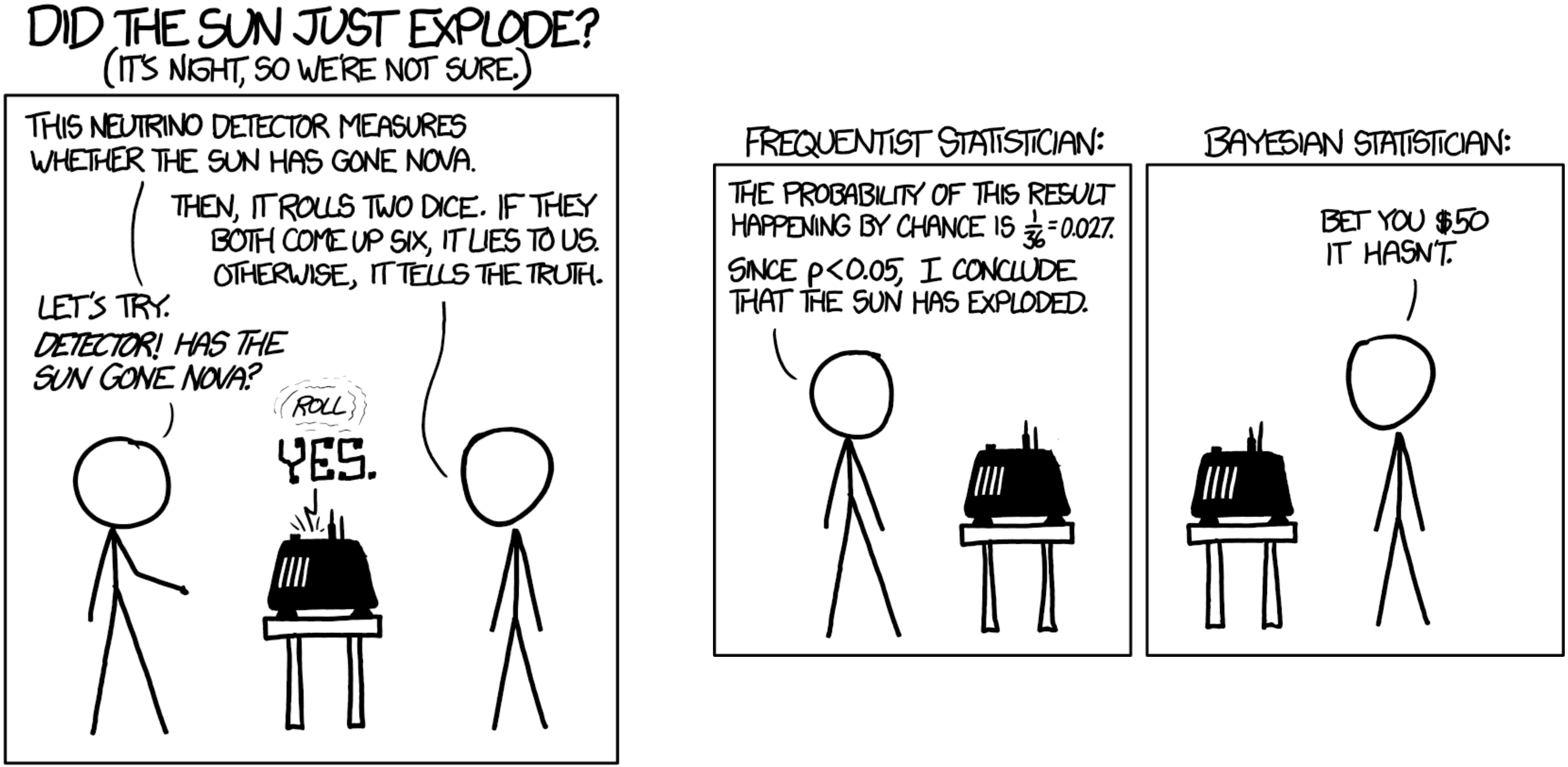


download these slides from: github.com/qtabs/compneuro/bayes_leipzig.{key, pdf}

Questions from last session

Q2.1 (Jun)

A neuron's receptive field describes how it responds to the world. Is this an objective property of the stimulus or a subjective model constructed by the brain? If different organisms have different RFs for the same sensory input, whose model is correct? What does this tell us about the nature of representation?

Q2.2 (Sebastian)

Why do some receptive fields, like the orientation selectivity of V1, have finite width? Why don't they, instead, respond selectively only to a single orientation value? What constraints or principles force the brain to use broader tuning?

Q2.4 (Izei)

Imagine you're designing a brain from scratch with no evolutionary constraints. Which properties would be desirable in the brain's internal representations of the visual input? For a few of the properties you list, try to formulate specific normative or phenomenological hypotheses on the kind of receptive fields we would expect in the visual pathways. How would you test such hypotheses?

Q2.5 (Shuping)

Take a close look at the beginning of the responses across cochlear frequency channels at the beginning of each tone. What are the first valleys in the response pattern encoding? Why do lower frequencies show longer delays and extended responses compared to higher frequencies?

Q2.7 (Jinweng)

Different measurement techniques (single-unit recordings, fMRI, EEG) reveal different aspects of neural representation. How does our choice of measurement tool shape our theories about how the brain represents information? What biases might this introduce?

The Bayesian Brain Hypothesis

Computational Neuroscience - Lecture 3

Alejandro Tabas

La théorie des probabilités n'est au fond que le bon sens réduit au calcul

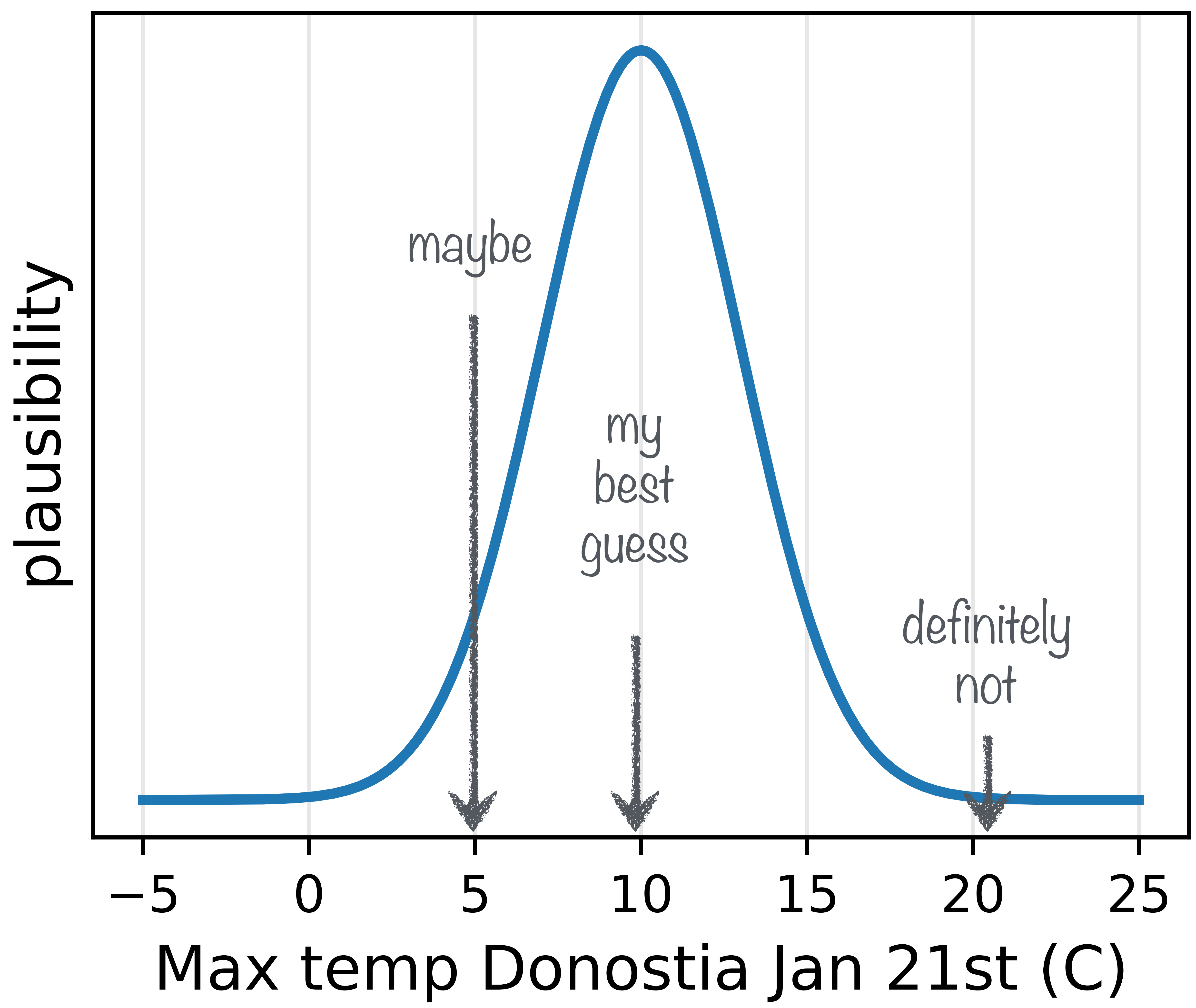
probability theory is basically just common sense reduced to calculation

DeepL

What will be the maximum temperature in Donosti today?

