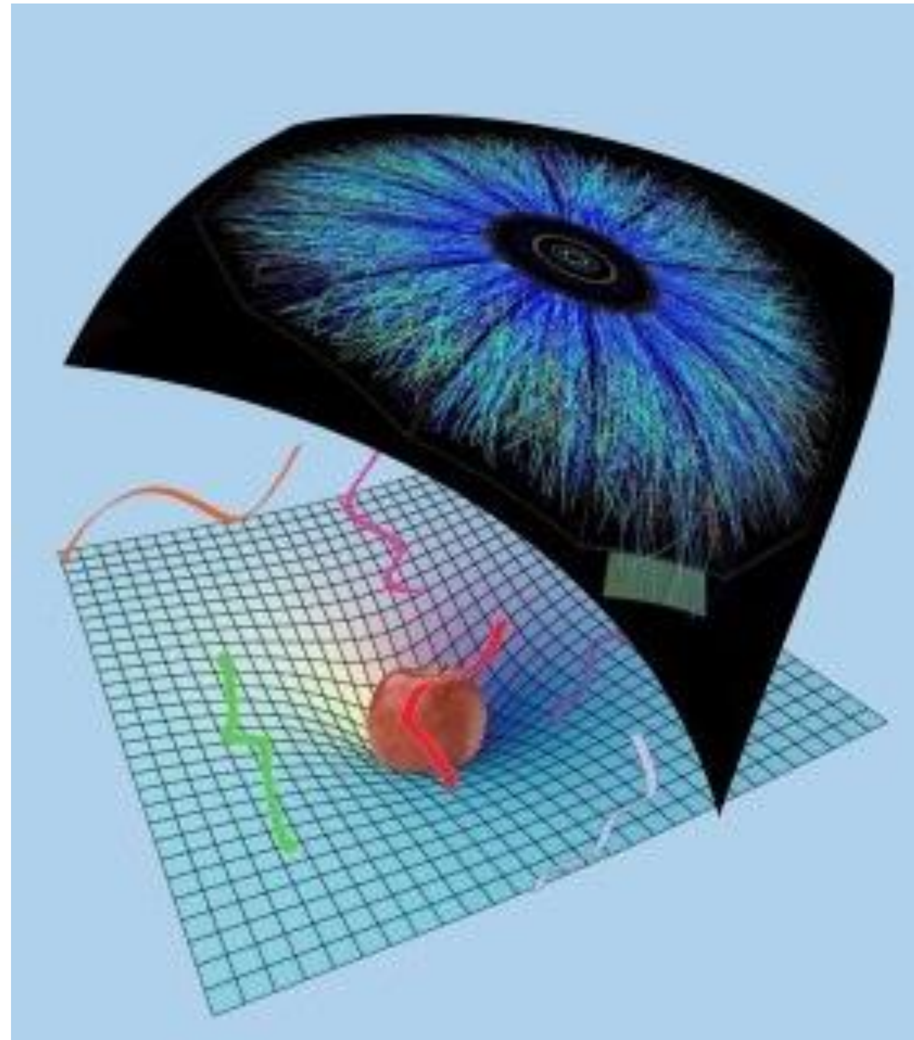
Annual EQuS workshop '15

A new bulk/boundary correspondence for quantum many-body systems



Sukhi Singh

Macquarie University

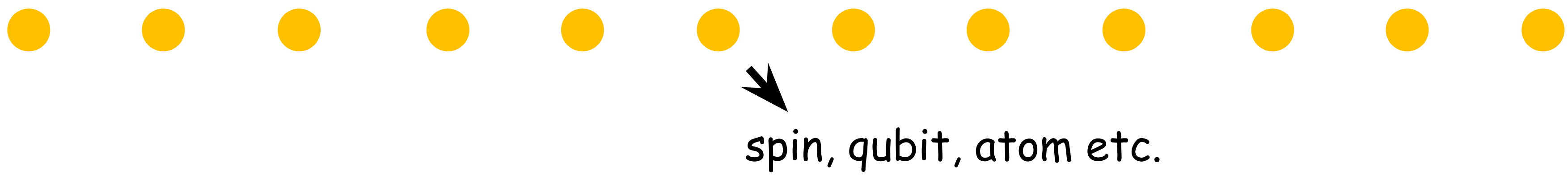
Ongoing work with

Gavin Brennen (Macquarie)

Nathan McMahon (PhD, UQ)

Quantum many-body systems

Lattice of *N* sites with a local Hamiltonian:



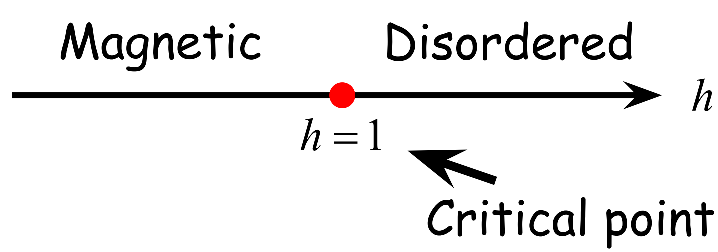
(a) Find the ground state phase diagram

(b) Ground state correlation functions/entanglement

1D quantum Ising model

$$H = \sum_k \sigma_x^k \sigma_x^{k+1} + h\ \sigma_z^k$$

$\sigma_{x,z}$: Pauli matrices



Key challenge

- Exponentially large Hilbert space.
- Have to diagonalize an exponentially large Hamiltonian
- Generic state specified by d^N complex numbers

(a) d=2, N=30 requires 17 GB memory
(b) d=3, N=20 requires 55 GB memory

But ground states of local Hamiltonians are atypical …

- Ground states of local hamiltonians have much less entanglement than generic states

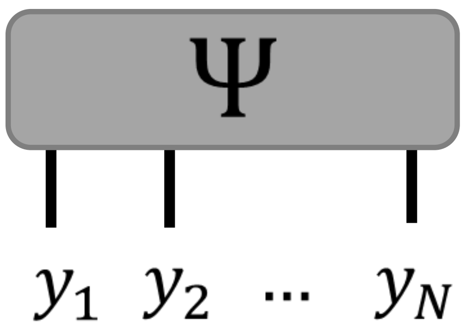
- One way this limited entanglement in ground states has been exploited is to build efficient representations of ground states.
  (Polynomial number of complex numbers.)

Ground state can be decomposed as a MERA

G. Vidal, PRL 99, 220405 (2007)

$$|\Psi_{ground}\rangle = \sum \Psi_{y_1 y_2 \ldots y_N} \; |y_1\rangle \otimes |y_2\rangle \otimes \cdots \otimes |y_N\rangle$$

(tensor)



