T2

1.

$$
\begin{aligned}
&\neg\ \exists x\ [P(x) \land Q(x)] \\
&\equiv \forall x\ \neg[P(x) \land Q(x)] \\
&\equiv \forall x\ [\neg P(x) \lor \neg Q(x)] \\
&\equiv \forall x\ [P(x) \Rightarrow \neg Q(x)] \\
&\therefore \neg\ \exists x\ [P(x) \land Q(x)] \iff \forall x\ [P(x) \Rightarrow \neg Q(x)]
\end{aligned}
$$

2.

$$
\begin{aligned}
&\neg\ \forall x\ [P(x) \Rightarrow Q(x)] \\
&\equiv \exists x\ \neg[\neg P(x) \lor Q(x)] \\
&\equiv \exists x\ [P(x) \land \neg Q(x)] \\
&\therefore \neg\ \forall x\ [P(x) \Rightarrow Q(x)] \iff \exists x\ [P(x) \land \neg Q(x)]
\end{aligned}
$$

3.

$$\neg\ \exists x\ [PN(x) < NN(x)]$$

$$\forall x\ [PN(x) >\ NN(x)]$$

$$PN(x) : positive\ number;\ NN(x) :\ negative\ number$$

$$
\begin{aligned}
&\neg\ \exists x\ [PN(x) < NN(x)] \\
&\equiv \forall x\ \neg[PN(x) < NN(x)] \\
&\equiv \forall x\ [PN(x) > NN(x)] \\
&\therefore \neg\ \exists x\ [PN(x) < NN(x)] \iff \forall x\ [PN(x) > NN(x)]
\end{aligned}
$$

4.

$$\neg\forall(x, y)\ [x == y \Rightarrow Diagonal(x, y)$$

$$\exists(x, y)\ [x == y\ \land\ \neg Diagnal(x, y)]$$

$$Diagnal(x, y) : x, y\ is\ diagnal$$

$$
\begin{aligned}
&\neg\forall(x, y)\ [x == y \Rightarrow Diagonal(x, y) \\
&\equiv \exists(x, y)\ \neg[(x! = y) \lor Diagonal(x, y)] \\
&\equiv \exists(x, y)\ [(x == y) \land \neg Diagnal(x, y)] \\
&\therefore \neg\forall(x, y)\ [x == y \Rightarrow Diagonal(x, y) \iff \exists(x, y)\ [(x == y) \land \neg Diagnal(x, y)]
\end{aligned}
$$

T4

前提：

$$\forall x\ [N(x) \Rightarrow (I(x) \land GZ(x))]$$

$$\forall x\ [I(x) \Rightarrow (O(x) \lor E(x))]$$

$$\forall x\ [S(E(x)) \Rightarrow I(x)]$$

目标：

$$\forall x\ [N(x) \Rightarrow (O(x) \lor S^{-1}(I(x)))]$$

证明：

原子语句：

$$
\begin{aligned}
&\neg N(x) \vee I(x) && 1\\
&\neg N(x) \vee GZ(x) && 2\\
&\neg I(x) \vee O(x) \vee E(x) && 3\\
&\neg S(E(x)) \vee I(x) && 4\\
&\neg S^{-1}(I(x)) && 5\\
&\neg O(x) && 6\\
&N(x) && 7\\
&[S^{-1}(I(x)) \iff E(x)]
\end{aligned}
$$

归结：

$$
\begin{aligned}
1,7 :&I(x)\\
3 :&E(x) \vee O(x)\\
6 :&E(x)\\
5 :&\text{空语句}\\
\text{原式得证}
\end{aligned}
$$

T1

1. $$\begin{aligned} &\exists x \,\{P(x) \wedge \forall y\,[\neg Q(y) \vee R(x,y)]\}\\ \equiv\ &\exists x\,\forall y\,\{P(x) \wedge [\neg Q(y) \vee R(x,y)]\}\end{aligned}$$

2.