- Assume you have 10€
- You need to distribute them as a bet across different values
- You get back five times what you bet
- How do you distribute the money?

Predictions are measures of plausibility



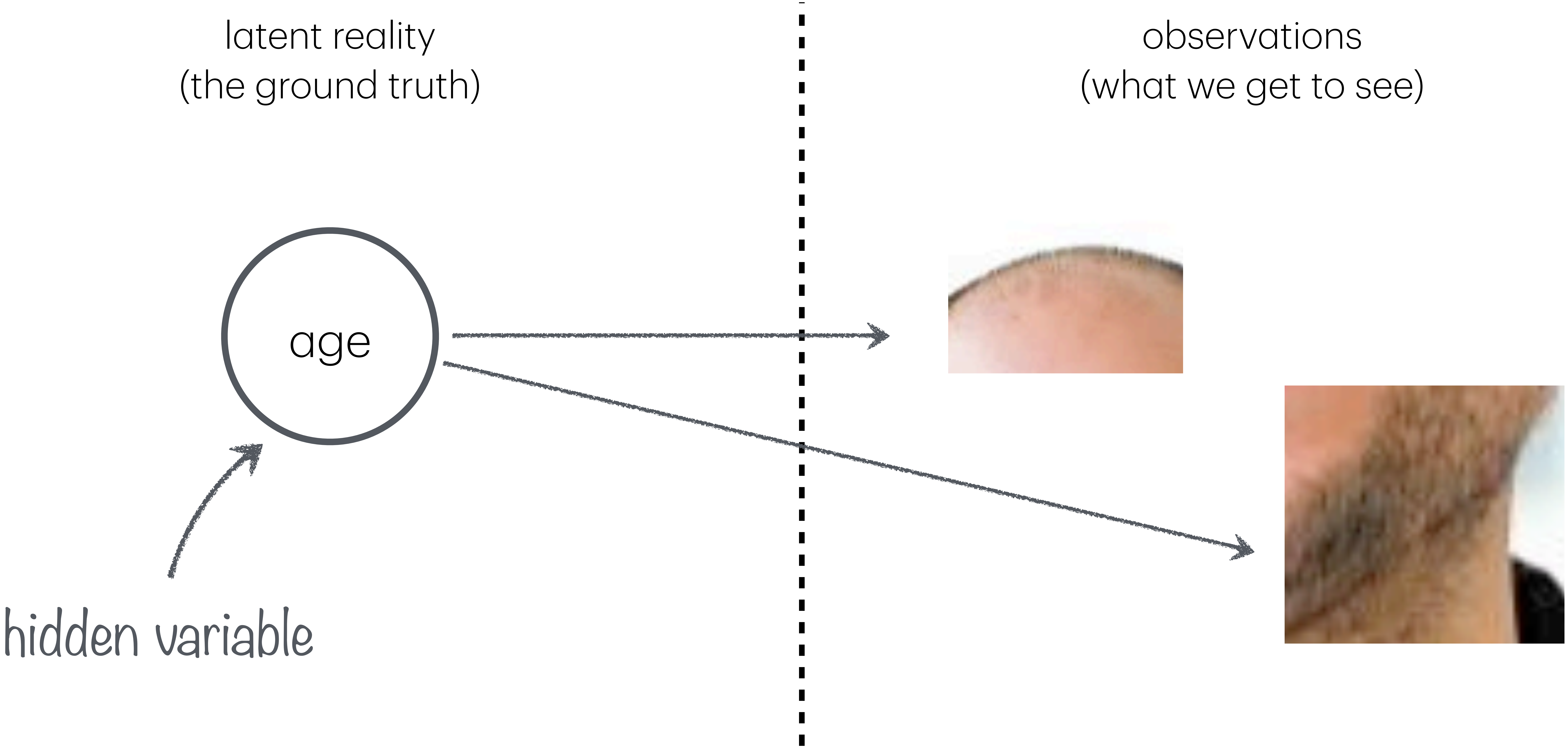
Predictions are measures of plausibility

- When we predict the future, we have **uncertainty**
- Our predictions are **internal beliefs on the plausibility** of different outcomes
- Internal beliefs are different for each person

What is Alex’s age?

this guy

Estimations are measures of plausibility

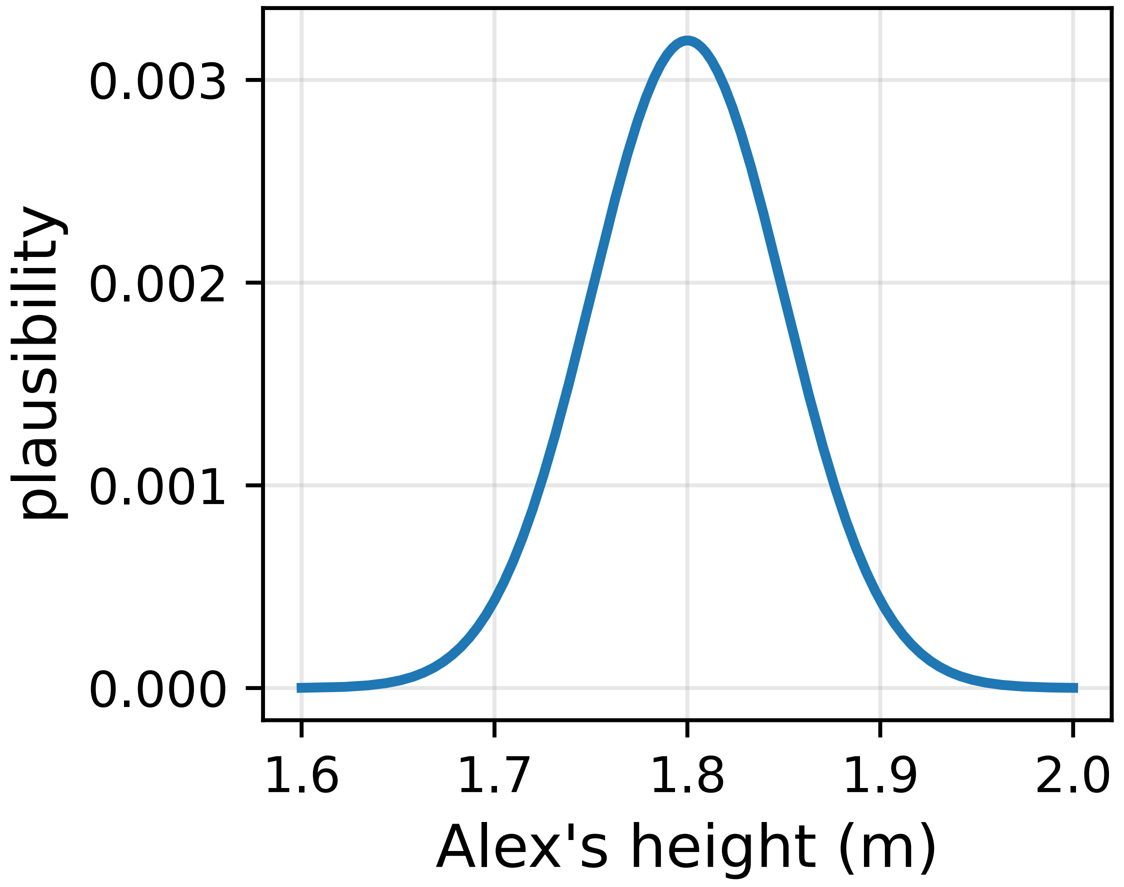


Estimations are measures of plausibility

- When we estimate hidden variables, we have **uncertainty**
- Our estimations are **internal beliefs on the plausibility** of possible hidden values
- Internal beliefs are different for each person

What is Alex’s heigh?

