

## T2

1. 
$$\begin{aligned} & \neg \exists x [P(x) \wedge Q(x)] \\ & \equiv \forall x \neg [P(x) \wedge Q(x)] \\ & \equiv \forall x [\neg P(x) \vee \neg Q(x)] \\ & \equiv \forall x [P(x) \Rightarrow \neg Q(x)] \\ & \therefore \neg \exists x [P(x) \wedge Q(x)] \iff \forall x [P(x) \Rightarrow \neg Q(x)] \end{aligned}$$
2. 
$$\begin{aligned} & \neg \forall x [P(x) \Rightarrow Q(x)] \\ & \equiv \exists x \neg [P(x) \Rightarrow Q(x)] \\ & \equiv \exists x [P(x) \wedge \neg Q(x)] \\ & \therefore \neg \forall x [P(x) \Rightarrow Q(x)] \iff \exists x [P(x) \wedge \neg Q(x)] \end{aligned}$$
3. 
$$\begin{aligned} & \neg \exists x [PN(x) < NN(x)] \\ & \quad \forall x [PN(x) > NN(x)] \\ & \text{PN}(x) : \text{positive number}; \text{NN}(x) : \text{negative number} \\ & \neg \exists x [PN(x) < NN(x)] \\ & \equiv \forall x \neg [PN(x) < NN(x)] \\ & \equiv \forall x [PN(x) > NN(x)] \\ & \therefore \neg \exists x [PN(x) < NN(x)] \iff \forall x [PN(x) > NN(x)] \end{aligned}$$
4. 
$$\begin{aligned} & \neg \forall (x, y) [x == y \Rightarrow \text{Diagonal}(x, y)] \\ & \exists (x, y) [x == y \wedge \neg \text{Diagonal}(x, y)] \\ & \quad \text{Diagonal}(x, y) : x, y \text{ is diagonal} \\ & \neg \forall (x, y) [x == y \Rightarrow \text{Diagonal}(x, y)] \\ & \equiv \exists (x, y) \neg [(x == y) \wedge \text{Diagonal}(x, y)] \\ & \equiv \exists (x, y) [(x == y) \wedge \neg \text{Diagonal}(x, y)] \\ & \therefore \neg \forall (x, y) [x == y \Rightarrow \text{Diagonal}(x, y)] \iff \exists (x, y) [(x == y) \wedge \neg \text{Diagonal}(x, y)] \end{aligned}$$

## T4

前提:

$$\begin{aligned} & \forall x [N(x) \Rightarrow (I(x) \wedge GZ(x))] \\ & \forall x [I(x) \Rightarrow (O(x) \vee E(x))] \\ & \forall x [S(E(x)) \Rightarrow I(x)] \end{aligned}$$

目标:

$$\forall x [N(x) \Rightarrow (O(x) \vee S^{-1}(I(x)))]$$

证明:

原子语句:

$\neg N(x) \vee I(x)$	1
$\neg N(x) \vee GZ(x)$	2
$\neg I(x) \vee O(x) \vee E(x)$	3
$\neg S(E(x)) \vee I(x)$	4
$\neg S^{-1}(I(x))$	5
$\neg O(x)$	6
$N(x)$	7
$[S^{-1}(I(x)) \iff E(x)]$	

归结:

$1, 7 : I(x)$   
 $3 : E(x) \vee O(x)$   
 $6 : E(x)$   
 $5 : \text{空语句}$   
 原式得证

## T1

- $$\exists x \{P(x) \wedge \forall y [\neg Q(y) \vee R(x, y)]\}$$

$$\equiv \exists x \forall y \{P(x) \wedge [\neg Q(y) \vee R(x, y)]\}$$
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