Kuantum Kaynak Teorilerine Giriş

MATEMATİKSEL FORMALİZM

$\langle$ QSB | KU $\rangle$

Dr. Onur Pusuluk

Koç Üniversitesi

10 Nisan 2021

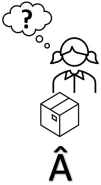
Gündem

- Fiziksel Sistemlerin Temsili
  - Kapalı Sistemler
  - Açık Sistemler
  - Bileşik Sistemler
- Fiziksel İşlemlerin Temsili
  - Tersinir İşlemler
  - Tersinmez İşlemler

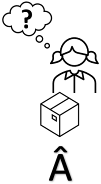
Ölçüm Problemi

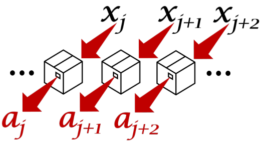
$\hat{A}$

Ölçüm Problemi

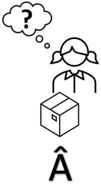


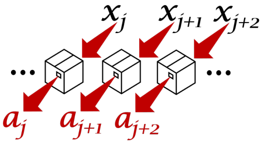
Ölçüm Problemi





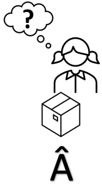
Ölçüm Problemi

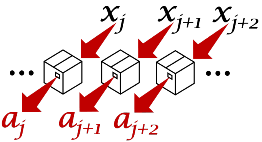




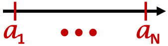
$$\langle \hat{\mathbf{A}} \rangle = \sum_i p_i \, a_i$$

Ölçüm Problemi

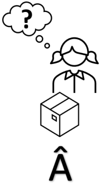


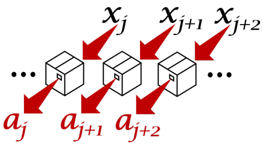


$$\langle \hat{\mathbf{A}} \rangle = \sum_i p_i \, a_i$$

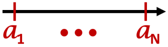


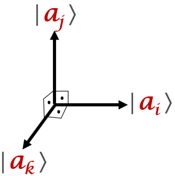
Ölçüm Problemi



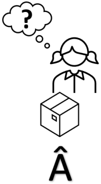


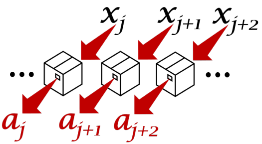
$$\langle \hat{\mathbf{A}} \rangle = \sum_i p_i \, a_i$$



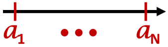


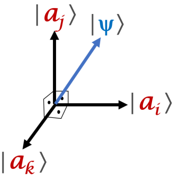
Ölçüm Problemi



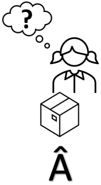


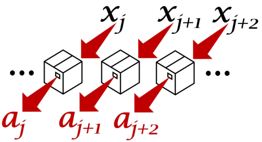
$$\langle \hat{\mathbf{A}} \rangle = \sum_i p_i \, a_i$$



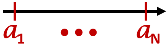


Ölçüm Problemi

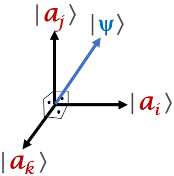




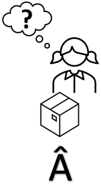
$$\langle \hat{\mathbf{A}} \rangle = \sum_i p_i \, a_i$$

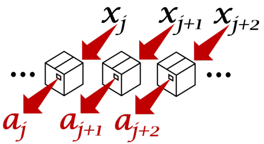


$$\hat{\mathbf{A}} \leftrightarrow \{a_i, |a_i\rangle\}$$

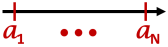


Ölçüm Problemi



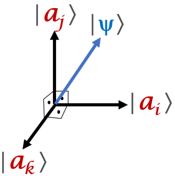


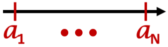
$$\langle\hat{\mathbf{A}}\rangle = \sum_i p_i\, a_i$$

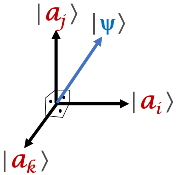


$$\hat{\mathbf{A}} \leftrightarrow \{a_i, |a_i\rangle\}$$

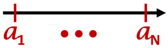
$$|\Psi\rangle \leftrightarrow \{p_i, |a_i\rangle\}$$