Percepts are measures of plausibility

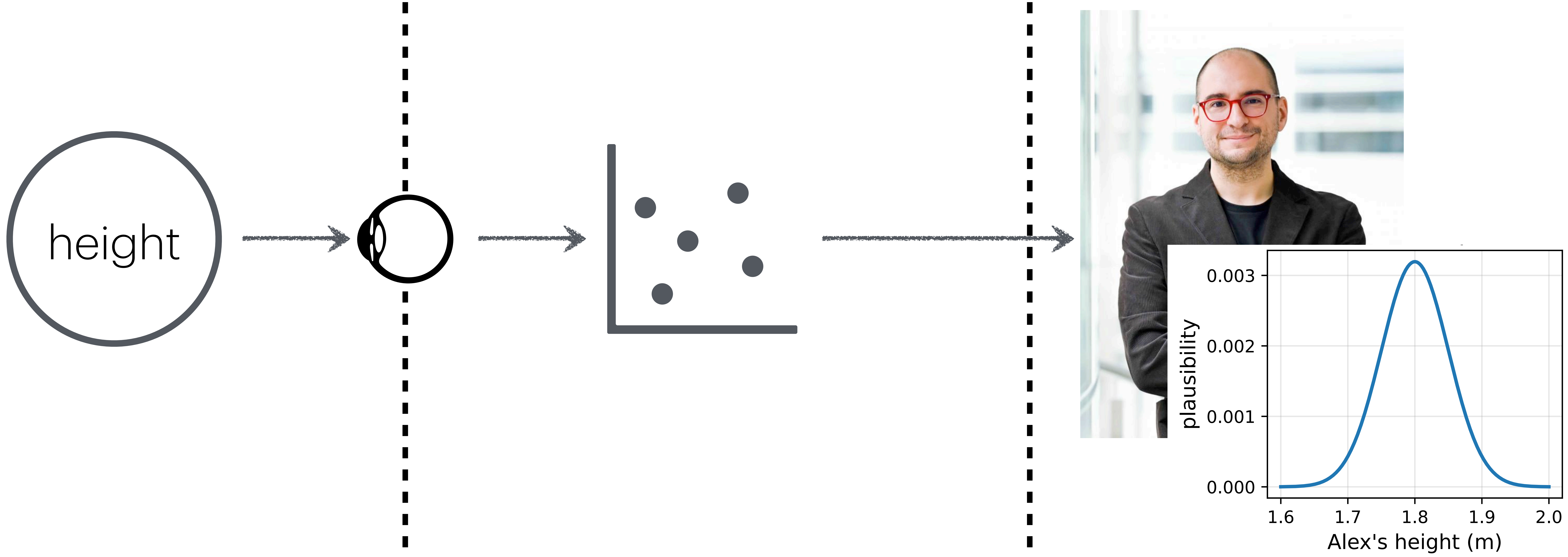
latent reality (the ground truth)	observations (what we get to see)	estimations (what we believe)
height		

Percepts are measures of plausibility

latent reality
(the ground truth)

observations
(what we get to see)

estimations
(what we believe)



Percepts are measures of plausibility

- When we estimate the latent variables responsible for what we see, we have **uncertainty**
- Our **percepts are internal beliefs on the plausibility** of the possible values of the latent variables responsible for what we observe
- Percepts are different for each person

The Bayesian Brain Hypothesis: First tenet*

Internal representations are measures of plausibility

The Bayesian Brain Hypothesis

1. First tenet: internal representations are probabilistic
2. Second tenet: cognition rests on priors
3. Third tenet: the brain performs Bayesian inference
4. Zeroth tenet: many aspects of cognition are optimal
5. Bayesian decision making
6. Empirical evidence
7. Conclusion

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What does p(side=3) = 1/6 really mean?

- Frequentist interpretation: $p(s=3)$ is the limit of the ratio of $s=3$ outcomes in N tosses

$$p(s=3) = \lim_{N \to \inf} \frac{n_{s=3}}{N}$$

- Bayesian interpretation: $p(s=3)$ is a measure of our internal belief on how plausible $s=3$ is

Can plausibility be quantified?

- Plausibility is bounded
  - There are top and bottom limits to it
  - Its quantification should have a minimum and a maximum value
- Plausibility is normalised
  - The sum of its quantification for all possible outcomes should equal the max value.
- Probability is both bounded and normalised

What does p(side=3) = 1/6 really mean?

- Frequentist interpretation: $p(s=3)$ is the limit of the ratio of $s=3$ outcomes in N tosses

$$p(s=3) = \lim_{N \to \inf} \frac{n_{s=3}}{N}$$

- Bayesian interpretation: $p(s=3)$ is a **measure of our internal belief** on how plausible $s=3$ is

A property of reality or a state of knowledge

Frequentist statistics assume that probability is a **property of reality**:

- $p(s = 3) = 1/6$ is a property of the die roll
- $p(\text{height} = x) = \mathcal{N}(x; \mu, \sigma^2)$ is a property of the human population of a country

Bayesian statistics assume that probability represents a **state of knowledge**

- $p(s = 3) = 1/6$ is a measure of our state of knowledge on the result of the die roll
- $p(\text{height} = x) = \mathcal{N}(x; \mu, \sigma^2)$ describes our internal belief about someone we don't know

Probabilities cannot be a property of reality

The laws of physics do not generally allow for identical independent measurements:

- A die rolled in a specific way in a specific environment will always land in the same side
- Skilled rollers can influence the landing side at will

Many events only happen once and are irreproducible:

- What is the limit frequency of asteroid 2024 YR4 hitting earth on the 22nd of December 2032?
- What is the limit frequency of Trump surviving the term?

Probabilities change with additional information:

- What was at 8:45 the probability that Alex would be late? And now?

Probabilities are subjective believes on plausibility

Different observers can have different believes on the outcome of a die roll:

- A trick roller can manipulate the outcome distribution of the roll
- Even if they know the roll is tricked, observers do not know which sides will be favoured

We often have believes on one-time only events:

- We currently believe asteroid 2024 YR4 will hitting earth in 2032 with $p = 0.013$

States of knowledge are naturally updated with new incoming information:

- Our believes on Alex being late were updated the moment he arrived

First tenet

Internal representations are probabilistic

- Internal representations of: predictions (future), estimates (hidden), percepts (present), …
- We can model them using Bayesian probability theory

The Bayesian Brain Hypothesis

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Internal beliefs depend on the information we have

We model dependency on information with **conditional probability**:

- $P(a = x \mid B)$ is the probability of a being equal to x given the information B

A note on notation:

- $A, B, \ldots$ are used for events, things that can only be either true or false
- $a, b, \ldots$ are used for variables that can take several values (side of a die s, height of a person h)
- $x, y, \ldots$ are used for specific values ($a = x, b = y, \ldots$)
- Sometimes we use $P(x \mid B)$ as a short simplified equivalent of $P(a = x \mid B)$

Internal beliefs depend on the information we have

$P(a = x \mid B)$ is the probability of a being equal to x given the information B

Some examples:

- $P(s = 2 \mid s \neq 3) = 1/5$
- $P(\text{Alex's age} < 25 \mid \text{Alex has a PhD}) < 0.1$

Beliefs can be conditioned on several things

- $P(30 < \text{Alex's age} < 40 \mid \text{Alex is bald, Alex's beard is black}) > 0.7$

In the Bayesian framework, **probabilities are almost always conditioned on something**

- We usually write $P(A) \equiv P(A \mid \text{Avalable information})$ or $P(A \mid I)$ for simplicity

Second tenet

Internal representations depend on our prior information

The Bayesian Brain Hypothesis

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Internal beliefs depend on the information we have

$P(a = x \mid B)$ is the probability of a being equal to x given the information B

Is $P(A \mid B) \stackrel{?}{=} P(B \mid A)$ correct?

One counter-example:

- $P(\text{Alex's age} < 25 \mid \text{Alex has a PhD}) \neq P(\text{Alex has a PhD} \mid \text{Alex's age} < 25)$

$$P(A \mid B) \neq P(B \mid A)$$

Joint probabilities

$P(a = x, b = y)$ is the probability that both, $a = x$ and $b = y$

Example:

- $P(s = 1, \text{said truth}) = 0$

Joint probabilities are normalised across all combinations:

- $P(s = 1, \text{said truth}) + P(s \neq 1, \text{said truth}) + P(s = 1, \text{lied}) + P(s \neq 1, \text{lied}) = 1$

- Joint probabilities can also be conditioned on some information::

- $P(s = 1, \text{lied} \,|\, s \neq 4) > P(s = 1, \text{lied})$

Marginals

$P(a = x, b = y)$ is the probability that $a = x$ and $b = y$ simultaneously

What is the plausibility of $s = 3$?

- $P(s = 3) = P(s = 3, \text{said truth}) + P(s = 3, \text{lied})$

$P(s = 3)$ is called the marginal

$P(\text{said truth})$ is also a marginal

Marginal probabilities

	Pass	Fail	Total	
Males	46	56	102	P(male) = 0.51
Females	68	30	98	P(female) = 0.49
Total	114	86	200	
	P(passed) = 0.57	P(failed = 0.43)		

Conditional expansion

$P(a = x, b = y)$ is the probability that both, $a = x$ and $b = y$

$P(A, B)$ is the probability that both, A and B

Can we rewrite this as: $P(A, B) \stackrel{?}{=} P(A)\, P(B)$

- NO: $P(s = 1) = 1/6,\ \ P(\text{said truth}) > 0$. However, $P(s = 1, \text{said truth}) = 0$

We are not taking into account all available information! Lets think more carefully:

- If B is true, then $P(A, B) = P(A \,|\, B)$
- If B is false, $P(a = x, b = y) = 0$

$$P(A, B) = P(A \,|\, B)\, P(B)$$

Symmetry

$P(a = x, b = y)$ is the probability that both, $a = x$ and $b = y$

$P(A, B)$ is the probability that both, A and B

$$P(A, B) = P(A \,|\, B)\, P(B) = P(B \,|\, A)\, P(A)$$

Note that this also follows from $P(A, B) = P(B, A)$

Bayes rule

$P(A, B) = P(A \mid B)\, P(B) = P(B \mid A)\, P(A)$

$P(A \mid B) \neq P(B \mid A)$

This identity allows to derive $P(A \mid B)$ based on $P(B \mid A)$ and vice versa:

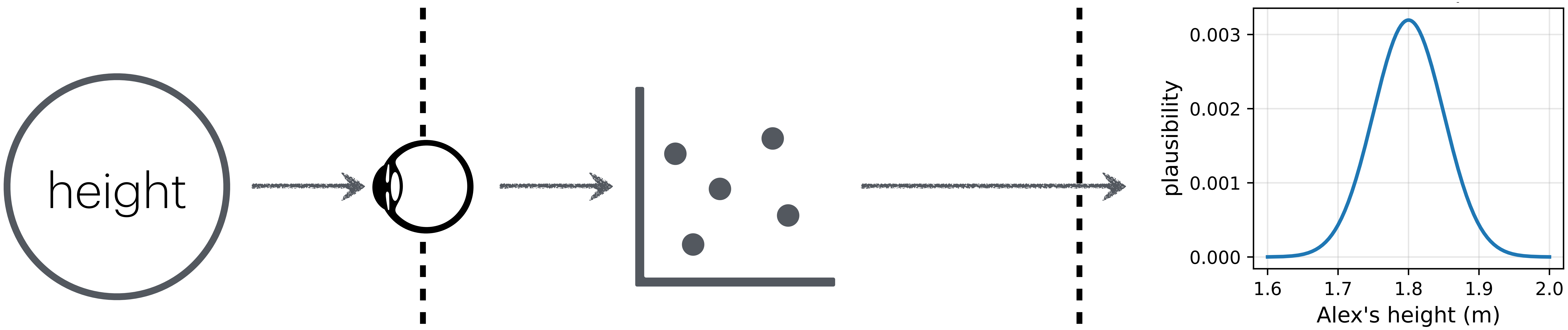
$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}$$

Estimating percepts based on observations

latent reality
(the ground truth)

observations
(what we get to see)

estimations
(what we believe)



$$x \longrightarrow y \longrightarrow P(x \mid y, I)$$

Bayesian inference

Likelihood
how plausible would
y be if latent was x

Prior
what we know
about x before seeing y
(knowing I)

Posterior
what we know
about x after seeing y

$$P(x \mid y) = \frac{P(y \mid x)\, P(x)}{P(y)}$$

Marginal
how plausible is y
(knowing I)

$$x \longrightarrow y \longrightarrow P(x \mid y, I)$$

Bayesian inference

$$P(A\,|\,B) = \frac{P(B\,|\,A)\,P(A)}{P(B)}$$

Examples:

$$P(\text{it rained}\,|\,\text{wet streets}) = \frac{P(\text{wet streets}\,|\,\text{it rained})\,P(\text{it rained}\,|\,I)}{P(\text{wet streets})}$$

$$P(s = 2\,|\,\text{said is }2) = \frac{P(\text{said is }2\,|\,s = 2)\,P(s = 2\,|\,I)}{P(\text{said is }2)} = \frac{P(\text{said truth})\,1/6}{P(\text{said is }2\,|\,s = 2) + P(\text{said is }2\,|\,s \neq 2)}$$

Third tenet

The brain performs Bayesian inference to infer the latent causes of the observations

Some nuances:

- Inference can be approximate, there's no claim that it's exact
- The tenet does not elaborate on the specific implementation
- Not all cognition needs to be Bayesian, there's no claim on scope

The Bayesian Brain Hypothesis

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Why would the brain do all that stuff?

it kind-of looks complicated?

$$P(x \mid y) = \frac{P(y \mid x)\, P(x)}{P(y)}$$

Formal logic

Assume $A \to B$. Then:

- If A is true, B should be true
- If B is false, A should be false

Example:

- My laptop runs out of battery $\longrightarrow$ it will not start

Formal logic does not tolerate uncertainty:

- What can we say about my laptop battery if it does not start?

Probability theory extends formal logic to uncertainty

What can we say about B if A makes B more plausible and it happens?

Example:

- Laptop does not start $\longrightarrow$ that it has run out of battery is now more plausible

We can say: $P(A \mid B) > P(A)$

Quantifying plausibility should be self-consistent

Any quantity that represents plausibility following the rational of formal logic should satisfy:

1. Degrees of plausibility are represented by real numbers
2. Degrees of plausibility qualitatively correspond with common sense
3. Consistency:
   - If a conclusion can be reasoned in several ways, every way must reach the same result
   - All relevant information must be taken into account (arbitrary omissions are not made)

Self-consistency rules → Bayesian probability theory

There is **only only one quantification system** following the consistency requirement.

It satisfies (among others):

- Plausibilities should* be mapped monotonically to numbers between 0 and 1
- $P(A,B) = P(A \mid B)\,P(A) = P(B,A)$
- Updating beliefs with new evidence must follow Bayes' rule.

* $0 \leq P \leq 1$ is not the only possible mapping, $-\infty \leq P \leq 0$ also work. No other mapping works.

Bayesian probability theory is optimal

Using Bayesian probability is the optimal way to make estimations under uncertainty because:

- uses all available information in a self-consistent way
- violating its rules leads to guaranteed losses or logical inconsistencies

As a result, if we use Bayesian probability to estimate x given some observations y:

- the mean of $P(X \mid Y)$ minimises mean squared error
- the median of $P(X \mid Y)$ minimises absolute error
- the value for x that maximises $P(X \mid Y)$ maximises the probability of being exactly right

Evolutionary pressure for optimality

Our ancestors might have evolutionarily benefit from optimality to

- Detect camouflaged predators
- Locating prey from weak cues (sound, smell, motion)
- Estimating resource abundance: inferring where food density is highest
- Judging terrain safety: estimating whether a surface will hold weight when stepping on it
- Predict weather for sheltering or hunting.
- Navigating under uncertain landmarks and cues
- Inferring social rank and trust
- Detecting sickness in others via subtle behaviuoral cues to avoid infection
- …

Zeroth tenet

Perception and other cognitive systems had evolutionary pressure to be Bayesian

This is not a human thing, is a life thing:

- Any living creature that needs to make decisions under uncertainty had evolutionary pressure to be optimal

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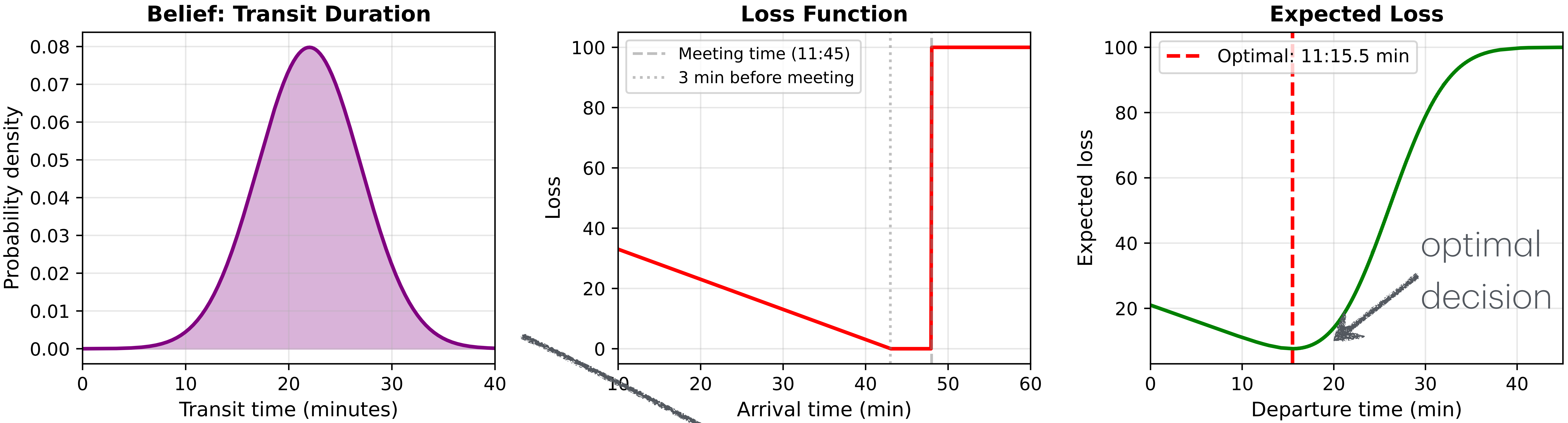
6. Empirical evidence
7. Conclusion

Help Alex decide what time he should leave from Antiguo:

- Alex has a meeting at 11:45 in Miramón
- There's around 22 minutes transit from Antiguo to Miramón
- Does Alex have time to eat a croqueta in the faculty’s cafeteria?

