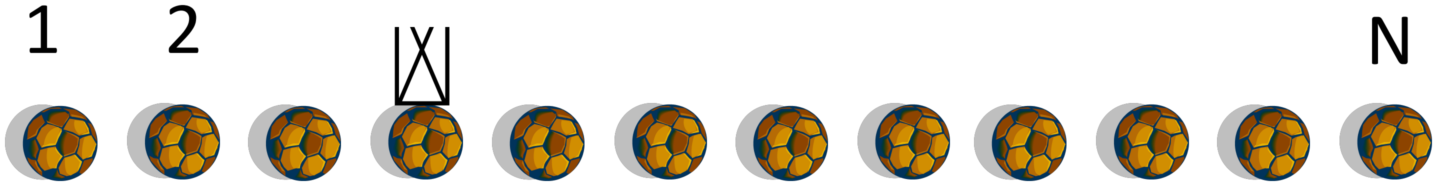
MERA, symmetries and AdS/CFT

Sukhi Singh (EQuS, Macquarie)



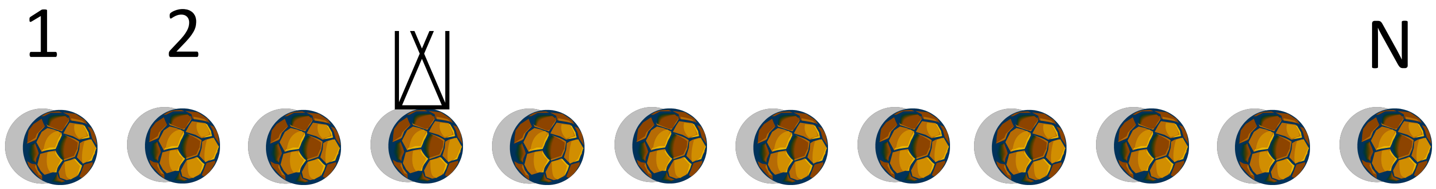
dim(V) = d

1) Hamiltonian :

A) Local

B) Critical/Gapless ($N \to \infty$) -> described by a CFT

$$e.g.\ \hat{H} = \sum_{k=1}^{N} \left(\sigma_X^k \sigma_X^{k+1} + \sigma_Y^k \sigma_Y^{k+1} + \sigma_Z^k \sigma_Z^{k+1} \right) \quad \text{(Heisenberg model)}$$



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2) Ground state: $\left| \Psi_{gs} \right\rangle = \sum_{i_1 i_2 \dots i_N} \hat{\Psi}_{i_1 i_2 \dots i_N} \left| i_1 \right\rangle \left| i_2 \right\rangle \dots \left| i_N \right\rangle$

Locality & criticality => Limited entanglement in GS

$$S_l = \frac{c}{3} \log l$$

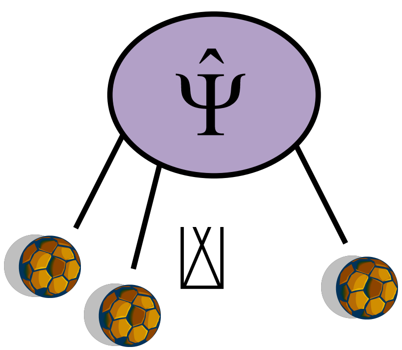
c : central charge of the underlying CFT

$$S_l = -\mathrm{tr}(\rho_l \log \rho_l)$$

$$\rho_l = \mathrm{tr}_{\bar{l}} \left(\left| \Psi_{gs} \right\rangle \left\langle \Psi_{gs} \right| \right)$$

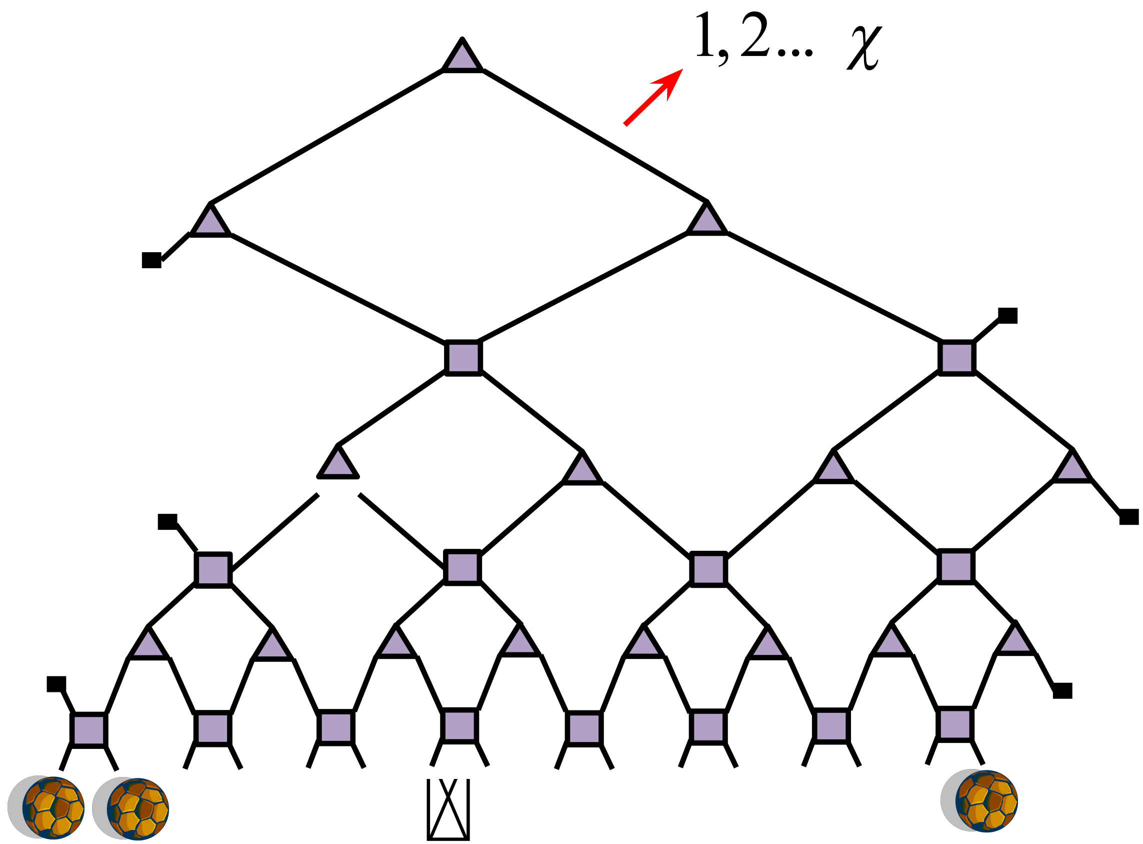
MERA

$$\left|\Psi_{gs}\right\rangle = \sum_{i_1 i_2 \dots i_N} \hat{\Psi}_{i_1 i_2 \dots i_N} \left|i_1\right\rangle\left|i_2\right\rangle \dots \left|i_N\right\rangle$$



d^L coefficients

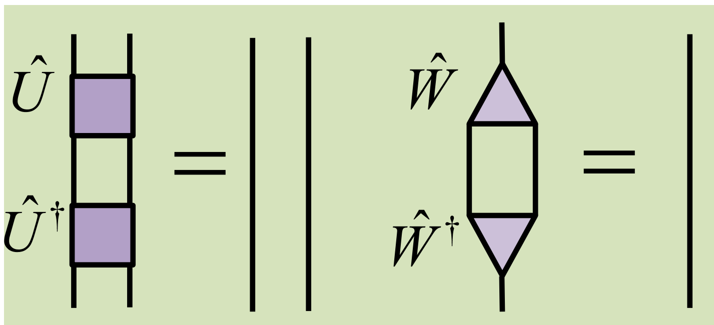
$\approx$

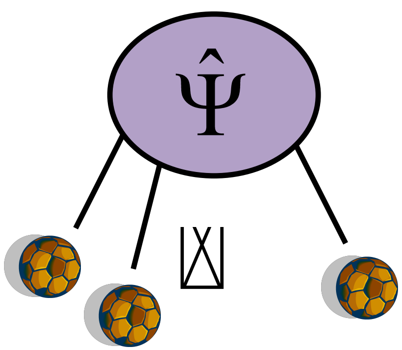


$O(L\chi^4)$ coefficients

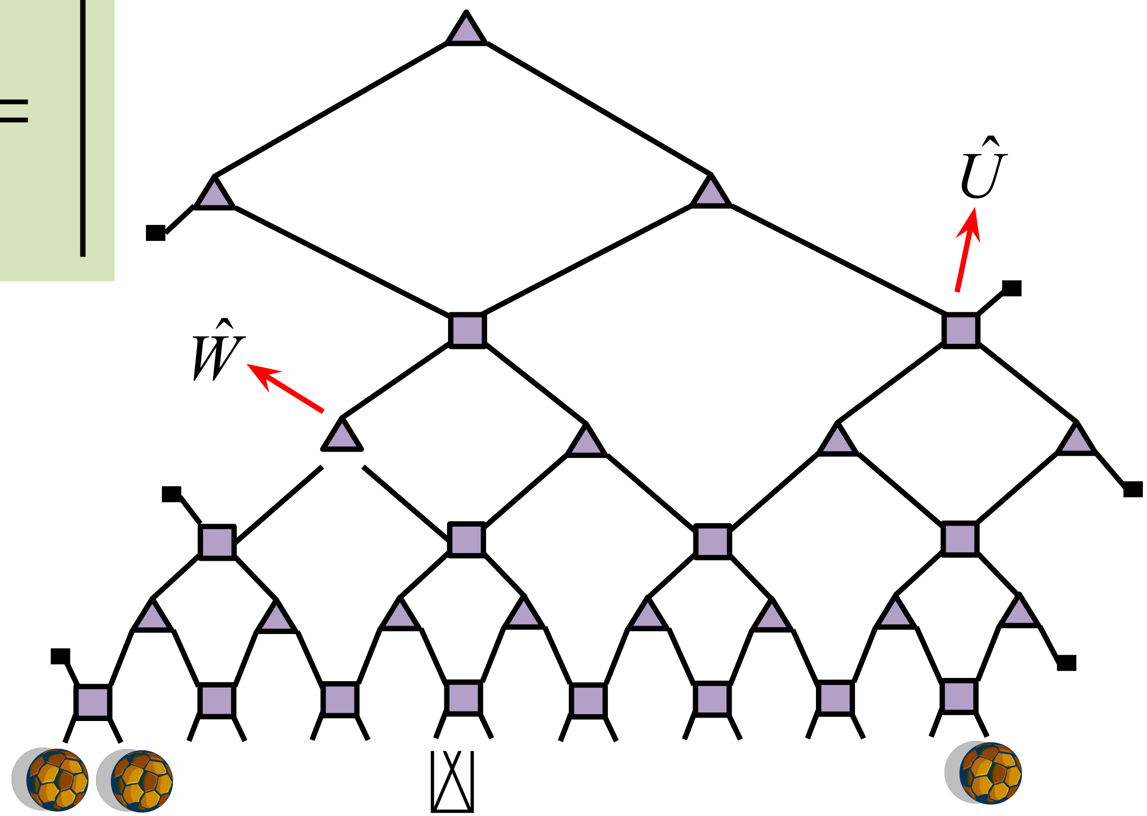
Disentanglers & isometries

$$\left|\Psi_{gs}\right\rangle = \sum_{i_1 i_2 \ldots i_N} \hat{\Psi}_{i_1 i_2 \ldots i_N} \left|i_1\right\rangle \left|i_2\right\rangle \ldots \left|i_N\right\rangle$$