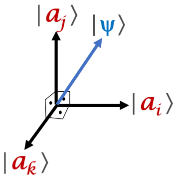

a_1
• • •
a_N


$|a_j\rangle$
$|\psi\rangle$
$|a_i\rangle$
$|a_k\rangle$

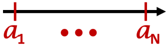
Fiziksel Gözlenebilirler



$$\boxed{\hat{\mathbf{A}} = \sum_i a_i |a_i\rangle\langle a_i|}$$

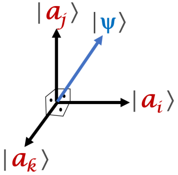


Fiziksel Gözlenebilirler

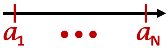


$$\hat{\mathbf{A}} = \sum_i a_i |a_i\rangle\langle a_i|$$

$$\langle a_i| \equiv (|a_i\rangle)^\dagger = \left((|a_i\rangle)^T\right)^*$$



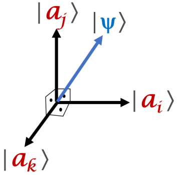
Fiziksel Gözlenebilirler



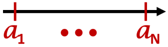
$$\hat{\mathbf{A}} = \sum_i a_i |a_i\rangle\langle a_i|$$

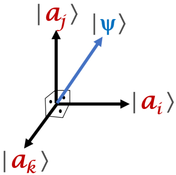
$$\langle a_i| \equiv (|a_i\rangle)^\dagger = \left((|a_i\rangle)^T\right)^*$$

$$\hat{\mathbf{A}}|a_i\rangle = a_i|a_i\rangle$$

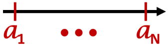


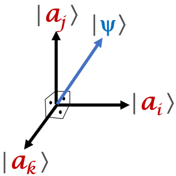
Fiziksel Durumlar



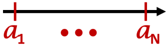


$$\boxed{|\Psi\rangle = \sum_i \Psi_i |a_i\rangle}$$



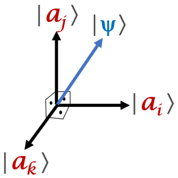


Fiziksel Durumlar

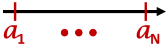


$$|\Psi\rangle = \sum_i \Psi_i |a_i\rangle$$

$$p_i \equiv p(a_i|\Psi) = |\Psi_i|^2$$



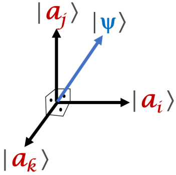
Fiziksel Durumlar



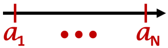
$$|\Psi\rangle = \sum_i \Psi_i |a_i\rangle$$

$$p_i \equiv p(a_i|\Psi) = |\Psi_i|^2$$

$$\rho = |\Psi\rangle\langle\Psi|$$



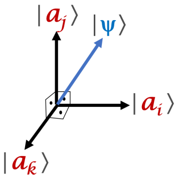
Fiziksel Durumlar



$$|\Psi\rangle = \sum_i \Psi_i |a_i\rangle$$

$$p_i \equiv p(a_i|\Psi) = |\Psi_i|^2$$

$$\rho = |\Psi\rangle\langle\Psi| \rightarrow \langle\hat{A}\rangle = \mathrm{tr}[\rho\,\hat{A}]$$



Gündem

Fiziksel Sistemlerin Temsili

- Kapalı Sistemler
- Açık Sistemler
- Bileşik Sistemler

Fiziksel İşlemlerin Temsili

- Tersinir İşlemler
- Tersinmez İşlemler

Saf Vs Karışık Durumlar

$$\{|0\rangle, |1\rangle\} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} , \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$$

Saf Vs Karışık Durumlar

$$\{|0\rangle, |1\rangle\} = \{\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}\}$$

- kuantum eşevreli üst üste binme

$$|\psi\rangle = \sum_{j=0}^{1} \psi_j |j\rangle = \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix}$$

Saf Vs Karışık Durumlar

$$\{|0\rangle, |1\rangle\} = \{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\}$$

- kuantum eşevreli üst üste binme

$$|\psi\rangle = \sum_{j=0}^{1} \psi_j |j\rangle = \begin{pmatrix}\psi_0\\ \psi_1\end{pmatrix}$$

