *Markov chains*, named after Andrey Markov, are mathematical systems that hop from one "state" (a situation or set of values) to another.[4] This sounds familiar as it is very similar to a dynamical system that evolves over some time steps. For instance, we can make a Markov chain model of a typical UofT student's behavior during the daytime. We might include "studying", "procrastinating", "eating", "sleeping", and "crying" as states, in other words, the behaviours that the student exhibits. We could find more behaviors to form a *state space*, a list of all possible states. Moreover, a Markov chain tells us the probability of hopping, or "transitioning" from one state to any other state, the chance that the UofT student currently studying will fall asleep in the next thirty minutes without crying first.[5] Below is a Markov chain of my Friday afternoon labeled with probabilities.

[1] Nykamp DQ, "An introduction to discrete dynamical systems." Math Insight, http://mathinsight.org/discrete_dynamical_system_introduction

[2] Nykamp DQ, "The idea of a dynamical system." Math Insight, http://mathinsight.org/dynamical_system_idea.

[3] Picture Source

[4] Powell, Lehe. "Markov Chains Explained Visually." Explained Visually, 7 Nov. 2014, setosa.io/ev/markov-chains/.




We can clearly see that it is not a productive afternoon; if I start to procrastinate, I have a 100% chance of continuing to procrastinate.

Some interesting applications of Markov chains are text generator and typing word prediction. They can predict upcoming words, generate dummy texts or produce large essays and compile speeches. They are also used in auto-completion and suggestions when texting.[5] Suppose we want to help our shy friend Tom to write a love letter that is similar to what he would write. We start by analyzing his writing or speech samples such as his dairy, essays, chat history with his friends and family members, etc. Using powerful coding language, we can easily count the number of times that a word appeared in his writing samples and turn them into a probability. Then, we get a list or Markov chain of words with the probability of their appearing in a sentence after the previous word.

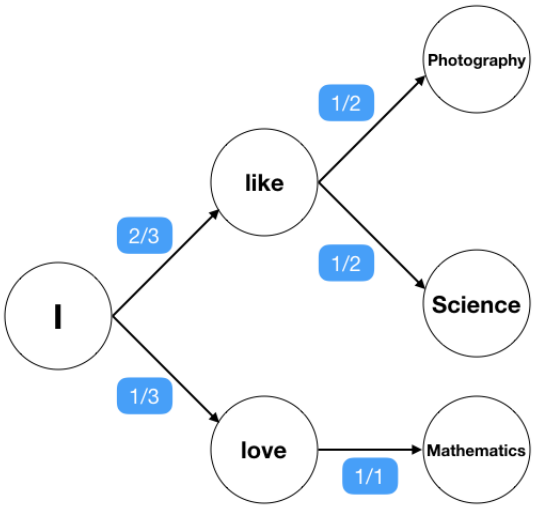



Figure 2: A naive example of what Tom's words Markov chain might look like[6]

[5]Lateef, Zulaikha. "A Brief Introduction To Markov Chains: Markov Chains In Python." Edureka, 2 July 2019, www.edureka.co/blog/introduction-to-markov-chains/.

[6]Picture Source

Starting with the word "I", we hop over the Markov chain of Tom's writings to create a love letter! It is probably a very dreadful one with lots of grammar errors. However, after few tweaks, switching around the words, and adding punctuation, we produce a writing piece that is similar to what Tom will write. If one can program a well-functioning text generator, then this essay would be easily generated by initializing with "Dynamical System" (maybe that is what I did). Typing word prediction and text generator are very convenient for communication. I rely heavily on auto-correction of spelling checks when typing. Another meaningful application of Markov chains is *Google PageRank*. It is an algorithm used by Google to rank web pages by the number of clicks in their search engine results. Every web page is a state and the links or references between these pages can be thought of as transitions with probabilities.[5] More important websites are likely to receive more links from other websites.[7] Google uses this to ensure it returns suitable and useful websites based on what people search.

Dynamical systems are extremely advantageous for modeling. We study properties of a Markov Chain for practical usage through dynamics. Scientific or Economic modelling involves building dynamical systems — to simplify the complicated reality into a set of equations and use computers to study the behaviour of the simulated system.[8] With this mathematical tool in our hands, dynamical systems help us to understand better how things work in the world, and develop innovative technologies to benefit the community at large.

[7] "PageRank." Wikipedia, Wikimedia Foundation, 18 Feb. 2020, https://en.wikipedia.org/wiki/PageRank.

[8] "Calculus, Analysis, and Dynamical Systems." Maths Careers, www.mathscareers.org.uk/article/calculus-analysis-dynamical-systems/