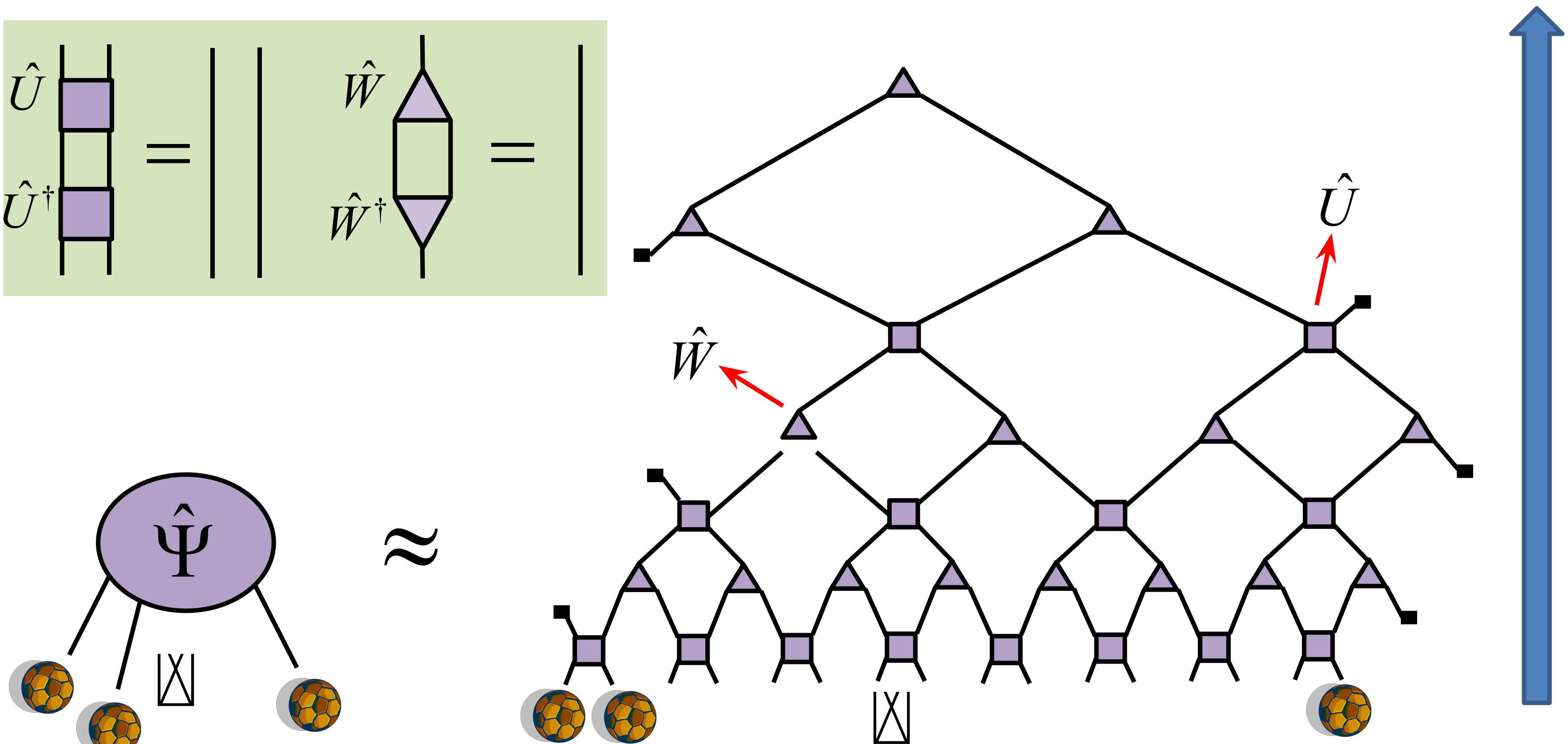
$\approx$



RG transformations

$$\left|\Psi_{gs}\right\rangle = \sum_{i_1 i_2 \ldots\, i_N} \hat{\Psi}_{i_1 i_2 \ldots\, i_N} \left|i_1\right\rangle \left|i_2\right\rangle \ldots \left|i_N\right\rangle$$

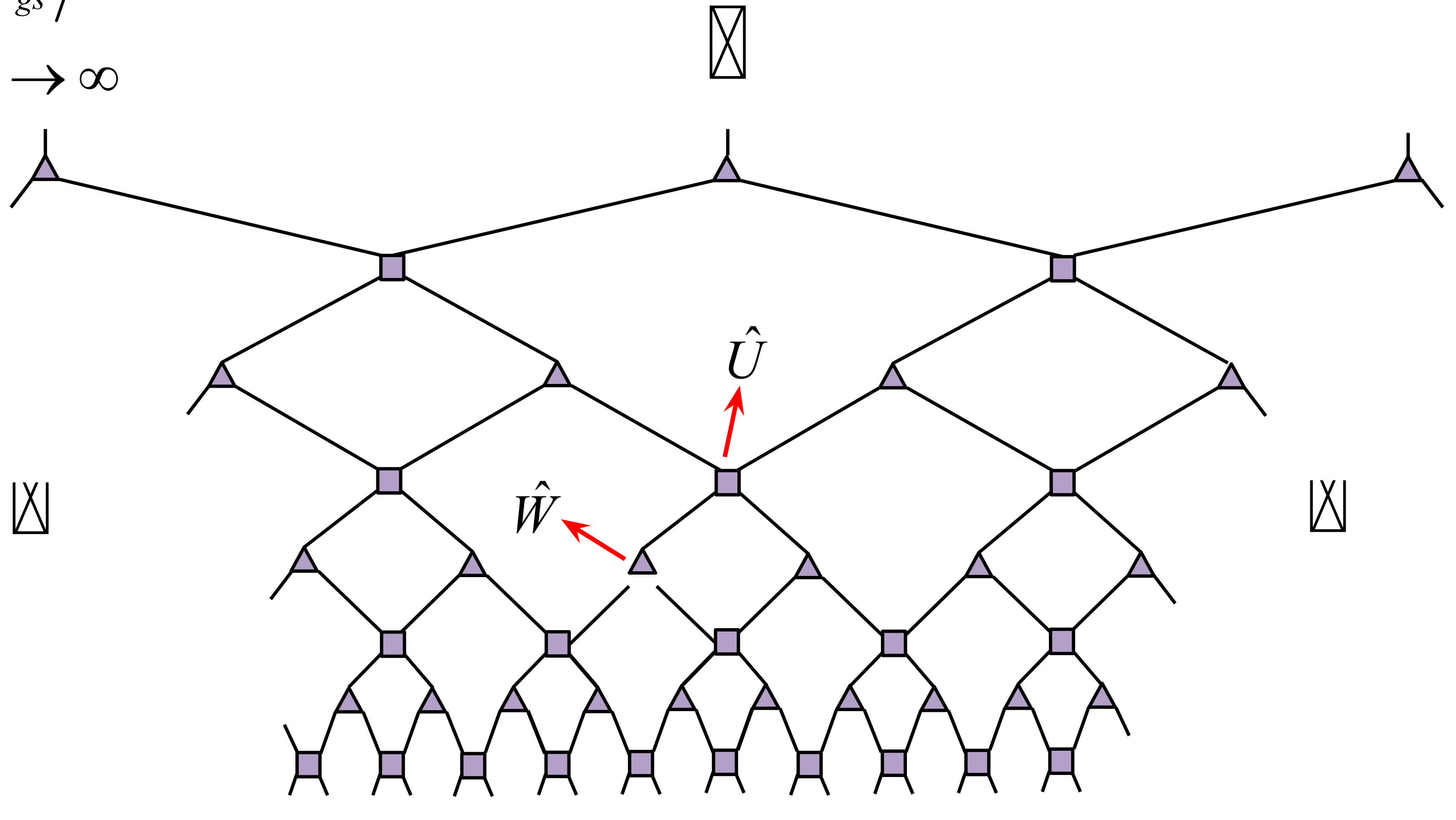
Length Scale



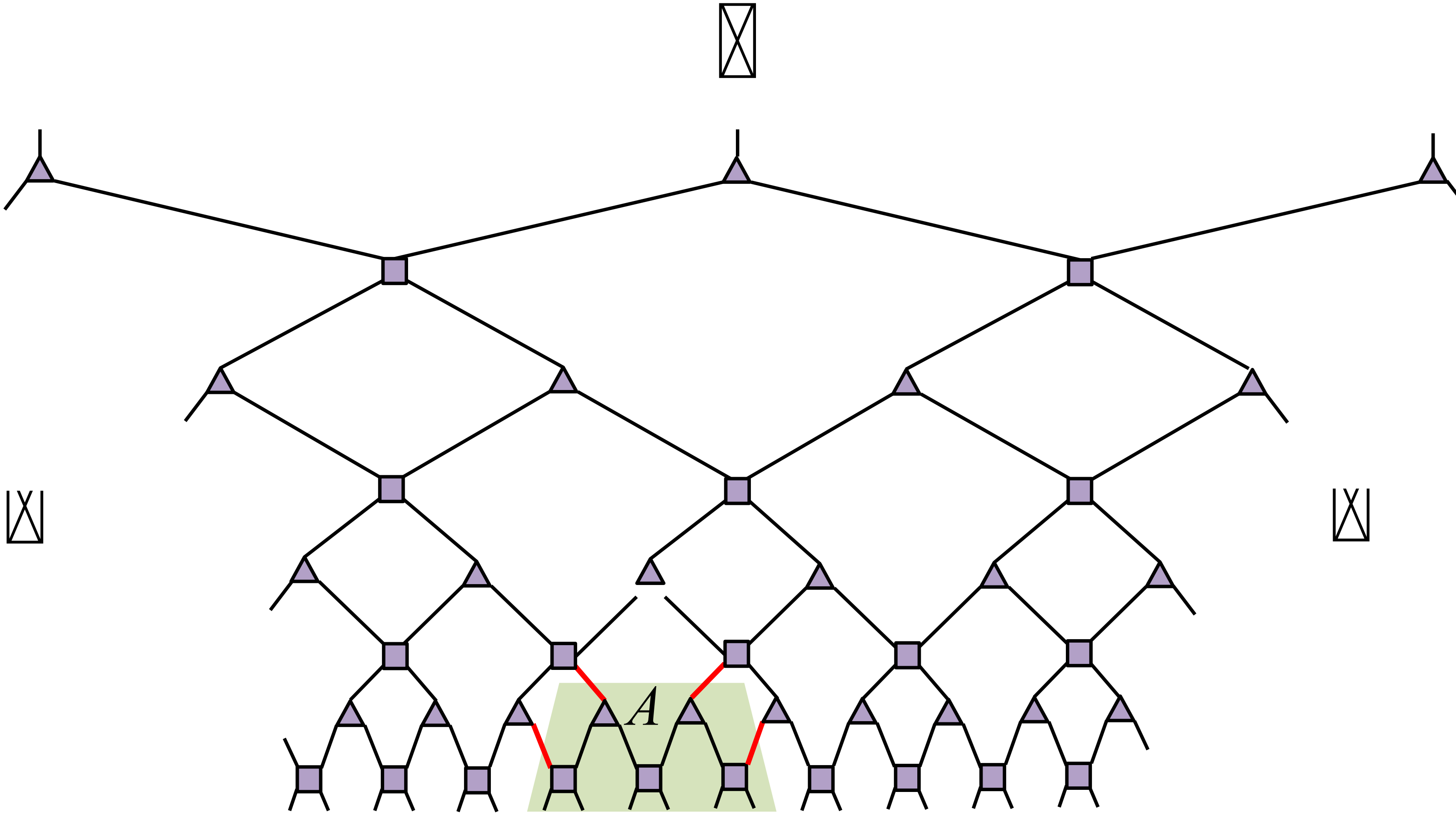
Scale-invariant MERA

$$\left|\Psi_{gs}\right\rangle \approx$$

$$N \to \infty$$



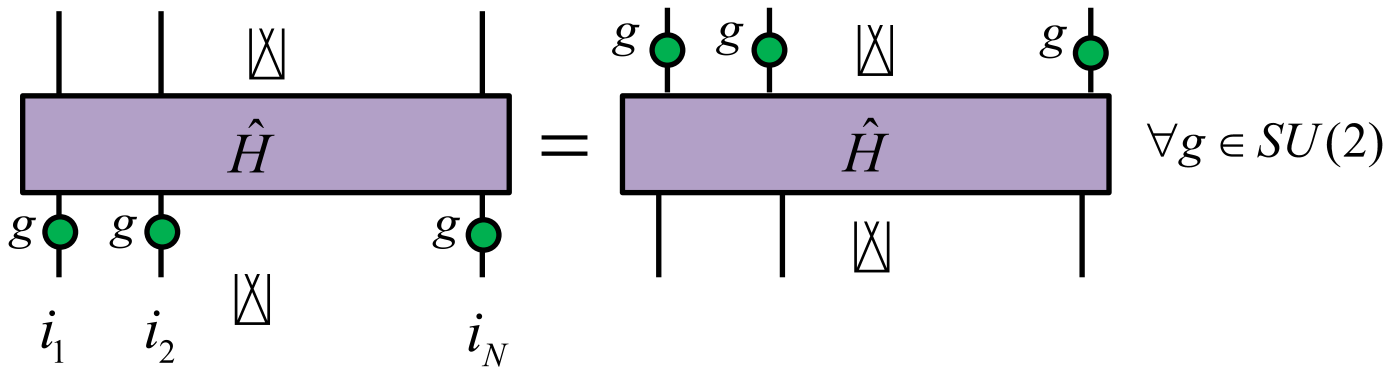
Entanglement from geometry in MERA



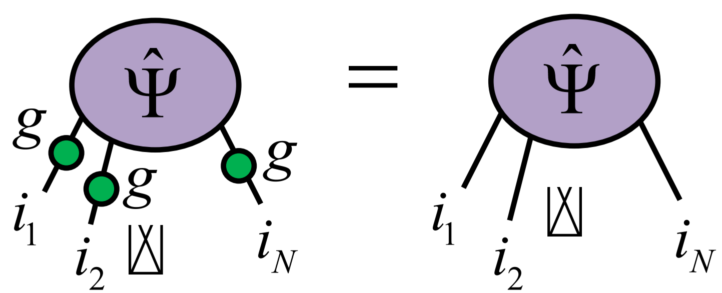
min number of connecting bonds $\approx \log l$

