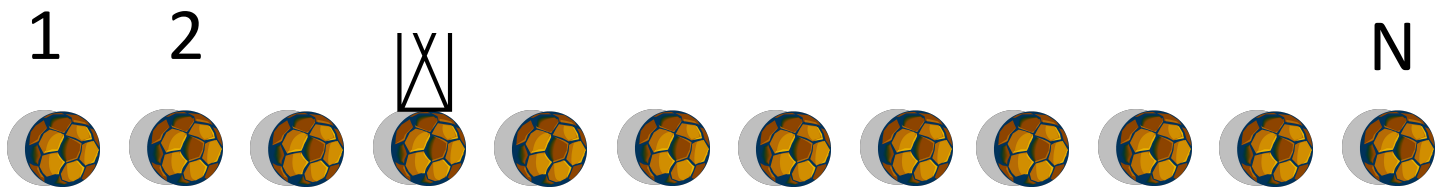


MERA, symmetries and AdS/CFT

Sukhi Singh (EQuS, Macquarie)

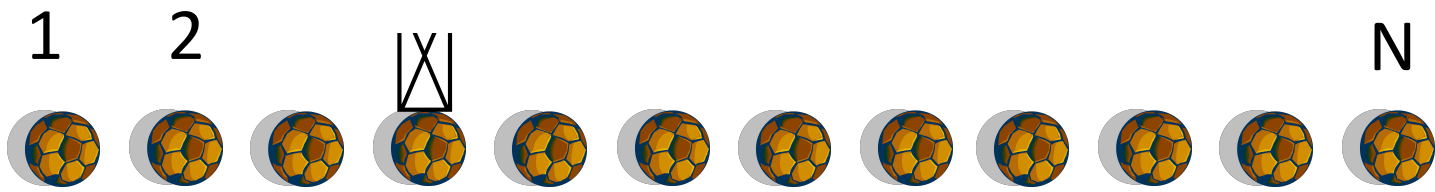


1) Hamiltonian :

A) Local

B) Critical/Gapless ($N \rightarrow \infty$) \rightarrow described by a CFT

e.g.
$$\hat{H} = \sum_{k=1}^N \left(\sigma_X^k \sigma_X^{k+1} + \sigma_Y^k \sigma_Y^{k+1} + \sigma_Z^k \sigma_Z^{k+1} \right) \quad (\text{Heisenberg model})$$



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2) Ground state:

$$|\Psi_{gs}\rangle = \sum_{i_1 i_2 \dots i_N} \hat{\Psi}_{i_1 i_2 \dots i_N} |i_1\rangle |i_2\rangle \dots |i_N\rangle$$

Locality & criticality \Rightarrow Limited entanglement in GS

$$S_l = \frac{c}{3} \log l$$

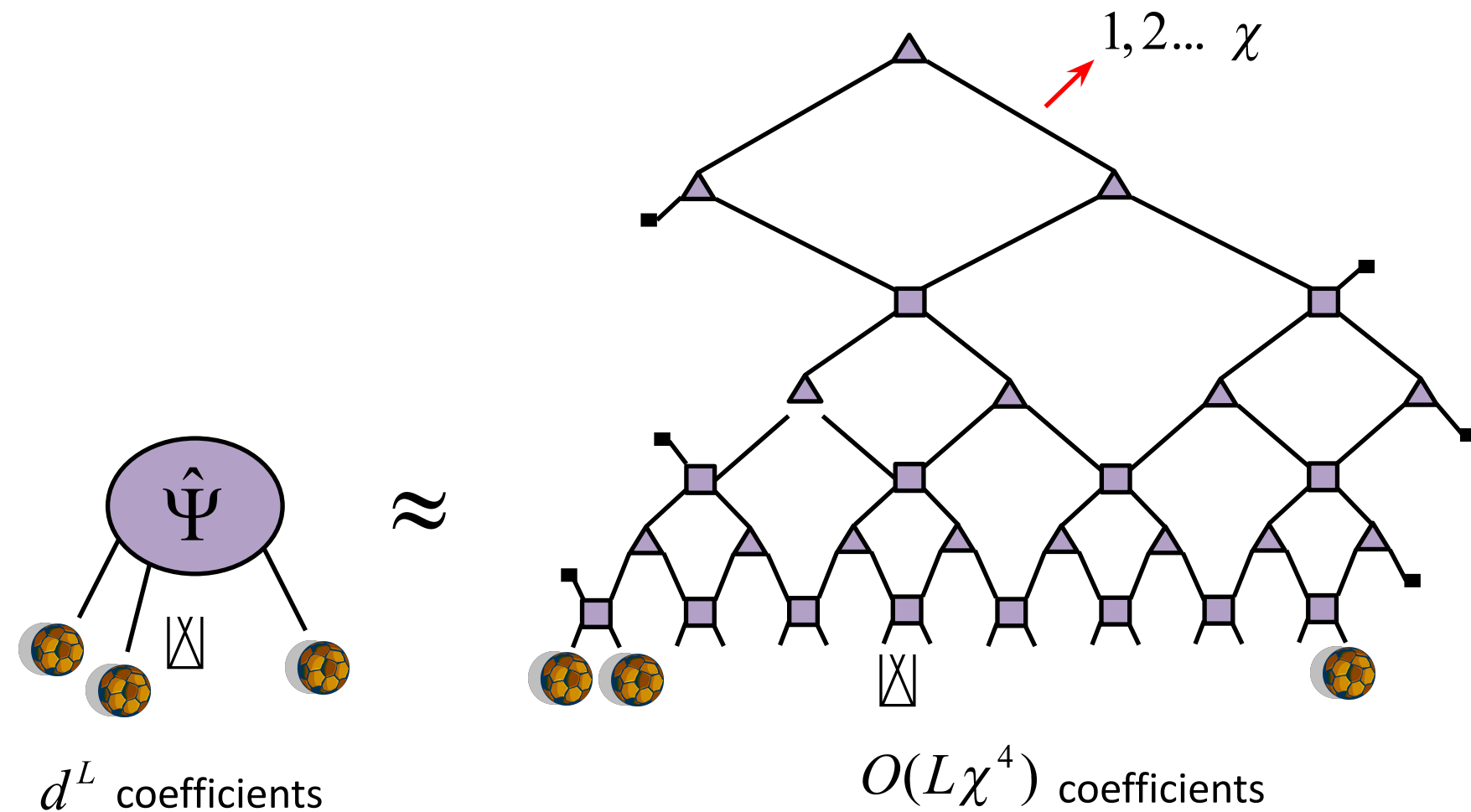
c : central charge of the underlying CFT

$$S_l = -\text{tr}(\rho_l \log \rho_l)$$

$$\rho_l = \text{tr}_{\bar{l}} \left(|\Psi_{gs}\rangle \langle \Psi_{gs}| \right)$$

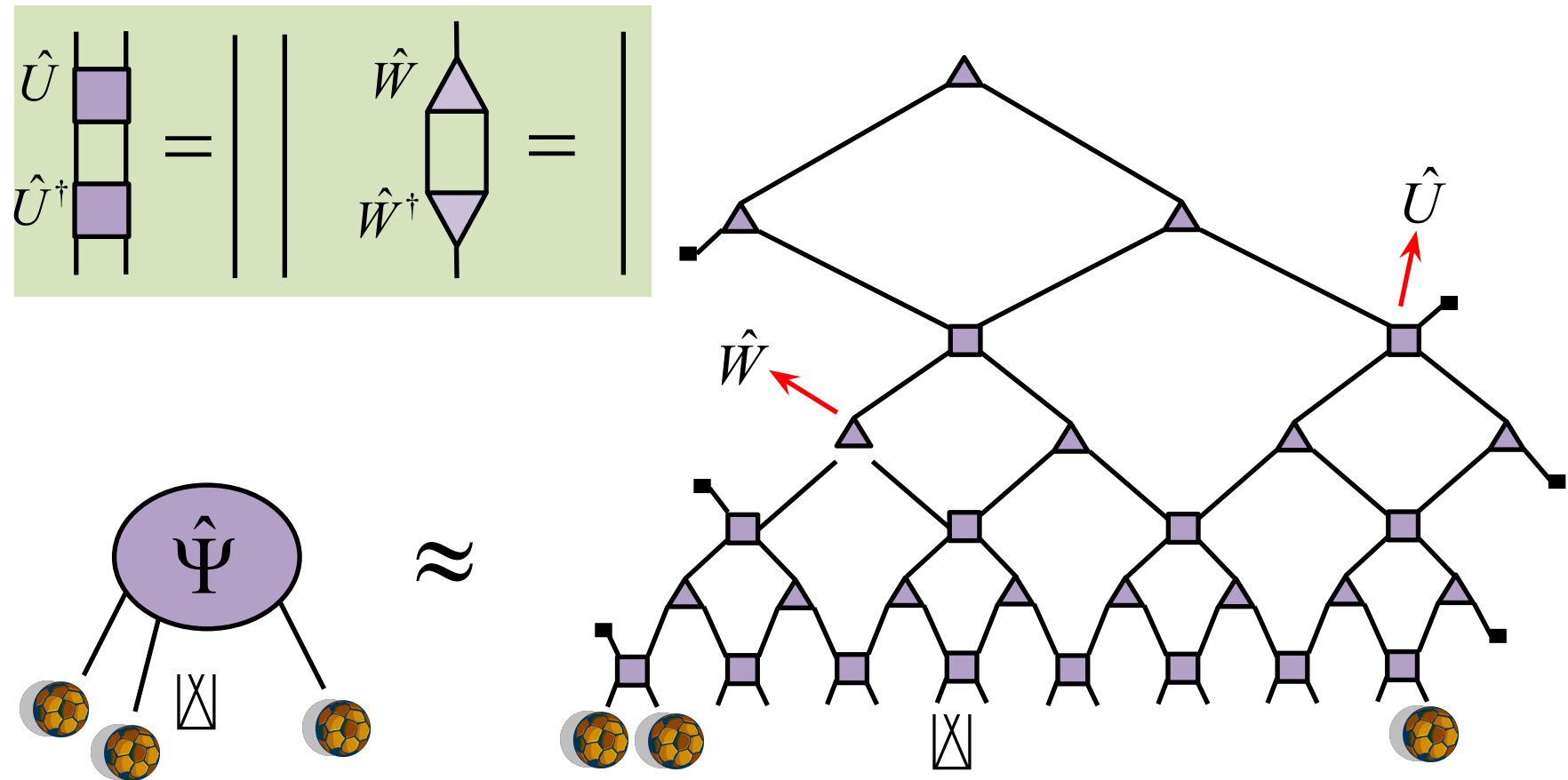
MERA

$$|\Psi_{gs}\rangle = \sum_{i_1 i_2 \dots i_N} \hat{\Psi}_{i_1 i_2 \dots i_N} |i_1\rangle |i_2\rangle \dots |i_N\rangle$$



