

Symbolic Execution as DPLL Modulo Theories

Quoc-Sang Phan

Queen Mary University of London

q.phan@qmul.ac.uk

Abstract

We show how Symbolic Execution can be understood as a variant of the DPLL(\mathcal{T}) algorithm, which is the dominant technique for the Satisfiability Modulo Theories (SMT) problem. In other words, Symbolic Executors are SMT solvers. This view enables us to use an SMT solver, with the ability of generating all models with respect to a set of Boolean atoms, to explore all symbolic paths of a program. This results in a more lightweight approach for Symbolic Execution.

1998 ACM Subject Classification D.2.4 Software/Program Verification

Keywords and phrases Symbolic Execution, Satisfiability Modulo Theories

Digital Object Identifier 10.4230/OASIcs.ICCSW.2014.58

1 Introduction

Symbolic Execution (SE) [11] is now popular. It is increasingly used not only in academic settings but also in industry, such as in Microsoft, NASA, IBM and Fujitsu [4]. In the success of SE, the efficiency of SMT solvers [9] is a key factor. In fact, while SE was introduced more than three decades ago, it had not been made practical until research in SMT made significant advances [5].

State-of-the-art SMT solvers, e.g. [8, 2], implement the DPLL(\mathcal{T}) algorithm [15] which is an integration of two components as follows. The first component is a propositional satisfiability (SAT) solver, based on the Davis-Putnam-Logemann-Loveland (DPLL) algorithm [7], to search on the *Boolean skeleton* of the formula. The second component is a decision procedure, called the \mathcal{T} -solver, to check the consistency w.r.t. the theory \mathcal{T} of conjunctions of literals, each literal is an atomic formula or the negation of an atomic formula. The path conditions generated by a Symbolic Executor, e.g. Symbolic PathFinder (SPF) [14], are also conjunctions of literals. Therefore, when an SMT solver checks such a path condition, only the \mathcal{T} -solver works on it, and the SAT component is not used.

On the other hand, a *classical* Symbolic Executor [11] can be divided into two components. The first component, dubbed as *Boolean Executor* hereafter, executes the instructions, and updates the path condition. The second component is a \mathcal{T} -solver (since the SAT solver is not used) to validate the consistency of the path condition. This paper shows that a Boolean Executor does the same work as the DPLL algorithm. Thus, SE is a variant of DPLL(\mathcal{T}). This view is important since it connects two communities and can give an insight for future research.

Based on this new insight, we propose a lightweight approach for SE which uses the DPLL component of an SMT solver to explore symbolic paths instead of a Boolean Executor. Our approach relies on an SMT solver with the ability of generating all models w.r.t. a set of Boolean atoms.



© Quoc-Sang Phan;

licensed under Creative Commons License CC-BY
Imperial College Computing Student Workshop (ICCSW'14).
Editors: Rumyana Neykova and Nicholas Ng; pp. 58–65



OpenAccess Series in Informatics

OASIcs Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

ICCSW

2 Background

Before showing the correspondence between a Symbolic Executor and an SMT solver, we recall some background on Symbolic Execution and the SMT problem.

2.1 Symbolic Execution

Symbolic Execution [11] (SE) is a programming analysis technique which executes programs on unspecified inputs, by using symbolic values instead of concrete data. For each executed program path, SE builds a path condition pc which represents the condition on the inputs for the execution to follow that path.

```
function EXECUTOR(Program P){
    PathCondition pc = TRUE;
    InstructionPointer i = NULL;
    update(P, i);
    if (i == RETURN) return;
    while (TRUE) {
        l = chooseLiteral(i);
        pc = pc ∧ l;
        update(P, i);
        if (i == RETURN)
            if (allStatesAreExplored())
                return;
            else backtrack(pc, i);
    }
}
```

Figure 1 A simplified Boolean Executor.