Global SU(2) symmetry

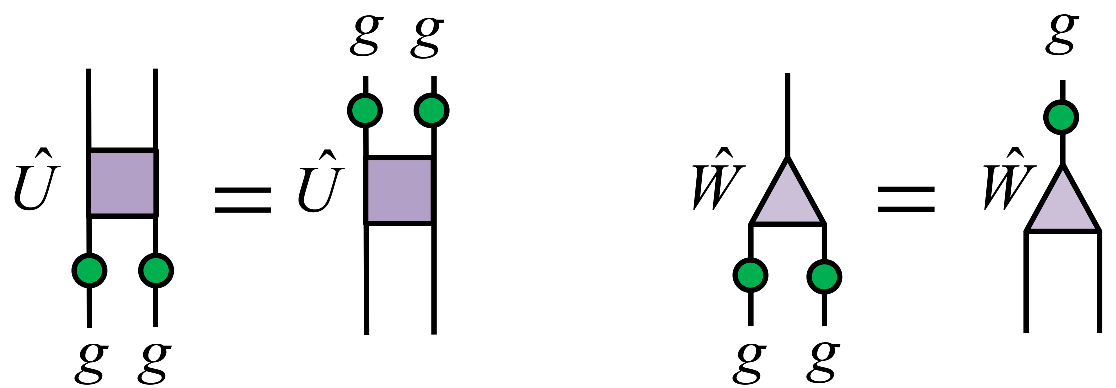
1) Hamiltonian :



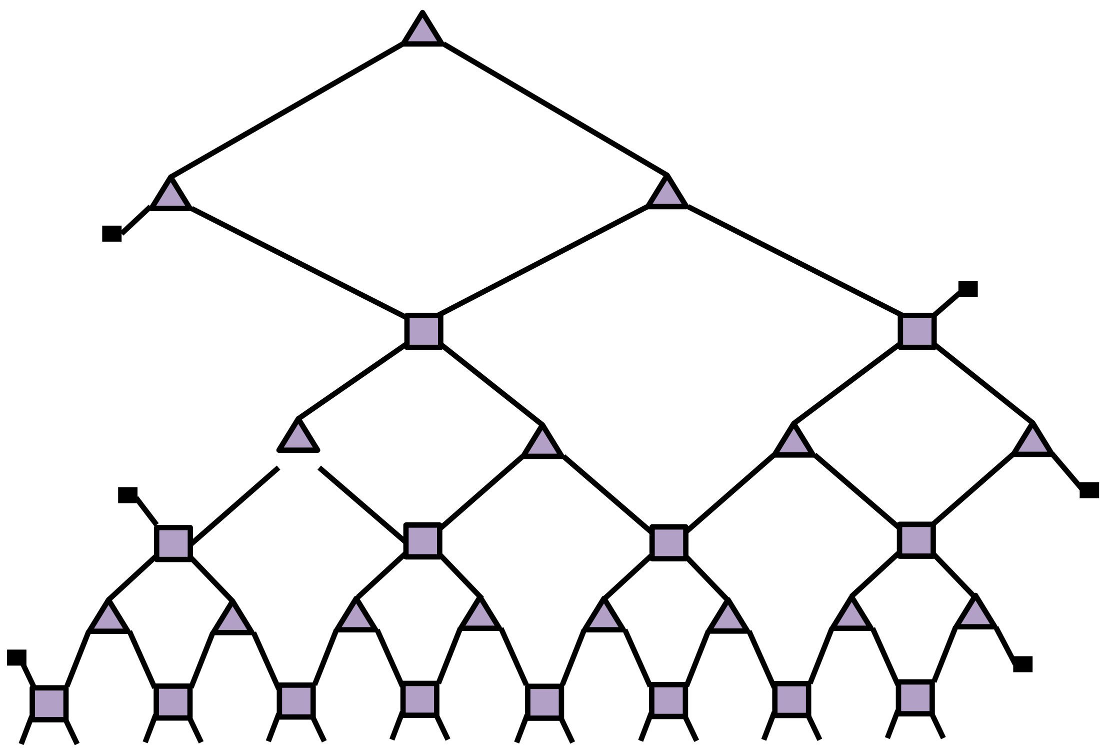
2) Ground state: total spin 0



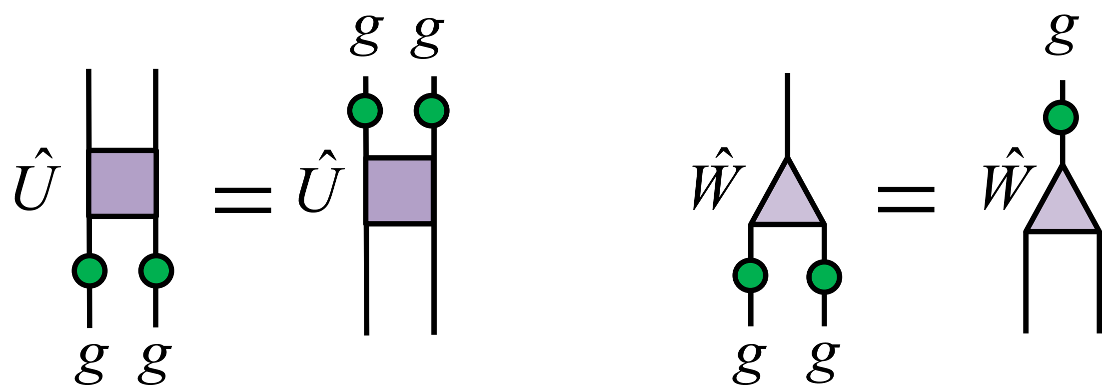
SU(2)-invariant MERA



$\forall g \in SU(2)$

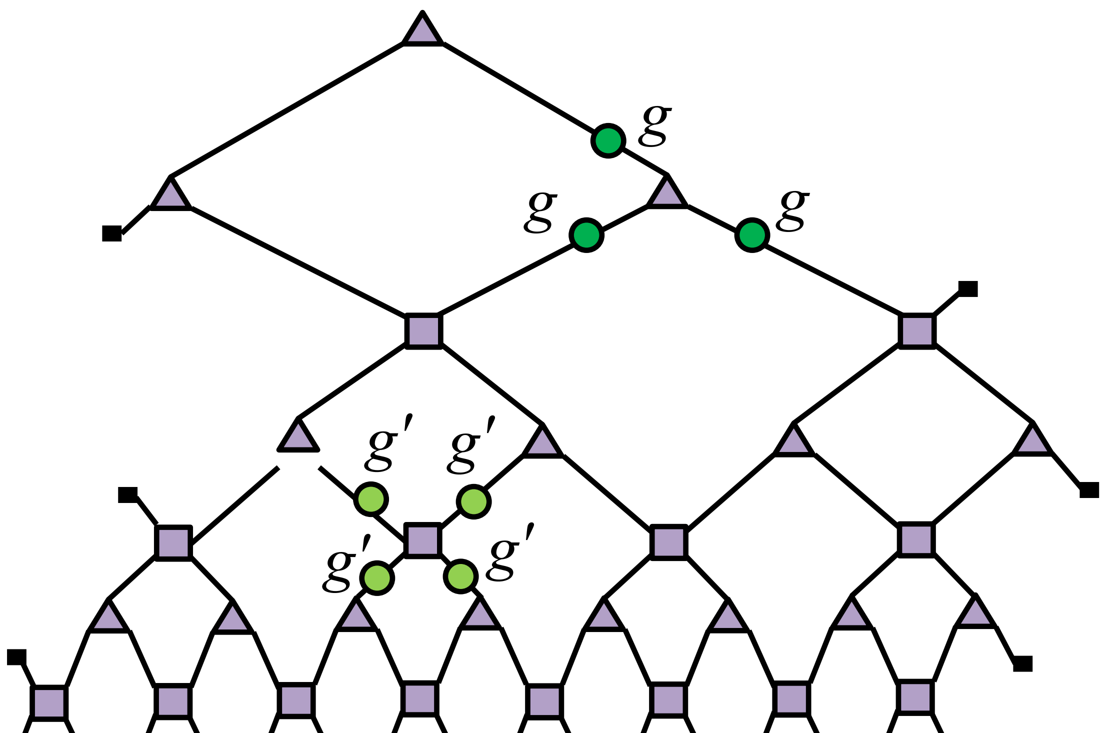


SU(2)-invariant MERA



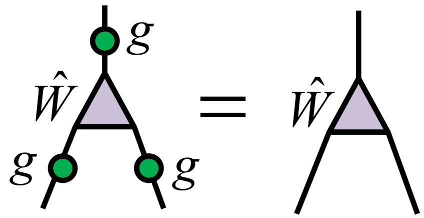
$\forall g \in SU(2)$

SU(2) gauge transformations



Decomposition into spin networks

$\forall g \in SU(2)$



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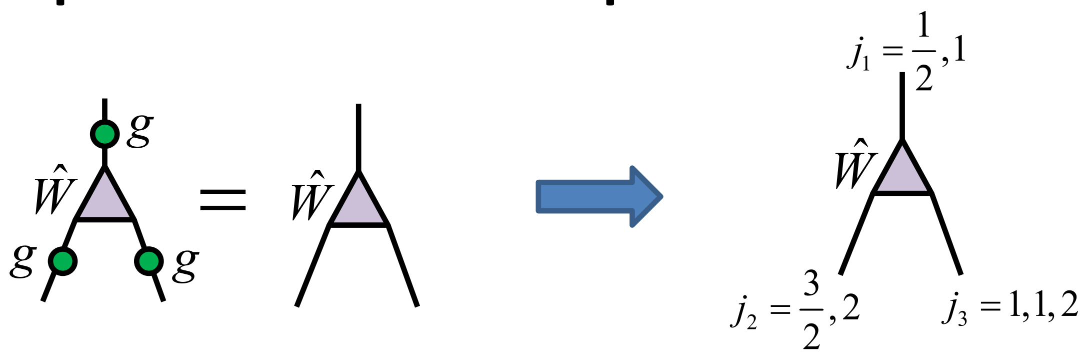
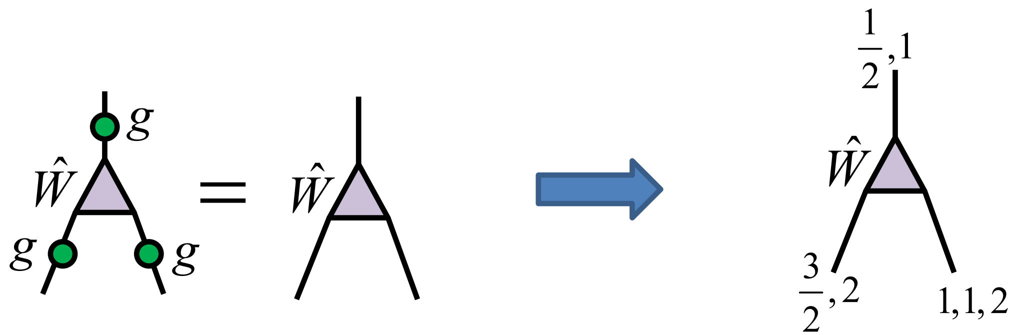


Spin conservation $|j_1 - j_2| \le j_3 \le j_1 + j_2$

Decomposition into spin networks

$\forall g \in SU(2)$



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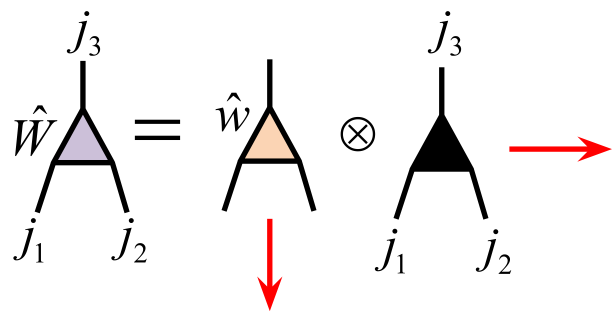
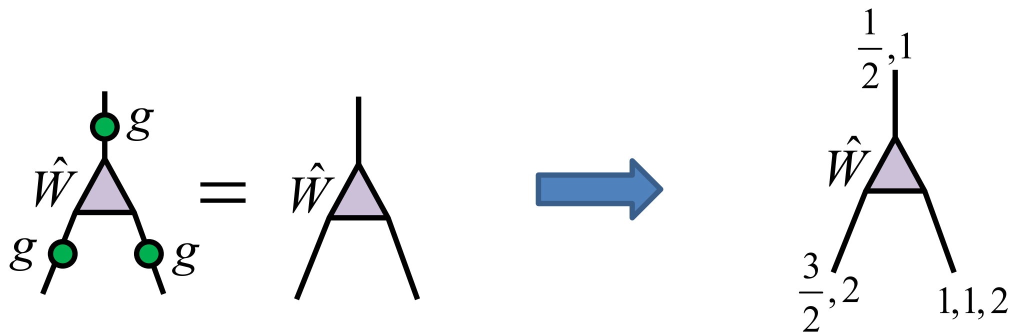