Disentanglers & isometries

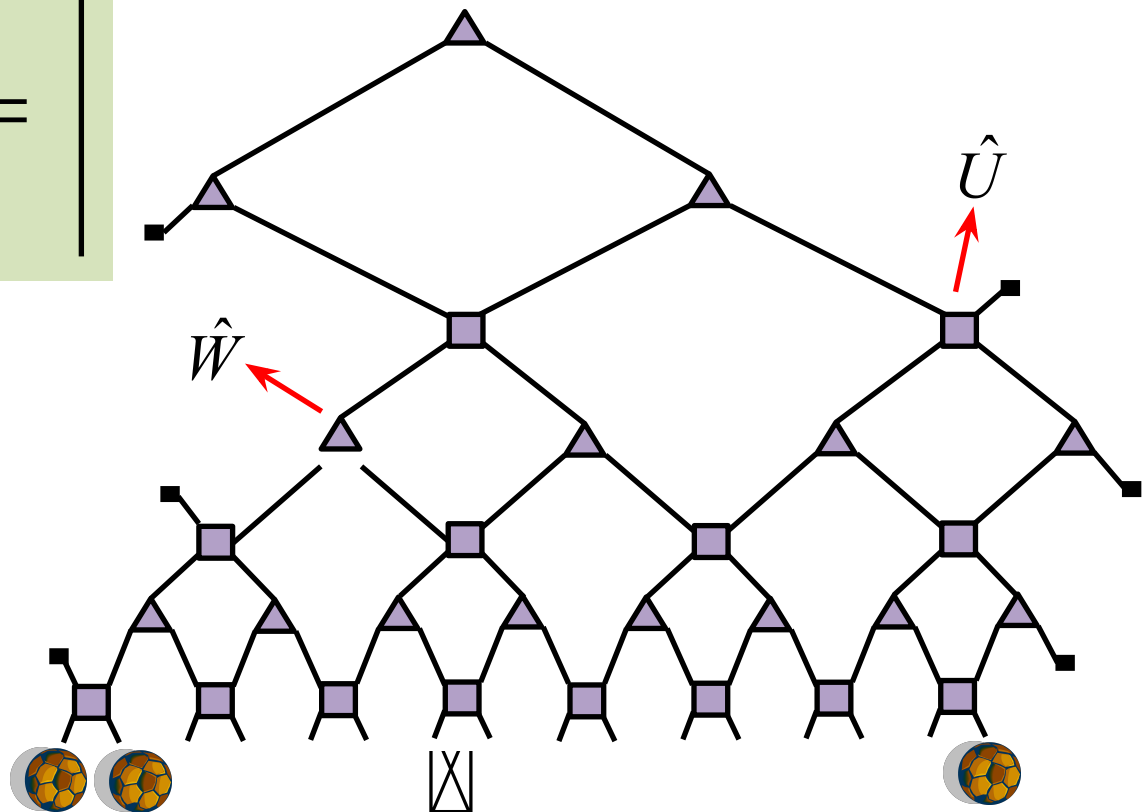
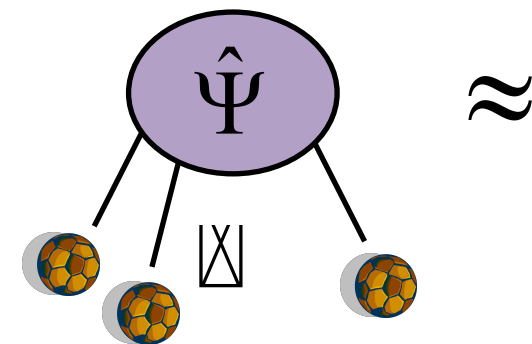
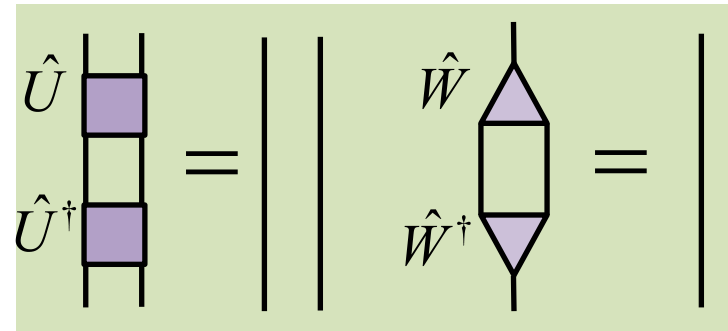
$$|\Psi_{gs}\rangle = \sum_{i_1 i_2 \dots i_N} \hat{\Psi}_{i_1 i_2 \dots i_N} |i_1\rangle |i_2\rangle \dots |i_N\rangle$$



RG transformations

$$|\Psi_{gs}\rangle = \sum_{i_1 i_2 \dots i_N} \hat{\Psi}_{i_1 i_2 \dots i_N} |i_1\rangle |i_2\rangle \dots |i_N\rangle$$

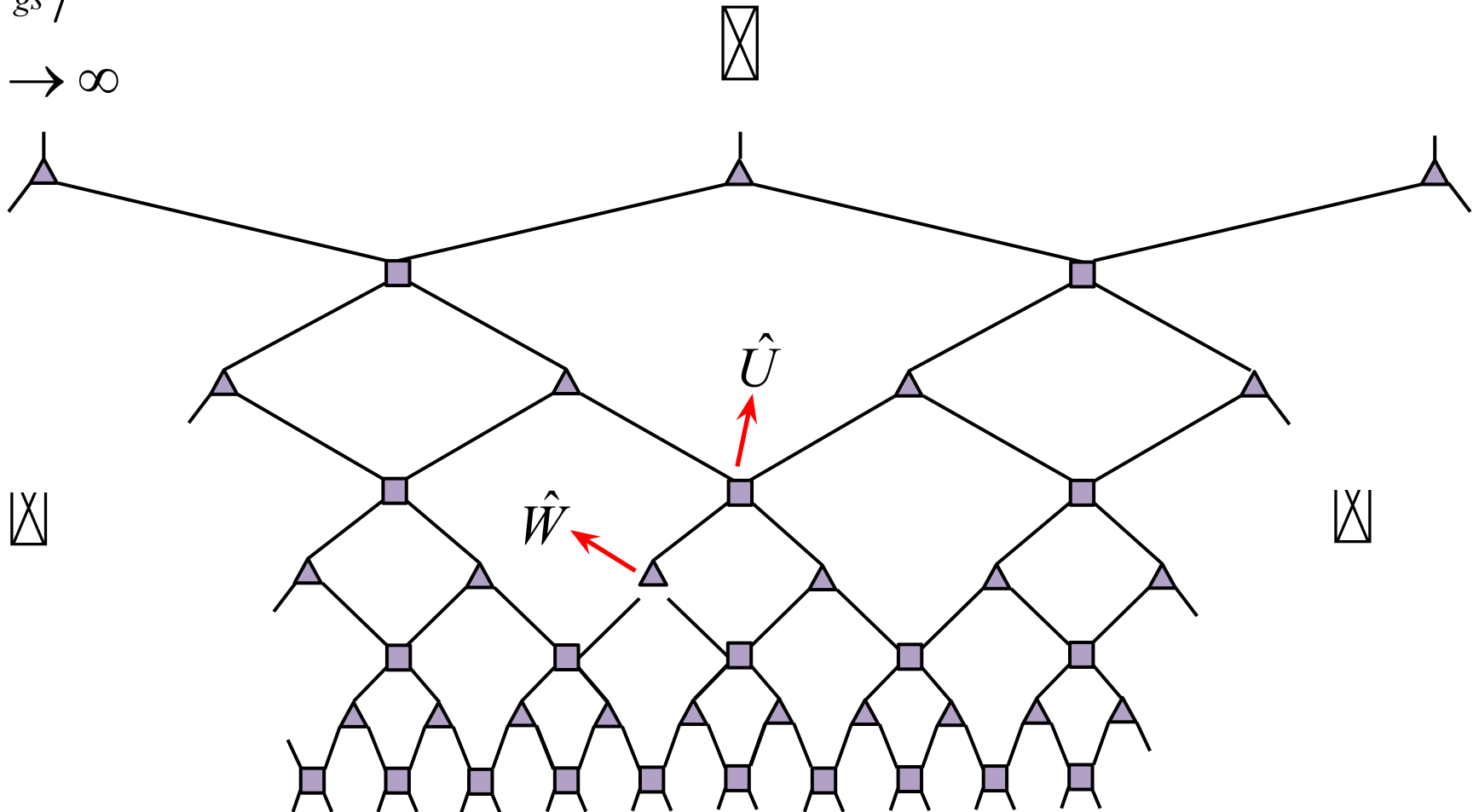
Length Scale



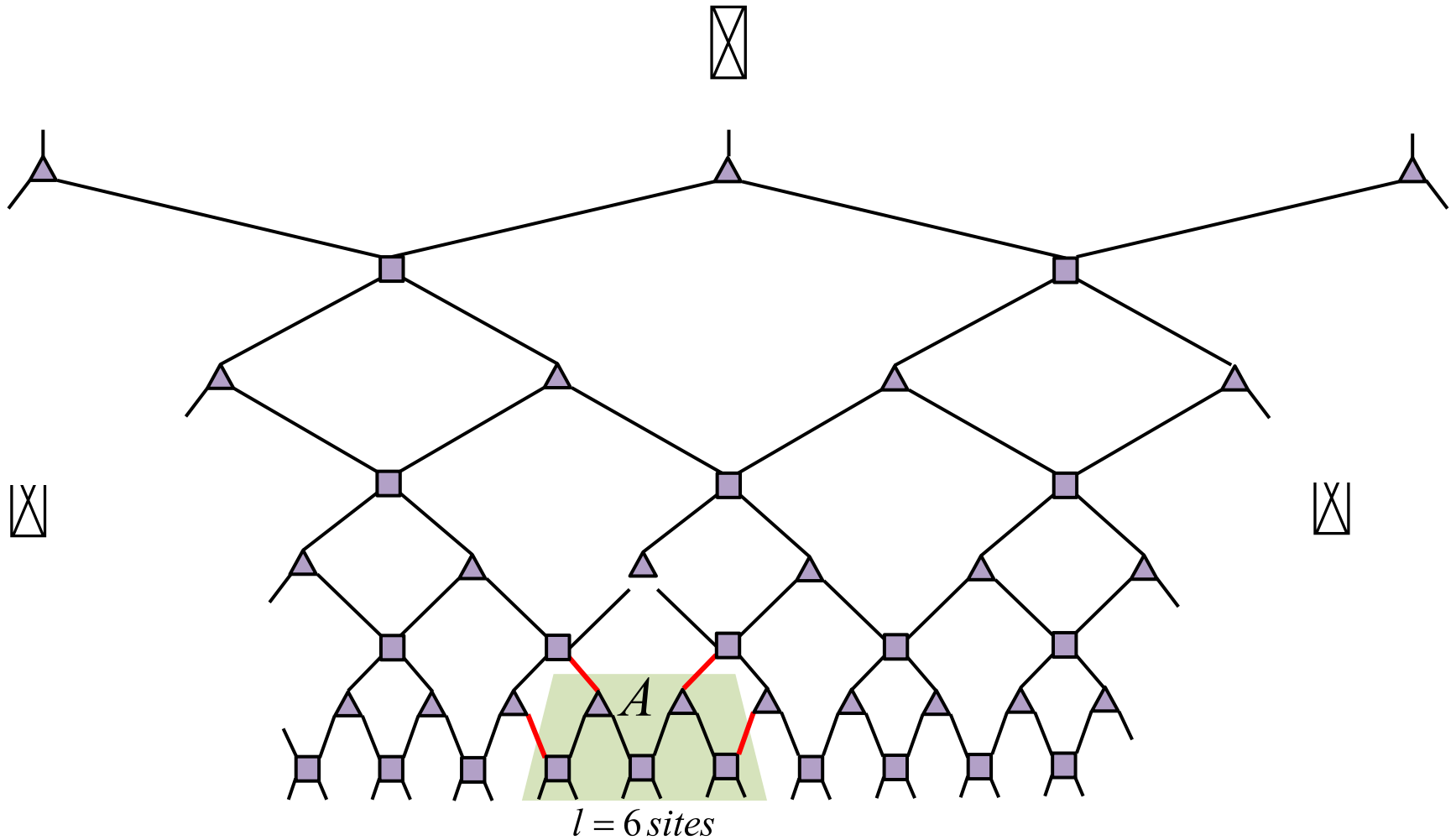
Scale-invariant MERA

$$|\Psi_{gs}\rangle \approx$$

$$N \rightarrow \infty$$



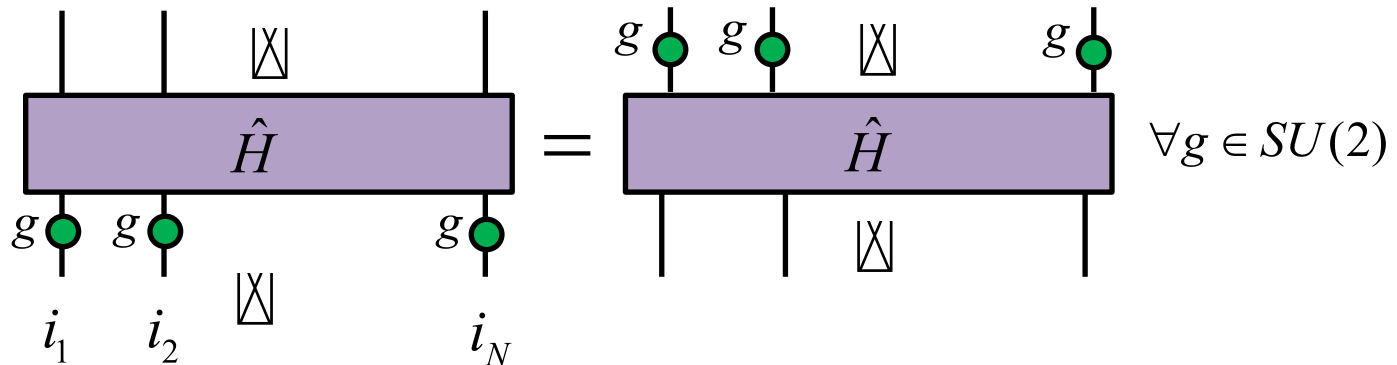
Entanglement from geometry in MERA



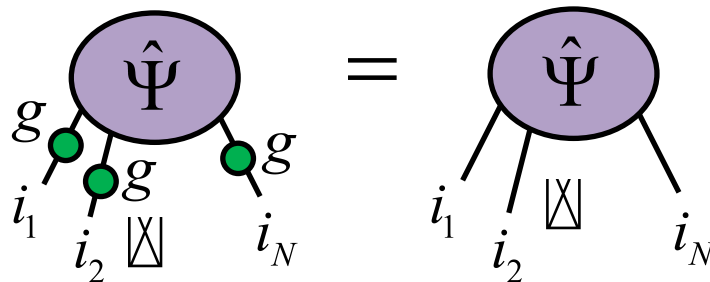
min number of connecting bonds $\approx \log l$