Posterior estimates + loss functions = expected loss



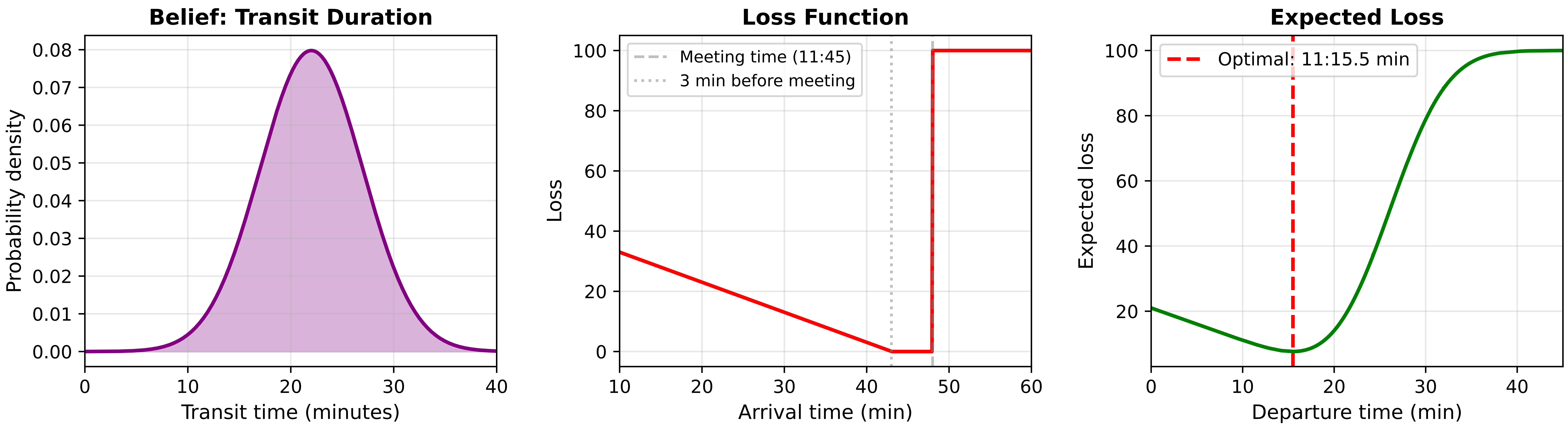
$$E[\text{loss}] = \sum_x p(x \mid I)\ \text{loss}(x)$$

Decisions are not distributions, they are points

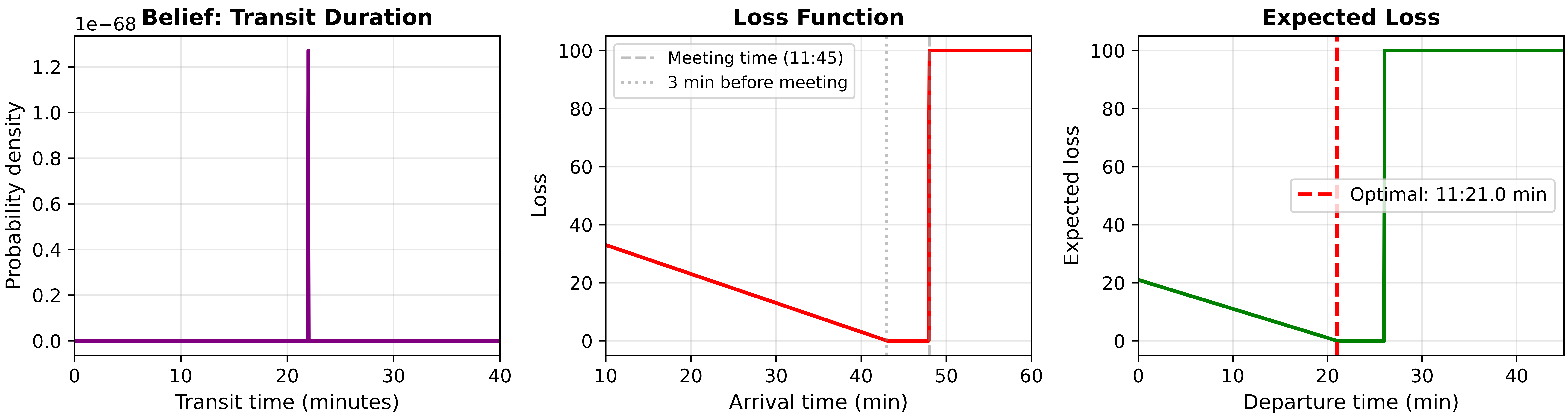
The outcome of a decision process needs to be actionable

- We find optimality as the solution with the lowest expected cost
- This is the reason why reports of perceptual decisions are always point estimates:
  - Reporting a perceptual decision is an action

Posterior estimates + loss functions = expected loss



Disregarding uncertainty = suboptimal decisions

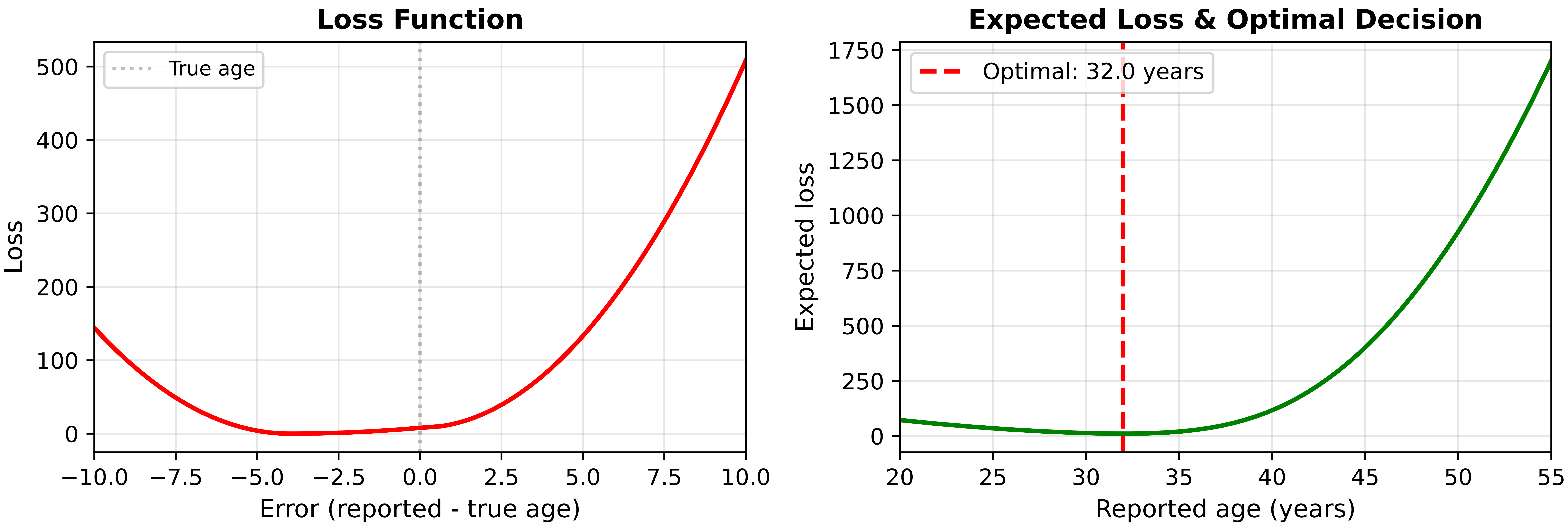


Cats as optimal Bayesian decision makers

Behaviour not always representative of optimal estimates

Suboptimal actions may actually be optimal under asymmetric costs:

- Criterion shifts in signal detection tasks
- Risk-sensitive behaviour
- ...



Zeroth tenet (revisited)

Evolutionary pressure is not on maximising our accuracy, is on minimising the loss:

- **living creatures strive to behave optimality under some loss function**
- different domains and contexts have different loss functions
- Loss functions are not defined solely on accuracy, also on:
  - metabolic costs of computation
  - constraints on the computational resources available
  - ...

Zeroth tenet (revisited)

Perception and other cognitive systems had evolutionary pressure to optimally minimise some loss function.

This often requires computing optimal estimates

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Empirical evidence for the Bayesian Brain Hypothesis

Vast evidence that at least some systems, under certain conditions, perform optimal inference

Here we will briefly mention a few examples on:

- behavioural responses
- human neuroimaging data
- non-human animal experiments
- cellular level computation