Intertwiner of SU(2)

Part not fixed by symmetry

Decomposition into spin networks

$\forall g \in SU(2)$



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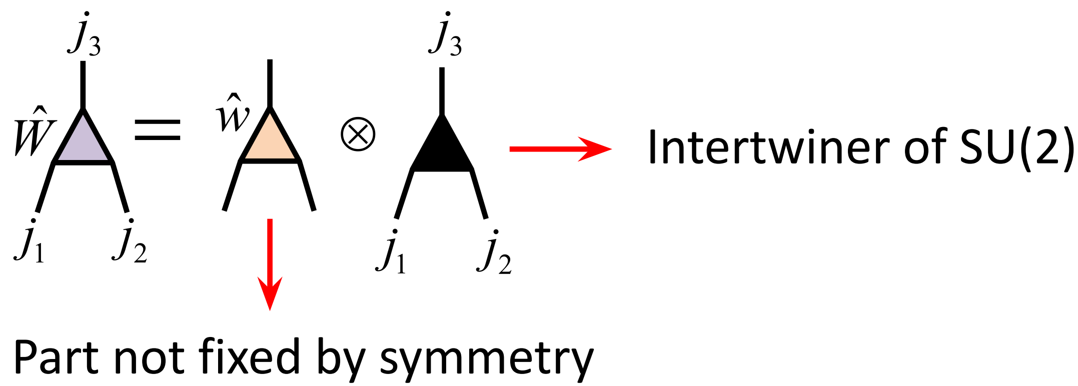
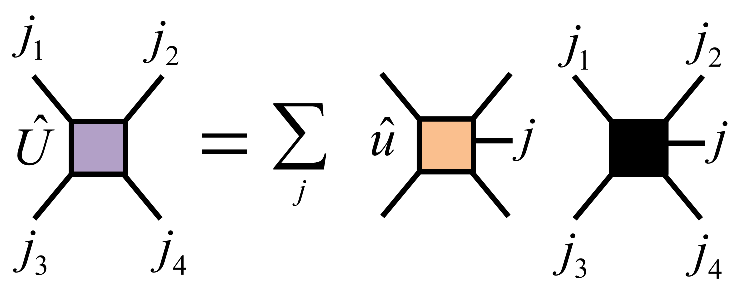
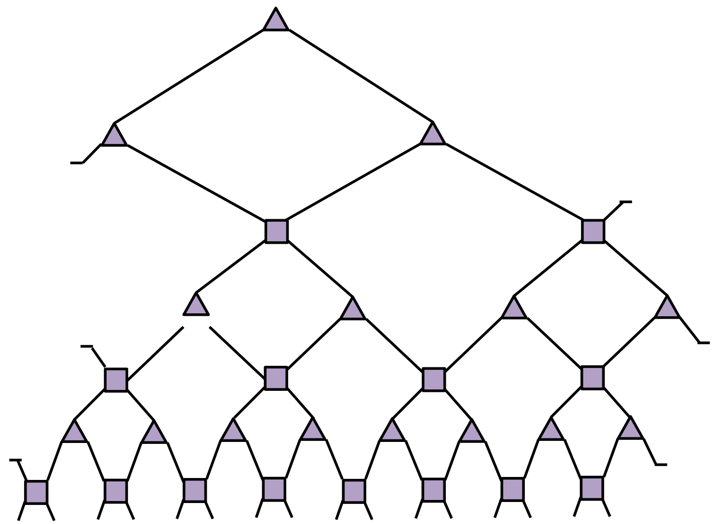
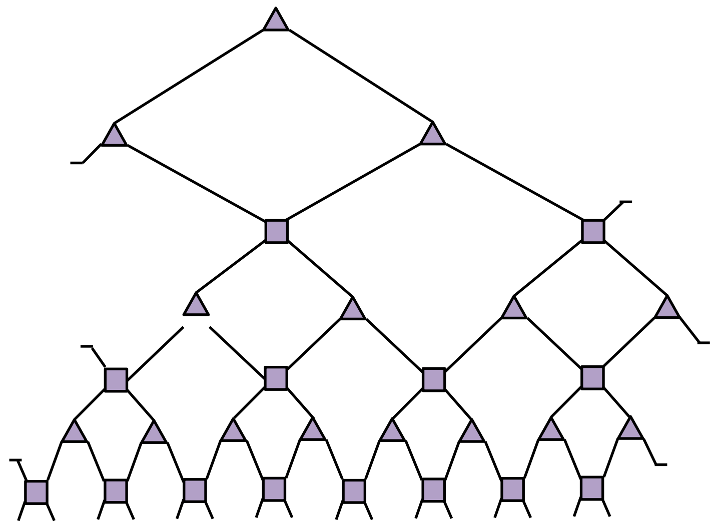