Global SU(2) symmetry

1) Hamiltonian :



2) Ground state: total spin 0

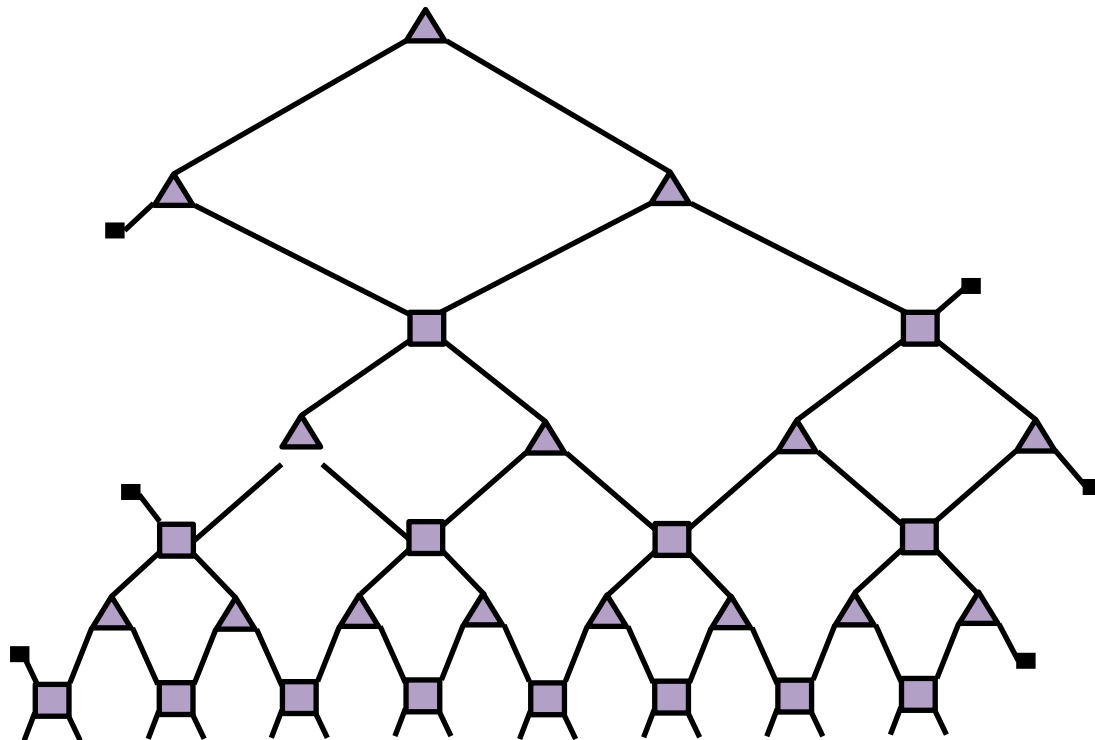


SU(2)-invariant MERA

$$\begin{array}{c} \hat{U} \end{array} \begin{array}{c} \text{Diagram: A purple square with two vertical lines passing through it. Each line has a green circle at the bottom, labeled } g. \end{array} = \begin{array}{c} \hat{U} \end{array} \begin{array}{c} \text{Diagram: A purple square with two vertical lines passing through it. Each line has a green circle at the top, labeled } g. \end{array}$$

$$\begin{array}{c} \hat{W} \end{array} \begin{array}{c} \text{Diagram: A purple triangle pointing up with a vertical line entering from the top and two lines exiting from the bottom, each with a green circle labeled } g. \end{array} = \begin{array}{c} \hat{W} \end{array} \begin{array}{c} \text{Diagram: A purple triangle pointing up with a vertical line entering from the top with a green circle labeled } g, and two lines exiting from the bottom. \end{array}$$

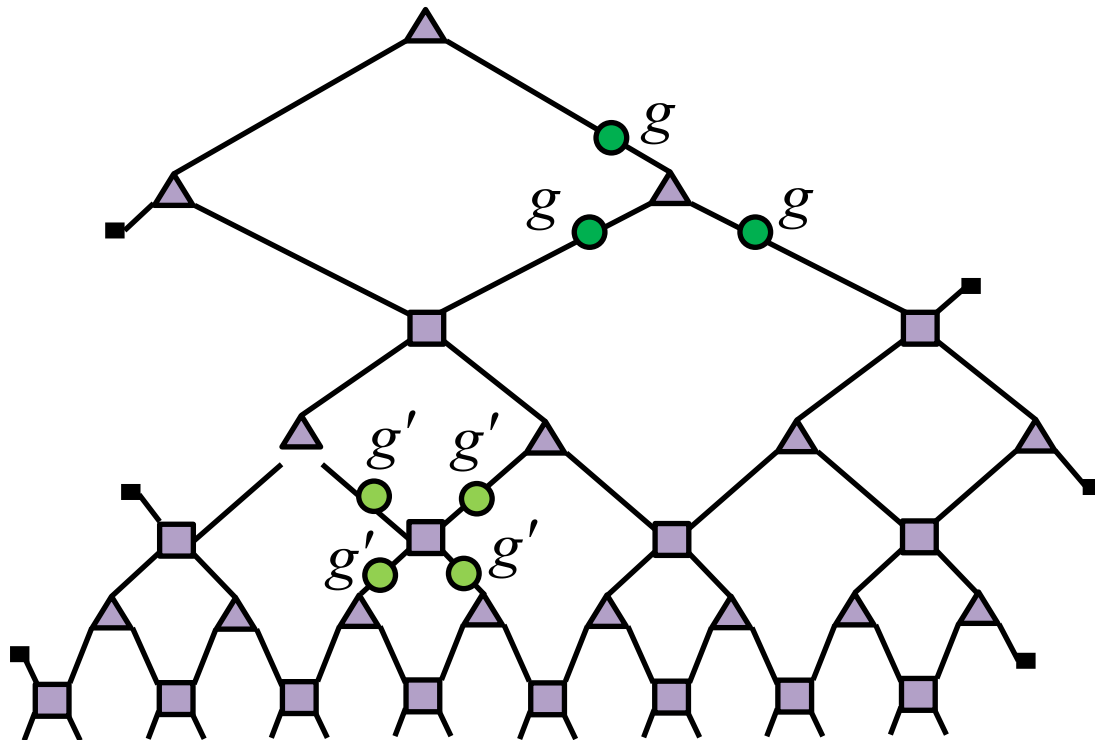
$$\forall g \in SU(2)$$



SU(2)-invariant MERA

$$\begin{array}{c} \hat{U} \end{array} \begin{array}{c} \text{diagram: square with two vertical lines, green circles at bottom} \end{array} = \begin{array}{c} \hat{U} \end{array} \begin{array}{c} \text{diagram: square with two vertical lines, green circles at top} \end{array} \quad \begin{array}{c} \hat{W} \end{array} \begin{array}{c} \text{diagram: triangle with two vertical lines, green circles at bottom} \end{array} = \begin{array}{c} \hat{W} \end{array} \begin{array}{c} \text{diagram: triangle with two vertical lines, green circle at top} \end{array} \quad \forall g \in SU(2)$$

SU(2) gauge transformations



Decomposition into spin networks

$$\forall g \in SU(2) \quad \begin{array}{c} \text{diagram with three green dots labeled } g \end{array} = \begin{array}{c} \text{diagram without dots} \end{array}$$

The diagram on the left shows a vertex labeled \hat{W} with three incident edges. Each edge has a green dot at the vertex, and each dot is labeled g . The diagram on the right shows the same vertex \hat{W} with three incident edges, but without the green dots.

Decomposition into spin networks

$$\forall g \in SU(2) \quad \begin{array}{c} \text{Diagram 1: A triangle with a purple shaded interior and a horizontal line across its middle. The top vertex has an incoming line from above with a green dot and label } g. \text{ The bottom-left vertex has an outgoing line to the bottom-left with a green dot and label } g. \text{ The bottom-right vertex has an outgoing line to the bottom-right with a green dot and label } g. \text{ The triangle is labeled } \hat{W} \text{ above it.} \\ \text{Diagram 2: A triangle with a purple shaded interior and a horizontal line across its middle. The top vertex has an incoming line from above. The bottom-left and bottom-right vertices have outgoing lines to the bottom-left and bottom-right respectively. The triangle is labeled } \hat{W} \text{ above it.} \end{array} = \begin{array}{c} \text{Diagram 3: A triangle with a purple shaded interior and a horizontal line across its middle. The top vertex has an incoming line from above labeled } j_1 = \frac{1}{2}, 1. \text{ The bottom-left vertex has an outgoing line to the bottom-left labeled } j_2 = \frac{3}{2}, 2. \text{ The bottom-right vertex has an outgoing line to the bottom-right labeled } j_3 = 1, 1, 2. \text{ The triangle is labeled } \hat{W} \text{ above it.} \end{array}$$

Spin conservation $|j_1 - j_2| \leq j_3 \leq j_1 + j_2$

Decomposition into spin networks

$$\forall g \in SU(2) \quad \begin{array}{c} \text{Diagram 1: Triangle with } \hat{W} \text{ and three } g \text{ labels} \end{array} = \begin{array}{c} \text{Diagram 2: Triangle with } \hat{W} \end{array} \Rightarrow \begin{array}{c} \text{Diagram 3: Triangle with } \hat{W} \text{ and labels } \frac{1}{2}, 1, \frac{3}{2}, 2, 1, 1, 2 \end{array}$$

Spin conservation $|j_1 - j_2| \leq j_3 \leq j_1 + j_2$

$$\begin{array}{c} \text{Diagram 4: Triangle with } \hat{W} \text{ and labels } j_1, j_2, j_3 \end{array} = \begin{array}{c} \text{Diagram 5: Triangle with } \hat{w} \text{ and labels } j_1, j_2 \end{array} \otimes \begin{array}{c} \text{Diagram 6: Triangle with labels } j_1, j_2, j_3 \end{array} \rightarrow \text{Intertwiner of } SU(2)$$

Part not fixed by symmetry

Decomposition into spin networks

$$\forall g \in SU(2) \quad \begin{array}{c} \text{Diagram 1: Triangle with } \hat{W} \text{ and three } g \text{ labels} \\ \hat{W} \end{array} = \begin{array}{c} \text{Diagram 2: Triangle with } \hat{W} \\ \hat{W} \end{array} \quad \longrightarrow \quad \begin{array}{c} \text{Diagram 3: Triangle with } \hat{W} \text{ and labels } \frac{1}{2}, 1, \frac{3}{2}, 2, 1, 1, 2 \\ \hat{W} \end{array}$$