For an **if** statement with condition c , there are three possible cases: **(i)** $pc \vdash c$: SE chooses the **then** path; **(ii)** $pc \vdash \neg c$: SE chooses the **else** path; **(iii)** $(pc \not\vdash c) \wedge (pc \not\vdash \neg c)$: SE executes both paths: in the **then** path, it updates the path condition $pc_1 = pc \wedge c$, in the **else** path it updates the path condition $pc_2 = pc \wedge \neg c$. The satisfiability of the path condition is checked by SMT solvers, for example the symbolic executor KLEE [3] uses the SMT solver STP [10], while SPF provides a parameter to select one of the SMT solvers: CVC3, Yices, Z3. In this way, only feasible program paths are explored. Test generation is performed by solving the path conditions.

2.2 SMT and DPLL(\mathcal{T})

Satisfiability Modulo Theories (SMT) is the problem of checking the satisfiability of logical formulas over one or more first-order theories \mathcal{T} .

Our setting is standard first-order logic. Boolean variables are called Boolean atoms or simply atoms, and atomic formulas are called theory atoms or \mathcal{T} -atoms. A *truth assignment* μ for a formula φ is a truth value assignment to the \mathcal{T} -atoms of φ . We define a bijective function \mathcal{BA} (*Boolean abstraction*) which maps Boolean atoms into themselves and \mathcal{T} -atoms into fresh Boolean atoms. The *Boolean refinement* function \mathcal{BR} is then defined as the inverse of \mathcal{BA} , which means $\mathcal{BR} = \mathcal{BA}^{-1}$.

At a high level, an SMT solver is the integration of two components: a SAT solver and \mathcal{T} -solvers. SMT solving can be viewed as the iteration of the two following steps. First, the

SAT solver searches on the Boolean abstraction of the formula, $\varphi^P = \mathcal{BA}(\varphi)$, and returns a (partial) truth assignment μ^P . The \mathcal{T} -solvers then check the Boolean refinement of the candidate, $\mathcal{BR}(\mu^P)$, whether it is consistent with the theories \mathcal{T} .

```

function DPLL(BooleanFormula  $\varphi$ ){
     $\mu$  = TRUE; status = propagate( $\varphi$ ,  $\mu$ );
    if (status == SAT) return SAT;
    else if (status == UNSAT) return UNSAT;
    while (TRUE) {
         $l$  = chooseLiteral( $\varphi$ );
         $\mu$  =  $\mu \wedge l$ ;
        status = propagate( $\varphi$ ,  $\mu$ );
        if (status == SAT) return SAT;
        else if (status == UNSAT)
            if (allStatesAreExplored())
                return UNSAT;
            else backtrack( $\varphi$ ,  $\mu$ );
    } }
```

Figure 2 DPLL algorithm.

The dominant approach for SAT solvers is the DPLL family of algorithms [7]. The simplest form of DPLL is depicted in Figure 2, its input is a propositional formula φ in Conjunctive Normal Form (CNF), which means φ takes the form:

$$\varphi = \bigwedge (l_1 \vee l_2 \cdots \vee l_k)$$

A (finite) disjunction of literals $(l_1 \vee l_2 \cdots \vee l_k)$ is called a clause, and a literal l_i is an atom or its negation. A clause that contains only one literal is called a *unit clause*. At a high level, DPLL is a stack-based depth-first search procedure which iteratively performs the two following steps: first choose a literal l_i from the remaining clause, and add it to the current truth assignment; then apply Boolean Constraint Propagation (BCP), backtracking if there is a conflict. These two steps are repeated until a model is found or all states are explored without finding a model.

The procedure BCP for a literal l_i removes all the clauses containing l_i , and removes $\neg l_i$ from the remaining clauses. If the removal results an empty clause, the search encounters a conflict.