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d^N coefficients

Ground state can be decomposed as a MERA

G. Vidal, PRL 99, 220405 (2007)

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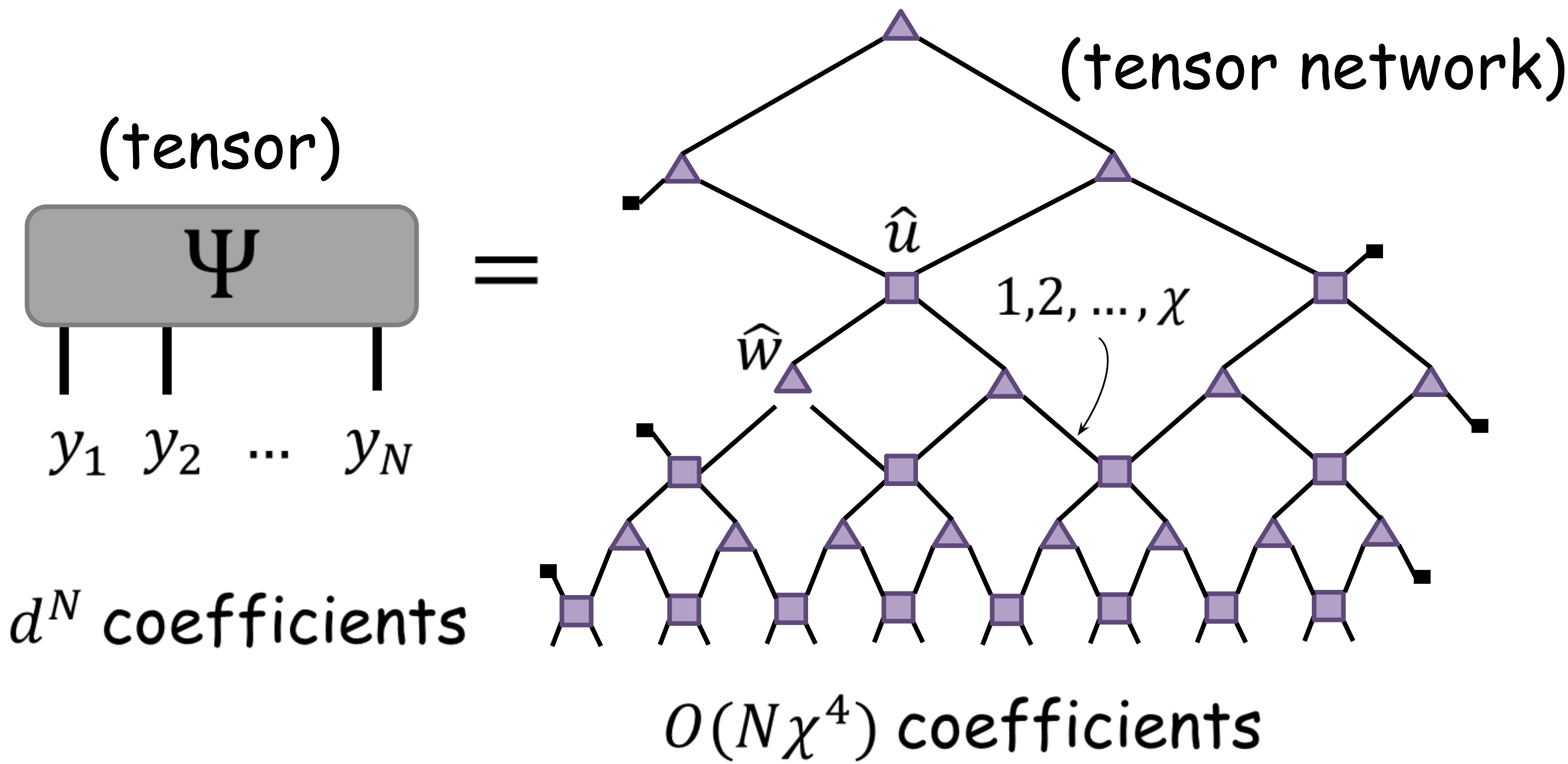
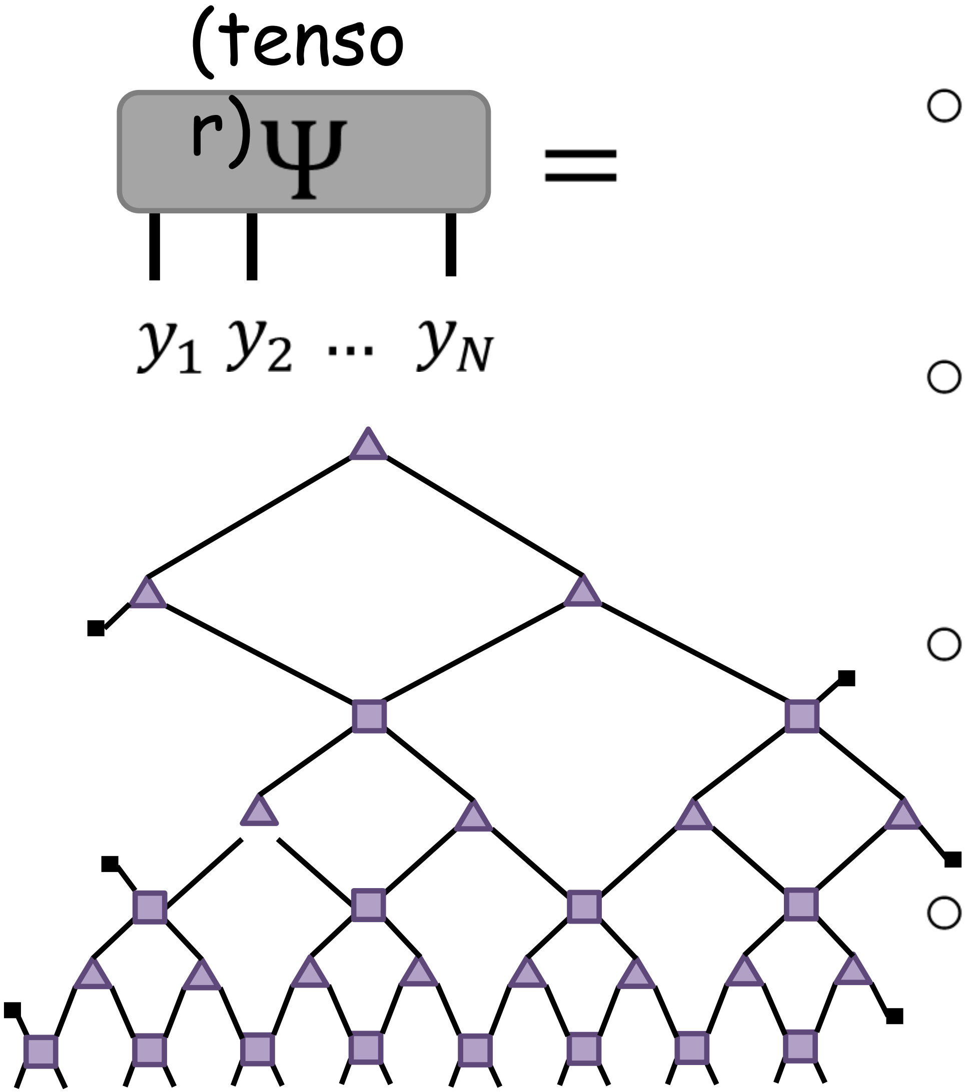


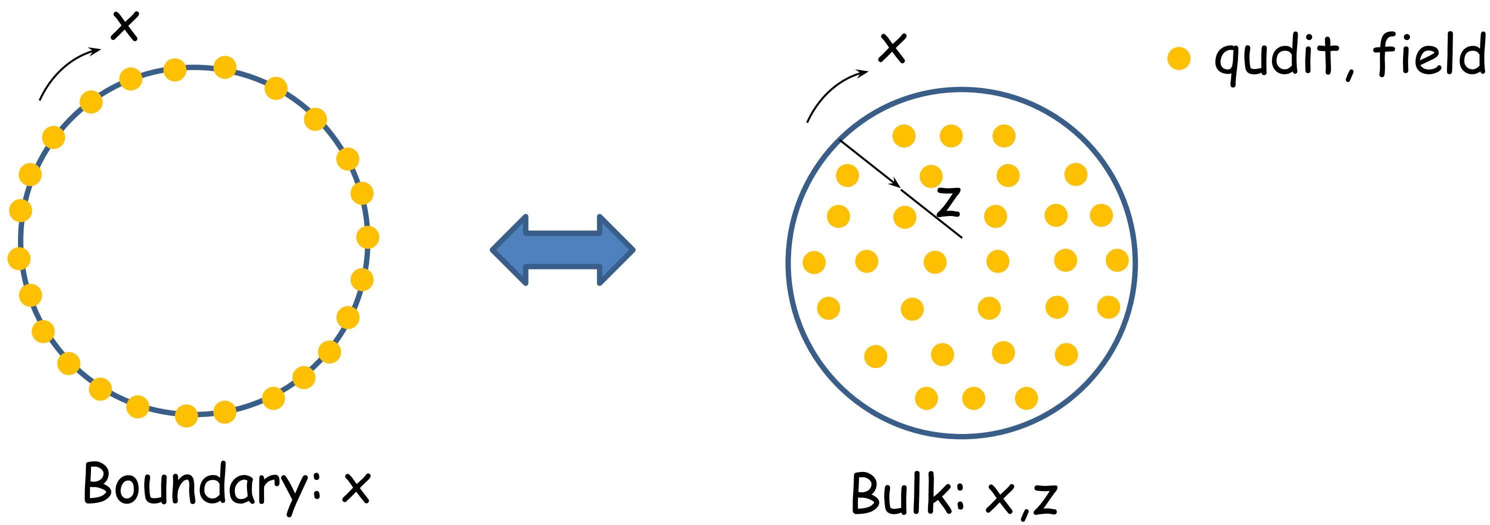
Features of the MERA representation



$O(N\chi^4)$ coefficients

- Ground state properties can be efficiently computed.
- Algorithms known for: $\widehat{H}$ → ground state as MERA
- Applied to several 1D & 2D quantum lattice models
- Works well for both gapped and critical ground states

Bulk/boundary correspondence



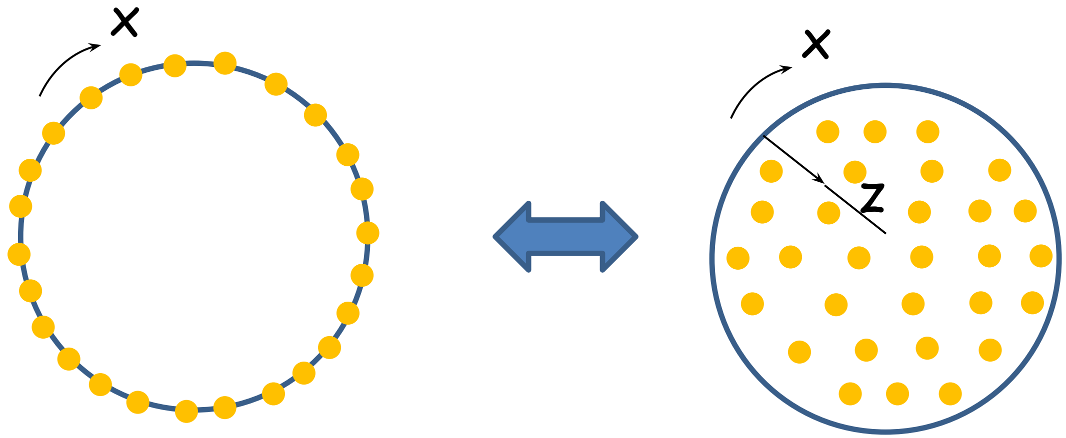
- Is a map between two different quantum systems
- One lives at the boundary of a given spacetime manifold
- The other lives inside the bulk
- Properties of the two systems are related in a given way

