T1. $\hat{y}=\hat{w}x+b$. 由最小二乘法定义知 $\sum(\hat{y}-y_i)^2$ 最小.

$\therefore 2\sum(\hat{w}x_i+b-y_i)=0 \qquad 2x_i\sum(\hat{w}x_i+b-y_i)=0$

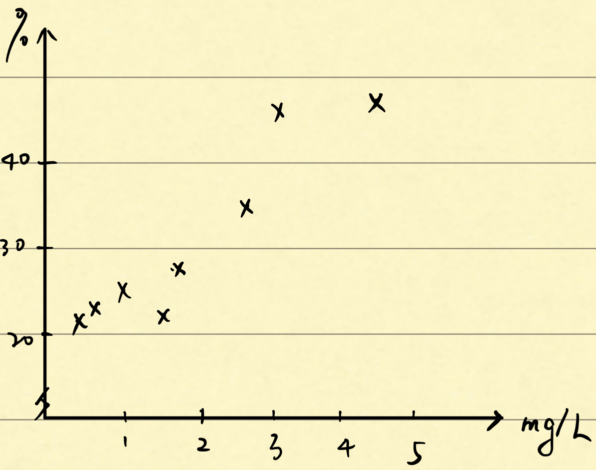
$\therefore \sum(\hat{y}_i-y_i)=0 \qquad \sum y_i(\hat{y}_i-y_i)=0$

$\therefore \sum(\bar{y}-y_i)(\hat{y}_i-y_i)=0$

原式左侧 $=\sum(y_i-\hat{y})^2+\sum(\hat{y}_i-\bar{y})^2+2\sum(y_i-\hat{y}_i)(\hat{y}_i-\bar{y})$

将上式代入 $=\sum(y_i-\hat{y}_i)^2+\sum(\hat{y}_i-\bar{y})^2$ = 右边，得证

T2.



$\bar{x}=1.995$. $\bar{y}=31.126$

$S_{xx}=\sum(x_i-\bar{x})^2=15.18$. $S_{xy}=\sum(x_i-\bar{x})(y_i-\bar{y})=97.48$

$S_{yy}=\sum(y_i-\bar{y})^2=718.03$

$\hat{w}=\frac{S_{xy}}{S_{xx}}=6.42 \qquad b=\bar{y}-\hat{w}\bar{x}=18.3$

$\therefore \hat{y}=6.42x+18.3 \qquad r^2=\frac{S_{xy}^2}{S_{xx}S_{yy}}=0.87$

T4. (1) $J(\beta)=(X\beta-Y)^T(X\beta-Y)+\lambda\beta^T\beta$

令 $\frac{\partial J(\beta)}{\partial\beta}=0 \qquad \therefore 2X^T(X\beta-Y)+2\lambda I\beta=0$

$\therefore (X^TX+\lambda I)\beta=X^TY \qquad \therefore \beta=(X^TX+\lambda I)^{-1}X^TY$

(2). $\lambda=1$ 时. $\beta=(0.6143 \quad 0.548 \quad 0.0662)^T$

$\lambda=5$ 时 $\beta=(0.3909 \quad 0.3721 \quad 0.0188)^T$

$\lambda=10$ 时 $\beta=(0.2687 \quad 0.2669 \quad 0.0019)^T$