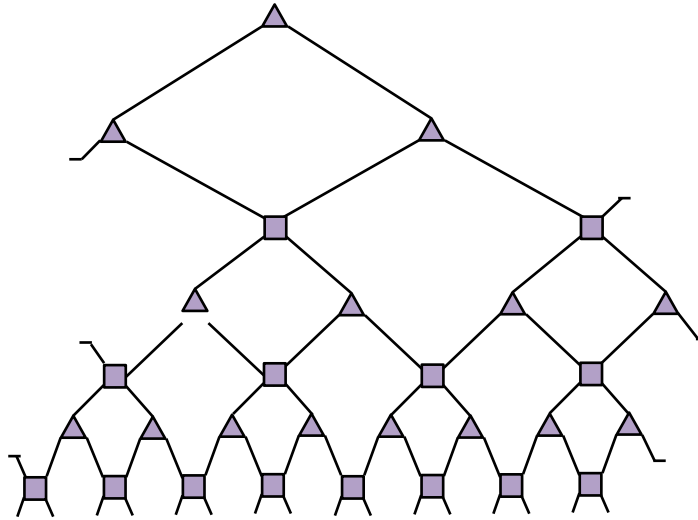
Spin conservation $|j_1 - j_2| \leq j_3 \leq j_1 + j_2$

$$\begin{array}{c} \text{Diagram 4: Triangle with } \hat{W} \text{ and labels } j_1, j_2, j_3 \\ \hat{W} \end{array} = \begin{array}{c} \text{Diagram 5: Triangle with } \hat{w} \text{ and labels } j_1, j_2 \\ \hat{w} \end{array} \otimes \begin{array}{c} \text{Diagram 6: Triangle with labels } j_1, j_2, j_3 \\ \end{array} \quad \longrightarrow \quad \text{Intertwiner of } SU(2)$$

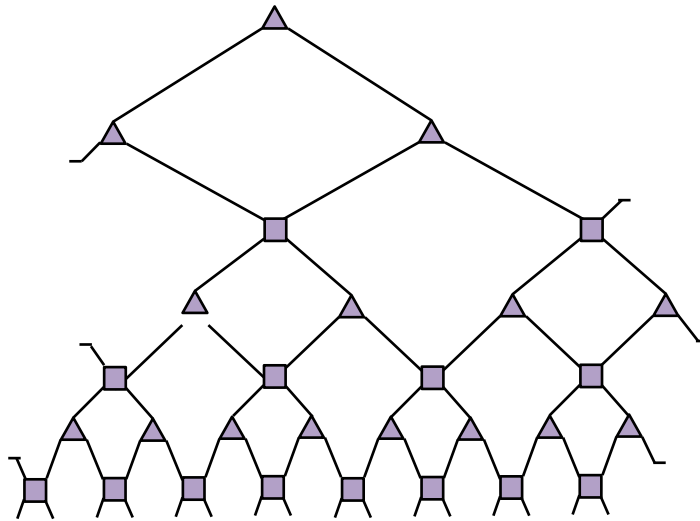
Part not fixed by symmetry

$$\begin{array}{c} \text{Diagram 7: Square with } \hat{U} \text{ and labels } j_1, j_2, j_3, j_4 \\ \hat{U} \end{array} = \sum_j \begin{array}{c} \text{Diagram 8: Square with } \hat{u} \text{ and labels } j_1, j_2, j_3, j_4 \\ \hat{u} \end{array} \otimes \begin{array}{c} \text{Diagram 9: Square with labels } j_1, j_2, j_3, j_4 \\ \end{array}$$

