Homework

(1) Suppose that the change in the value of a portfolio over a 10-day time horizon is normally distributed with a mean of zero and a standard deviation of 20 million. Compute the 10-day 99% VaR and ES for the portfolio.

(2) Consider two independent random variables L_1 and L_2 with the following distributions:
- $P(L_1 = 0) = 0.98$, $P(L_1 = 10) = 0.02$
- $P(L_2 = 0) = 0.98$, $P(L_2 = 10) = 0.02$
 1. Prove that $\text{VaR}_{0.98}$ is not subadditive.
 2. Determine whether $\text{VaR}_{0.99}$ is subadditive.
 3. Find $\text{ES}_{0.98}$ for $L_1, L_2, L_1 + L_2$, respectively.

(3) A bank has two 10 million one-year loans. The probabilities of default are as follows:
- $\mathbb{P}(\text{Neither loan defaults}) = 0.975$
- $\mathbb{P}(\text{Loan 1 defaults, Loan 2 does not default}) = 1.25\%$
- $\mathbb{P}(\text{Loan 2 defaults, Loan 1 does not default}) = 1.25\%$
- $\mathbb{P}(\text{Both loans default}) = 0\%$

If a default occurs, all losses between 0% and 100% of the principal are equally likely. If the loan does not default, a profit of 0.2 million is made.
 1. Compute one-year 99% VaR for the Loan 1.
 2. Compute one-year 99% VaR for the bank loan portfolio. Is it subadditive?
 3. Compute the one-year 99% expected shortfall (ES) for the Loan 1 and bank loan portfolio, respectively.

(4) VaR monotonicity says that If $L_1 \leq L_2$, then $\text{VaR}_\alpha(L_1) \leq \text{VaR}_\alpha(L_2)$. Justify the following statement:

If $L_1 < L_2$ almost surely, then $\text{VaR}_\alpha(L_1) < \text{VaR}_\alpha(L_2)$ for any $\alpha < 1$.

(5) Prove that if f is an increasing function and $X = f(Z)$ for some random variable Z, then $\text{VaR}_\alpha(X) = f(\text{VaR}_\alpha(Z))$.

(6) Prove the monotonicity of Expected Shortfall (ES).

(7) The loss variables X and Y are given by:
$$X = Z + \rho\epsilon, \quad Y = Z + 2\rho\epsilon$$

where $Z, \epsilon \sim N(0,1)$ are independent, and ρ is some constant.
 (a) Is X, Y positively or negatively dependent?
 (b) For $\alpha \in (0,1)$, prove that
$$\text{VaR}_\alpha(X + Y) \leq \text{VaR}_\alpha(X) + \text{VaR}_\alpha(Y).$$
 (c) Are X, Y comonotonic?