This talk: 1d boundary ⟺ 2d bulk

Example: AdS/CFT correspondence

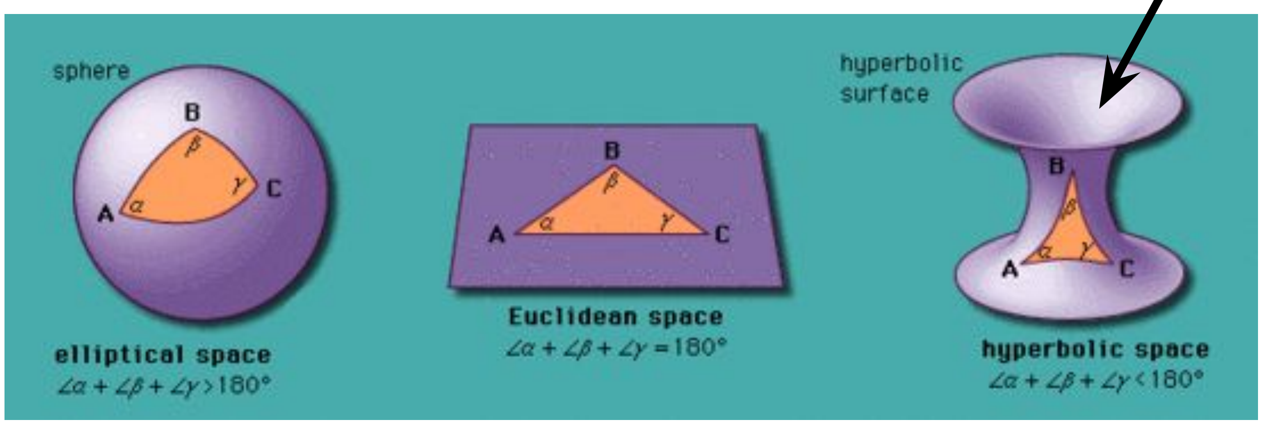
J. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998)

CFT ground state



Gauge fields in Hyperbolic space

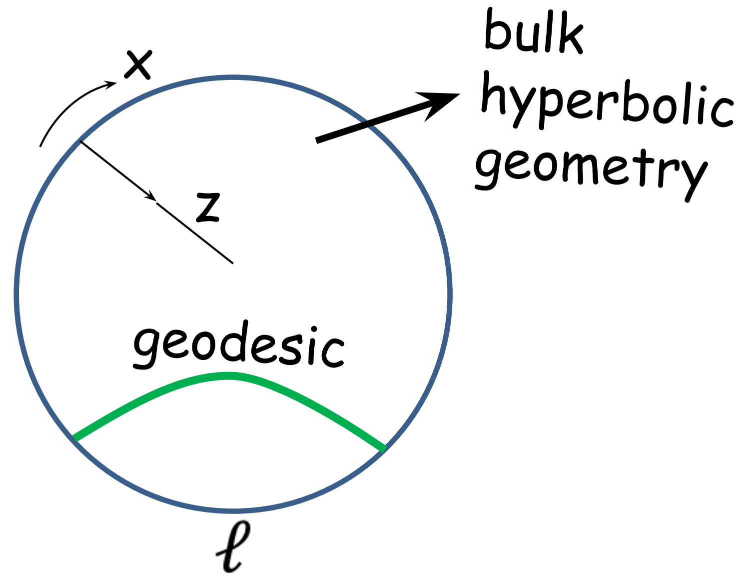
Central charge, c = Radius of curvature, R



AdS/CFT : Ryu-Takayanagi formula

S. Ryu & T. Takayanagi, PRL 96, 181602 (2006)

CFT ground state $\Leftrightarrow$

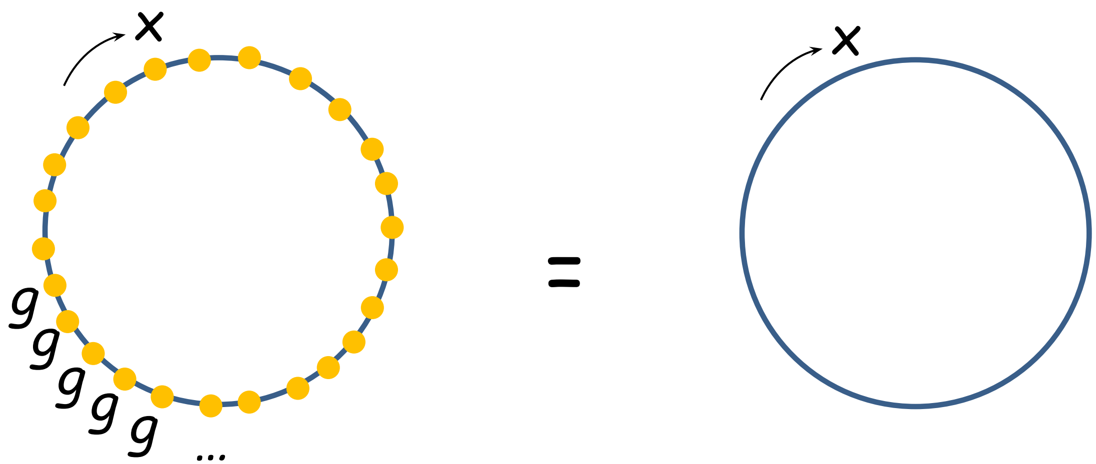


Entanglement entropy = Length of geodesic

$(\propto c \log \ell)$ $(\propto R \log \ell)$

AdS/CFT : Symmetries

If boundary state has a global symmetry G = {g,g’,g’’,…}



$\forall\, g \in G$

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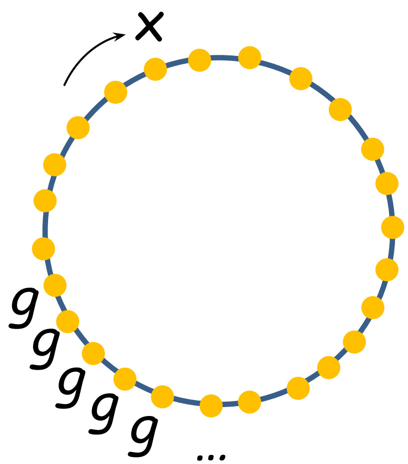

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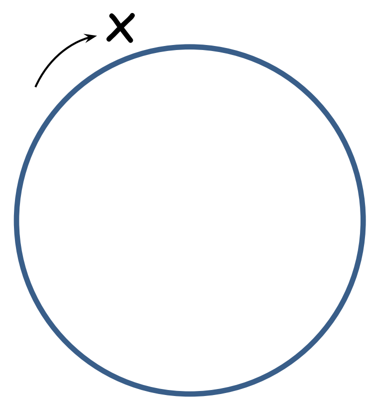
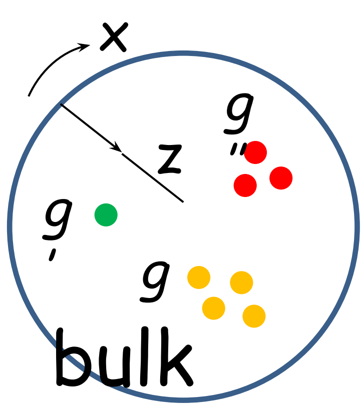


$\forall\, g \in G$

Then the bulk state has a local symmetry



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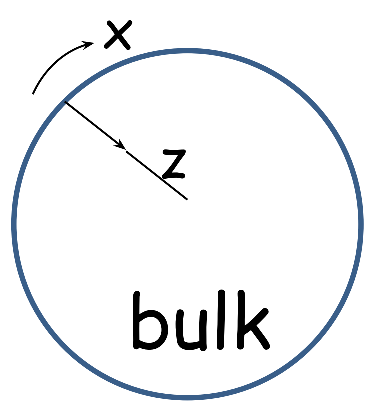