3 Illustration of DPLL(\mathcal{T})

A complete formal description of first-order theories and the DPLL(\mathcal{T}) algorithm can be found in, e.g., [15]. Here we briefly introduce necessary preliminaries via a running example as follows.

$$\begin{aligned} \varphi := & (\neg(x_0 > 5) \vee T_1) \wedge ((x_0 > 5) \vee T_2) \wedge (\neg(x_0 > 5) \vee (x_1 = x_0 + 1)) \wedge \\ & (\neg(x_1 < 3) \vee T_3) \wedge (\neg(x_1 < 3) \vee (x_2 = x_1 - 1)) \wedge \\ & ((x_1 < 3) \vee T_4) \wedge ((x_1 < 3) \vee (y_1 = x_1 + 1)) \end{aligned} \quad (1)$$

φ is a Linear Arithmetic formula. Boolean variables, $T_1 \dots T_4$, are called Boolean atoms, and atomic formulas, e.g. $(x_0 > 5)$, are called theory atoms or \mathcal{T} -atoms. A first-order formula can

be abstracted into a Boolean skeleton by replacing all the \mathcal{T} -atoms with new Boolean atoms, which is often called *Boolean abstraction*. For the example above, we define new Boolean variables G_1, G_2, A_1, A_2, A_3 for the Boolean abstraction of \mathcal{T} -atoms, and the abstraction can be expressed as:

$$\begin{aligned} \mathcal{BA} := G_1 &= (x_0 > 5) \wedge G_2 = (x_1 < 3) \wedge \\ A_1 &= (x_1 = x_0 + 1) \wedge A_2 = (x_2 = x_1 - 1) \wedge A_3 = (y_1 = x_1 + 1) \end{aligned} \quad (2)$$

As the result, we obtain a formula φ^P (P stands for propositional) as the Boolean skeleton of φ . Obviously, φ is logically equivalent to $\varphi^P \wedge \mathcal{BA}$.

$$\begin{aligned} \varphi^P := (\neg G_1 \vee T_1) \wedge (G_1 \vee T_2) \wedge (\neg G_1 \vee A_1) \wedge \\ (\neg G_2 \vee T_3) \wedge (\neg G_2 \vee A_2) \wedge \\ (G_2 \vee T_4) \wedge (G_2 \vee A_3) \end{aligned} \quad (3)$$

The DPLL(\mathcal{T}) algorithm is the integration of the DPLL algorithm with a \mathcal{T} -solver. The DPLL algorithm searches on φ^P , returning a conjunction of Boolean literal μ^P . Replacing all the new Boolean atoms, G_i and A_i , in μ^P with their corresponding \mathcal{T} -atoms, we obtain the conjunction μ in \mathcal{T} . The \mathcal{T} -solver then checks whether μ is consistent with the theory \mathcal{T} . Below is the illustration of DPLL(\mathcal{T}) on φ (for the limit of space, only decision literals are shown in μ^P):

0. $\mu^P = \text{True}$	φ^P
1. $\mu^P = G_1$	$\varphi^P = (\neg G_2 \vee T_3) \wedge (\neg G_2 \vee A_2) \wedge (G_2 \vee T_4) \wedge (G_2 \vee A_3)$
2. $\mu^P = G_1 \wedge G_2$	$\varphi^P = \text{True} ; \mathcal{T}\text{-solver}(\mu) = \text{Inconsistent}$
3. $\mu^P = G_1$	$\varphi^P = (\neg G_2 \vee T_3) \wedge (\neg G_2 \vee A_2) \wedge (G_2 \vee T_4) \wedge (G_2 \vee A_3)$
4. $\mu^P = G_1 \wedge \neg G_2$	$\varphi^P = \text{True} ; \mathcal{T}\text{-solver}(\mu) = \text{Consistent}$

The DPLL algorithm tries to build a model using three main operations: `decide`, `propagate`, and `backtrack` [9]. The operation `decide` heuristically chooses a literal l (which is an unassigned Boolean atom or its negation) for branching. The operation `propagate` then removes all the clauses containing l , and deletes all occurrences of $\neg l$ in the formula; this procedure is also called *Boolean Constraint Propagation* (BCP). If after deleting a literal from a clause, the clause only has only one literal left (*unit clause*), BCP assigns this literal to `True`. If deleting a literal from a clause results in an empty clause, this is called a conflict. In this case, the DPLL procedure must `backtrack` and try a different branch value.