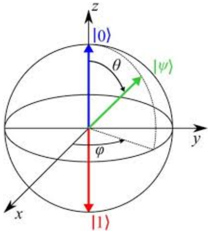
$$\rho = |\psi\rangle\langle\psi| = \begin{pmatrix}\psi_0\\ \psi_1\end{pmatrix}\begin{pmatrix}\psi_0^* & \psi_1^*\end{pmatrix} = \begin{pmatrix}|\psi_0|^2 & \psi_0\psi_1^*\\ \psi_0^*\psi_1 & |\psi_1|^2\end{pmatrix}$$

$$\{|0\rangle, |1\rangle\} = \{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\}$$

- kuantum eşevreli üst üste binme

$$|\psi\rangle = \sum_{j=0}^{1} \psi_j |j\rangle = \begin{pmatrix}\psi_0\\\psi_1\end{pmatrix}$$

$$\rho = |\psi\rangle\langle\psi| = \begin{pmatrix}\psi_0\\\psi_1\end{pmatrix} \begin{pmatrix}\psi_0^* & \psi_1^*\end{pmatrix} = \begin{pmatrix}|\psi_0|^2 & \psi_0\psi_1^*\\ \psi_0^*\psi_1 & |\psi_1|^2\end{pmatrix}$$



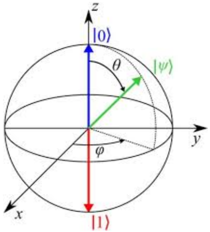
Saf Vs Karışık Durumlar

$$\{|0\rangle, |1\rangle\} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} , \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$$

- kuantum eşevreli üst üste binme

$$|\psi\rangle = \sum_{j=0}^{1} \psi_j |j\rangle = \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix}$$

$$\rho = |\psi\rangle\langle\psi| = \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix} \begin{pmatrix} \psi_0^* & \psi_1^* \end{pmatrix} = \begin{pmatrix} |\psi_0|^2 & \psi_0 \psi_1^* \\ \psi_0^* \psi_1 & |\psi_1|^2 \end{pmatrix}$$



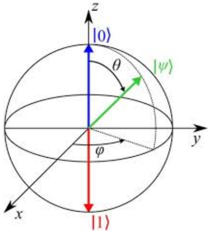
- kuantum eşevresiz üst üste binme

$$|\psi\rangle = \{q_0, |0\rangle; q_1, |1\rangle\}$$

Saf Vs Karışık Durumlar

$$\{|0\rangle, |1\rangle\} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$$

- kuantum eşevreli üst üste binme



$$|\psi\rangle = \sum_{j=0}^{1} \psi_j |j\rangle = \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix}$$

$$\rho = |\psi\rangle\langle\psi| = \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix} \begin{pmatrix} \psi_0^* & \psi_1^* \end{pmatrix} = \begin{pmatrix} |\psi_0|^2 & \psi_0\psi_1^* \\ \psi_0^*\psi_1 & |\psi_1|^2 \end{pmatrix}$$

- kuantum eşevresiz üst üste binme

$$|\psi\rangle = \{q_0, |0\rangle; q_1, |1\rangle\} \to \rho = \sum_{j=0}^{1} q_j |j\rangle\langle j| = \begin{pmatrix} q_0 & 0 \\ 0 & q_1 \end{pmatrix}$$

Gündem

Tensör Çarpımı

- $|\Psi\rangle_A = |\psi\rangle$
- $|\Psi\rangle_B = |\phi\rangle$

Tensör Çarpımı

- $|\Psi\rangle_A = |\psi\rangle$
- $|\Psi\rangle_B = |\phi\rangle$

$$|\Psi\rangle_{AB}$$

Tensör Çarpımı

- $|\Psi\rangle_A = |\psi\rangle$
- $|\Psi\rangle_B = |\phi\rangle$

$$|\Psi\rangle_{AB} \equiv |\psi\rangle \otimes |\phi\rangle$$

Tensör Çarpımı

- $|\Psi\rangle_A = |\psi\rangle$
- $|\Psi\rangle_B = |\phi\rangle$

$$\boxed{|\Psi\rangle_{AB} \equiv |\psi\rangle \otimes |\phi\rangle} = \begin{pmatrix} \psi_1 |\phi\rangle \\ \psi_2 |\phi\rangle \\ \vdots \\ \psi_n |\phi\rangle \end{pmatrix}$$