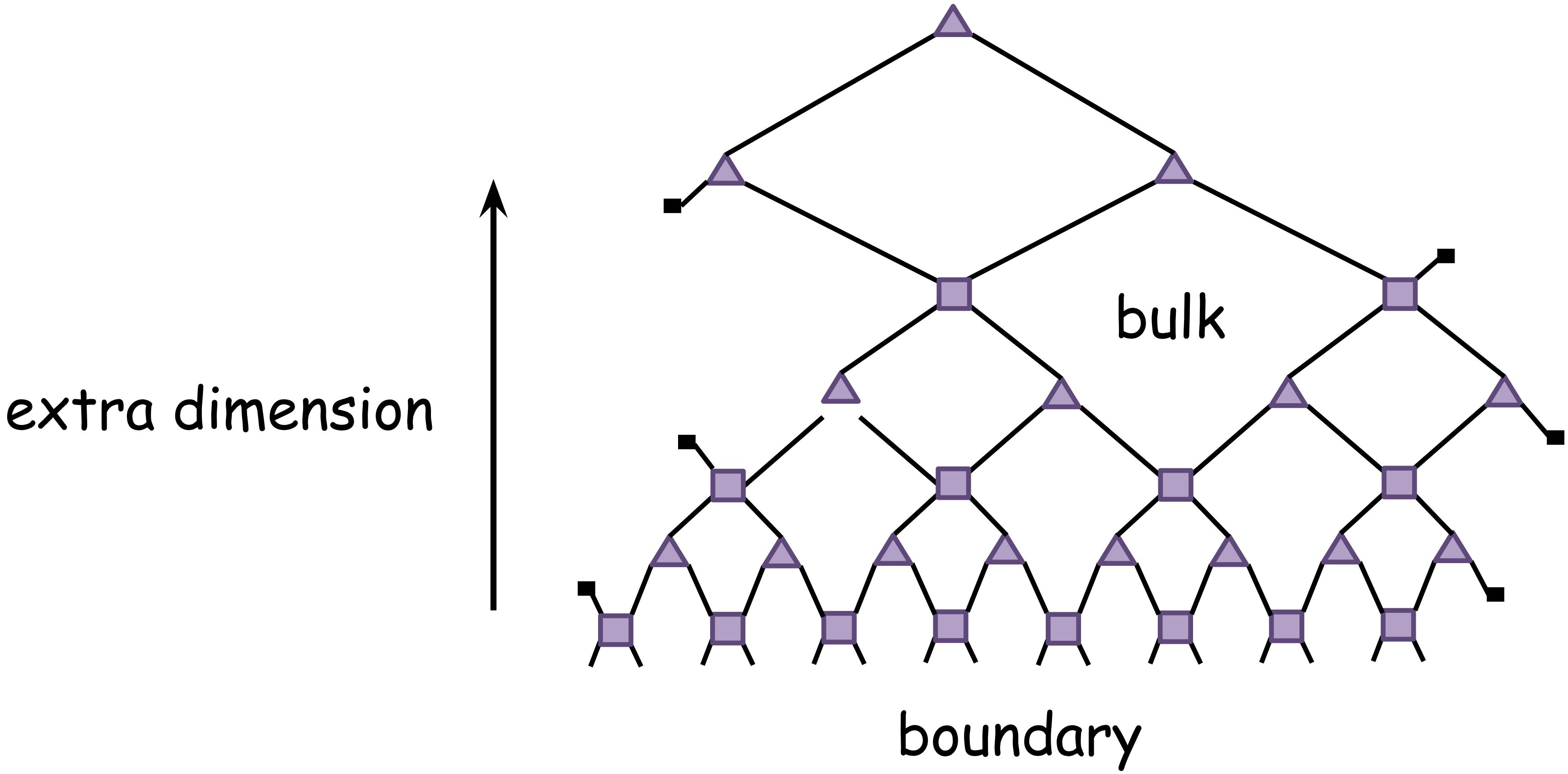
$\forall\, g, g', g'' \in G$

Applications in condensed matter physics

S. Sachdev, Annual Review of Condensed Matter Physics 3, 9 (2012)

- Originated in String theory and quantum field theory but now applied in condensed matter physics
- Critical points □ described by CFTs in the continuum e.g. critical point at the phase transition in Ising model

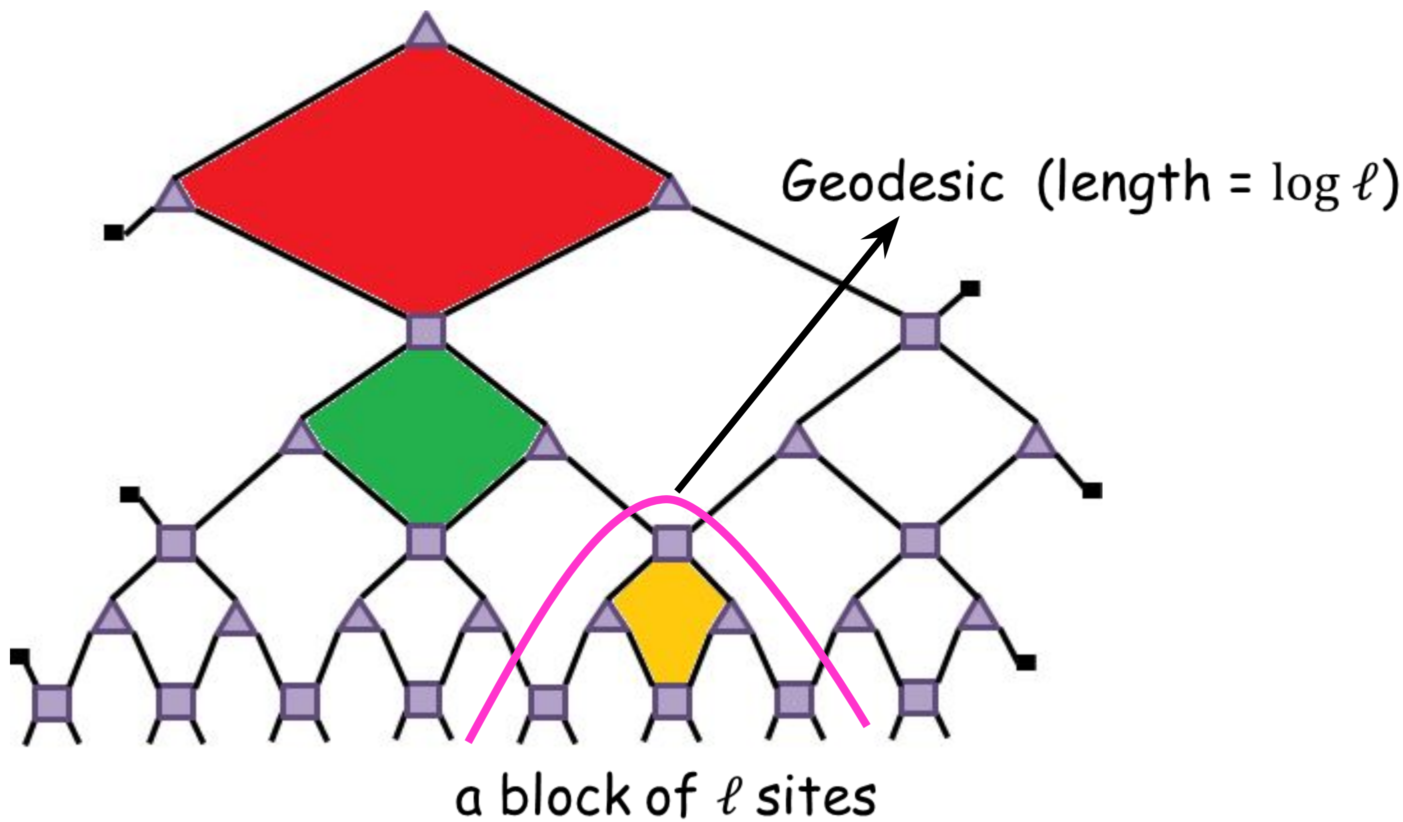
A bulk/boundary correspondence from the MERA



MERA can be viewed as a tiling of the hyperbolic plane

B. Swingle, Phys. Rev. D 86, 065007 (2012)

The red, green and yellow tiles have the same area!



MERA related to AdS/CFT correspondence?

- MERA is a 2D representation of a 1D ground state and has a hyperbolic geometry, reminds of AdS/CFT

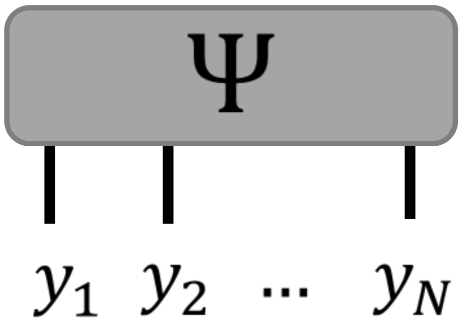
- Several other features of the AdS/CFT correspondence have been realized using the MERA