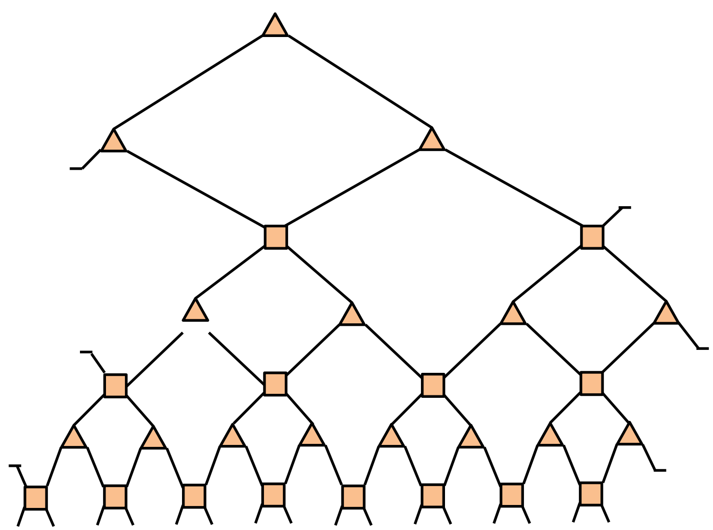
Decomposition into spin networks



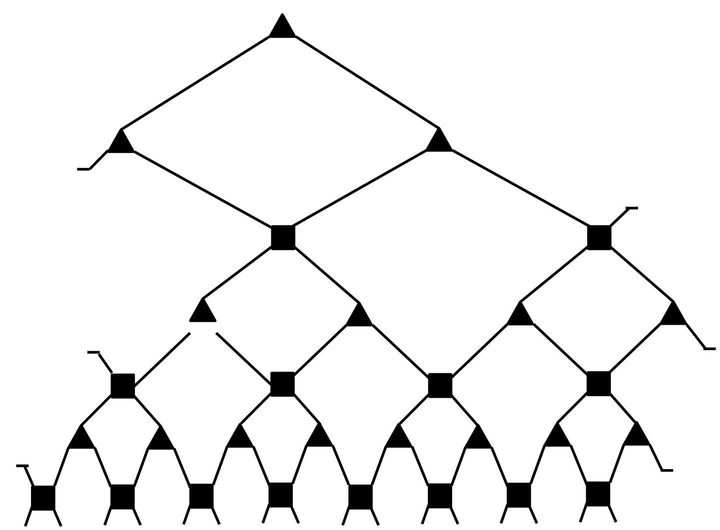
Decomposition into spin networks



$$= \sum_j$$

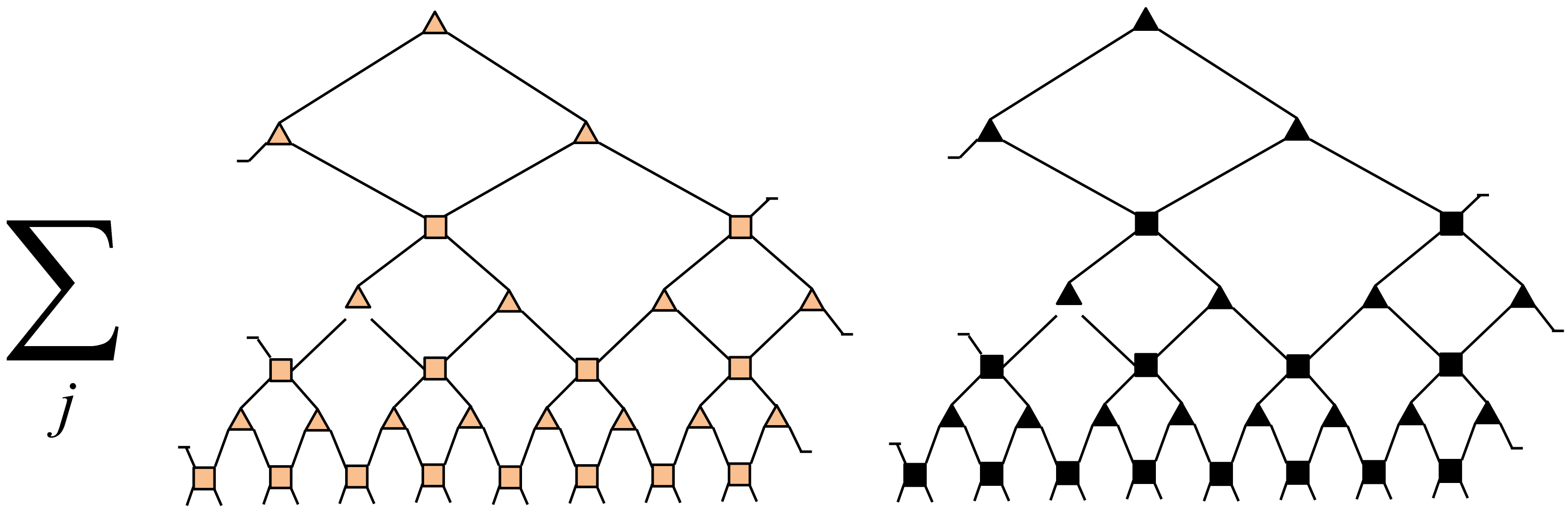


Part not fixed by the symmetry



Spin network

Computational advantages



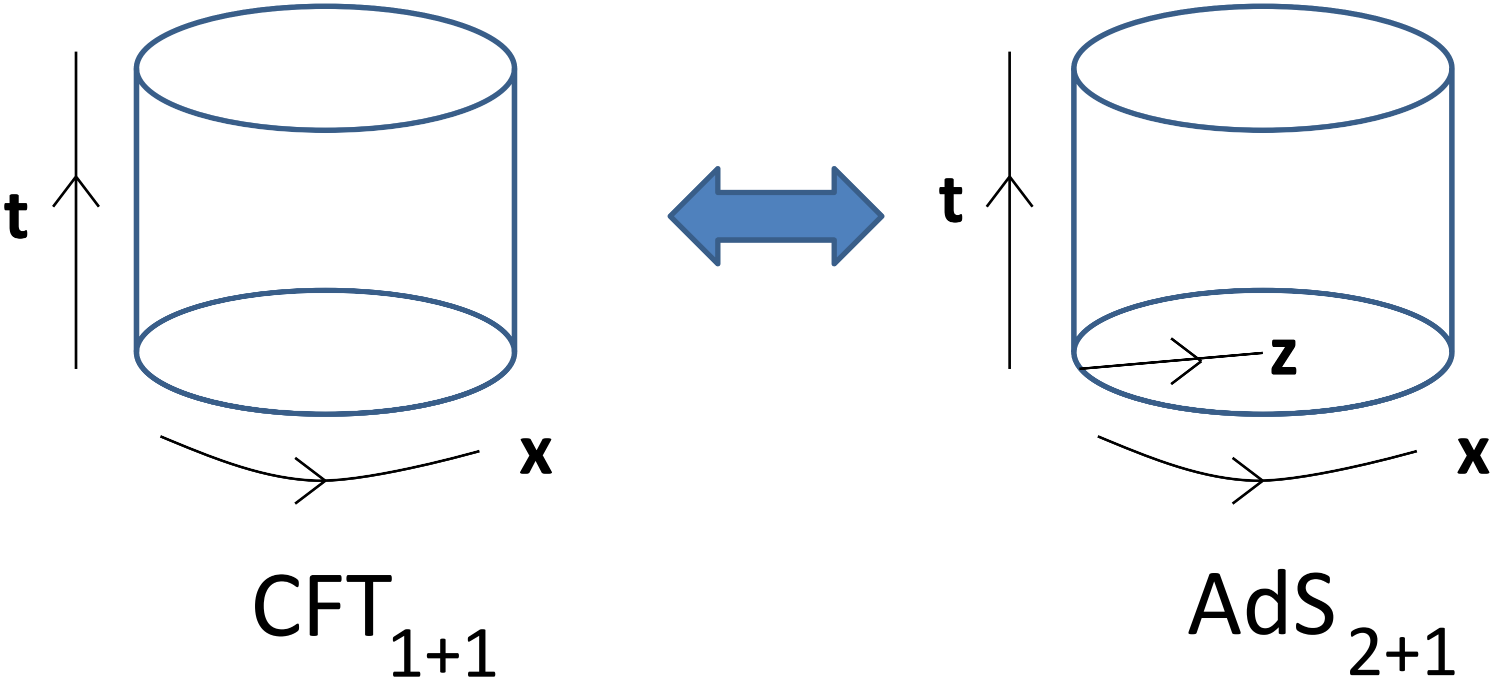
1) Compactified description

2) Faster tensor network manipulations

3) Can target specific symmetry sectors

MERA and AdS/CFT

AdS/CFT correspondence



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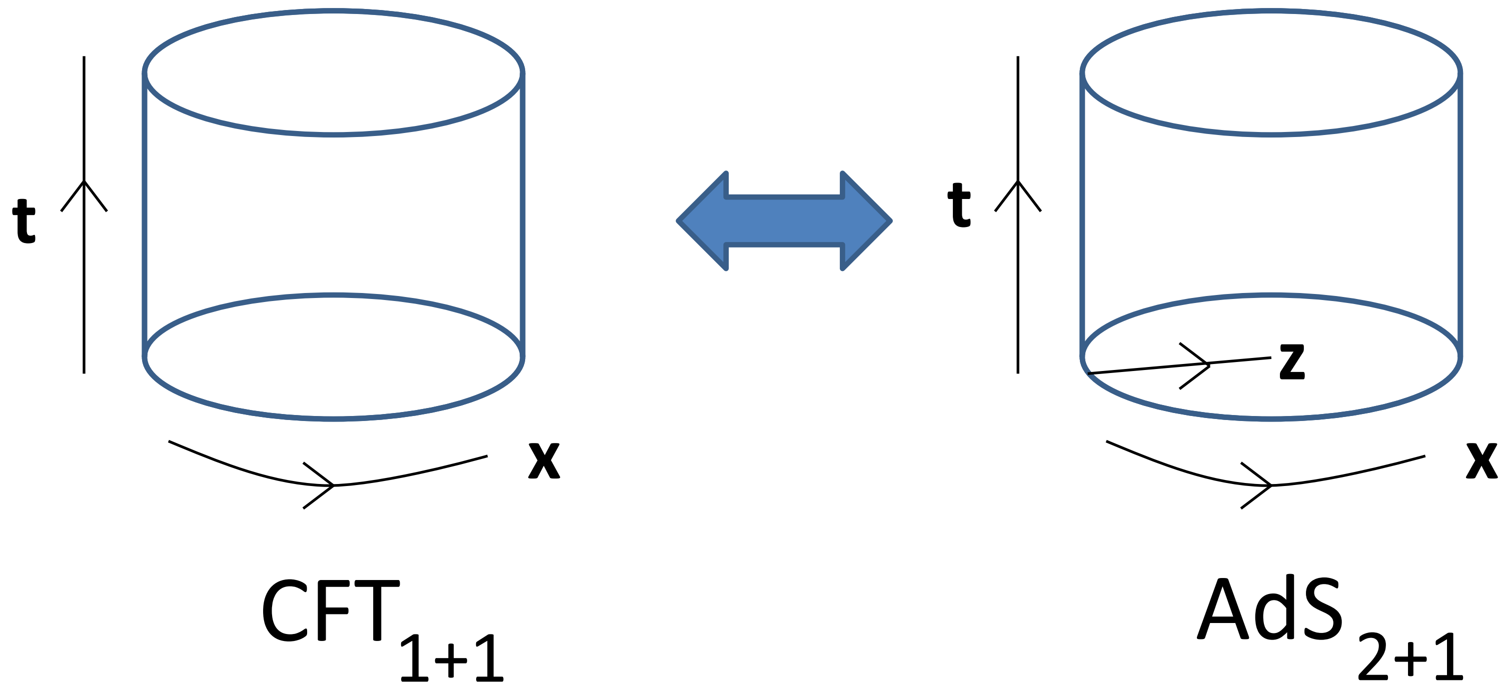


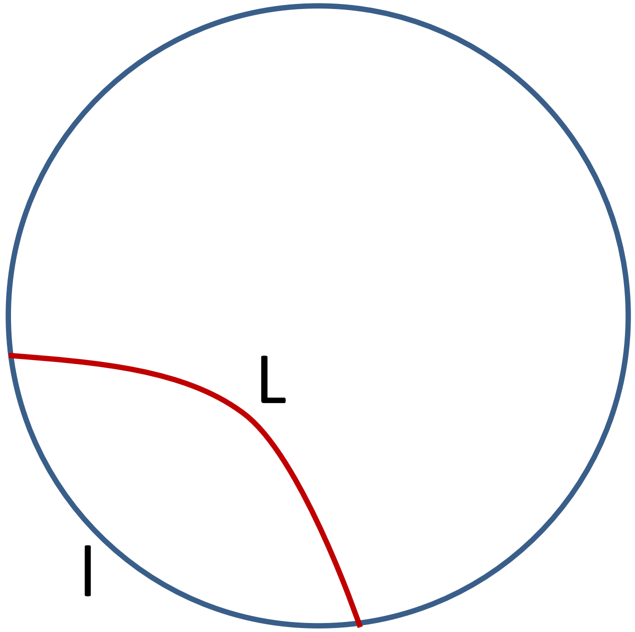
$$c = \frac{3R}{2G}$$ [Brown & Henneaux ‘86]

Small c => quantum geometry in the bulk?

Gravity dual of the critical ising model, c=1/2

[Castro et. al, PRD ‘12]

Gravitational dual of entanglement entropy



L : Length of geodesic in AdS

$$L \approx \frac{R}{2G} \log l$$

$$S_l \approx L$$

[Ryu & Takayanagi PRL '06]

MERA and AdS/CFT

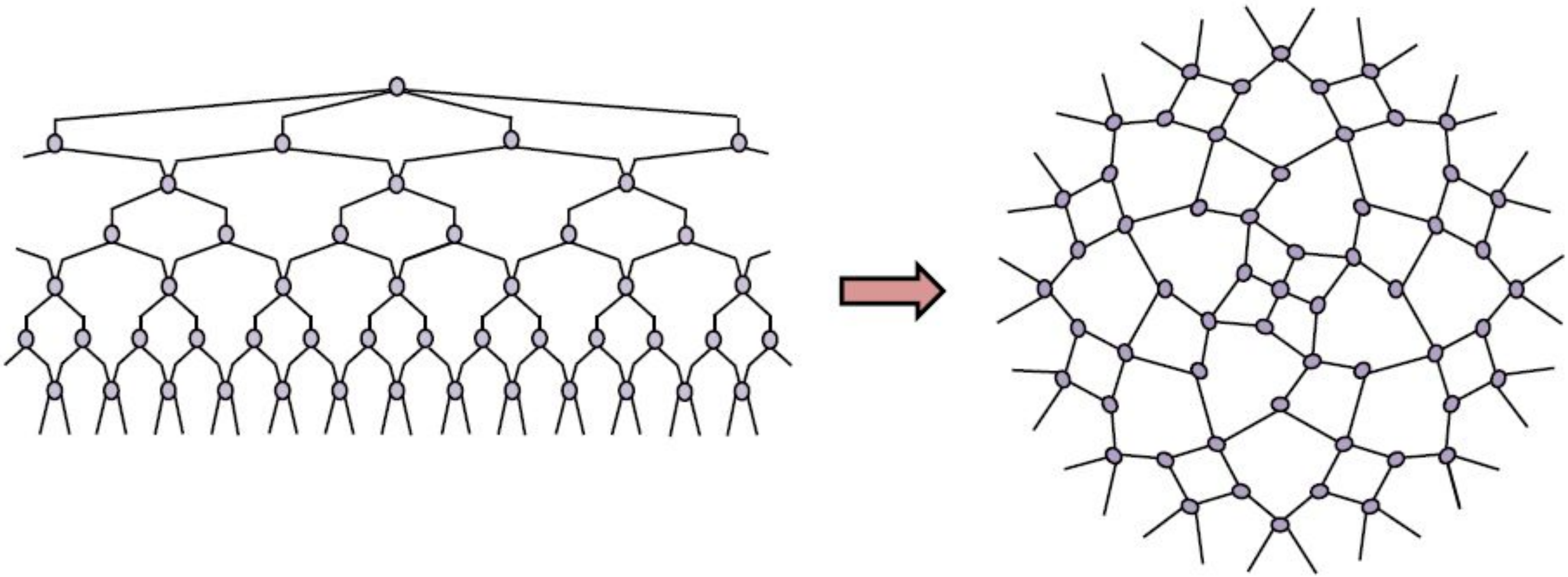


Fig. source: Evenbly, Vidal (2011)

