Let (X, η) and (Y, ρ) be two Polish spaces. $C(X, Y)$ is the set of all continuous mappings $f : X \mapsto Y$. For $f, g \in C(X, Y)$, we define

$$d(f, g) = \sup_{x \in X} \rho(f(x), g(x)).$$

(1) Prove that $(C(X, Y), d)$ is a Polish space.
(2) If $K \subset Y$ are compact, is $C(X, K)$ compact in $C(X, Y)$?