  Mostly qualitative observations

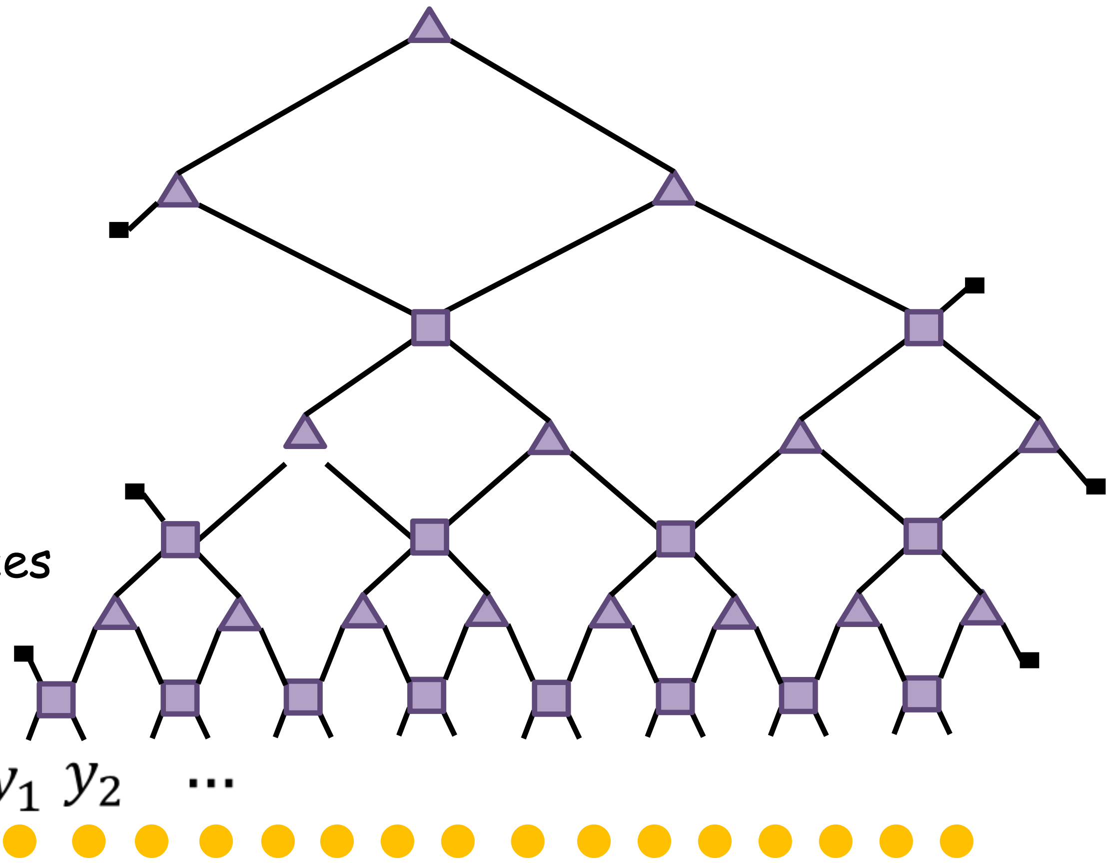
- One of the goals: Attempt a concrete connection to AdS/CFT by constructing a bulk state from the MERA.

A bulk/boundary correspondence from the MERA

$$|\Psi_{ground}\rangle = \sum \Psi_{y_1 y_2 \ldots y_N} \; |y_1\rangle \otimes |y_2\rangle \otimes \cdots \otimes |y_N\rangle$$

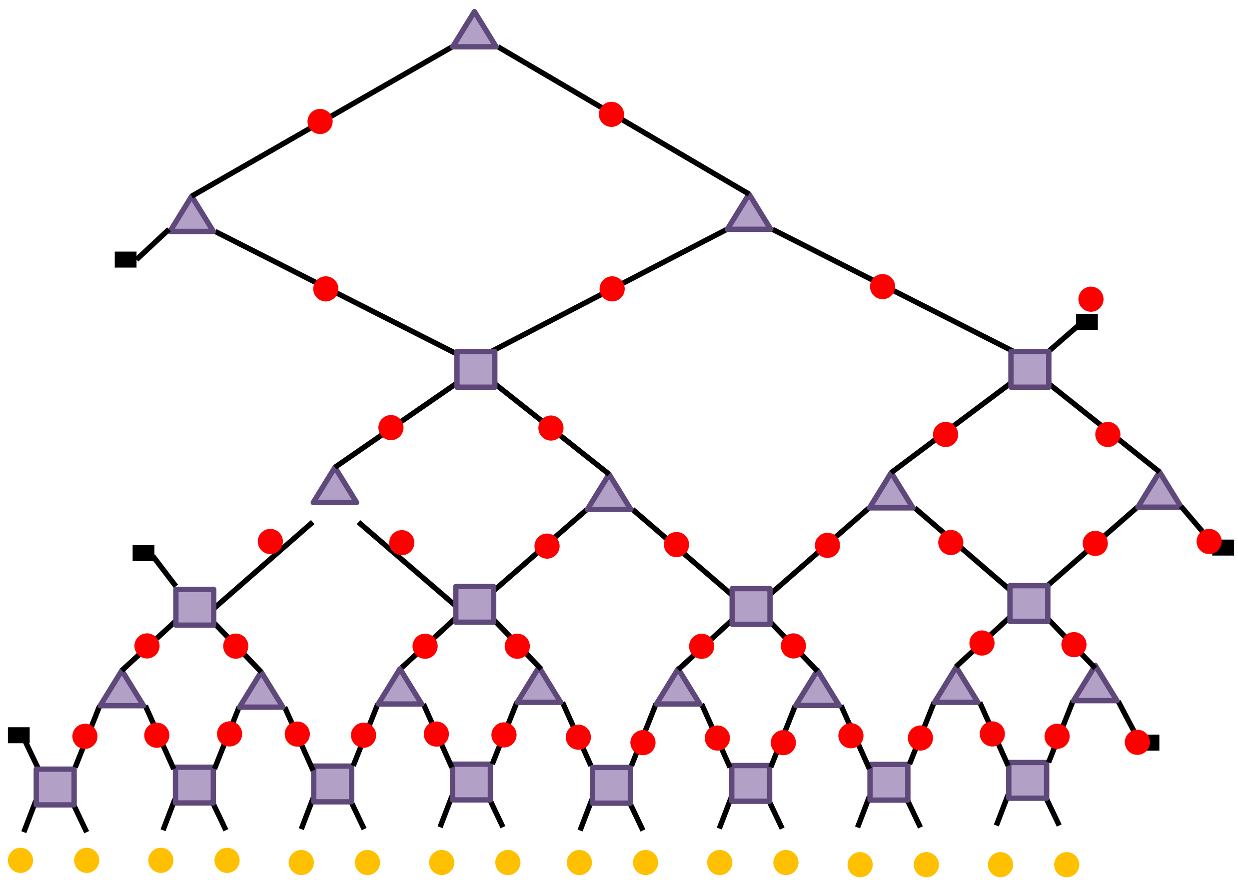


$$= \sum_{\text{All bond indices}}$$



Original lattice

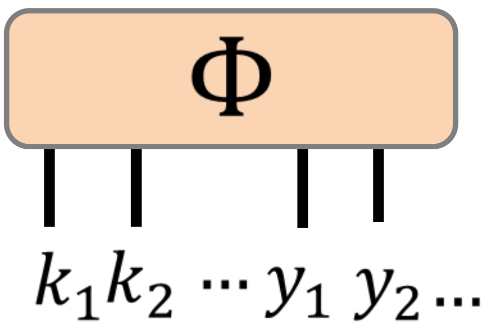
A bulk/boundary correspondence from the MERA



A bulk/boundary correspondence from the MERA

$$|\Phi\rangle = \sum \Phi_{k_1,k_2,\ldots y_1 y_2\ldots}\ |k_1\rangle \otimes |k_2\rangle \cdots \otimes |y_1\rangle \otimes |y_2\rangle \cdots$$

$$\langle k'|k\rangle = \delta_{k'k}$$



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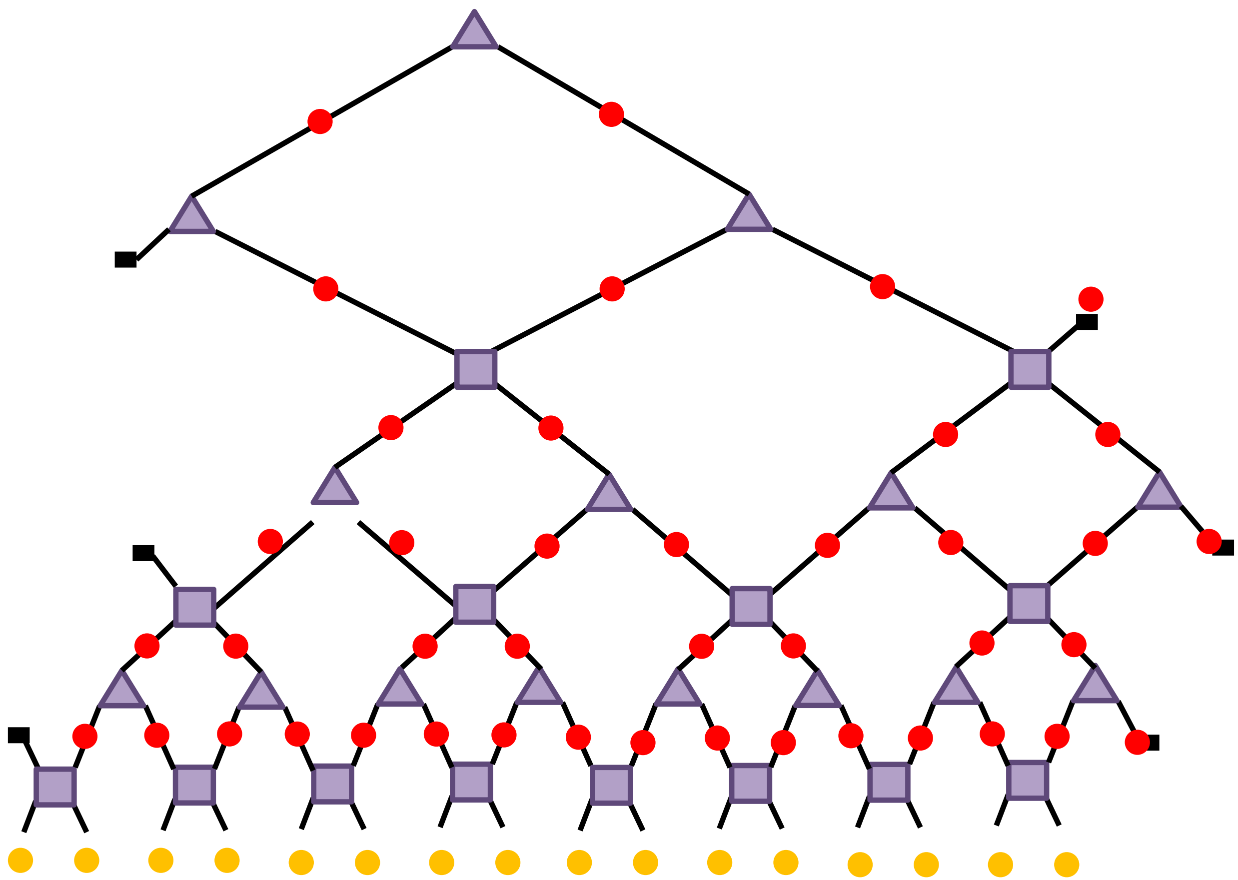