Empirical evidence from behavioural experiments

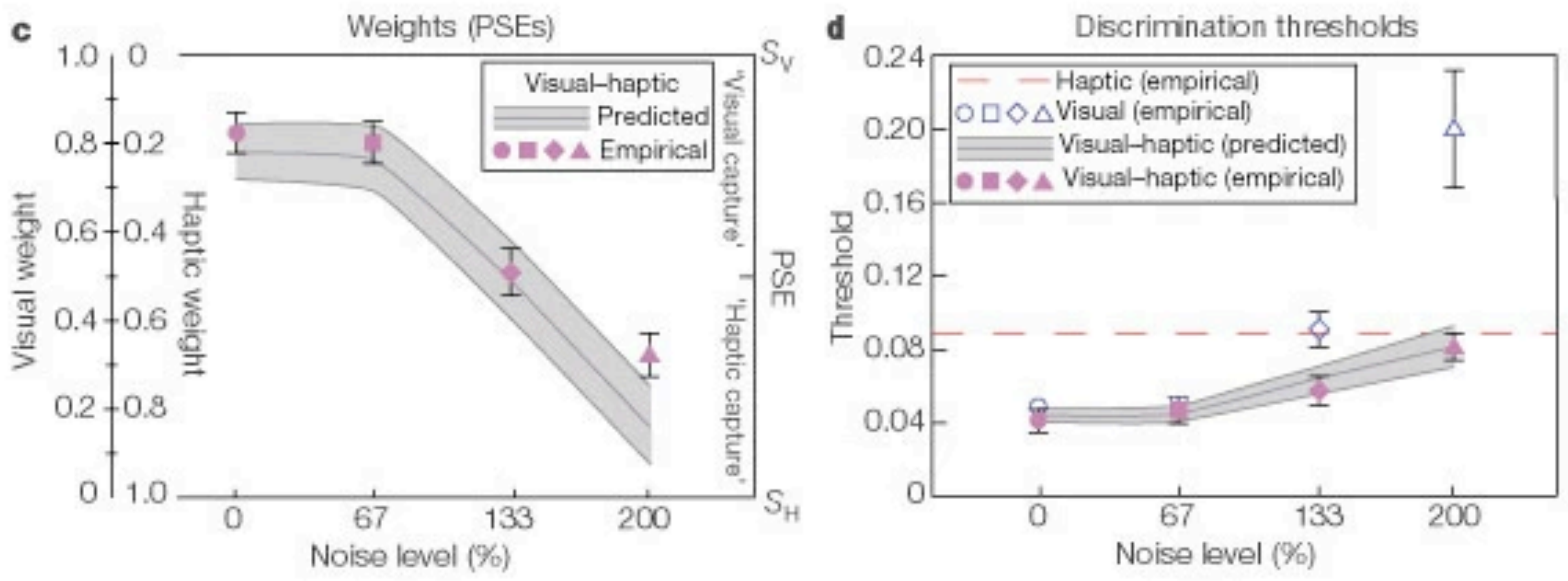
Example 1: Humans **combine cues in a nearly posterior mean = precision-weighted average**

Humans integrate visual and haptic information in a statistically optimal fashion

Marc O. Ernst* & Martin S. Banks

Vision Science Program/School of Optometry, University of California, Berkeley 94720-2020, USA

When a person looks at an object while exploring it with their hand, vision and touch both provide information for estimating the properties of the object. Vision frequently dominates the integrated visual–haptic percept, for example when judging size, shape or position[1–3], but in some circumstances the percept is clearly affected by haptics[4–7]. Here we propose that a general principle, which minimizes variance in the final estimate, determines the degree to which vision or haptics dominates. This principle is realized by using maximum-likelihood estimation[8–15] to combine the inputs. To investigate cue combination quantitatively, we first measured the variances associated with visual and haptic estimation of height. We then used these measurements to construct a maximum-likelihood integrator. This model behaved very similarly to humans in a visual–haptic task. Thus, the nervous system seems to combine visual and haptic information in a fashion that is similar to a maximum-likelihood integrator.



[Ernst & Banks 2002, Nature]

Empirical evidence from behavioural experiments

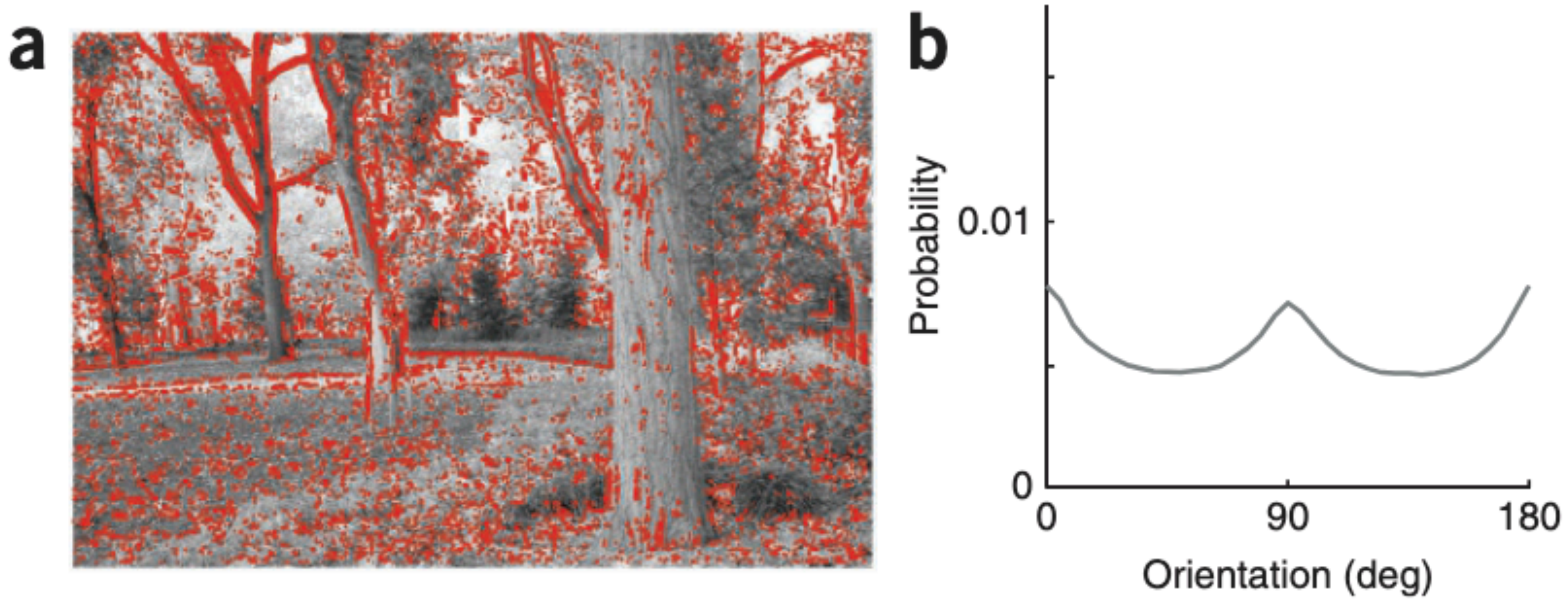
Example 2: **Priors shape perception**

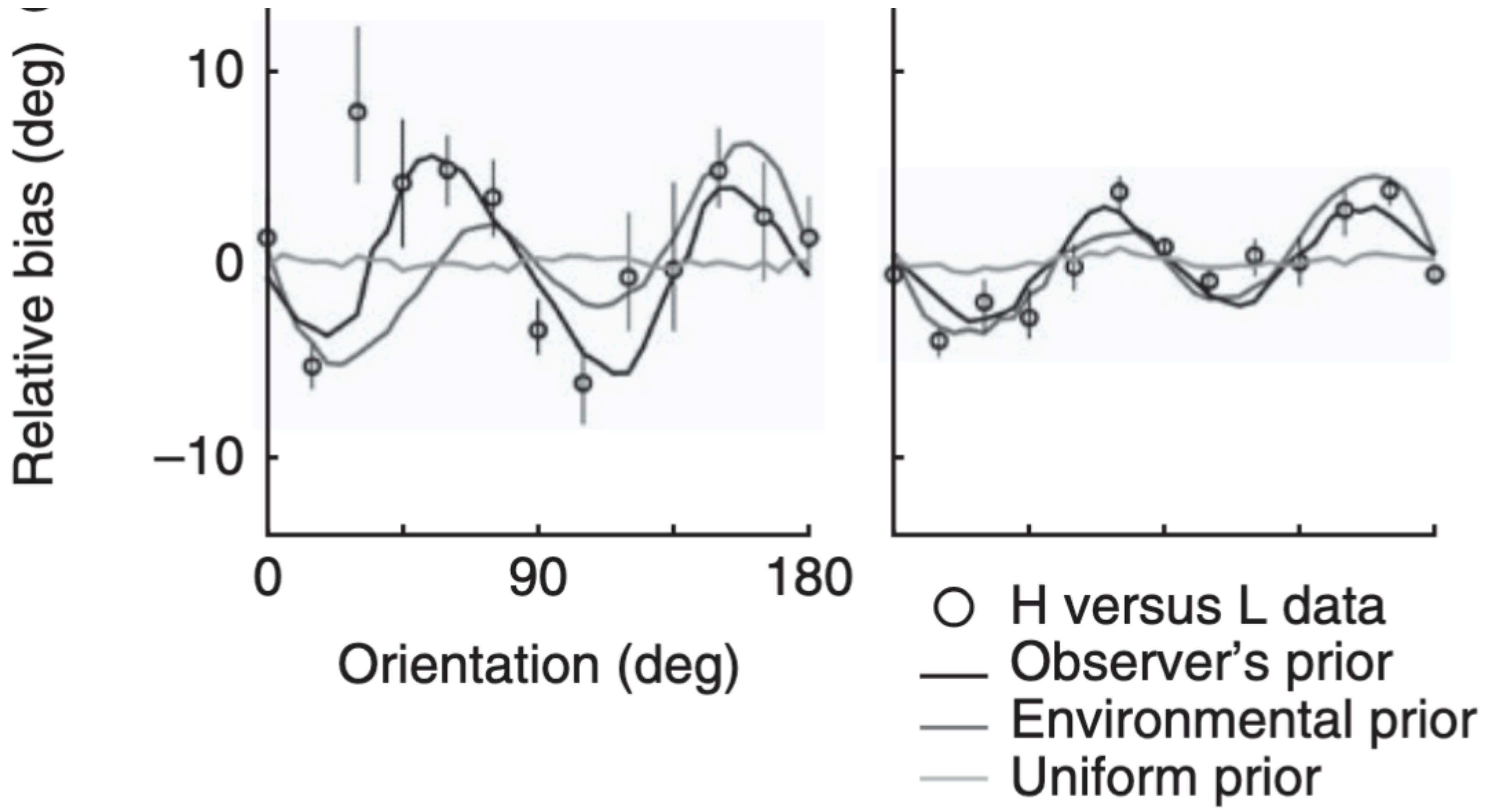
Cardinal rules: visual orientation perception reflects knowledge of environmental statistics