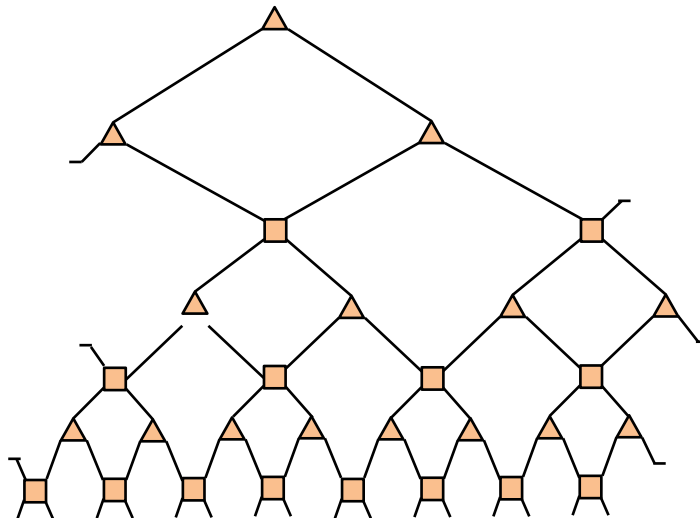
Decomposition into spin networks



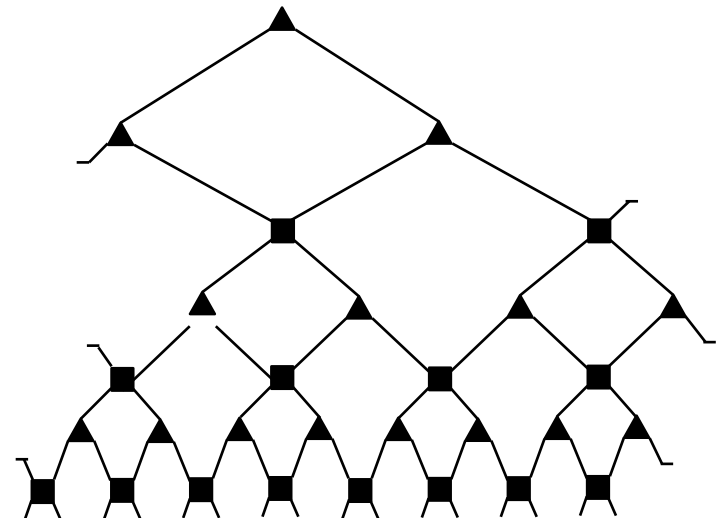
Decomposition into spin networks



$= \sum_j$

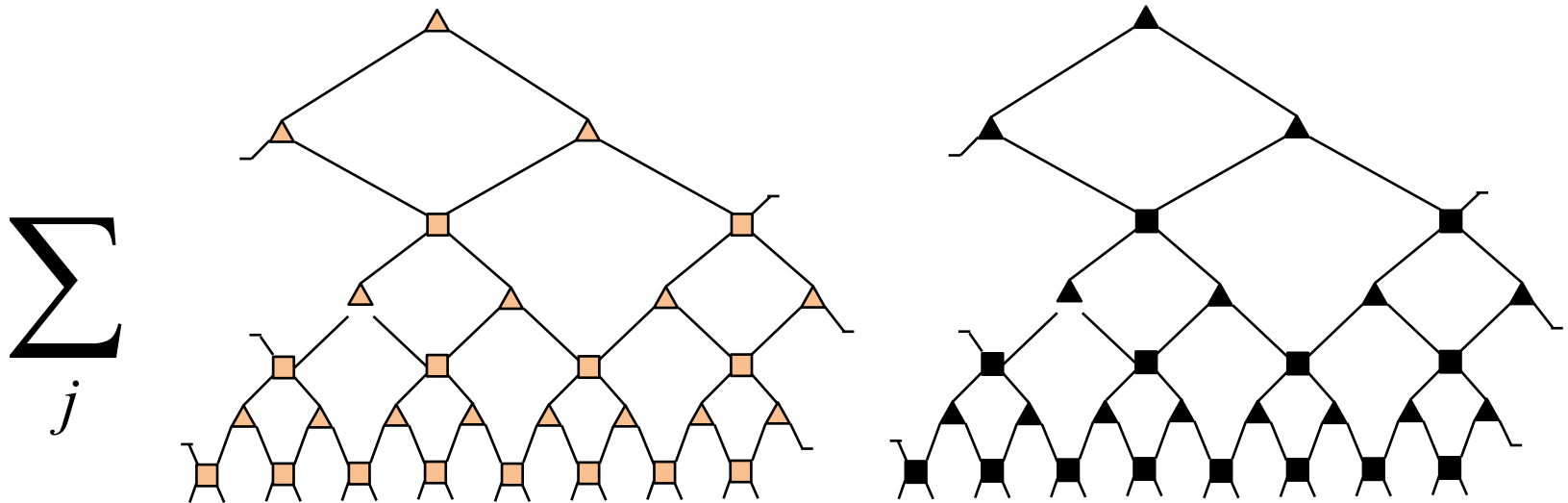


Part not fixed by the symmetry



Spin network

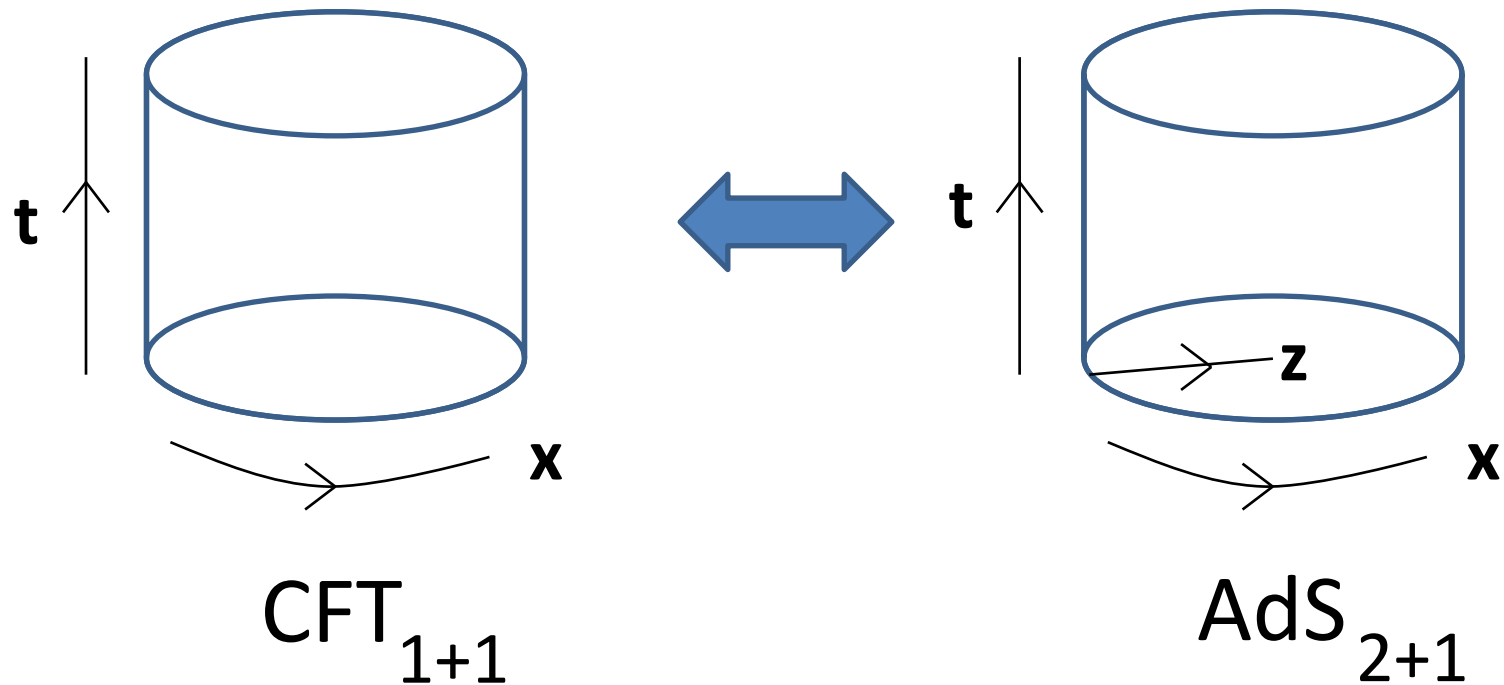
Computational advantages



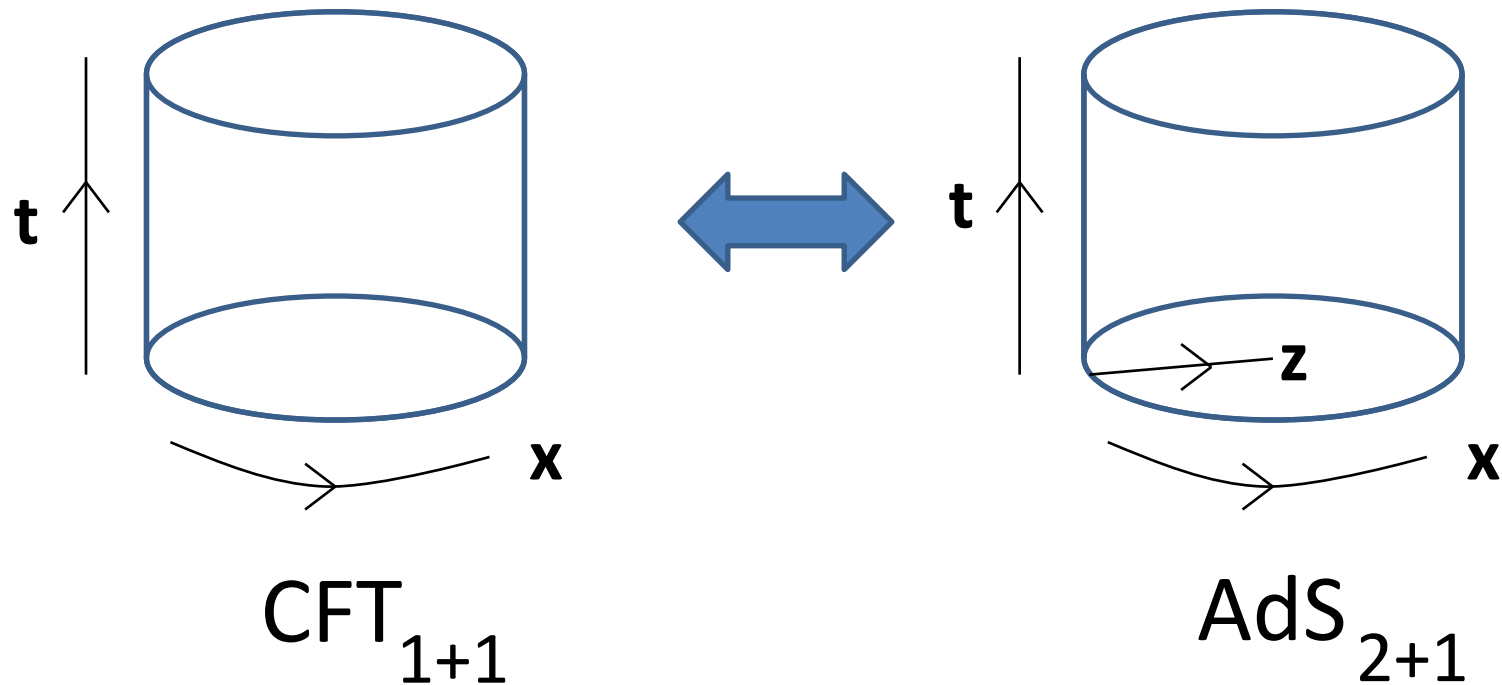
- 1) Compactified description
- 2) Faster tensor network manipulations
- 3) Can target specific symmetry sectors

MERA and AdS/CFT

AdS/CFT correspondence



AdS/CFT correspondence



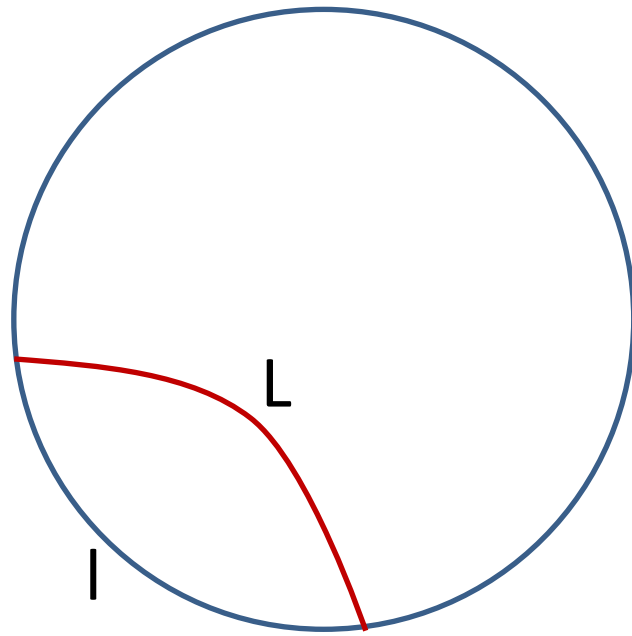
$$c = \frac{3R}{2G} \quad [\text{Brown \& Henneaux '86}]$$

Small $c \Rightarrow$ quantum geometry in the bulk?

Gravity dual of the critical ising model, $c=1/2$

[Castro et. al, PRD '12]

Gravitational dual of entanglement entropy



L : Length of geodesic in AdS

$$L \approx \frac{R}{2G} \log l$$

$$S_l \approx L$$

[Ryu & Takayanagi PRL '06]

MERA and AdS/CFT

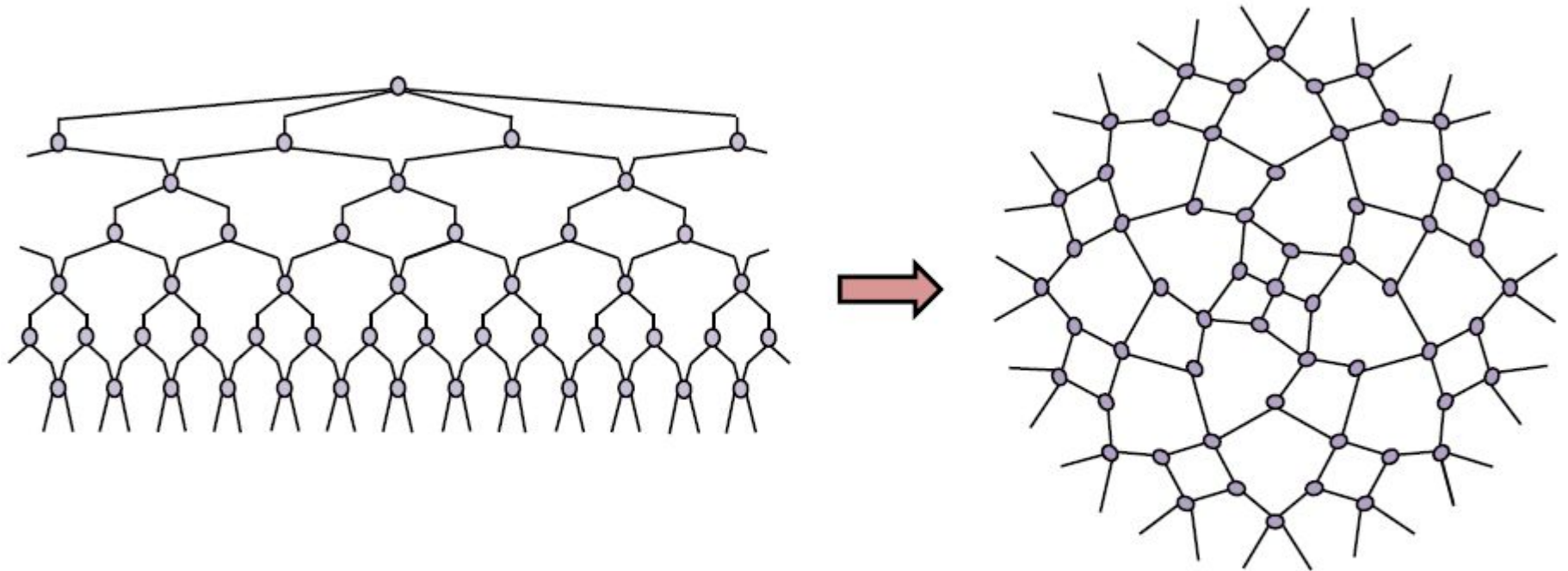


Fig. source: Evenbly, Vidal (2011)