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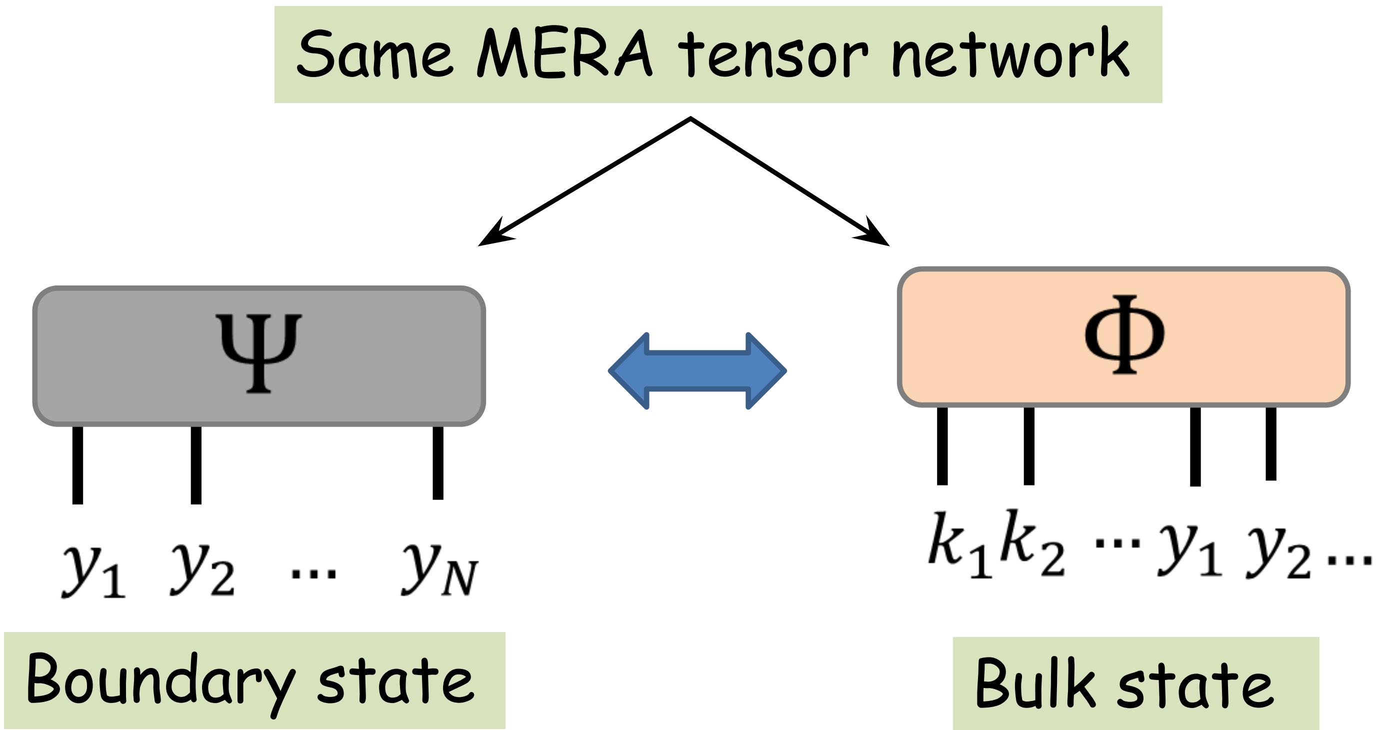
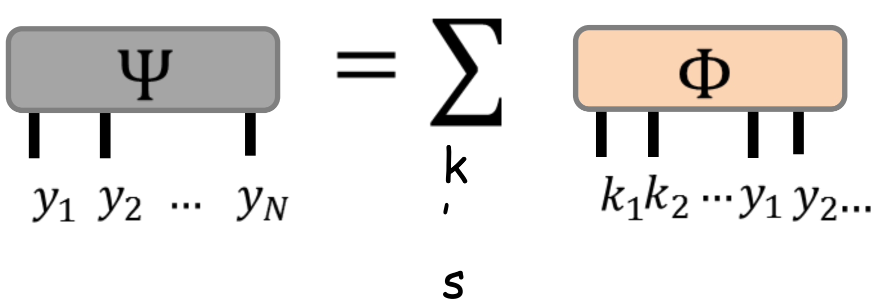
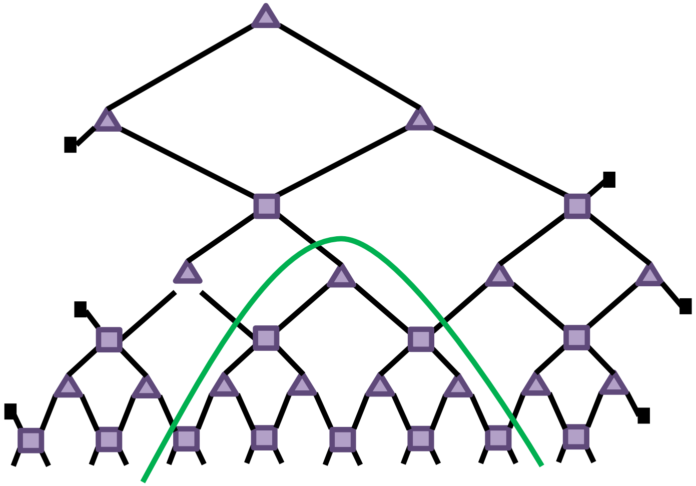




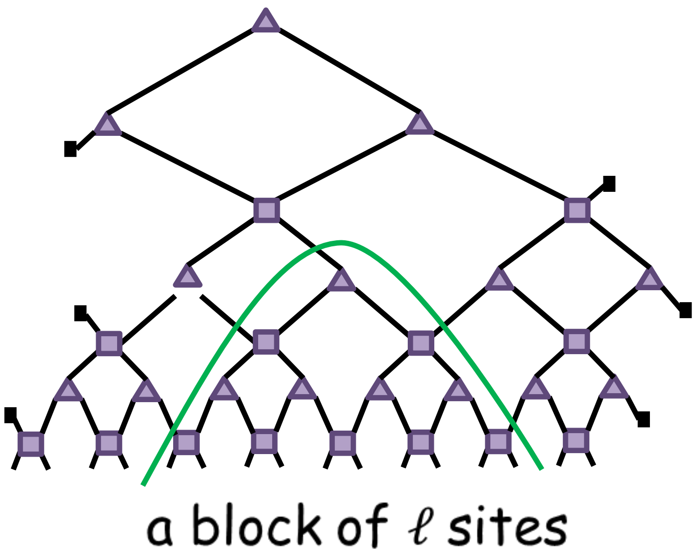
Project each bulk site to $|+\rangle = |0\rangle + |1\rangle + \cdots |\chi\rangle$