At step 1, G_1 is decided to be the branching literal, and the \mathcal{T} -solver validates that $(x_0 > 5)$ is consistent. BCP removes the clause $(G_1 \vee T_2)$, and deletes all occurrences of $\neg G_1$. This results in two unit clauses T_1 and A_1 , so they are assigned to `True`, which means $\mu^P = G_1 \wedge T_1 \wedge A_1$. Similarly, at step 2 G_2 is chosen, i.e. $\mu^P = G_1 \wedge T_1 \wedge A_1 \wedge G_2$. The \mathcal{T} -solver checks the conjunction: $\mu = (x_0 > 5) \wedge T_1 \wedge (x_1 < 3) \wedge (x_1 = x_0 + 1)$. This is obviously inconsistent, thus DPLL(\mathcal{T}) backtracks and tries $\neg G_2$, which leads to a consistent model.

Note that DPLL(\mathcal{T}) refers to various procedures integrating DPLL and a \mathcal{T} -solver. There are procedures with an integration schema different from what we have described here. The interested reader is pointed to [15] for further references.

4 Symbolic Execution as DPLL(\mathcal{T})

Intuitively, a program can be encoded into a (first-order) formula whose models correspond to program traces. Symbolic Executors explore all program traces w.r.t. the set of program conditions, therefore they can be viewed as SMT solvers that return all (partial) models w.r.t. a set of Boolean atoms.

In this paper we only consider bounded programs, since this is the class of programs that SE can analyse. This means every loop can be unwound into a sequence `if` statements. In order to encode a program into a formula, all program variables are renamed in the manner of Static Single Assignment form [6]: each variable is assigned exactly once, and it is renamed into a new variable when being reassigned. In this way, assignments such as $x = x + 1$ will not be encoded into an unsatisfiable atomic formula. Under these settings, a program P can be modelled by a *Symbolic Transition System* (STS) as follows:

$$P \equiv (S, I, G, A, T)$$

S is the set of program states, $I \subseteq S$ is the set of initial states; each state in the STS models the computer memory at a program point. G is the set of guards and A is the set of actions; guards and actions are first-order formulas. An action models the effect of an instruction on the computer memory. Actions that do not update the computer memory (e.g. conditional jumps) are Boolean atoms, the others are \mathcal{T} -atoms. $T \subseteq S \times G \times A \times S$ is the transition function, $t_{ij} = \langle s_i, g_{ij}, a_{ij}, s_j \rangle \in T$ models a transition from state s_i to state s_j by taking action a_{ij} under the guard g_{ij} . After a transition $t_{ij} : s_i \rightarrow s_j$, the state s_j is exactly as the state s_i apart from the variable updated by the action a_{ij} .

One way to encode a transition t_{ij} into a first-order formula is to present it in the form: $t_{ij} \equiv g_{ij} \rightarrow a_{ij}$, or equally $t_{ij} \equiv \neg g_{ij} \vee a_{ij}$. This encoding expresses that satisfying the guard g_{ij} implies that the action a_{ij} is performed. In this way, a program trace is defined as a sequence of transitions:

$$t_{01} \wedge t_{12} \wedge \cdots \wedge t_{(k-1)k} = (\neg g_{01} \vee a_{01}) \wedge (\neg g_{12} \vee a_{12}) \cdots \wedge (\neg g_{(k-1)k} \vee a_{(k-1)k})$$

The semantics of the program is then defined as the set of all possible traces, or equally the set of all possible transitions, which can be represented as the following formula:

$$\varphi = \bigwedge_{t_{ij} \in T} t_{ij} = \bigwedge_{t_{ij} \in T} (\neg g_{ij} \vee a_{ij}) \tag{4}$$

Fig. 3 depicts a simple example program and its associated STS. Encoding this STS following (4) results in the formula (1) that we have illustrated with DPLL(\mathcal{T}) in the previous section. We now illustrate this example with SE.