MERA and AdS/CFT

Brian Swingle 2009

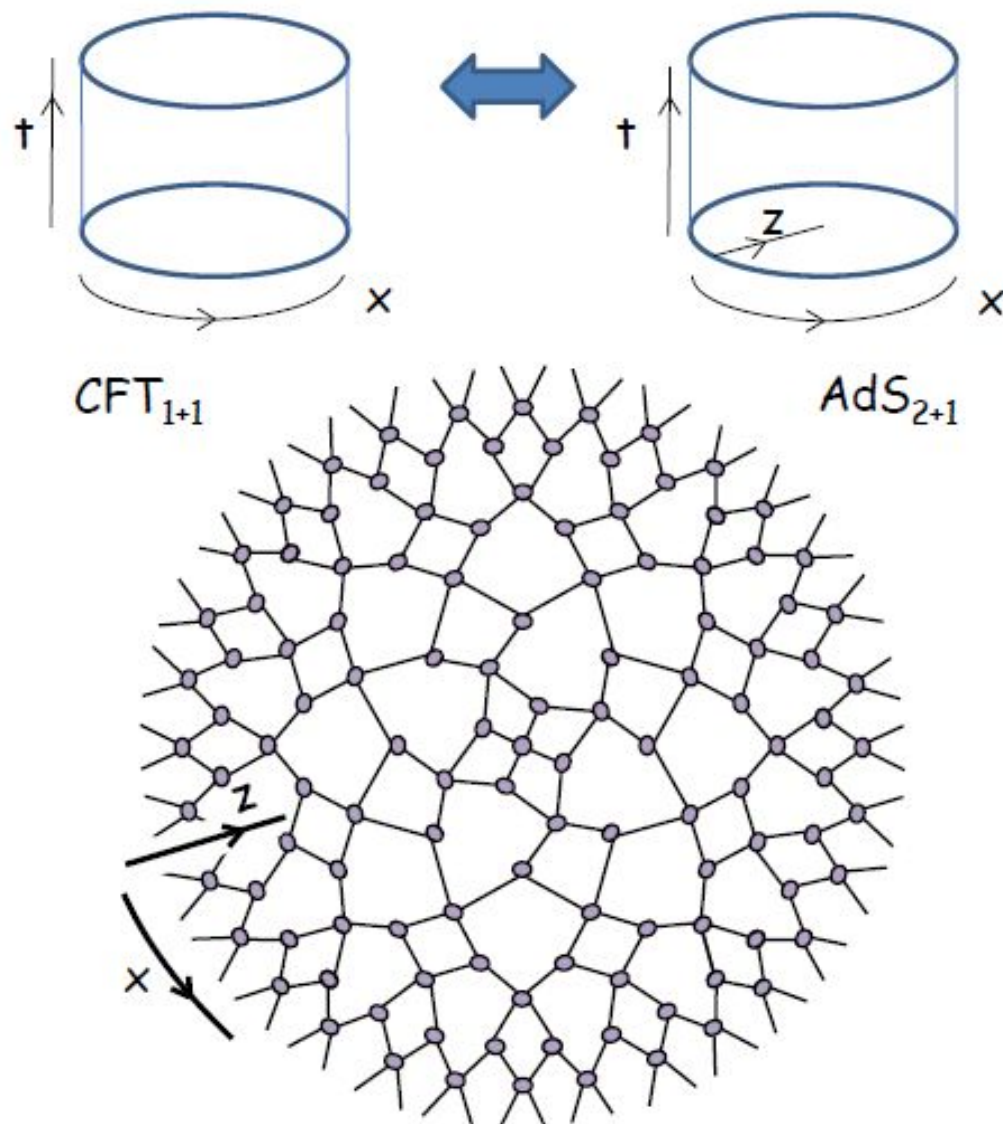
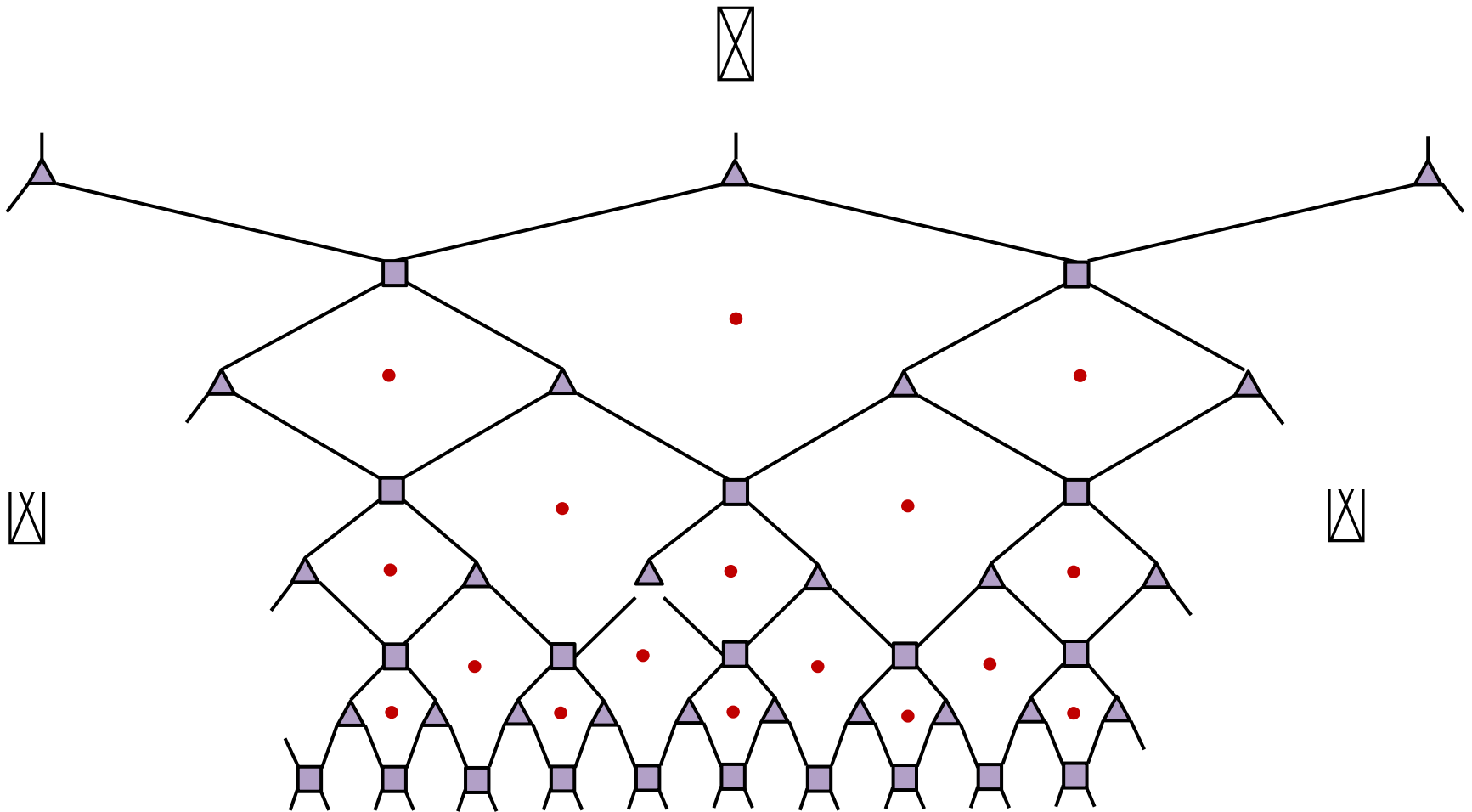
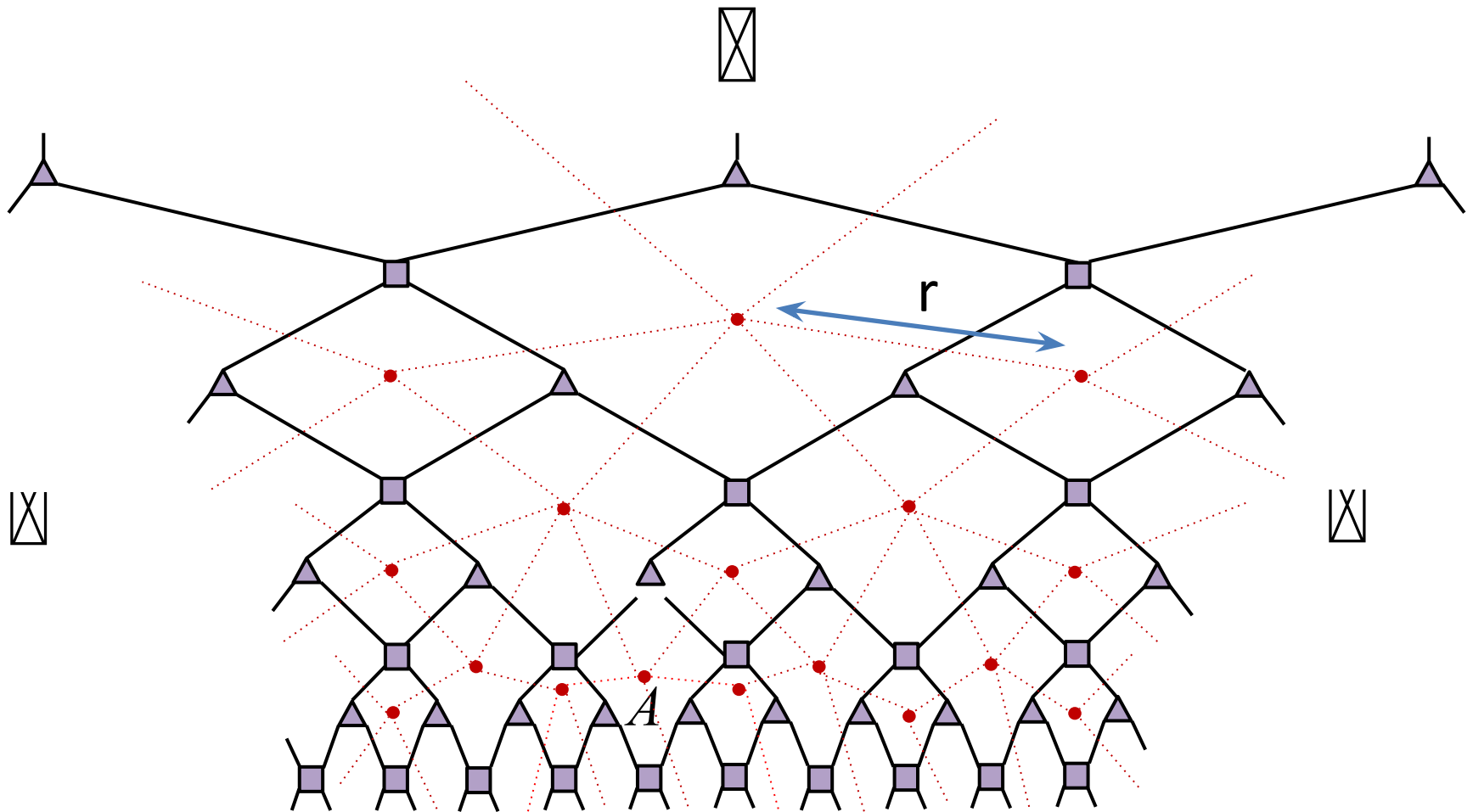


Fig. source: Evenbly, Vidal (2011)