Ahna R Girshick[1,2], Michael S Landy[1,2] & Eero P Simoncelli[1–4]

Humans are good at performing visual tasks, but experimental measurements have revealed substantial biases in the perception of basic visual attributes. An appealing hypothesis is that these biases arise through a process of statistical inference, in which information from noisy measurements is fused with a probabilistic model of the environment. However, such inference is optimal only if the observer's internal model matches the environment. We found this to be the case. We measured performance in an orientation-estimation task and found that orientation judgments were more accurate at cardinal (horizontal and vertical) orientations. Judgments made under conditions of uncertainty were strongly biased toward cardinal orientations. We estimated observers' internal models for orientation and found that they matched the local orientation distribution measured in photographs. In addition, we determined how a neural population could embed probabilistic information responsible for such biases.

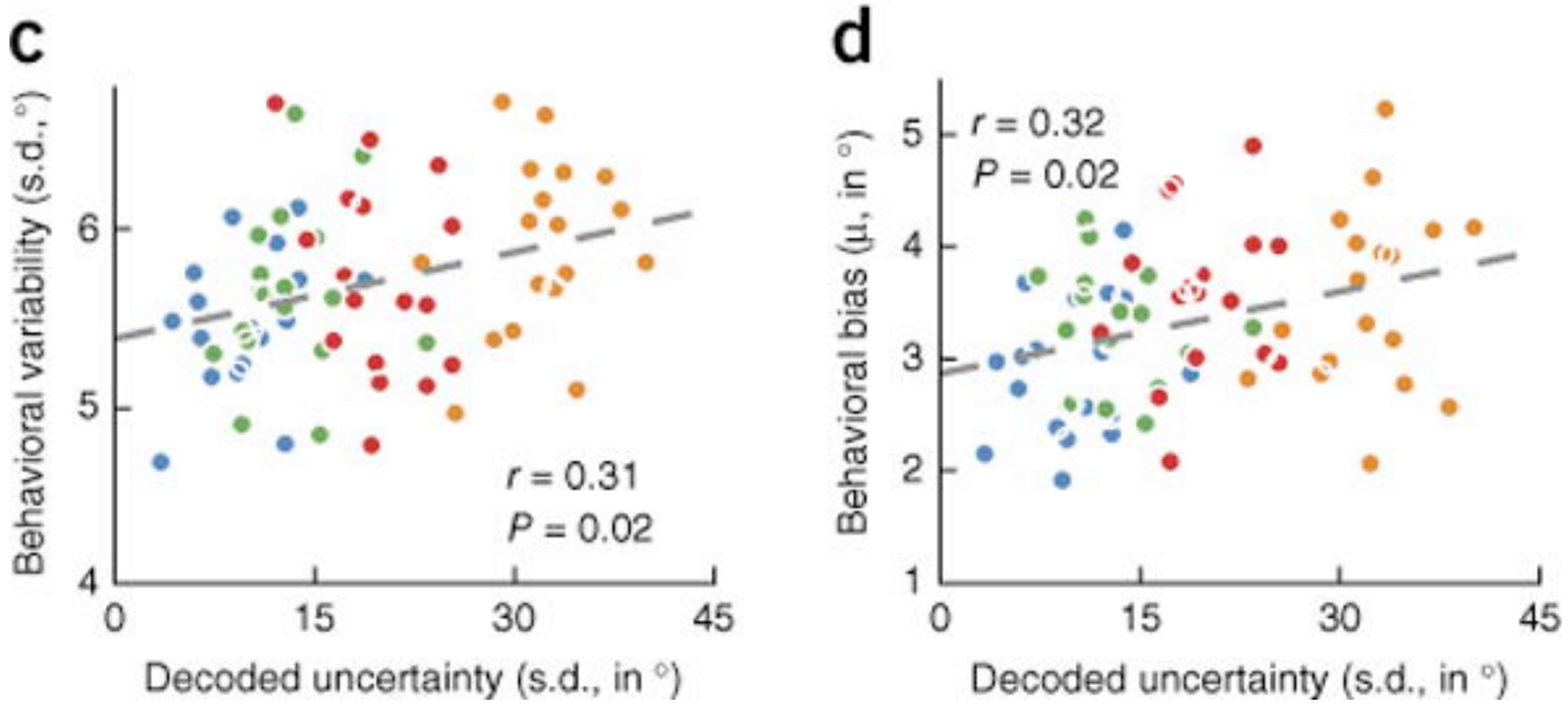
[Girshick, Landy & Simoncelli, 2011, Nature Neuroscience]





Empirical evidence from human neuroimaging

Example 1: **Sensory cortices encode uncertainty**



[van Bergen, Ji Ma, Pratte & Jehee, 2015, Nature Neuroscience]

nature > nature neuroscience > brief communications > article

Brief Communication | Published: 26 October 2015

Sensory uncertainty decoded from visual cortex predicts behavior

Ruben S van Bergen, Wei Ji Ma, Michael S Pratte & Janneke F M Jehee

Nature Neuroscience **18**, 1728–1730 (2015) | Cite this article

Abstract

Bayesian theories of neural coding propose that sensory uncertainty is represented by a probability distribution encoded in neural population activity, but direct neural evidence supporting this hypothesis is currently lacking. Using fMRI in combination with a generative model-based analysis, we found that probability distributions reflecting sensory uncertainty could reliably be estimated from human visual cortex and, moreover, that observers appeared to use knowledge of this uncertainty in their perceptual decisions.

Empirical evidence from human neuroimaging

Example 2: **Sensory cortices encode optimal priors**

A Neural Representation of Prior Information during Perceptual Inference

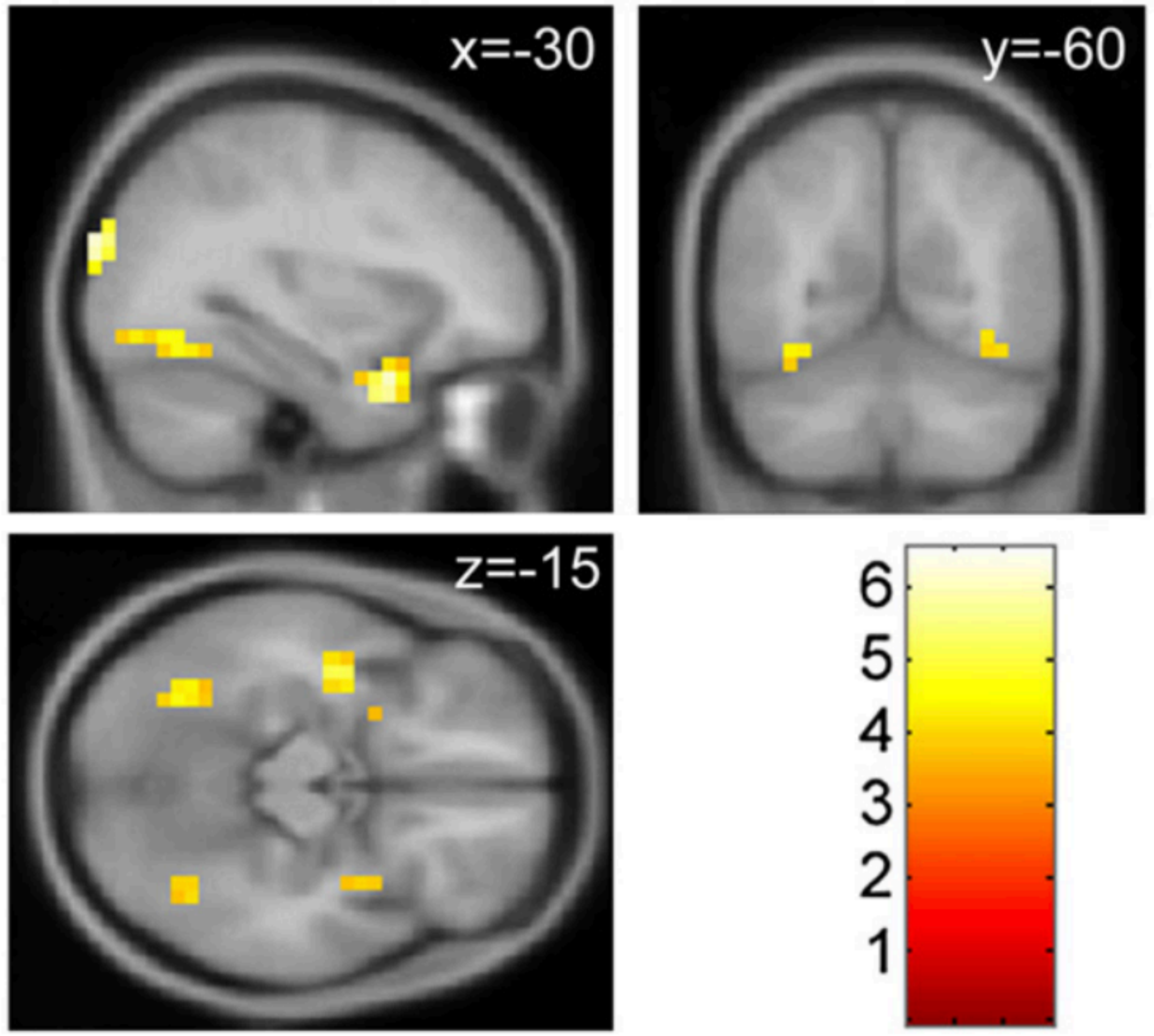
Christopher Summerfield[1,*] and Etienne Koechlin[1,2]
[1]Institut National de la Santé et de la Recherche Médicale, Département des Etudes Cognitives, Ecole Normale Supérieure, 29, rue d'Ulm, Paris 75005, France
[2]Center for NeuroImaging Research, University Pierre and Marie Curie (Paris 6), Groupe Hospitalier Pitie-Salpetriere, Paris 75005, France
*Correspondence: summerfd@paradox.columbia.edu
DOI 10.1016/j.neuron.2008.05.021

