T2

1.

$$\begin{aligned}
&\neg\ \exists x\ [P(x) \wedge Q(x)] \\
&\equiv \forall x\ \neg[P(x) \wedge Q(x)] \\
&\equiv \forall x\ [\neg P(x) \vee \neg Q(x)] \\
&\equiv \forall x\ [P(x) \Rightarrow \neg Q(x)] \\
&\therefore \neg\ \exists x\ [P(x) \wedge Q(x)] \iff \forall x\ [P(x) \Rightarrow \neg Q(x)]
\end{aligned}$$

2.

$$\begin{aligned}
&\neg\ \forall x\ [P(x) \Rightarrow Q(x)] \\
&\equiv \exists x\ \neg[\neg P(x) \vee Q(x)] \\
&\equiv \exists x\ [P(x) \wedge \neg Q(x)] \\
&\therefore \neg\ \forall x\ [P(x) \Rightarrow Q(x)] \iff \exists x\ [P(x) \wedge \neg Q(x)]
\end{aligned}$$

3.

$$\neg\ \exists x\ [PN(x) < NN(x)]$$
$$\forall x\ [PN(x) > \ NN(x)]$$
$$PN(x) : positive\ number;\ NN(x) :\ negative\ number$$
$$for\ example : PN(x) = |x|, NN(x) = -|x|$$

$$\begin{aligned}
&\neg\ \exists x\ [PN(x) < NN(x)] \\
&\equiv \forall x\ \neg[PN(x) < NN(x)] \\
&\equiv \forall x\ [PN(x) > NN(x)] \\
&\therefore \neg\ \exists x\ [PN(x) < NN(x)] \iff \forall x\ [PN(x) > NN(x)]
\end{aligned}$$

4.

$$\neg\forall(x, y)\ [x == y \Rightarrow Diagonal(x, y)$$
$$\exists(x, y)\ [x == y\ \wedge\ \neg Diagnal(x, y)]$$
$$Diagnal(x, y) : x, y\ is\ diagnal$$

$$\begin{aligned}
&\neg\forall(x, y)\ [x == y \Rightarrow Diagonal(x, y) \\
&\equiv \exists(x, y)\ \neg[(x! = y) \vee Diagonal(x, y)] \\
&\equiv \exists(x, y)\ [(x == y) \wedge \neg Diagnal(x, y)] \\
&\therefore \neg\forall(x, y)\ [x == y \Rightarrow Diagonal(x, y) \iff \exists(x, y)\ [(x == y) \wedge \neg Diagnal(x, y)]
\end{aligned}$$

T4

前提：

$$\forall x\ [N(x) \Rightarrow (I(x) \wedge GZ(x))]$$
$$\forall x\ [I(x) \Rightarrow (O(x) \vee E(x))]$$
$$\forall x\ [E(x) \Rightarrow I(S(x)]$$

目标：

$$\forall x\ [N(x) \Rightarrow (O(x) \vee I(S(x)))]$$

证明：

原子语句：

$$
\begin{aligned}
&\neg N(x) \vee I(x) && 1 \\
&\neg N(x) \vee GZ(x) && 2 \\
&\neg I(x) \vee O(x) \vee E(x) && 3 \\
&\neg E(x) \vee I(S(x)) && 4 \\
&\neg I(S(x)) && 5 \\
&\neg O(x) && 6 \\
&N(x_0) && 7 \\
&[S^{-1}(I(x_0)) \iff E(x_0)]
\end{aligned}
$$

归结:

$$
\begin{aligned}
1,7 :&I(x_0) \\
3 :&E(x_0) \vee O(x_0) \\
6 :&E(x_0) \\
4 :&I(s(x_0)) \\
5 :&\text{空语句}
\end{aligned}
$$

原式得证

T1

1. $$\begin{aligned} &\exists x \, \{P(x) \wedge \forall y \, [\neg Q(y) \vee R(x, y)]\} \\ \equiv &\exists x \, \forall y \, \{P(x) \wedge [\neg Q(y) \vee R(x, y)]\} \end{aligned}$$

2.