Tensör Çarpımı

- $|\Psi\rangle_A = |\psi\rangle$
- $|\Psi\rangle_B = |\phi\rangle$

$$\boxed{|\Psi\rangle_{AB} \equiv |\psi\rangle \otimes |\phi\rangle} = \begin{pmatrix} \psi_1 |\phi\rangle \\ \psi_2 |\phi\rangle \\ \vdots \\ \psi_n |\phi\rangle \end{pmatrix} = \begin{pmatrix} \psi_1 \phi_1 \\ \vdots \\ \psi_1 \phi_m \\ \psi_2 \phi_1 \\ \vdots \\ \psi_2 \phi_m \\ \vdots \\ \psi_n \phi_1 \\ \vdots \\ \psi_n \phi_m \end{pmatrix}$$

Tensör Çarpımı- Devam

$\rho \otimes \sigma$

Tensör Çarpımı- Devam

$$\rho \otimes \sigma = \begin{pmatrix} \rho_{11}\sigma & \cdots & \rho_{1n}\sigma \\ \vdots & \ddots & \vdots \\ \rho_{n1}\sigma & \cdots & \rho_{nn}\sigma \end{pmatrix}$$

Tensör Çarpımı- Devam

$$
\begin{aligned}
\rho \otimes \sigma &= \begin{pmatrix} \rho_{11}\sigma & \cdots & \rho_{1n}\sigma \\ \vdots & \ddots & \vdots \\ \rho_{n1}\sigma & \cdots & \rho_{nn}\sigma \end{pmatrix} \\
&= \begin{pmatrix}
\rho_{11}\sigma_{11} & \cdots & \rho_{11}\sigma_{1m} & \cdots & \rho_{1n}\sigma_{11} & \cdots & \rho_{1n}\sigma_{1m} \\
\vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\
\rho_{11}\sigma_{m1} & \cdots & \rho_{11}\sigma_{mm} & \cdots & \rho_{1n}\sigma_{m1} & \cdots & \rho_{1n}\sigma_{mm} \\
\vdots & & \vdots & \ddots & \vdots & & \vdots \\
\rho_{n1}\sigma_{11} & \cdots & \rho_{n1}\sigma_{1m} & \cdots & \rho_{nn}\sigma_{11} & \cdots & \rho_{nn}\sigma_{1m} \\
\vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\
\rho_{n1}\sigma_{m1} & \cdots & \rho_{n1}\sigma_{mm} & \cdots & \rho_{nn}\sigma_{m1} & \cdots & \rho_{nn}\sigma_{mm}
\end{pmatrix}
\end{aligned}
$$

Kısmi İz

$$\boxed{\rho_A \equiv \mathrm{tr}_B[\rho_{AB}]}$$

Kısmi İz

$$\boxed{\rho_A \equiv \mathrm{tr}_B[\rho_{AB}]} = \sum_j \left(\mathbb{I}^{(A)} \otimes \langle j_B|\right) \rho_{AB} \left(\mathbb{I}^{(A)} \otimes |j_B\rangle\right)$$

Gündem

Fiziksel Sistemlerin Temsili

- Kapalı Sistemler
- Açık Sistemler
- Bileşik Sistemler

Fiziksel İşlemlerin Temsili

- Tersinir İşlemler
- Tersinmez İşlemler

Birimcil/Üniter Operatörler

$$\boxed{|\psi(t)\rangle = U(t,t_0)|\psi(t_0)\rangle}$$

Birimcil/Üniter Operatörler

$$|\psi(t)\rangle = U(t,t_0)|\psi(t_0)\rangle$$

$$U^\dagger U = U U^\dagger = \mathbb{I}$$

Birimcil/Üniter Operatörler

$$|\psi(t)\rangle = U(t,t_0)|\psi(t_0)\rangle$$

$$U^\dagger U = U U^\dagger = \mathbb{I}$$

$$\sum_i p_i(t) = \langle\psi(t)|\psi(t)\rangle$$

Birimcil/Üniter Operatörler

$$\boxed{|\psi(t)\rangle = U(t,t_0)|\psi(t_0)\rangle}$$

$$\boxed{U^\dagger\, U = U\, U^\dagger = \mathbb{I}}$$

$$\begin{aligned}\sum_i p_i(t) &= \langle\psi(t)|\psi(t)\rangle \\ &= \big(\langle\psi(t_0)|U^\dagger(t,t_0)\big)\big(U(t,t_0)|\psi(t_0)\rangle\big)\end{aligned}$$