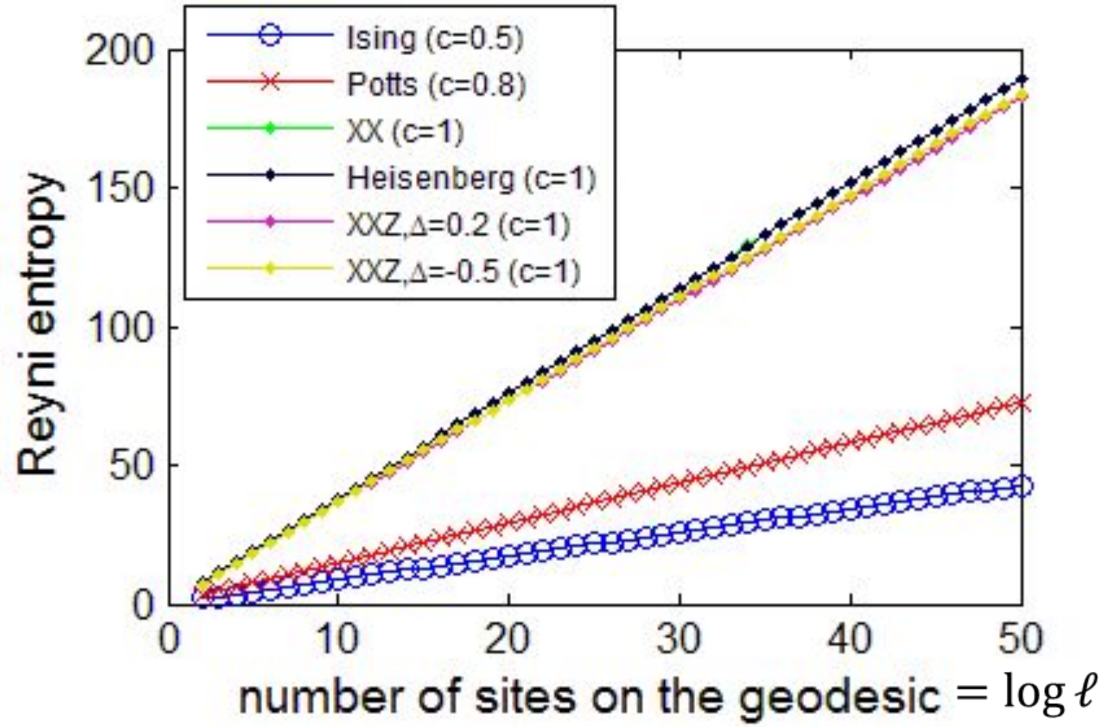
Example 1: critical boundary state



a block of ℓ sites

Example 1: critical boundary state





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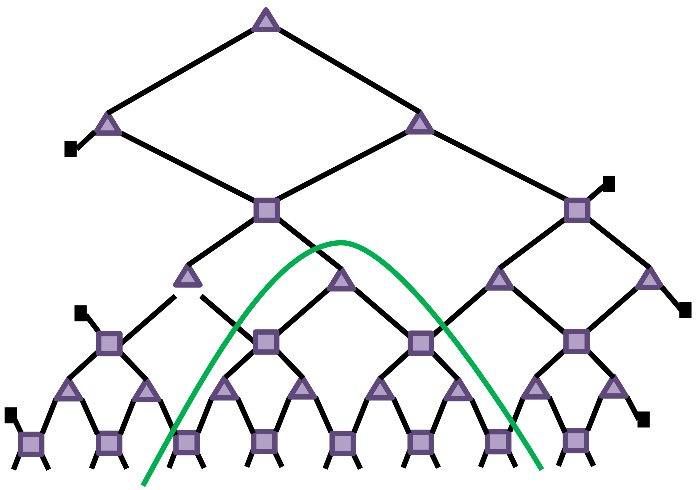
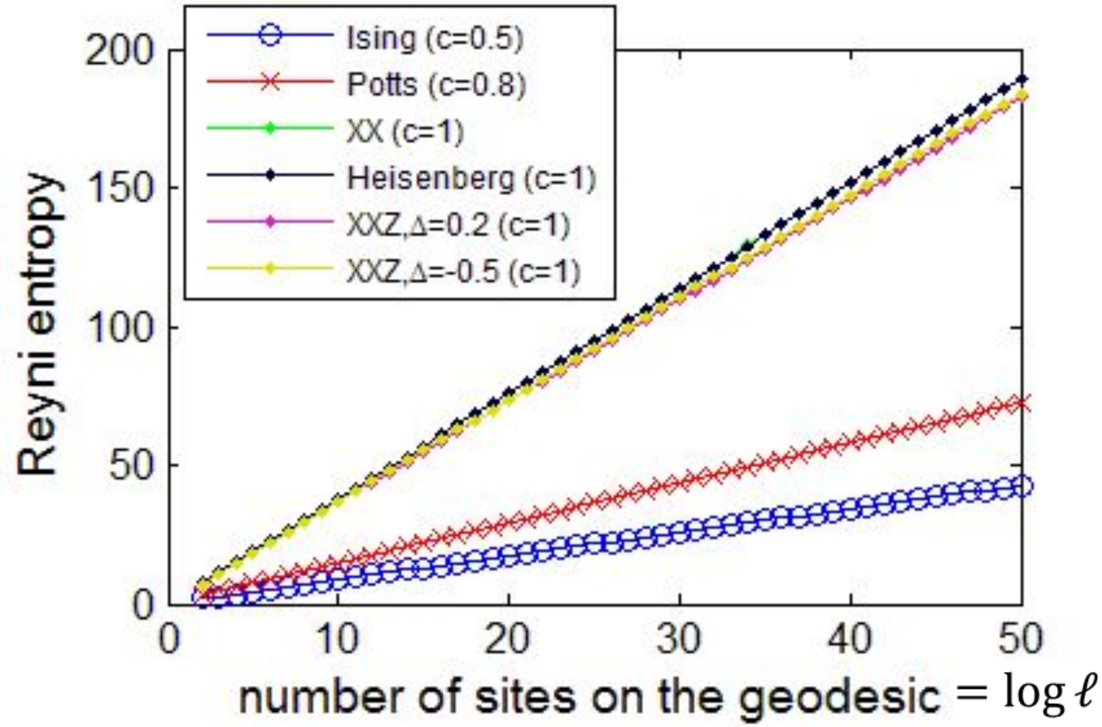




- We find that the entanglement entropy scales as log L with a slope that increases with central charge.
- And that for different models with the same central charge the slope is approximately equal.
- This suggests that the bulk states "knows" the central charge of the boundary.

- From 3 different value of *c* we could not ascertain a functional dependence of the slope on the central charge.
- But if the entanglement entropy of the geodesic sites indeed scales as log L with a prefactor that depends on the central charge, and we know that the entanglement entropy of the critical boundary state scales as c/3 log L (from CFT), then we can equate the two quantities together under this correspondence:

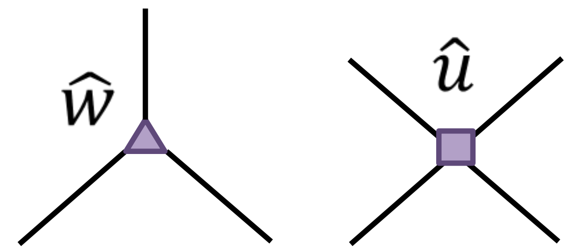
$$S^{boundary}(boundary\ sites) \approx S^{bulk}(geodesic\ sites)$$

- This looks like the Ryu-Takayanagi formula where we have replaced the length of the geodesic with entanglement entropy of sites lying on the geodesic.

Example 2: bulk state with topological order

- Consider a $\chi = 2$ MERA made of following tensors:

$$|0\rangle, |1\rangle \rightarrow irreps\ of\ Z_2 = \{I, X\}$$



$$\hat{w}^{0}_{00} = \hat{w}^{0}_{11} = \hat{w}^{1}_{01} = \hat{w}^{1}_{10} = 1$$

$$\hat{u}^{00}_{00} = \hat{u}^{00}_{11} = \hat{u}^{11}_{00} = \hat{u}^{01}_{01} = \hat{u}^{01}_{10} = \hat{u}^{10}_{01} = \hat{u}^{10}_{10} = 1$$

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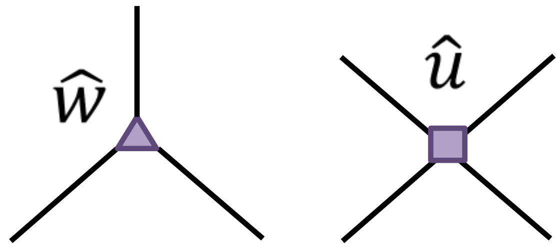

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- Bulk state belongs to a topological phase (ground state of Z_2 lattice gauge theory on a hyperbolic lattice)

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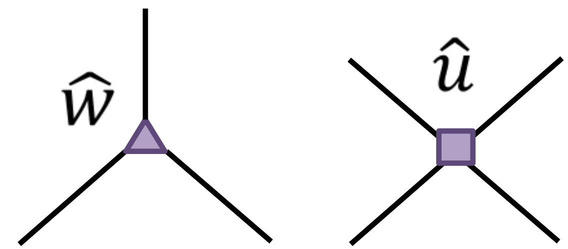

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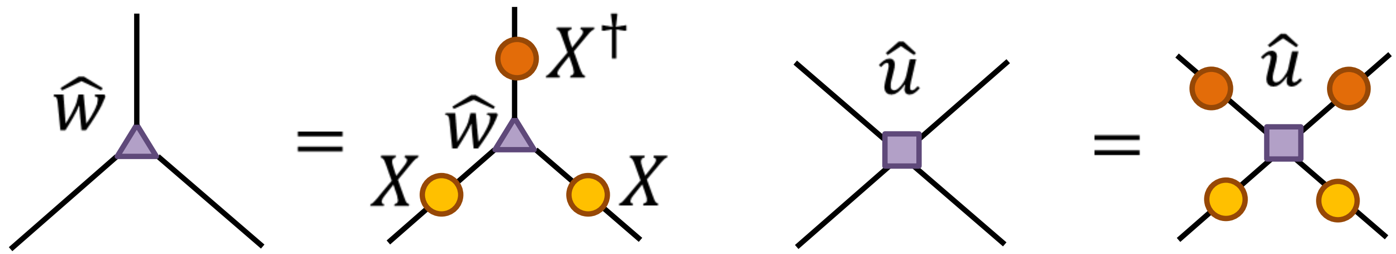
- Boundary state obtained by multiplying all the tensors, $|\Psi\rangle = |+ + \cdots\rangle + |- - - \cdots\rangle$, $\qquad |\pm\rangle = |0\rangle \pm |1\rangle$ (GHZ state)