MERA and AdS/CFT

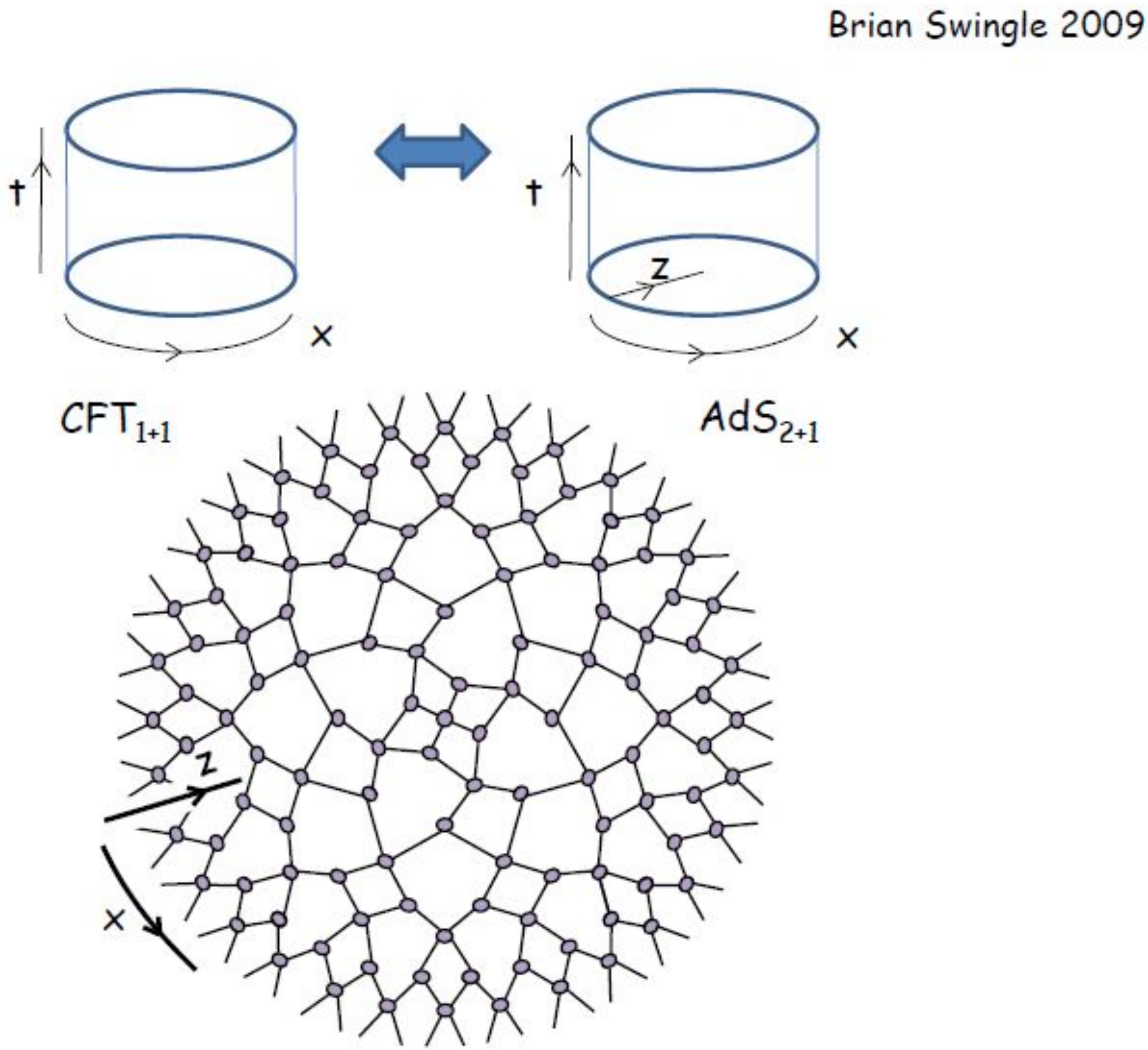
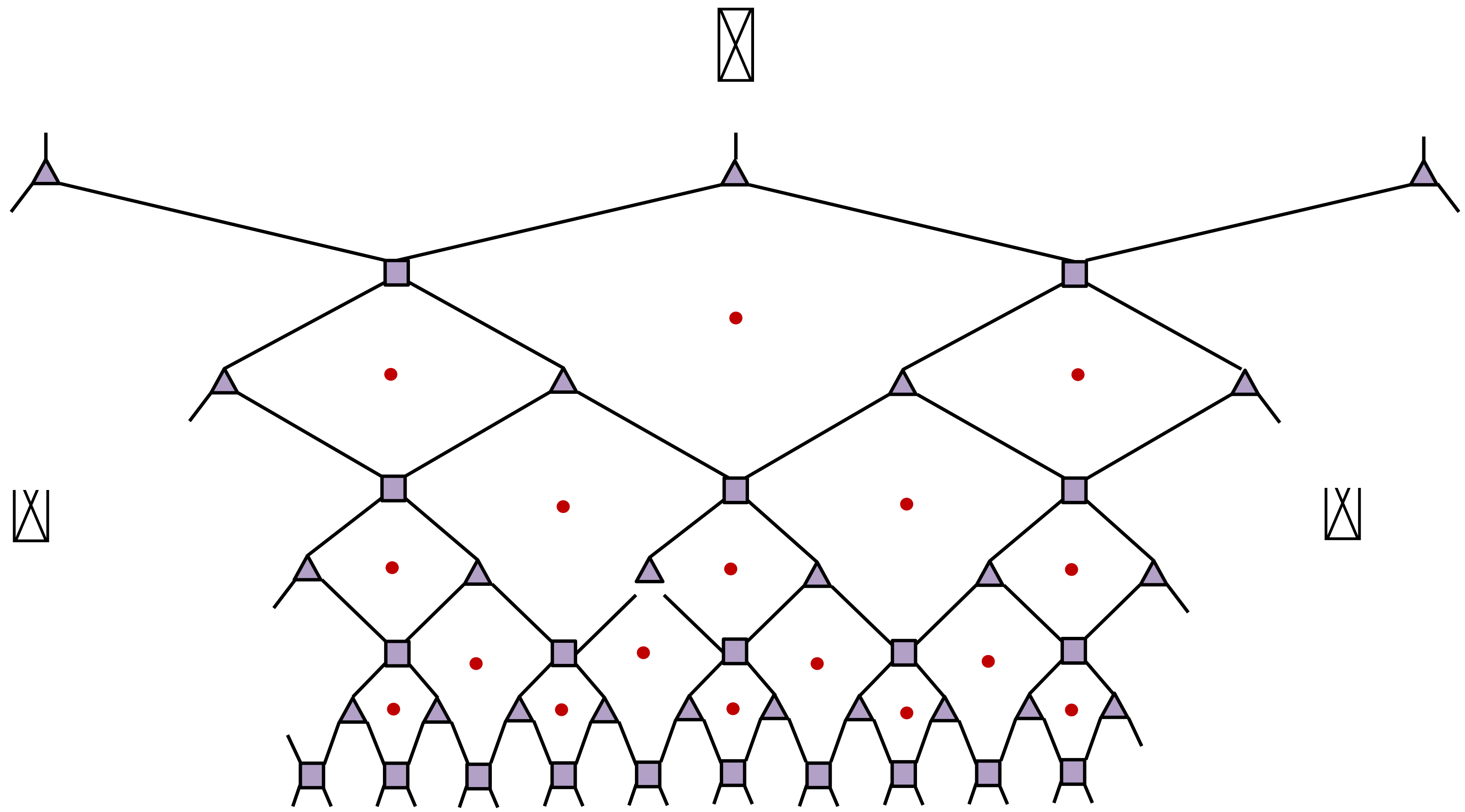
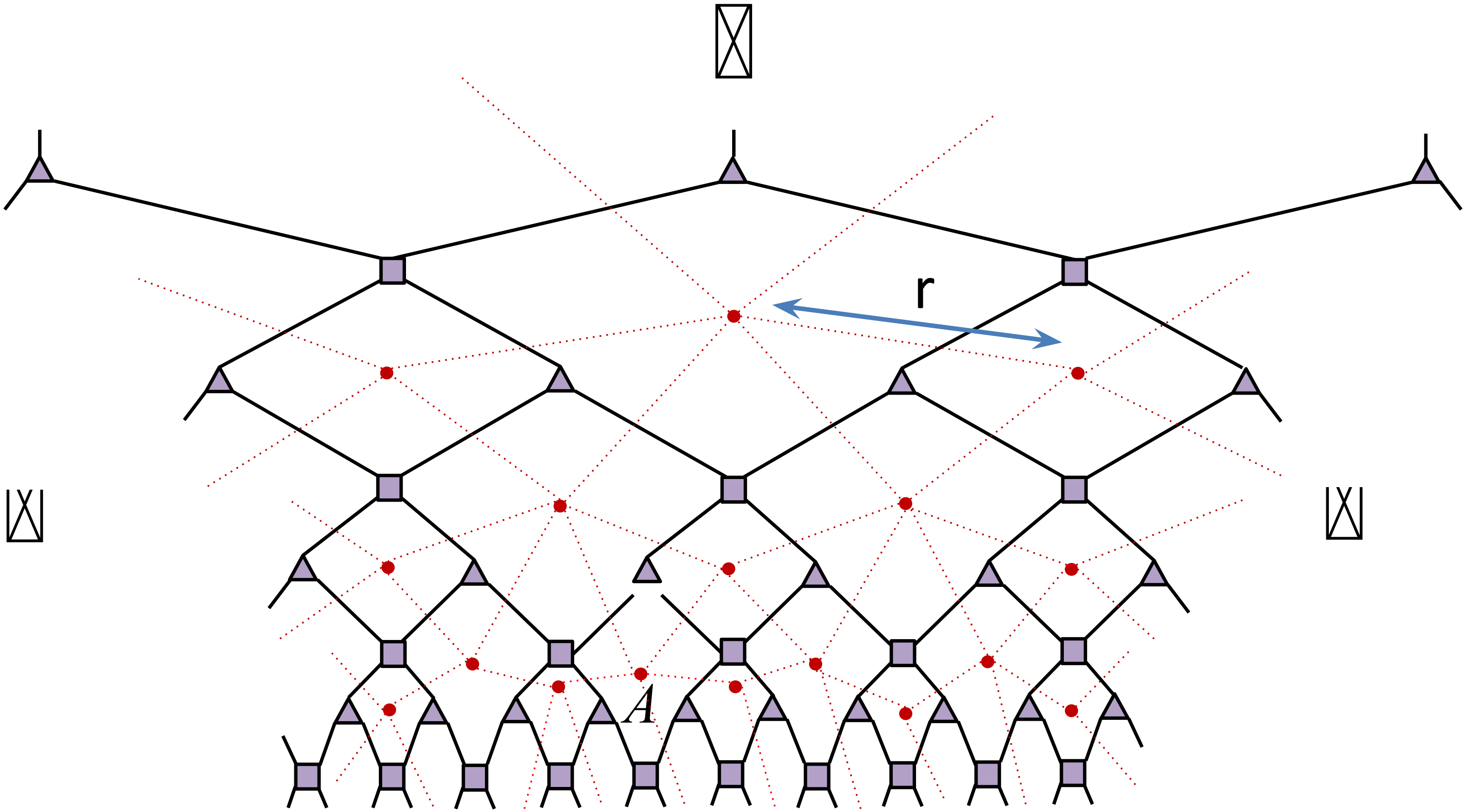


Fig. source: Evenbly, Vidal (2011)