SUMMARY

Perceptual inference is biased by foreknowledge about what is probable or possible. How prior expectations are neurally represented during visual perception, however, remains unknown. We used functional magnetic resonance imaging to measure

elementary constancies that influence our perception of shape and color (Land, 1977), innate statistical biases in the processing of natural images (Kersten et al., 2004), as well as the effects of learning (Gilbert et al., 2001), local environmental context (Bar, 2004; Palmer, 1975), or task set (O'Craven et al., 1999) on the processing of visual objects and scenes.

Although it is well established that prior information influences perceptual inference, consensus has yet to emerge on how this



[Summerfield & Koechlin 2008, Neuron]

The human brain is not the only Bayesian brain

- Monkeys show reliability-weighted integration of motion cues (doi: 10.1523/JNEUROSCI.2574-09.2009)
- Rats integrate noisy sensory evidence consistently with optimal Bayesian evidence accumulation (doi: 10.1126/science.1233912)
- Coral-reef fish adapt visual discrimination decisions via Bayesian updating under uncertainty (doi: 10.1101/2022.08.28.505588)
- Bees use Bayesian-style spatial inference for navigation under noisy sensory input (doi: 10.1073/pnas.0408550102)
- Fish shoals make collective decisions via Bayesian estimation (doi: 10.1371/journal.pcbi.1002282)

The human brain is not the only Bayesian brain

- Spontaneous cortical activity reveals encodes an optimal internal model of the environment (10.1126/science.1195870)
- Synapses with short-term plasticity are optimal estimators of presynaptic membrane potentials (10.1038/nn.2640)
- Conductance-based dendrites implement Bayes-optimal cue integration (10.1371/journal.pcbi.1012047)
- Neural Variability encodes perceptual uncertain in the visual cortex (10.1016/j.neuron.2016.09.038)

The Bayesian Brain Hypothesis

1. First tenet: internal representations are probabilistic
2. Second tenet: cognition rests on priors
3. Third tenet: the brain performs Bayesian inference
4. Zeroth tenet: many aspects of cognition are optimal
5. Bayesian decision making
6. Empirical evidence
7. **Conclusion**

Take home message: the four tenets

0. Cognition was **evolutionary pressured to be optimal**

1. Our **internal representations are probabilistic** and have uncertainty
    - because ignoring uncertainty is dramatically suboptimal

2. Our internal representations of reality **depend on our priors**
    - because ignoring available information is dramatically suboptimal

3. The brain performs **Bayesian inference** to infer the latent causes of the observations
    - because not doing so leads to inconsistent reasoning

Some clinical applications

Several disorders are currently being studied under the Bayesian Brain framework:

- **Psychosis:** a disproportionate weight on the priors during evidence integration
- **Dyslexia:** incorrect estimates for the priors and/or the likelihood
- **Autism:** dysfunctional generation of priors that puts an overwhelming emphasis on the sensory input

Some open questions

- Not every behaviour is optimally Bayesian: which behaviours are? Why?
- How does the brain deal with computational complexity of the calculations of posteriors? How does it approximate inference?
- What are the neural mechanisms underlying inference? Is there a unified mechanisms, or a plethora of them?
- What are the internal generative models of the world we humans have? How much do they differ across individuals? What determines them?

Criticisms of the Bayesian Brain Hypothesis

There is still no consensus on whether the theory satisfactorily explains brain function

Three main families of criticisms / controversies:

- Theoretical criticisms
- Mechanistic / biological criticisms
- Empirical criticisms

Theoretical criticisms of the Bayesian Brain Hypothesis

- Normative optimality does guarantee that biological systems implement it
- Priors are hard to measure, constrain, or justify empirically
- Circular logic 1:
  - we don't know what approximations participants us so we derive them from the data
  - the approximations show that the brain approximates Bayesian inference
- Circular logic 2:
  - we don't know what loss functions the participants use so we fit them from the data
  - the fits show that the brain optimises that loss function

Mechanistic criticisms of the Bayesian Brain Hypothesis

- No consensus on how neurons represent full probability distributions
- No consensus on uncertainty-coding mechanisms
- Exact inference is intractable; approximate schemes introduce arbitrary choices
- Neural energy and time constraints challenge optimality assumptions

Empirical criticisms of the Bayesian Brain Hypothesis

- Humans show systematic non-Bayesian biases in many tasks
- Strong fits occur mainly in simple psychophysics, but not in naturalistic behaviour
- Bayesian models can overfit data by adjusting priors and noise assumptions
- Competing non-Bayesian models are often not rigorously compared

Conclusion

The Bayesian Brain Hypothesis is a highly influential **normative theory of cognition**

- It assumes the brain performs approximate optimal inference in many aspects of cognition
- It makes no claims on:
    - the exact approximations used to make inference tractable
    - the specific domains where cognition is optimal
    - the mechanisms implementing inference
    - the representations used to encode uncertainty
- There is behavioural and physiological **evidence in favour** of the theory
- The acceptance of the theory is **not fully established** in the neuroscientific community

Questions for next session

Questions for next session: Q3.1

According to the Bayesian Brain Hypothesis, our only access to reality is via our observations y. Can we be sure there's a reality, and that that reality is common to everyone else, without observing it?

Questions for next session: Q3.2

Calibration is a measure of the match between the uncertainty of a belief and the actual variability of the events. How could miscalibration affect perception? Find some situations in which humans are often miscalibrated?

Questions for next session: Q3.3

We use Gaussian distributions a lot because of two reasons: 1) they are easy to operate with; 2) they are maximum entropy distributions for continuous variables. Find out what (2) means and why that's a desirable property. What would be the maximum entropy distribution for a discrete variable? What's common between that distribution and the Gaussian distribution?

Questions for next session: Q3.4

If different individuals have different priors, whose perception is "correct"? Does the Bayesian framework imply that perception might be constructed? What would then be the role of our culture and upbringing in our perception?

Questions for next session: Q3.5

Where do our priors come from? Evolution? Development? Culture? Can we possibly determine what is the source of our priors?

Questions for next session: Q3.6

Some people model stereotyped beliefs as reasonings based on the wrong assumption that $P(A|B) = P(B|A)$. Try to use this model to explain some stereotypes. What are the limits for the model? Why would humans, if the Bayesian Brain Hypothesis is correct, often make that mistake (hint: start by considering when does $P(A|B) = P(B|A)$)?

Questions for next session: Q3.7

Bayes' rule is typically named "Bayes Theorem". Dig into the history of it and tell us who came up with it and why it has that name.

Questions for next session: Q3.8

Use Bayes Theorem to reason what would happen if we encounter something for which we have a prior of 0. Imagine for example that you have a p(seeing a unicorn | I) = 0 and you run into one. What implications has this result for our own understanding of perception?

Questions for next session: Q3.9

Bayesian statistics is the only optimal solution to the extended logic problem. What are the consequences of this for classical frequentist statistics?

Questions for next session: Q3.10

Visual illusions are usually explained as a result of Bayesian integration of priors with the sensory input. Find two that can be explained by it, and one that can't.

Questions for next session: Q3.11

Do Bayesian decision making processes always have a single optimal solution? If you think this is possible: Can you think of a problem where multiple optimal solutions exists? How could an agent take a decision in this case?

All course materials:

github.com/qtabs/compneuro4cogneuros