Let X and Y be two Banach spaces. $C(X,Y)$ is the set of all continuous mappings $f : X \mapsto Y$. For $f, g \in C(X,Y)$, we define

$$\|f - g\| = \sup_{x \in X} \|f(x) - g(x)\|.$$

(1) Prove that $C(X,Y)$ is a Banach space.
(2) If X and Y are compact, is $C(X,Y)$ compact?