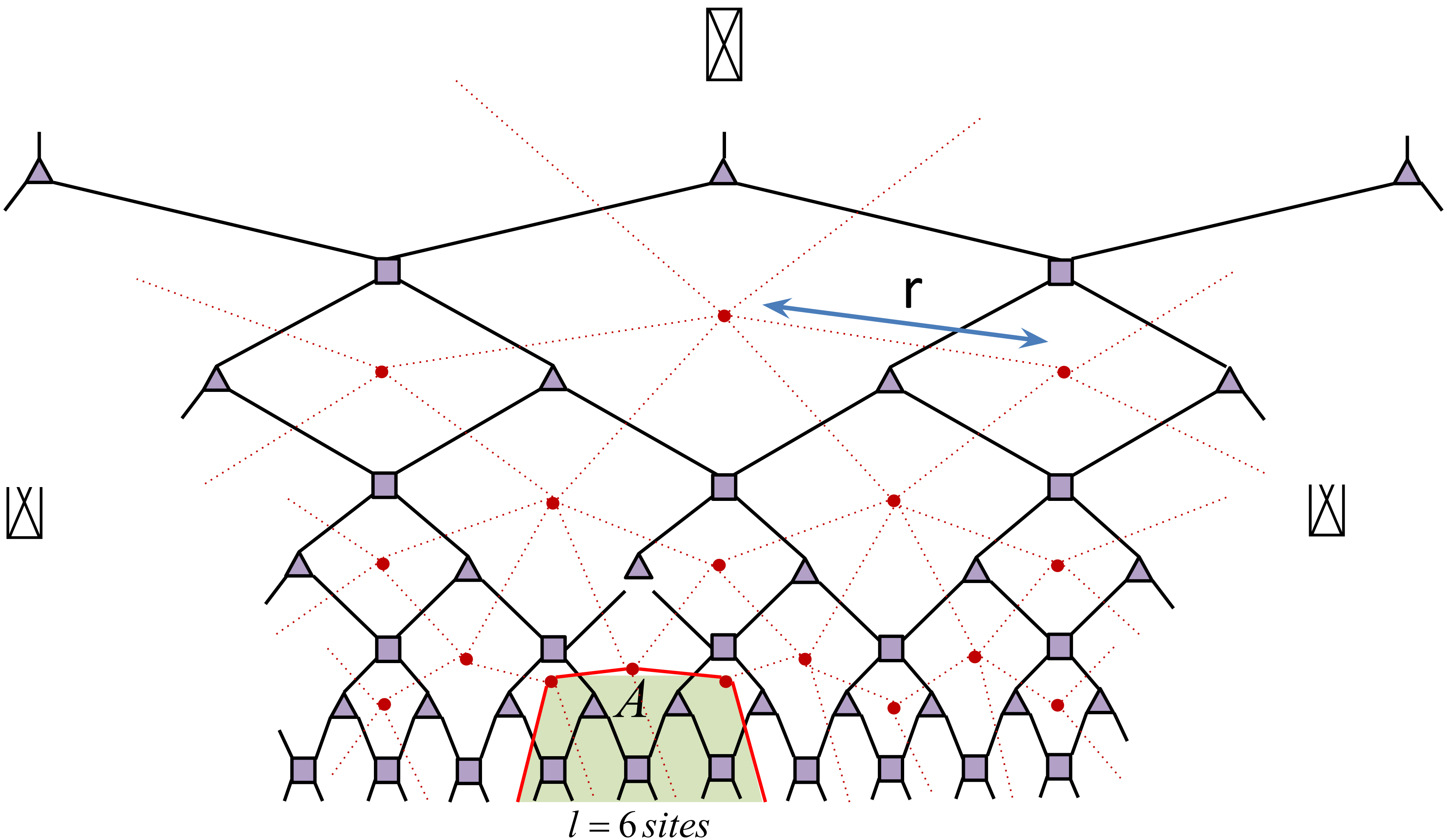
MERA as discrete AdS space



MERA as discrete AdS space



MERA as discrete AdS space

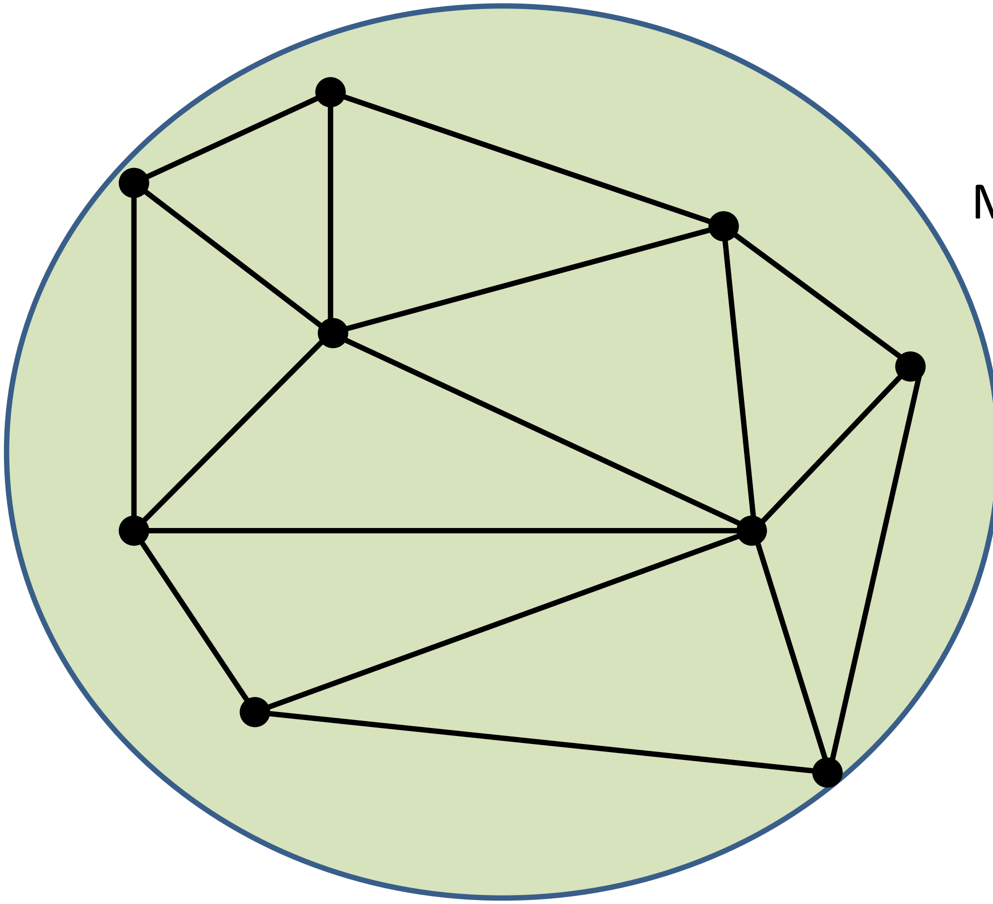


Length of geodesic in the dual graph = $r \log l$

$$S_l \approx r \log l \,? \qquad r = \frac{c}{3}\,?$$

Spin networks and quantum geometry

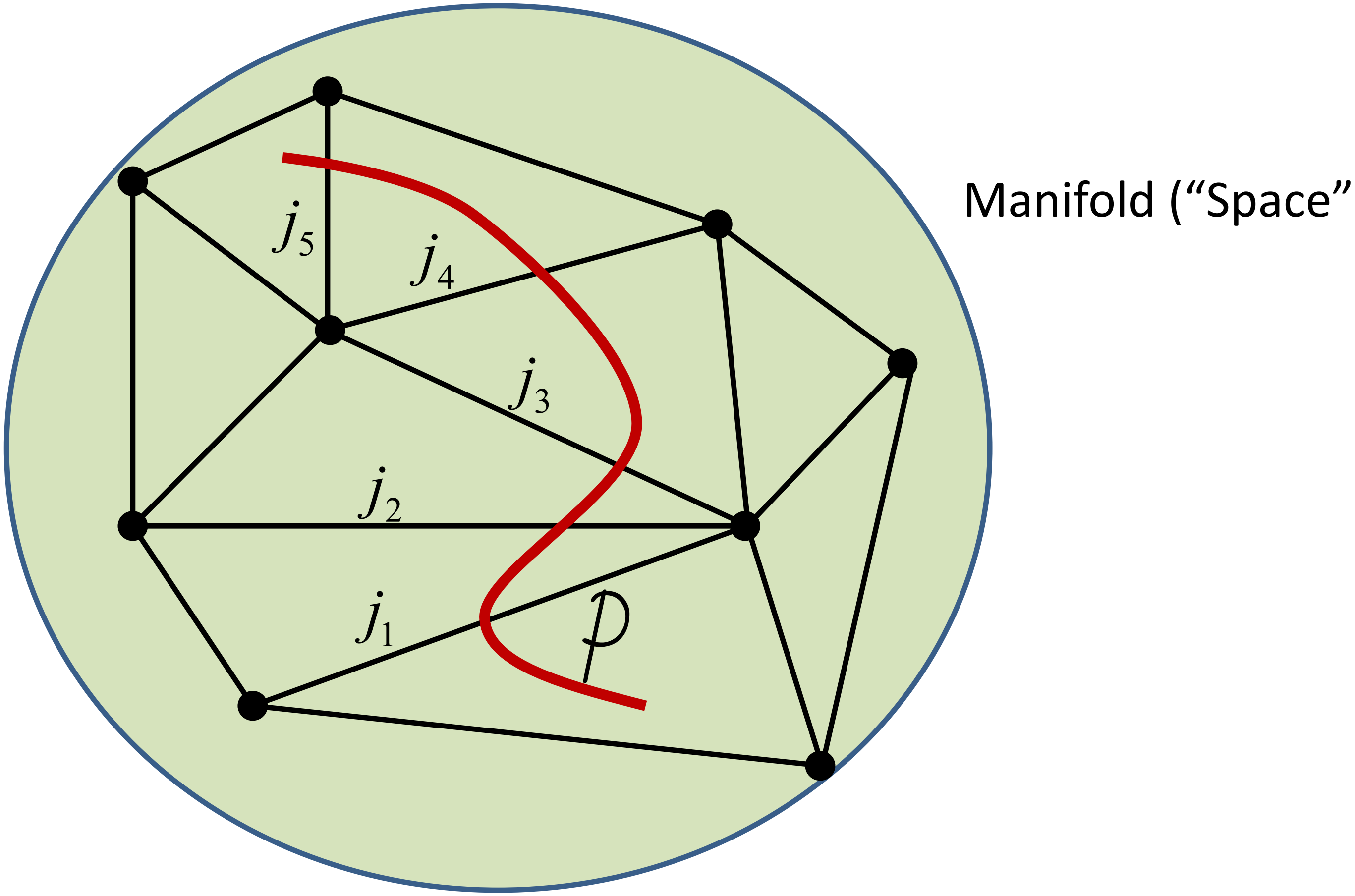
$$\left|\Psi_{QGeometry}\right\rangle =$$



Manifold ("Space")

Spin networks and quantum geometry

$$\left|\Psi_{QGeometry}\right\rangle =$$



$$\hat{L}(P) = \sum_{e \in P} \hat{r}_e, \qquad \hat{r}_e \left|\Psi_{QGeometry}\right\rangle = \sqrt{j_e(j_e+1)}\left|\Psi_{QGeometry}\right\rangle$$

Spin networks and quantum geometry

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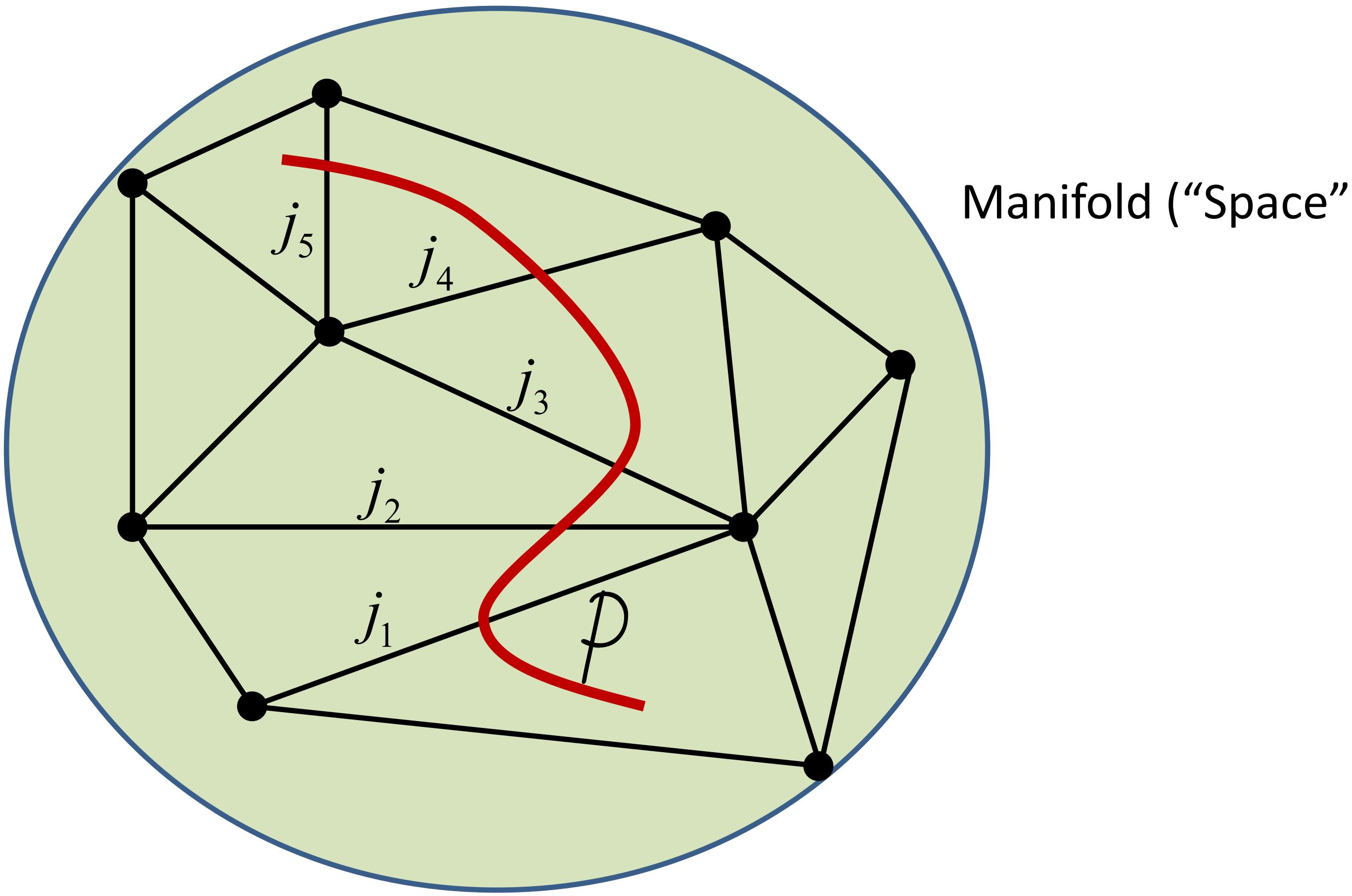

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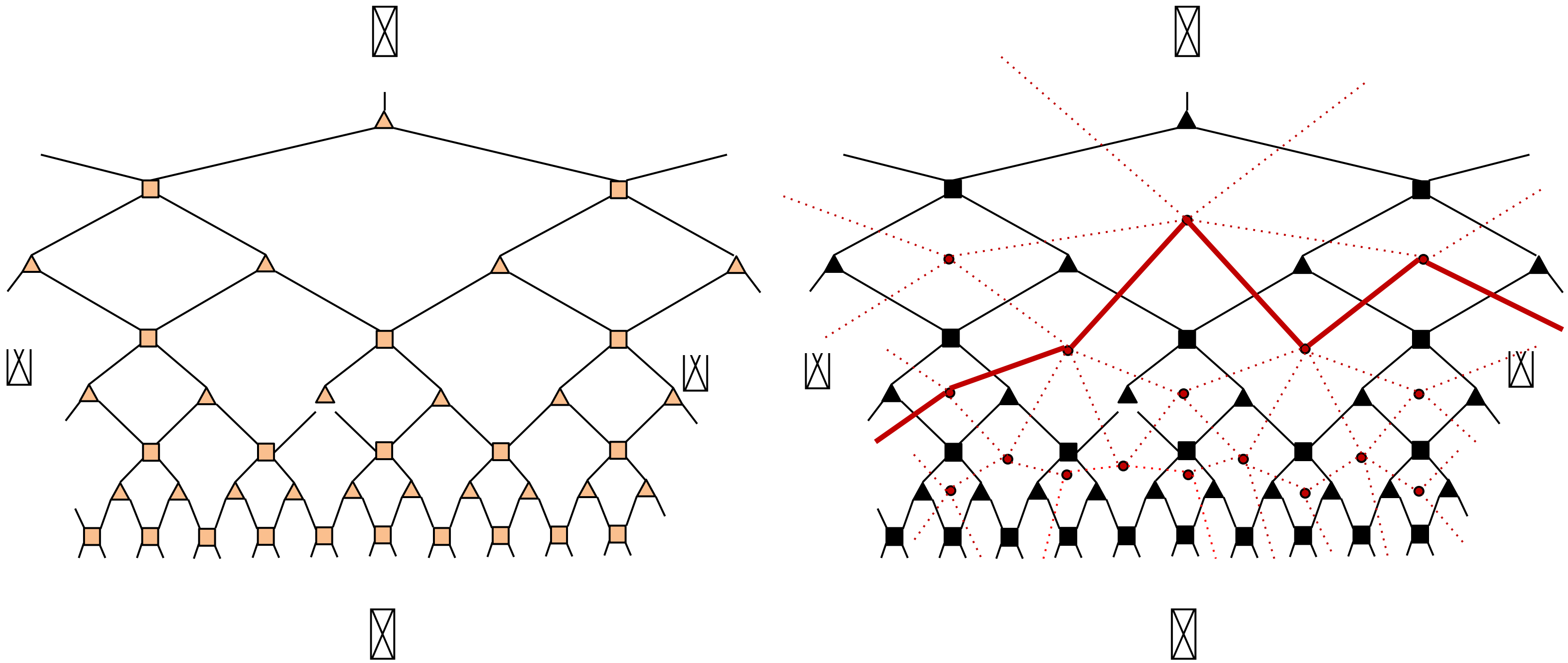
$$\left|\Psi_{QGeometry}\right\rangle \in H_{QG} \qquad \left\langle \hat{L}(P) \right\rangle = \frac{\left\langle \Psi_{QGeometry} \middle| \hat{L}(P) \middle| \Psi_{QGeometry} \right\rangle}{\left\langle \Psi_{QGeometry} \middle\| \Psi_{QGeometry} \right\rangle}$$