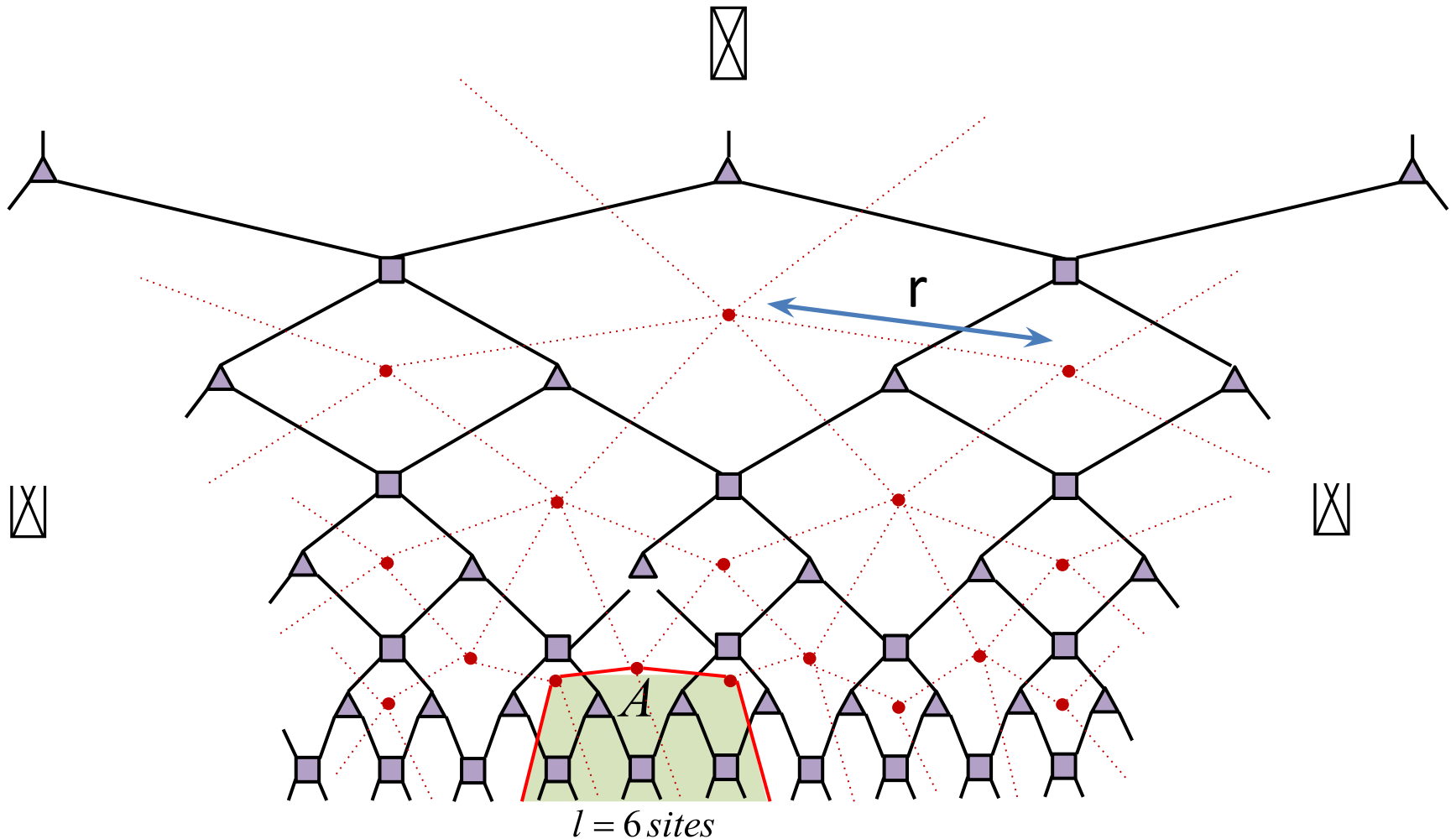
MERA as discrete AdS space



MERA as discrete AdS space



MERA as discrete AdS space

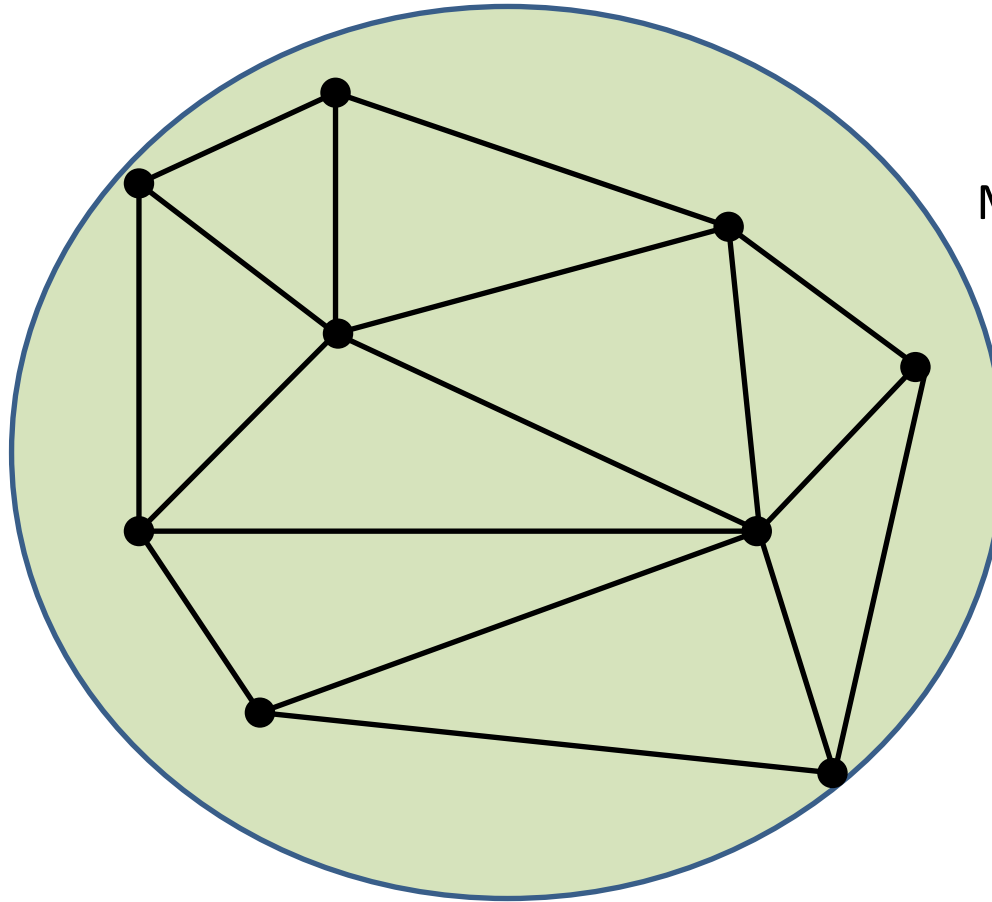


Length of geodesic in the dual graph = $r \log l$

$$S_l \approx r \log l? \quad r = \frac{c}{3}?$$

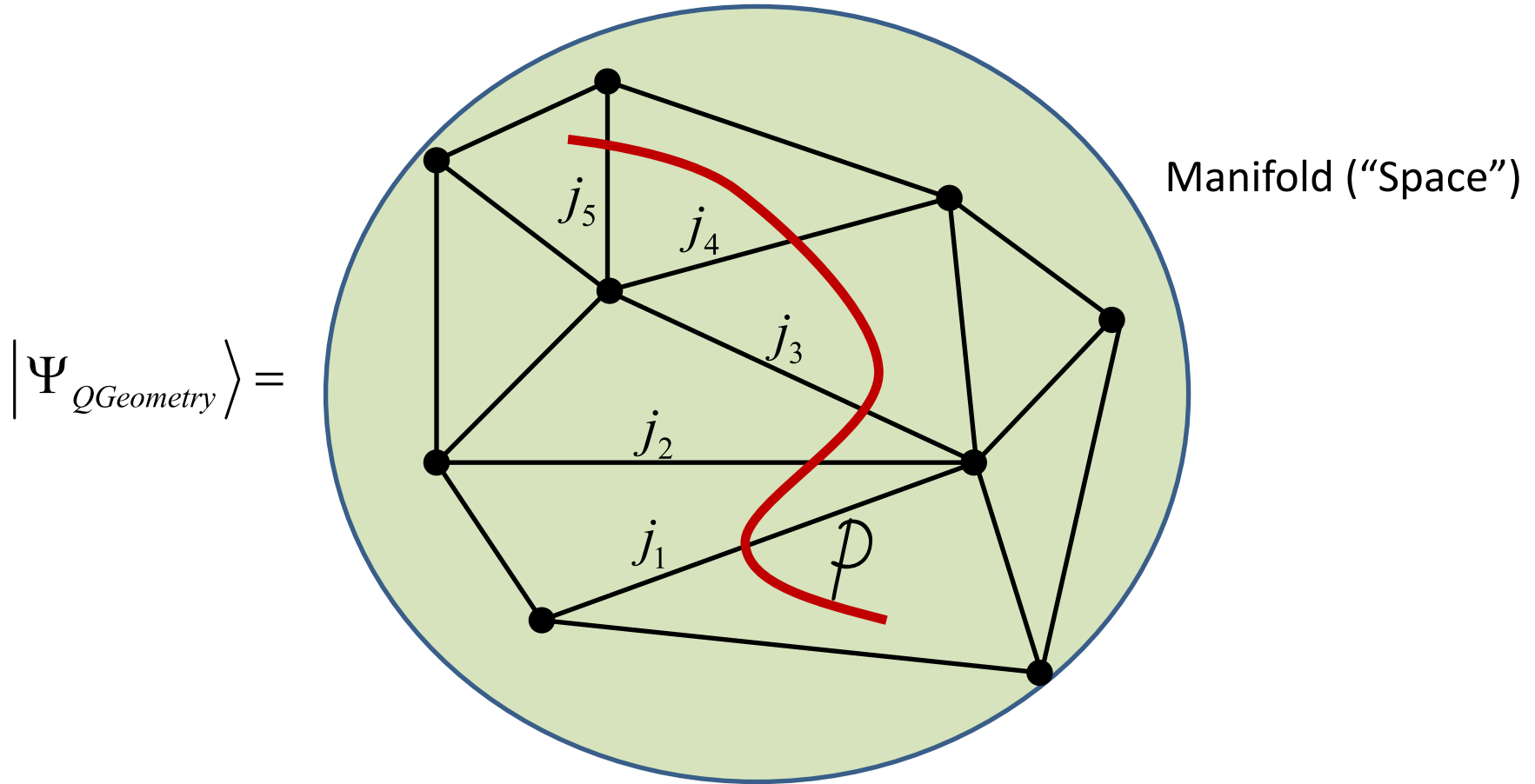
Spin networks and quantum geometry

$$|\Psi_{QGeometry}\rangle =$$



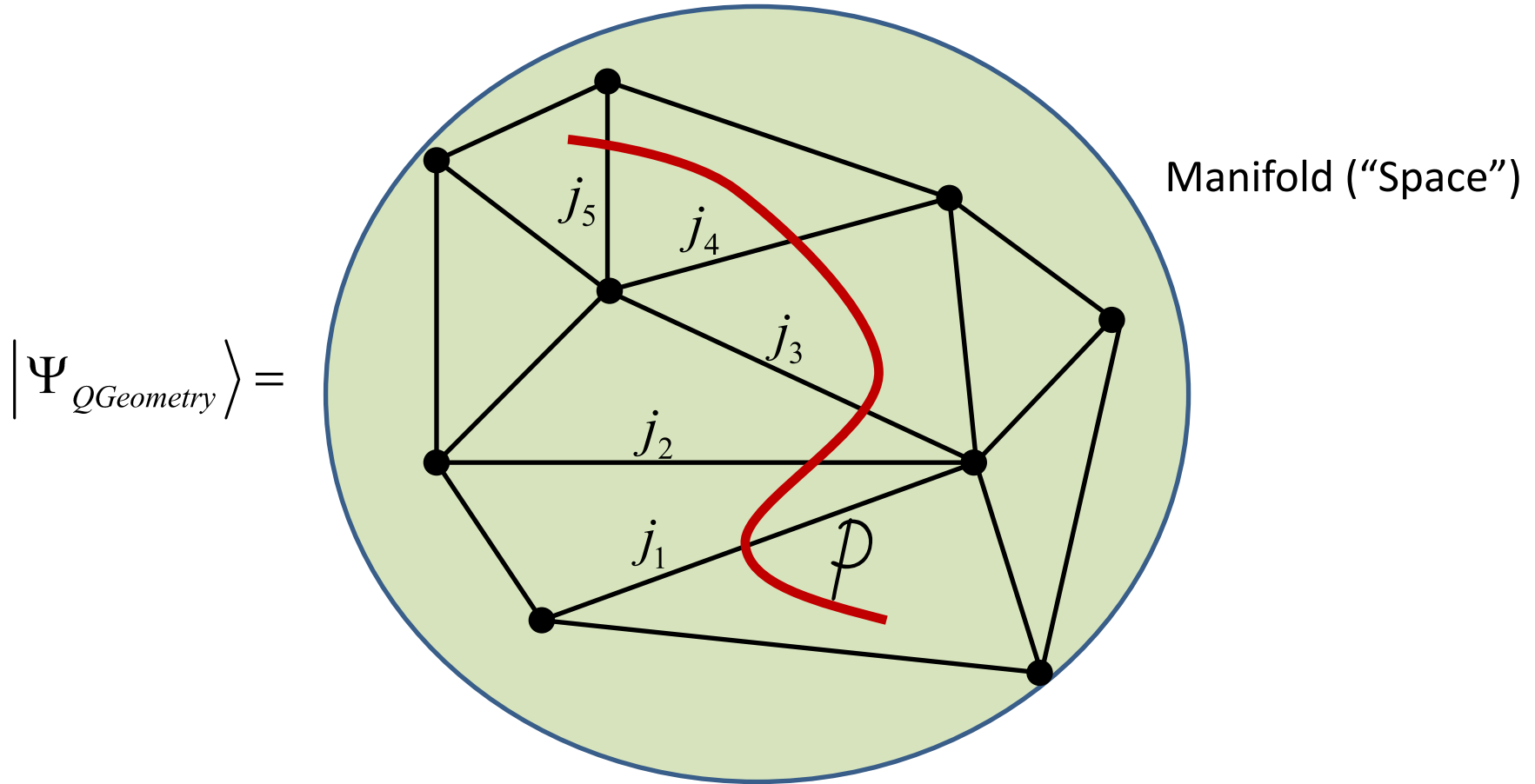
Manifold ("Space")

Spin networks and quantum geometry



$$\hat{L}(P) = \sum_{e \in P} \hat{r}_e, \quad \hat{r}_e |\Psi_{QGeometry}\rangle = \sqrt{j_e(j_e + 1)} |\Psi_{QGeometry}\rangle$$