Example 2: bulk state with topological order

- Tensors $\hat{u}, \widehat{w}$ commute with Z_2



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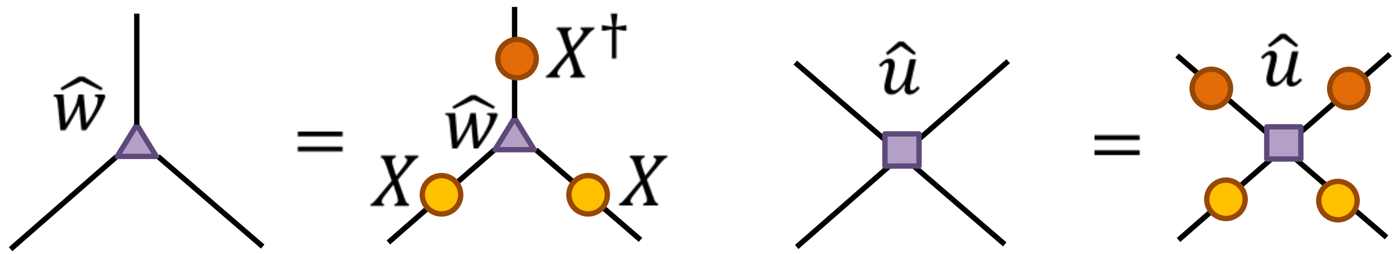
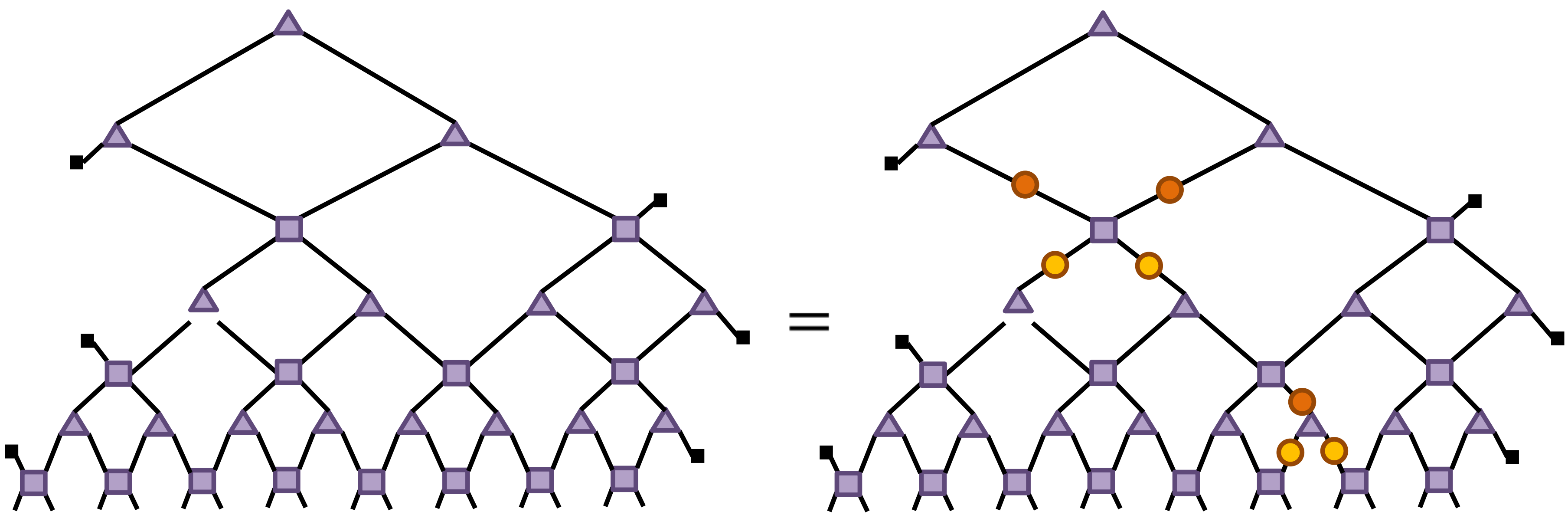
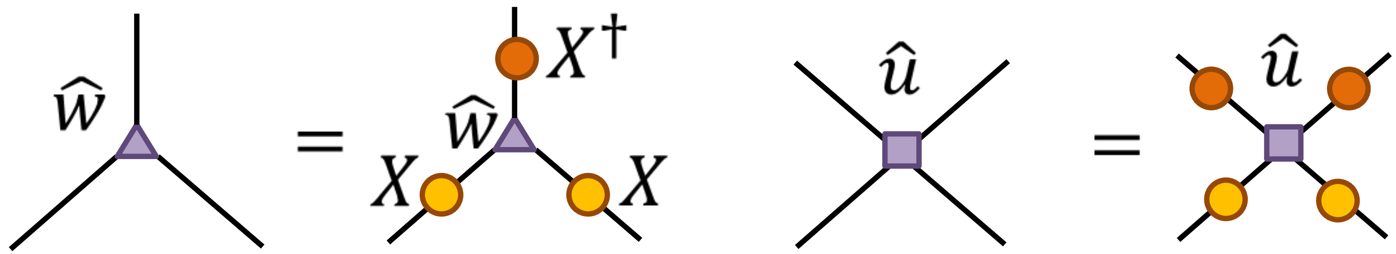


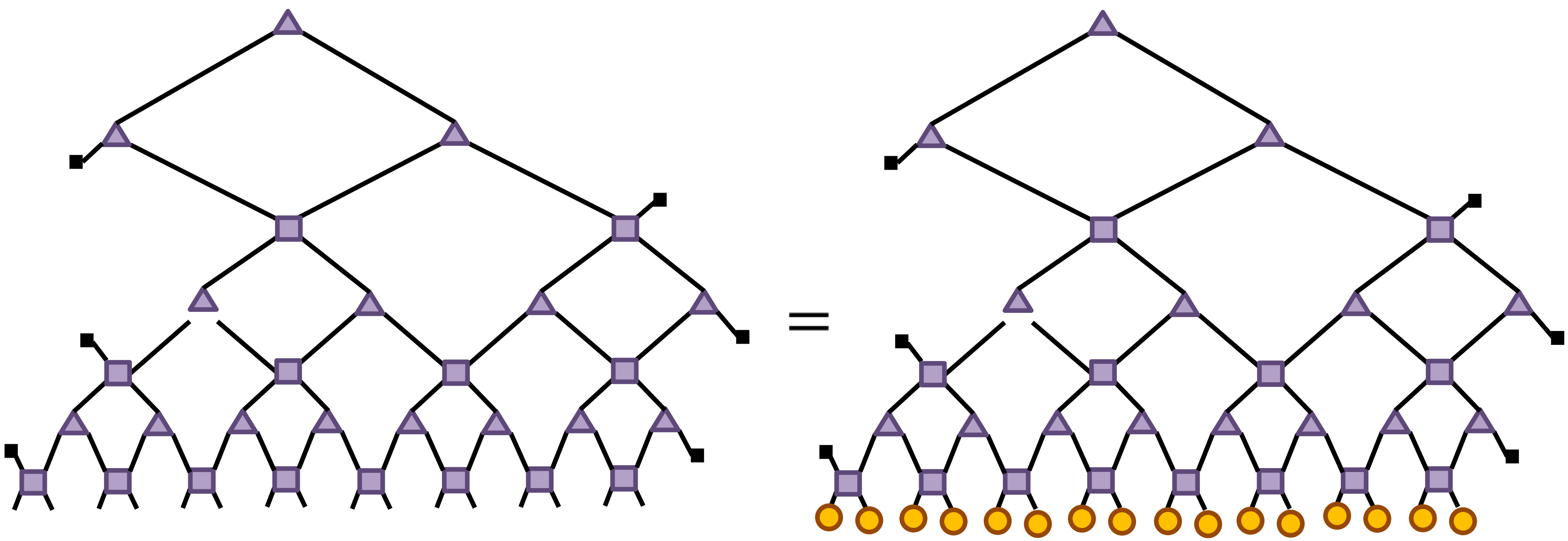


The bulk state has a local Z_2 symmetry
(ground state of a lattice gauge theory)

Example 2: bulk state with topological order

- Tensors $\hat{u}, \widehat{w}$ commute with Z_2





The boundary GHZ state has a global Z_2 symmetry bulk

Bulk ground state with Z_2 topological order

boundary

GHZ state

Topological phase of matter

?

Symmetry breaking phase of matter

e.g. fractional quantum hall system

e.g. Ising model, superfluids

Outlook

- MERA is a successful approach for practical simulations of ground states.
- Here we have argued that it also leads to a bulk/boundary correspondence.
- As a toy model to explore the AdS/CFT correspondence? (AdS/CFT of interest in quantum gravity + condensed matter physics.)
- And possibly to relate together interesting ground states e.g. different types of quantum phases?

Thanks