T1. $\hat{y} = \hat{w}x + b$. 由最小二乘法定义知 $\sum(\hat{y} - y_i)^2$ 最小.

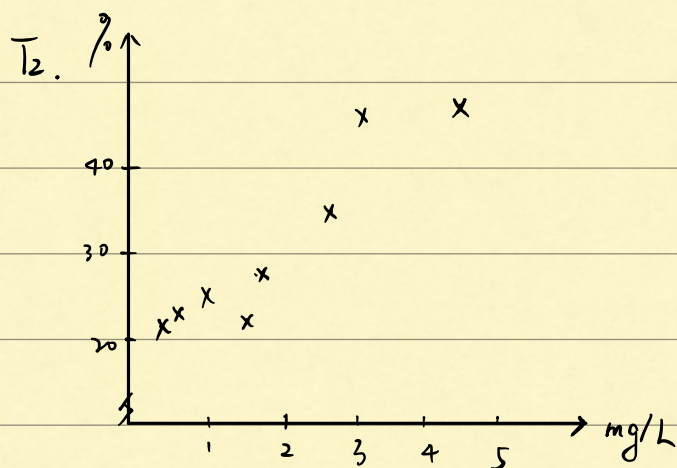$\therefore 2\sum(\hat{w}x_i + b - y_i) = 0$ $2x_i\sum(\hat{w}x_i + b - y_i) = 0$

$\therefore \sum(\hat{y}_i - y_i) = 0$ $\sum y_i(\hat{y}_i - y_i) = 0$

$\therefore \sum(\bar{y} - y_i)(\hat{y}_i - y_i) = 0$

原式左例 $= \sum(y_i - \hat{y}_i)^2 + \sum(\hat{y}_i - \bar{y})^2 + 2\sum(y_i - \hat{y}_i)(\hat{y}_i - \bar{y})$

将上式代入 $= \sum(y_i - \hat{y}_i)^2 + \sum(\hat{y}_i - \bar{y})^2$ $=$ 右边. 得证

T2.

$\bar{x} = 1.995$. $\bar{y} = 31.126$

$S_{xx} = \sum(x_i - \bar{x})^2 = 15.179$. $S_{xy} = \sum(x_i - \bar{x})(y_i - \bar{y}) = 97.4772$

$S_{yy} = \sum(y_i - \bar{y})^2 = 718.0282$

$\hat{w} = \dfrac{S_{xy}}{S_{xx}} = 6.4218$ $b = \bar{y} - \hat{w}\bar{x} = 18.3148$

$\therefore \hat{y} = 6.4218x + 18.3148$ $r^2 = \dfrac{S_{xy}^2}{S_{xx}S_{yy}} = 0.8718$

T4. (1) $J(\beta) = (X\beta - Y)^T(X\beta - Y) + \lambda\beta^T\beta$

令 $\dfrac{\partial J(\beta)}{\partial \beta} = 0$ $\therefore 2X^T(X\beta - Y) + 2\lambda I\beta = 0$

$\therefore (X^TX + \lambda I)\beta = X^TY$ $\therefore \beta = (X^TX + \lambda I)^{-1}X^TY$

(2). $\lambda = 1$ 时. $\beta = (0.6143 \quad 0.548 \quad 0.0662)^T$

$\lambda = 5$ 时 $\beta = (0.3909 \quad 0.3721 \quad 0.0188)^T$

$\lambda = 10$ 时 $\beta = (0.2687 \quad 0.2669 \quad 0.0019)^T$