SU(2)-invariant MERA as quantum geometry

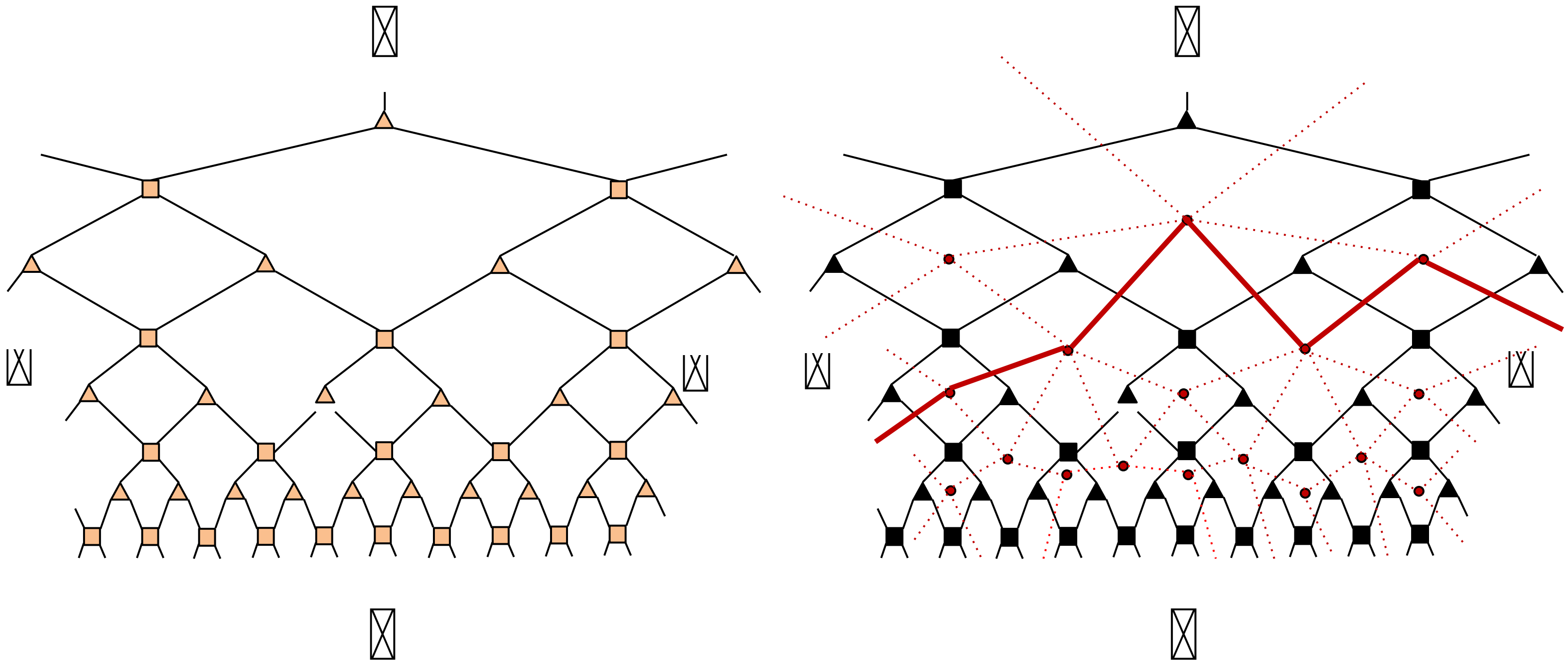
$$\sum_j$$



$$\left|\Psi_{gs}\right\rangle \in H_{LQG}$$

SU(2)-invariant MERA as quantum geometry

$$\sum_j$$



$$\left|\Psi_{gs}\right\rangle \in H_{LQG}$$

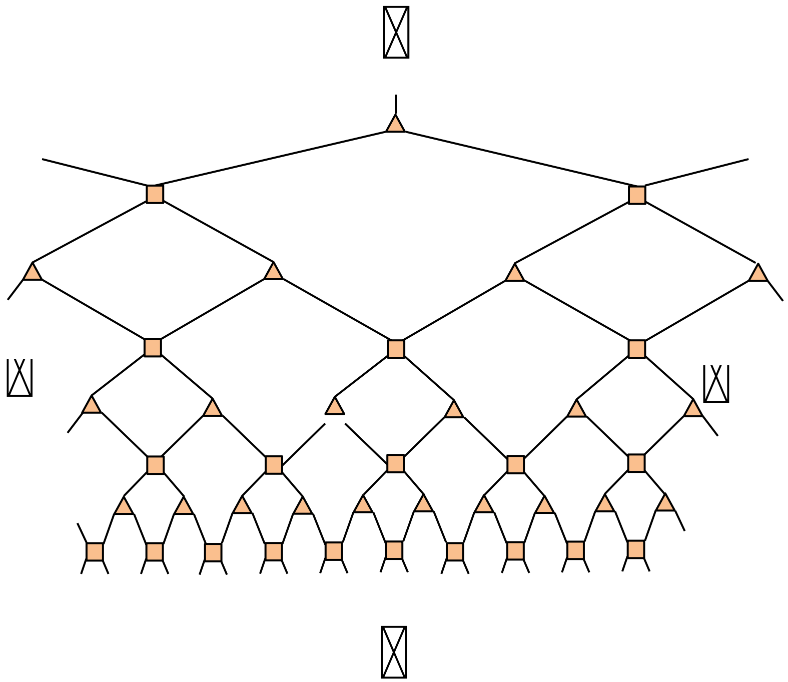
$$\left\langle \hat{L} \right\rangle = \frac{\left\langle \Psi_{gs} \middle| \hat{L} \middle| \Psi_{gs} \right\rangle}{\left\langle \Psi_{gs} \middle\| \Psi_{gs} \right\rangle} = \left\langle \hat{r}_1 \right\rangle + \left\langle \hat{r}_2 \right\rangle + \ldots + \left\langle \hat{r}_n \right\rangle$$

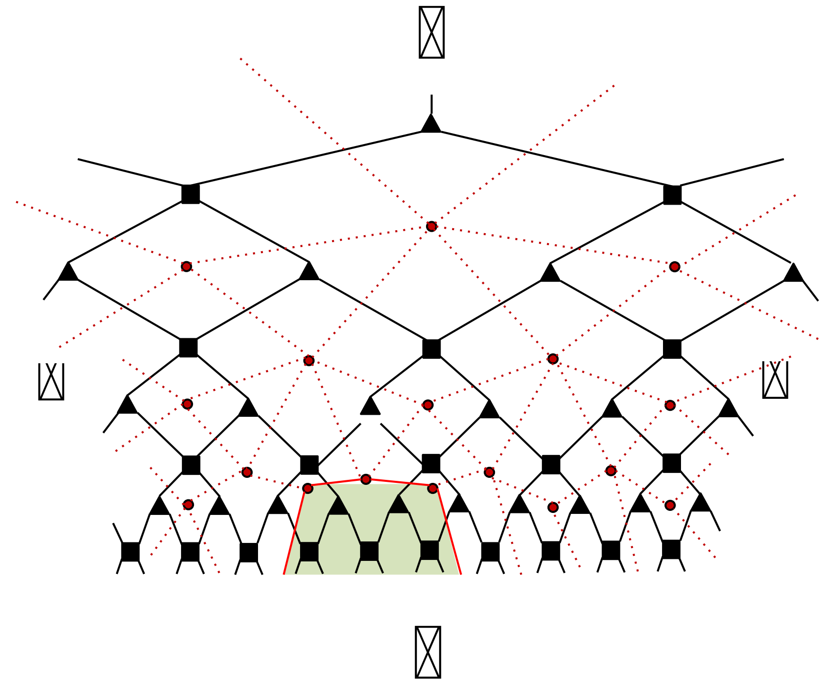
$$\left\langle \hat{r}_1 \right\rangle = \left\langle \hat{r}_2 \right\rangle = \ldots = \left\langle \hat{r}_n \right\rangle \quad \text{(Scale invariance)}$$

$$\left\langle \hat{L} \right\rangle = n \left\langle \hat{r} \right\rangle$$

SU(2)-invariant MERA as quantum geometry

$$\sum_j$$



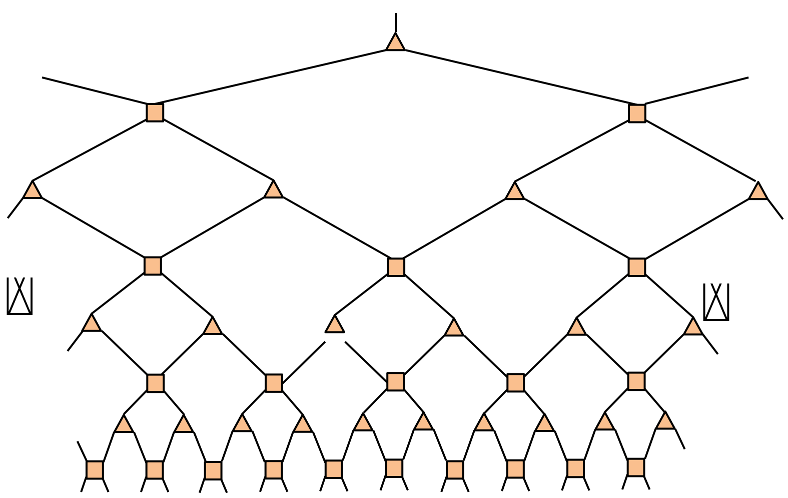


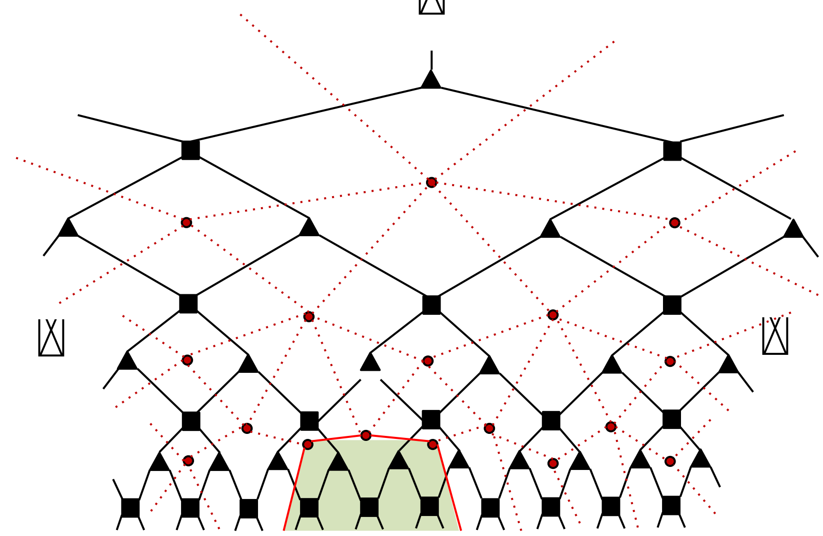
$$\left|\Psi_{gs}\right\rangle \in H_{QG}$$

$$\left\langle \hat{L} \right\rangle \approx \log l \left\langle \hat{r} \right\rangle$$

SU(2)-invariant MERA as quantum geometry

$$\sum_j$$





$$\left|\Psi_{gs}\right\rangle \in H_{QG}$$

$$\left\langle \hat{L} \right\rangle \approx \log l \left\langle \hat{r} \right\rangle$$

$$S_l \approx \left\langle \hat{L} \right\rangle ? \qquad \frac{\left\langle \Psi_{gs} \middle| \hat{r} \middle| \Psi_{gs} \right\rangle}{\left\langle \Psi_{gs} \middle\| \Psi_{gs} \right\rangle} \approx c\,?$$

Summary

SU(2)-invariant MERA as a realization of AdS/CFT?

$$\left|\Psi_{gs}\right\rangle \in V^{\otimes N} \Longleftrightarrow \left|\Psi_{gs}\right\rangle \in H_{LQG}$$

A) Have a description of bulk geometry
B) Quantum geometry, expected for small c?
C) Global symmetry $\Longleftrightarrow$ Gauge symmetry

Can explore :

A) AdS/CFT dictionary?
B) other geometric quantities? – holonomy, curvature
C) different gauge groups? e.g. SU(2)_k

Thanks!