Appendix

F Source Code - List of Contents

This document explains which figures, tables et cetera in the doctoral thesis Safety Verification of Real-Time Control Systems with Flexible Timing – M. Gaukler, 2023 correspond to which source and output files in the output.tar.gz archive.

The archive contains the following:

- a README.md file with an overview of how to run the program
- the source code
- the output files, so you don't have to run the code yourself.

The archive can be downloaded at https://doi.org/10.5281/zenodo.6373637 or created by running ./make_full_archive.sh according to the README.md instructions.

If you are looking for the hybrid-automaton system model as a SpaceEx XML file, skip to Appendix F.7.

The output includes raw plot images as produced by the tools. In contrast, the images shown in the thesis have been postprocessed to adjust colors, make the axis clearly visible and repair missing filling of outlines.

F.1 Problem Statement (Chapter 3)

Figure 11: Timing Simulation The simulation reuses the simulation infrastructure of direct reachability analysis, which is explained later.

```
Command See Appendix F.2

Output folder output/reachability_arch19/demo/

Plot images o %: D3_0_ONLY_SIMULATION___quadrotor_attitude_three_casts_large_jitter_3.png
10, 20, 40 %: D3_10_..., D3_20_..., D3_40_...
```

Figure 15: Bouncing Ball Simulation The plot data is created with MAT-LAB R2018b from the Simulink model notes/simulink_bouncingBall.slx.

F.2 Direct Reachability Analysis (Section 4.2.10)

Figure 26: Reachability analysis and simulation The computation can be started according to README.md. Output files are already contained in the archive, as noted earlier.

Command ./run_experiments_docker.sh arch19

Main source file src/qronos/reachability/experiments/arch19.py

Output folder output/reachability_arch19/

The output folder contents are grouped into several sub-folders, such as solved_with_spaceex/stable. The names of these subfolders are historical and were left unchanged for consistency with the older publication Gaukler and Ulbrich, 2019. Every system is sorted into one of these subfolders. The following files are created, where XX denotes the system, such as A1_1 for example A1.

- Randomized simulation (first column of Figure 26):
 - Input for HyST to generate the PySim file:

```
XX__for_pysim__..spaceex.xml,....cfg
```

• Plotted image:

```
XX__for_pysim__..spaceex.xml_pysim_plot_xp1_over_t.png
```

- Reachable set over time: (Second column of Figure 26)
 - Input for HyST to generate the SpaceEx file: XX_reachability_with_time_.spaceex.xml,cfg
 - The processing with HyST/SpaceEx is described later.
 - Plotted image:

```
XX__reachability_with_time_.spaceex.xml__
spaceex_plot_xp1_over_t.png
```

- Reachable set on infinite time horizon (Mark ✓/ ✗ in Figure 26):
 - System definition in SpaceEx format: XX.spaceex.xml,cfg
 This system model can be used as a benchmark for other reachability tools.
 - The system definition is preprocessed by HyST and sent to SpaceEx.
 The preprocessing by HyST is not strictly necessary but merely a result of the HybridPy toolchain.

• The result (success or not) is stored in results.tex in the column "SpaceEx".

To avoid errors on certain CPUs, the GMP library version 10.2 included in SpaceEx was replaced with a newer version 10.3.2. Compared to the unmodified version used in Gaukler and Ulbrich, 2019, no significant deviation of the results was found.

F.3 LIS Derivations

Theorem 5.1.4: Splitting The numerical experiment for Theorem 5.1.4 is given in src/qronos/lis/test_lis.py, function test_Ak_delta_to_nominal.

Remark 5.1.18: Extremal *P*-ellipsoidal Norm Symbolic and numeric computations for the example are given as a MATLAB R2018b script in notes/matlab_counterexample_for_existence_of_extreme_P.m.

F.4 LIS Results

The values of Table 1, except for the "Reach" column, are generated by the following command:

```
Command ./run_experiments_docker.sh ifac20

Main source file src/qronos/lis/experiments/timing_stability_ifac20.py

Output file output/lis_stability_ifac20/results.tex
```

Results are printed to the commandline and saved in the log file logfile.txt. There is no separate result file. No plots are generated.

The reachability result ("Reach" column) is computed separately as described in Appendix F.2. The value (success or not) is stored in the SpaceEx column of output/reachability_arch19/results.tex.

The run-time measurements in Table 1 are taken from the publication Gaukler, Roppenecker, et al., 2020. The output file in the archive has slightly different times since it was re-created later. The reason for the re-creation is to ensure that the output consistently matches the code.

The implementation of interval arithmetic uses a modified version of *mpmath*. Unfortunately, the maintainers of this software are very busy, so it was not possible to merge all the modifications into the main *mpmath* codebase. The GIT repository corresponding to the modified *mpmath* can be found at https://github.com/maxgaukler/mpmath/tree/current, branch "current".

F.5 Continuization

The continuization experiment of Figure 35 corresponds to the following files:

Command ./run_experiments_docker.sh cont

Main source file src/qronos/reachability/experiments/continuization.py

Output folder output/continuization_arch20/output_4

continuization

Left plot XX__reachability_with_time_.spaceex.xml__spaceex_plot_4

xp1_over_t.png

Center plot XX__for_pysim__.spaceex.xml_pysim_plot_xp1_over_t.png
Right plot XX__for_pysim__.spaceex.xml_pysim_plot_xd1_over_t.png

Rows Replace XX with C4_orig, C4_continuized, C5_orig,

C5_continuized.

HA file Hybrid automaton in SpaceEx format:

Original: C4_orig.spaceex.xml, C5_orig...

Continuized: C4_continuized.spaceex.xml, C5_continuized...

F.6 Dynamic Resource Management (Section 7.5)

The simulation experiment from Section 7.5 and its output can be found at the following location:

Command ./run_experiments_docker.sh abs

Main source file src/qronos/lis/experiments/abstraction_timing.py

Output folder output/abstractions/

Output files XX.pdf: PDF plot

XX/plot.tex LaTeX source code for plotting

XX/....tsv: raw data

In these file names, XX refers to the corresponding figure:

Figure	Filename
Figure 38	abstraction-timing-nominal_caseperfect_timingno_skips
Figure 39	abstraction-timing-skipsfixedshort,
	abstraction-timing-skips_fixed_long
Figure 40	abstraction-timing-skipslow_probability
Figure 41	abstraction-timing-skipshigh_probability,
	$\verb abstraction-timing-skips_high_probability_+ \leftarrow$
	ASSUMING_BETTER_ABSTRACTIONUNPROVEN
Figure 43	abstraction-timing-timing_forced_max
Figure 44	${\tt abstraction-timing-timing_deviation_low_variance},$
	abstraction-timing-timing_deviationmedium_variance

F.7 Example systems (Appendix A)

Definition of the Example Systems The example systems are defined in the source code at src/qronos/examples.py. As detailed in Appendix F.2, the hybrid automata are output in SpaceEx format at output/reachability_arch19/.../XX.spaceex.xml, where XX refers to the specific example. For instance, example A1 can be found at code/template/output/solved_with_spaceex/stable/~A1_1.spaceex.xml.

Figure 45: Nominal Case of Example C The plot is generated with MAT-LAB/Simulink R2018b from the files in notes/example-c-nominal-case/.