Analysis and solution of dynamics in interaction processes

As with the end-effector of an industrial robotic arm, the hand of the upper limb prosthesis remains the main part responsible for the movement communication between the prosthesis and the external environment. The kinetic analysis and solution not only allow better control of the interaction between the prosthesis and the external environment but also enables us to track the desired trajectory faster and more accurately. Let the mass distribution of linkage I be J_i :

$$J_i = \int_i {}^i r \, {}^i r^T dm \tag{1}$$

Where ${}^i r = \begin{bmatrix} {}^i x & {}^i y & {}^i z & 1 \end{bmatrix}^T$, is the flush coordinate of any elementary mass in the connecting rod i in the coordinate system i. Therefore, J_i is a symmetric constant value matrix.

Then the kinetic energy of the connecting rod i is:

$$K_i = \frac{1}{2}\int_i tr\left[\sum_{j=1}^{i}\sum_{k=1}^{i}\frac{\partial T_i}{\partial q_j}\,{}^i r\,{}^i r^T\left(\frac{\partial T_i}{\partial q_k}\right)^T \dot{q}_j \dot{q}_k\right]dm = \frac{1}{2}\sum_{j=1}^{i}\sum_{k=1}^{i} tr\left[\frac{\partial T_i}{\partial q_j} J_i \left(\frac{\partial T_i}{\partial q_k}\right)^T\right]\dot{q}_j \dot{q}_k \tag{2}$$

T_i is the transformation matrix of the coordinate system i with respect to the base coordinate system, then the total kinetic energy of the system is:

$$K = \sum_{i=1}^{7} K_i = \frac{1}{2}\sum_{i=1}^{7} tr\left[\sum_{j=1}^{i}\sum_{k=1}^{i}\frac{\partial T_i}{\partial q_j} J_i \left(\frac{\partial T_i}{\partial q_k}\right)^T \dot{q}_j \dot{q}_k\right] \tag{3}$$

Introduce the robot arm inertia matrix $H(q) = [h_{jk}]$.

$$h_{jk} = \sum_{i=\max\{j,k\}}^{7} tr\left[\frac{\partial T_i}{\partial q_j} J_i \frac{\partial T^T{}_i}{\partial q_k}\right] \tag{4}$$

Then equation (3) can be abbreviated as:

$$K = \frac{1}{2}\dot{q}^T H(q)\dot{q} \tag{5}$$

Let the expression of the flush coordinates of the center of mass of the connecting rod i in the system i be ${}^i r_{Ci}$, then its expression in the base coordinate system is $T_i {}^i r_{Ci}$, so the potential energy of the connecting rod i is:

$$V_i = -m_i g^T T_i \, {}^i r_{Ci} \tag{6}$$

Where $g^T \triangleq [g \quad 0]$ and g is the expression of the gravitational acceleration vector in the base coordinate system. The total potential energy of the upper limb prosthesis is:

$$V = \sum_{i=1}^{7} V_i = -\sum_{i=1}^{7} m_i g^T T_i \, {}^i r_{Ci} \tag{7}$$

The Lagrangian function of the upper limb prosthesis is thus obtained as:

$$L(q_i, \dot{q}_i) = K - V = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \sum_{i=1}^{7} m_i g^T T_i \, {}^i r_{Ci} \tag{8}$$

Then the kinetic equation of the upper limb prosthesis is:

$$\tau = H(q)\ddot{q} + v(q,\dot{q}) + g(q) \tag{9}$$

Where $H(q) = [h_{jk}]$, $v(q,\dot{q})$ is the vector associated with the position and velocity of the connecting rod, and $g(q)$ is the gravity term. The kinetic equations for linkage i can also be written as:

$$\tau_i = \sum_{j=1}^{7}\sum_{k=1}^{j} tr\left(\frac{\partial T_j}{\partial q_k} J_j \left(\frac{\partial T_j}{\partial q_i}\right)^T\right)\ddot{q}_k + \sum_{j=1}^{7}\sum_{k=1}^{j}\sum_{m=1}^{j} tr\left(\frac{\partial^2 T_i}{\partial q_k \partial q_m} J_j \left(\frac{\partial T_j}{\partial q_i}\right)^T\right)\dot{q}_k \dot{q}_m - \sum_{j=1}^{7} m_i g^T \frac{\partial T_i}{\partial q_j} \, {}^i r_{Ci} \tag{10}$$

The forces and moments to be applied to the connecting rod j in contact with the environment during the action consist of three components.

$$^{j}F_{e} = f_{h} + f_{g} + f_{c} \tag{11}$$

Where f_h is the force and moment of manipulation by the wrist joint to maintain the stability of the hand, which is a static force transfer; f_g is the force and moment of gravity on the remaining part of the rear end of the linkage; and f_c is the contact force between the linkage j and the environment.

Where the gravitational force on the remaining part of the rear end of the linkage is vertically downward regardless of any movement performed by the robot arm, so f_g can be expressed in the world coordinate system as:

$$^{W}f_{g} = \begin{bmatrix} 0 & 0 & -G \end{bmatrix}^{T} \tag{12}$$

Converting it to the coordinate system of the connecting rod j yields:

$$^{j}f_{g} = {}^{j}_{W}R\,{}^{W}f_{g} \tag{13}$$

Where ${}^{j}_{W}R$ is the rotation transformation matrix of the world coordinate system W with respect to the connecting rod j coordinate system.

The torque is generated by its weight $^{j}\tau_{g} = r \times {}^{j}f_{g}$, where r is the vector-matrix from the origin of the connecting rod j coordinate system to the center of gravity of the remaining part of the rear end. Thus the forces and moments generated by gravity are:

$$f_{g} = \begin{bmatrix} ^{j}f_{g} & ^{j}\tau_{g} \end{bmatrix}^{T} \tag{14}$$

For the manipulation forces and moments that maintain hand stability by the wrist joint, from the equilibrium state:

$$\begin{cases} ^{j}f_{h} = {}^{j+1}f_{h} \\ ^{j}\tau_{h} = {}^{j+1}\tau_{h} + r \times {}^{j}f_{h} \end{cases} \tag{15}$$

The Jacobi matrix in the force domain is usually expressed as a transfer matrix from the joint space to the operation space. Introduce the Jacobi matrix $\boldsymbol{J} = J(\theta, q)$. Ten the equivalent joint moment generated during the interaction is:

$$\tau_{e} = J^{T}\,{}^{j}F_{e} \tag{16}$$

Considering the existence of higher-order coupling terms in the kinetic equations of the 7-DOF upper limb prosthetic system, which cannot be solved directly using the analytical method, the Runge-Kutta method (RKM) is considered. The Runge-Kutta method uses linear combinations of function values at certain special points to estimate the average slope of higher-order single-step methods and is widely used in engineering for the numerical solution of differential equations. The more common fourth-order Longe-Kutta method has the formula:

$$\begin{cases} y_{n+1} = y_{n} + \dfrac{h}{6}(k_{1} + 2k_{2} + 2k_{3} + k_{4}) \\ k_{1} = f(x_{n}, y_{n}) \\ k_{2} = f(x_{n} + \dfrac{h}{2}, y_{n} + \dfrac{h}{2}k_{1}) \\ k_{3} = f(x_{n} + \dfrac{h}{2}, y_{n} + \dfrac{h}{2}k_{2}) \\ k_{4} = f(x_{n} + h, y_{n} + hk_{3}) \end{cases} \tag{17}$$