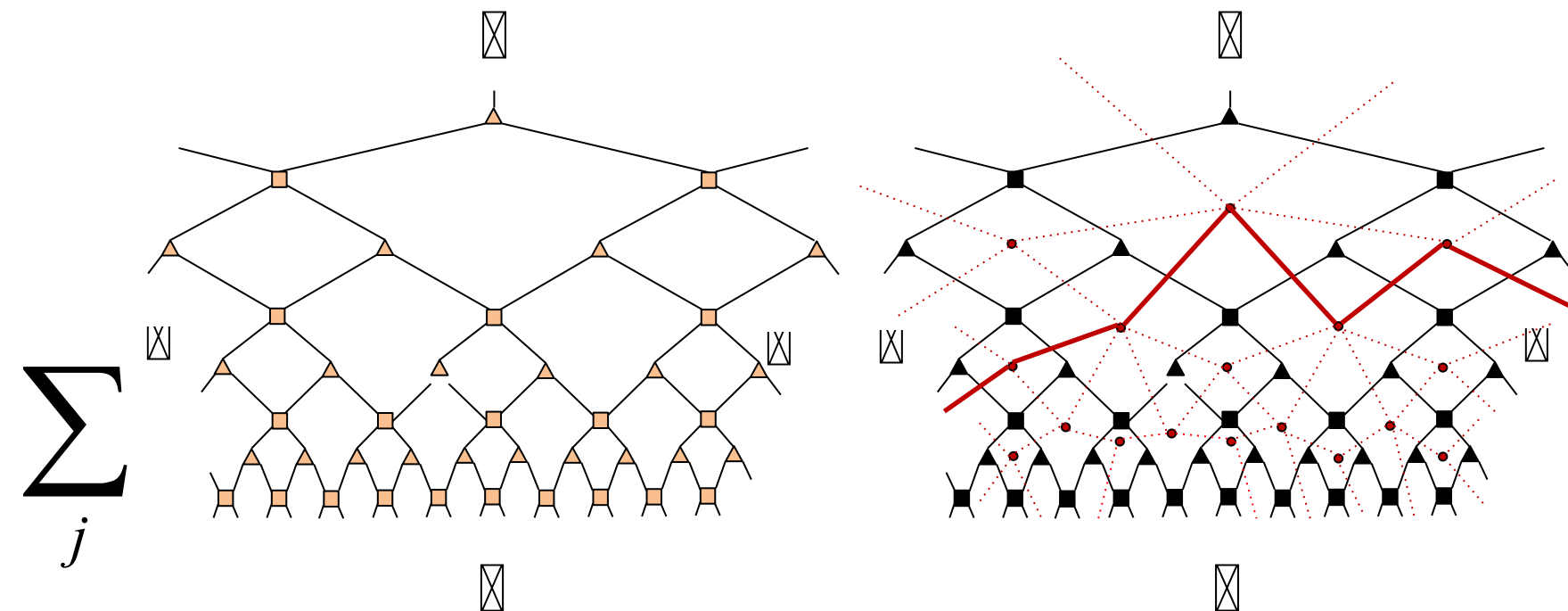
Spin networks and quantum geometry



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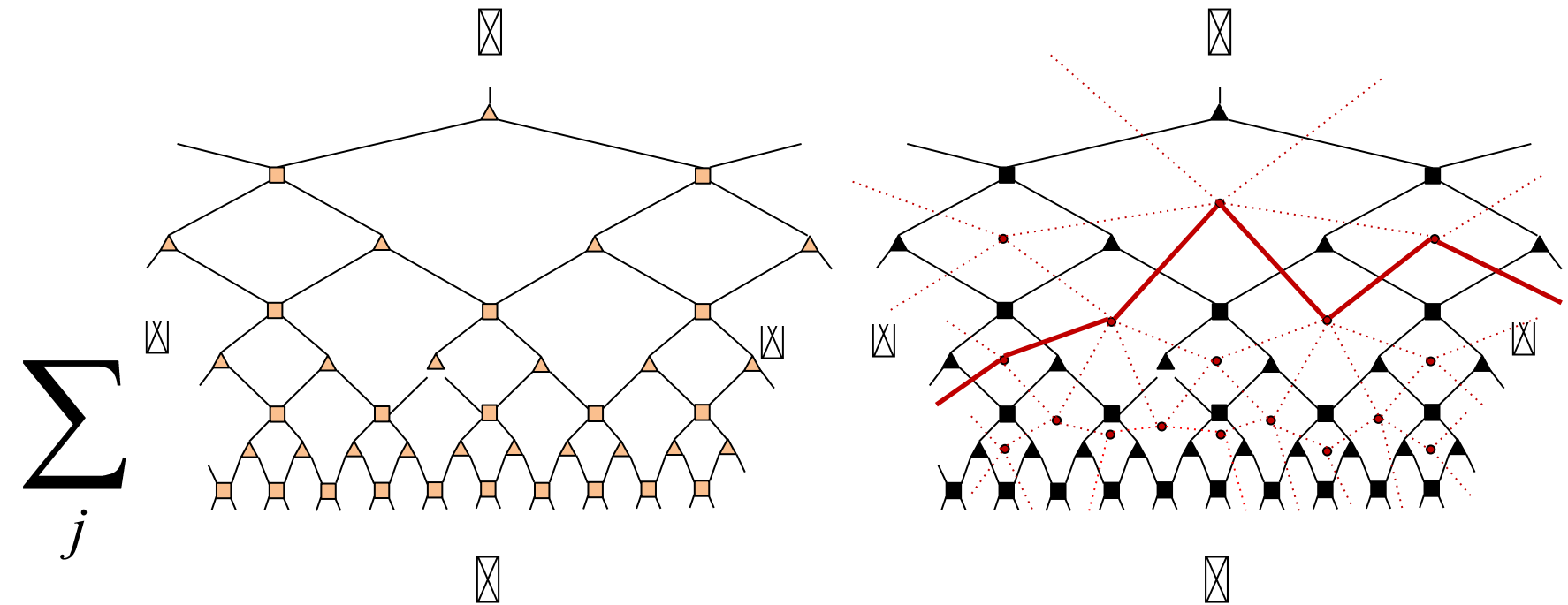
$$|\Psi_{QGeometry}\rangle \in H_{QG} \quad \langle \hat{L}(P) \rangle = \frac{\langle \Psi_{QGeometry} | \hat{L}(P) | \Psi_{QGeometry} \rangle}{\langle \Psi_{QGeometry} | \Psi_{QGeometry} \rangle}$$

SU(2)-invariant MERA as quantum geometry



$$|\Psi_{gs}\rangle \in H_{LQG}$$

SU(2)-invariant MERA as quantum geometry

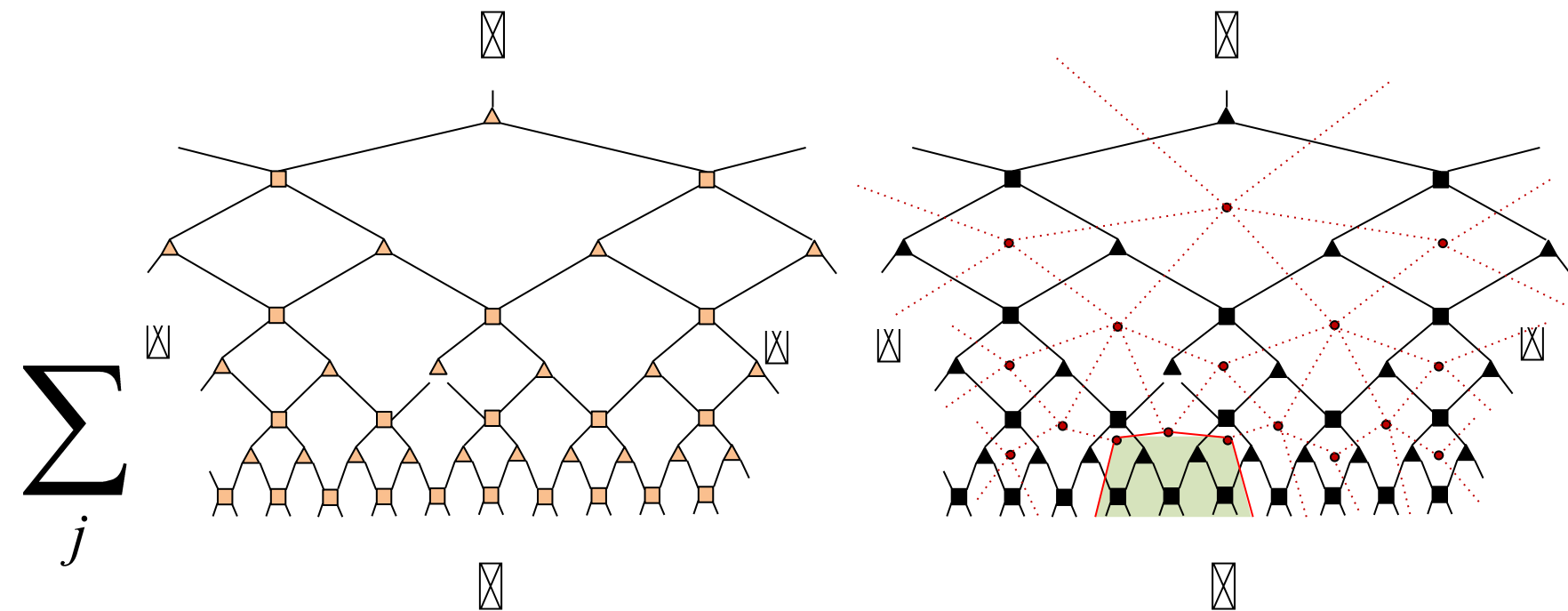


$$|\Psi_{gs}\rangle \in H_{LQG} \quad \langle \hat{L} \rangle = \frac{\langle \Psi_{gs} | \hat{L} | \Psi_{gs} \rangle}{\langle \Psi_{gs} | \Psi_{gs} \rangle} = \langle \hat{r}_1 \rangle + \langle \hat{r}_2 \rangle + \dots + \langle \hat{r}_n \rangle$$

$$\langle \hat{r}_1 \rangle = \langle \hat{r}_2 \rangle = \dots = \langle \hat{r}_n \rangle \quad (\text{Scale invariance})$$

$$\langle \hat{L} \rangle = n \langle \hat{r} \rangle$$

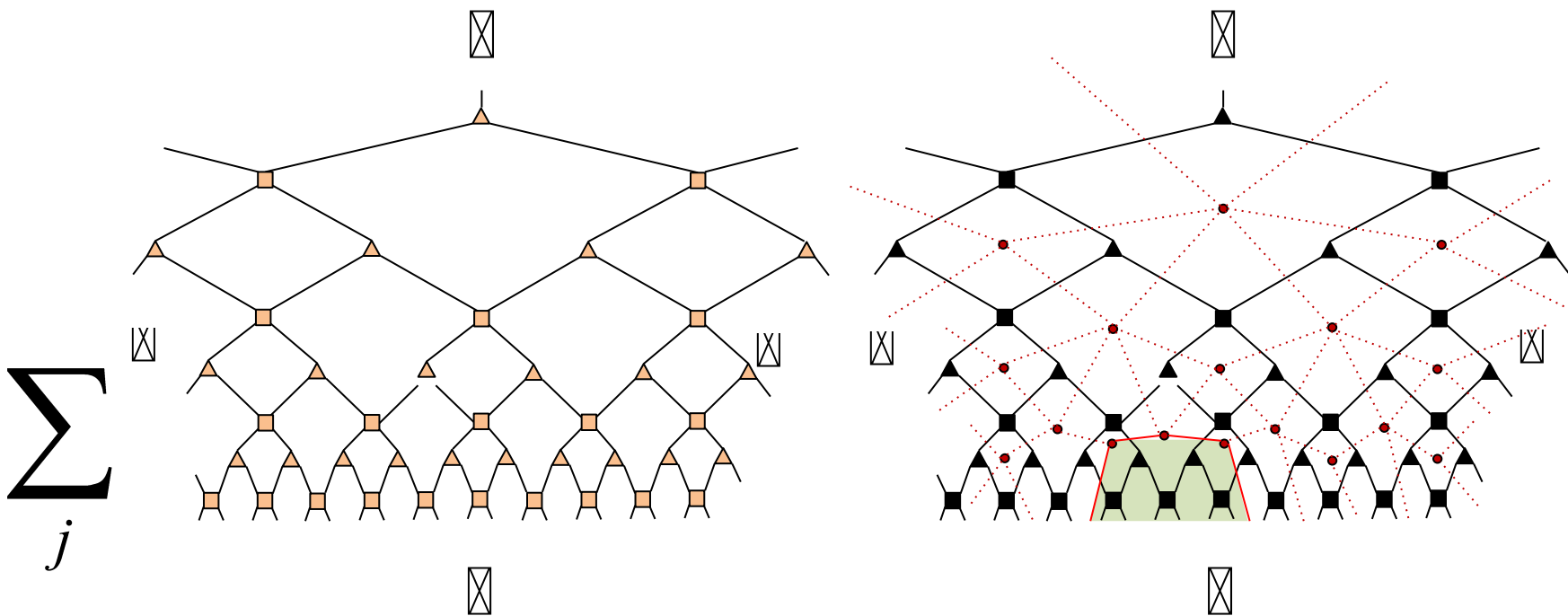
SU(2)-invariant MERA as quantum geometry



$$|\Psi_{gs}\rangle \in H_{QG}$$

$$\langle \hat{L} \rangle \approx \log l \langle \hat{r} \rangle$$

SU(2)-invariant MERA as quantum geometry



$$|\Psi_{gs}\rangle \in H_{QG}$$

$$\langle \hat{L} \rangle \approx \log l \langle \hat{r} \rangle$$

$$S_l \approx \langle \hat{L} \rangle? \quad \frac{\langle \Psi_{gs} | \hat{r} | \Psi_{gs} \rangle}{\langle \Psi_{gs} || \Psi_{gs} \rangle} \approx c?$$

Summary

SU(2)-invariant MERA as a realization of AdS/CFT?

$$|\Psi_{gs}\rangle \in V^{\otimes N} \longleftrightarrow |\Psi_{gs}\rangle \in H_{LQG}$$

- A) Have a description of bulk geometry
- B) Quantum geometry, expected for small c ?
- C) Global symmetry \longleftrightarrow Gauge symmetry

Can explore :

- A) AdS/CFT dictionary?
- B) other geometric quantities? – holonomy, curvature
- C) different gauge groups? e.g. SU(2)_k

Thanks!