At a high level, a Symbolic Executor can be considered as the integration of two components: a Boolean Executor (BE) to execute the instructions and a \mathcal{T} -solver to check the feasibility of path conditions. For example, SPF has a parameter `symbolic.dp` to customize which decision procedure to use. If we set this parameter with the option `no_solver` then SPF solely works on the BE.

A BE can be described as trying to build *all* path conditions using three main operations: `decide`, `update` and `backtrack`. The operation `decide` chooses a literal l , a condition (or its negation) of an `if` statement, for branching, adding it to the path condition. The operation `update` then symbolically executes a block of statements, i.e. no branching statement presents, updating the computer memory. When the BE reaches the end of a symbolic path,

```

void test(int x, int y){
    if(x > 5){
        x++;
        if (x < 3)
            x--;
        else
            y = x + 1;
    }
}

```

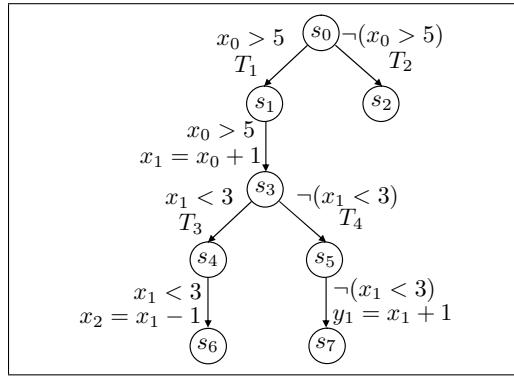


Figure 3 A simple program and its associated STS. The first `if` statement is modelled by two transitions $\langle s_0, (x_0 > 5), T_1, s_1 \rangle$ and $\langle s_0, \neg(x_0 > 5), T_2, s_2 \rangle$; the assignment `x++` is modelled by $\langle s_1, (x_0 > 5), x_1 = x_0 + 1, s_3 \rangle$; similarly for the rest of the program.

it backtracks to explore other paths. A Symbolic Executor, which is the integration of a BE and a \mathcal{T} -solver, backtracks if the path condition is not satisfied.

Both DPLL and BE rely on Depth-First Search, they are similar in the way they `decide` and `backtrack`¹. After choosing a literal, e.g. $(x_0 > 5)$, BE executes the block it guards, i.e. T_1 and $x_1 = x_0 + 1$. This is exactly the same as in DPLL: after choosing g_{ij} , for all the clauses $(\neg g_{ij} \vee a_{ij})$, BCP deletes $\neg g_{ij}$, assigning a_{ij} to `True`. Therefore, the operation `update` does the same work as BCP, we can view a BE as implementing the DPLL algorithm, and SE as $DPLL(\mathcal{T})$.

5 A lightweight approach for Symbolic Execution using All-SMT

While both Symbolic Executors and SMT solvers implement $DPLL(\mathcal{T})$, the main difference between them is the models they return. Symbolic Executors explore all path conditions, each path condition is identified by the values of the Boolean abstractions of its condition constraints. For instance, although there are many values of x_0 satisfying the path $(x_0 < 5) \wedge \neg(x_1 < 3)$, it is identified by $G_1 = \text{True}$ and $G_2 = \text{False}$. On the other hand, SMT solvers in general stop searching when a model is found. However, the solver MathSAT provides a functionality, called All-SMT [2], that given a formula and a set of Boolean atoms, it returns *all* models of the formula w.r.t. this set. Hence, by asking MathSAT to find all models w.r.t. the set of guards, we can explore all symbolic paths of the program.

We illustrate the approach with our running example in Fig. 3. The formula φ in (1) is expressed in SMT-LIB v2 format [1] as follows (for the limit of space variable declarations are omitted):

1 (assert (= (> x0 5) G1))	8 (assert (or (not G1) A1))
2 (assert (= (< x1 3) G2))	9 (assert (or (not G2) T3))
3 (assert (= (= x1 (+ x0 1)) A1))	10 (assert (or (not G2) A2))
4 (assert (= (= x2 (- x1 1)) A2))	11 (assert (or G2 T4))
5 (assert (= (= y1 (+ x1 1)) A2))	12 (assert (or G2 A3))
6 (assert (or (not G1) T1))	13 (check-allsat (G1 G2))
7 (assert (or G1 T2))	

¹ We consider DPLL in its simplest form, without non-chronological backtracking.

Recall that $\varphi = \mathcal{BA} \wedge \varphi^P$ as defined in (2) and (3). Assertions from 1 to 5 are for \mathcal{BA} , and each assertion from 6 to 12 represents a clause of φ^P . The final command, `check-allsat`, asks the solver to return all models w.r.t. the set (G_1, G_2) . Note that `check-allsat` is not included in standard SMT-LIB, and it is not supported by other solvers, e.g. Z3 [8].

Executing MathSAT on the example above, we obtain three models w.r.t. the set (G_1, G_2) as follows: `(True,False)`, `(False,True)` and `(False,False)`, which correspond to two feasible symbolic paths: $(x_0 > 5) \wedge \neg(x_1 < 3)$ and $\neg(x_0 > 5)$. As we did not model integer overflow rules, the path (G_1, G_2) as `(True,True)`, i.e. $(x_0 > 5) \wedge (x_1 < 3)$, is not feasible (this path is feasible when the overflow in the assignment $x_1 = x_0 + 1$ changes a big integer into a small one). The path $\neg(x_0 > 5)$ is listed twice, since in this case the value of G_2 , i.e. $(x_1 < 3)$, is irrelevant, but the solver still considered it in two cases `True` and `False`.

6 Discussions and Future Work

In this paper, we show the correspondence between a Symbolic Executor and an SMT solver implementing the DPLL(\mathcal{T}) framework [15]. Therefore, the claims in this paper do not apply for the bit vector theory, since its solvers do not implement the DPLL(\mathcal{T}) framework. Bit vector formulas are solved by flattening into propositional formulas, then the resulting formulas are checked by a SAT solver [10].

A common problem for both SE and SMT is that there is a considerable amount of redundancy in the queries to the \mathcal{T} -solver. Because the BE and its counterpart in SMT, the SAT component, build the conjunctions of literals in an incremental manner. To address this problem, previous work in SE and SMT used different approaches. In SE, the BE often uses the \mathcal{T} -solver as a black box, thus most research on constraint redundancy problem has focused on caching techniques. For example, KLEE [3] has a counterexample cache that maps of sets of constraints to counterexamples, Green [16] can store the constraints offline, reusing them in different runs of the Symbolic Executor. In SMT solver, there is a tight synergy between the SAT component and the \mathcal{T} -solver, and both are incremental and backtrackable. Therefore, the \mathcal{T} -solver do not start from scratch if the constraint is similar to the previous query.

An immediate direction for investigation is whether the caching techniques in KLEE and Green can improve the efficiency of SMT solvers. Conversely, it would be interesting to investigate if techniques developed for SMT can be exploited for SE, for example to develop a concurrent Symbolic Executor based on previous work in concurrent SMT solver [17].

7 Conclusion

We show the correspondence between Symbolic Execution and Satisfiability Modulo Theories. This correspondence is important, as it enables us to migrate techniques developed in one community to the other. Moreover, it can give an insight for future research. In our previous work, we proposed a DPLL-based algorithm to tackle the quantitative information flow problem, and implemented it using SPF [13, 12].

Based on the correspondence above, we propose a lightweight approach for SE, using the DPLL component of an SMT solver to explore symbolic paths. The main limitation of our approach is path redundancies, which is because the off-the-shelf All-SMT functionality is not tailored for SE. Hence, future work also includes extending the open-source SMT solver Z3 so that it can function as SE.

References

- 1 Clark Barrett, Aaron Stump, and Cesare Tinelli. The smt-lib standard: Version 2.0. In *SMT Workshop*, 2010.
- 2 Roberto Bruttomesso, Alessandro Cimatti, Anders Franzén, Alberto Griggio, and Roberto Sebastiani. The mathsat 4 smt solver. In *Proceedings of the 20th international conference on Computer Aided Verification*, CAV '08, pages 299–303, Berlin, Heidelberg, 2008. Springer-Verlag.
- 3 Cristian Cadar, Daniel Dunbar, and Dawson Engler. Klee: unassisted and automatic generation of high-coverage tests for complex systems programs. OSDI'08, pages 209–224.
- 4 Cristian Cadar, Patrice Godefroid, Sarfraz Khurshid, Corina S. Păsăreanu, Koushik Sen, Nikolai Tillmann, and Willem Visser. Symbolic execution for software testing in practice: Preliminary assessment. In *Proceedings of the 33rd International Conference on Software Engineering*, ICSE '11, pages 1066–1071, New York, NY, USA, 2011. ACM.
- 5 Cristian Cadar and Koushik Sen. Symbolic execution for software testing: Three decades later. *Commun. ACM*, 56(2):82–90, February 2013.
- 6 R. Cytron, J. Ferrante, B. K. Rosen, M. N. Wegman, and F. K. Zadeck. An efficient method of computing static single assignment form. In *Proceedings of the 16th ACM SIGPLAN-SIGACT symposium on Principles of programming languages*, POPL '89, pages 25–35, New York, NY, USA, 1989. ACM.
- 7 Martin Davis, George Logemann, and Donald Loveland. A machine program for theorem-proving. *Commun. ACM*, 5(7):394–397, July 1962.
- 8 Leonardo De Moura and Nikolaj Bjørner. Z3: an efficient smt solver. In *Proceedings of the 14th international conference on Tools and algorithms for the construction and analysis of systems*, TACAS'08, pages 337–340, Berlin, Heidelberg, 2008. Springer-Verlag.
- 9 Leonardo De Moura and Nikolaj Bjørner. Satisfiability modulo theories: introduction and applications. *Commun. ACM*, 54(9):69–77, September 2011.
- 10 Vijay Ganesh and David L. Dill. A decision procedure for bit-vectors and arrays. In *Proceedings of the 19th international conference on Computer aided verification*, CAV'07, pages 519–531, Berlin, Heidelberg, 2007. Springer-Verlag.
- 11 James C. King. Symbolic execution and program testing. *Commun. ACM*, 19(7):385–394, July 1976.
- 12 Quoc-Sang Phan and Pasquale Malacaria. Abstract model counting: A novel approach for quantification of information leaks. In *Proceedings of the 9th ACM Symposium on Information, Computer and Communications Security*, ASIA CCS '14, pages 283–292, New York, NY, USA, 2014. ACM.
- 13 Quoc-Sang Phan, Pasquale Malacaria, Oksana Tkachuk, and Corina S. Păsăreanu. Symbolic quantitative information flow. *SIGSOFT Softw. Eng. Notes*, 37(6):1–5, November 2012.
- 14 Corina S. Păsăreanu and Neha Rungta. Symbolic pathfinder: symbolic execution of java bytecode. In *Proceedings of the IEEE/ACM international conference on Automated software engineering*, ASE '10, pages 179–180, New York, NY, USA, 2010. ACM.
- 15 Roberto Sebastiani. Lazy Satisfiability Modulo Theories. *Journal on Satisfiability, Boolean Modeling and Computation*, 3:141–224, 2007.
- 16 Willem Visser, Jaco Geldenhuys, and Matthew B. Dwyer. Green: reducing, reusing and recycling constraints in program analysis. In *Proceedings of the ACM SIGSOFT 20th International Symposium on the Foundations of Software Engineering*, FSE '12, pages 58:1–58:11, New York, NY, USA, 2012. ACM.
- 17 Christoph M. Wintersteiger, Youssef Hamadi, and Leonardo Mendonça de Moura. A concurrent portfolio approach to smt solving. In *CAV*, pages 715–720, 2009.