Birimcil/Üniter Operatörler

$$|\psi(t)\rangle = U(t,t_0)|\psi(t_0)\rangle$$

$$U^\dagger U = U U^\dagger = \mathbb{I}$$

$$\begin{aligned}\sum_i p_i(t) &= \langle\psi(t)|\psi(t)\rangle \\ &= \big(\langle\psi(t_0)|U^\dagger(t,t_0)\big)\big(U(t,t_0)|\psi(t_0)\rangle\big) \\ &= \langle\psi(t_0)|\big(U^\dagger(t,t_0)U(t,t_0)\big)|\psi(t_0)\rangle\end{aligned}$$

Birimcil/Üniter Operatörler

$$|\psi(t)\rangle = U(t,t_0)|\psi(t_0)\rangle$$

$$U^\dagger U = U U^\dagger = \mathbb{I}$$

$$\begin{aligned}
\sum_i p_i(t) &= \langle\psi(t)|\psi(t)\rangle \\
&= \big(\langle\psi(t_0)|U^\dagger(t,t_0)\big)\big(U(t,t_0)|\psi(t_0)\rangle\big) \\
&= \langle\psi(t_0)|\big(U^\dagger(t,t_0)U(t,t_0)\big)|\psi(t_0)\rangle \\
&= \langle\psi(t_0)|\psi(t_0)\rangle
\end{aligned}$$

Birimcil/Üniter Operatörler

$$|\psi(t)\rangle = U(t,t_0)|\psi(t_0)\rangle$$

$$U^\dagger\, U = U\, U^\dagger = \mathbb{I}$$

$$\begin{aligned}
\sum_i p_i(t) &= \langle\psi(t)|\psi(t)\rangle \\
&= \big(\langle\psi(t_0)|U^\dagger(t,t_0)\big)\big(U(t,t_0)|\psi(t_0)\rangle\big) \\
&= \langle\psi(t_0)|\big(U^\dagger(t,t_0)U(t,t_0)\big)|\psi(t_0)\rangle \\
&= \langle\psi(t_0)|\psi(t_0)\rangle = \sum_i p_i(t_0) = 1
\end{aligned}$$

Gündem

Fiziksel Sistemlerin Temsili
- Kapalı Sistemler
- Açık Sistemler
- Bileşik Sistemler

Fiziksel İşlemlerin Temsili
- Tersinir İşlemler
- Tersinmez İşlemler

Dinamik Gönderimler

$$
\begin{tikzcd}
\rho_{\mathcal{SE}}(t_0) = \rho_{\mathcal{S}}(t_0) \otimes \rho_{\mathcal{E}}(t_0) \arrow[r, "\text{üniter evrim}"] \arrow[d, "\mathrm{tr}_{\mathcal{E}}"'] & \rho_{\mathcal{SE}}(t) = U_{\mathcal{SE}}(t,t_0)\,\rho_{\mathcal{SE}}(t_0)\,U^{\dagger}_{\mathcal{SE}}(t,t_0) \arrow[d, "\mathrm{tr}_{\mathcal{E}}"] \\
\rho_{\mathcal{S}}(t_0) \arrow[r, "\text{dinamik gönderim}"'] & \rho_{\mathcal{S}}(t) = \mathcal{E}_t[\rho_{\mathcal{S}}(t_0)]
\end{tikzcd}
$$

Operatör-Toplam Temsili

Kraus Operatörleri

$$\rho_{\mathcal{S}}(t) = \mathcal{E}_t[\rho_{\mathcal{S}}(t_0)]$$

$$\rho_{\mathcal{S}}(t) = \mathcal{E}_t[\rho_{\mathcal{S}}(t_0)] = \sum_j K_j(t,t_0)\,\rho_{\mathcal{S}}(t_0)\,K_j^{\dagger}(t,t_0)$$

$$\boxed{\rho_{\mathcal{S}}(t) = \mathcal{E}_t[\rho_{\mathcal{S}}(t_0)] = \sum_j K_j(t,t_0)\,\rho_{\mathcal{S}}(t_0)\,K_j^\dagger(t,t_0)}$$

$$1 = \mathrm{tr}[\rho_{\mathcal{S}}(t)]$$

$$\boxed{\rho_{\mathcal{S}}(t) = \mathcal{E}_t[\rho_{\mathcal{S}}(t_0)] = \sum_j K_j(t,t_0)\,\rho_{\mathcal{S}}(t_0)\,K_j^\dagger(t,t_0)}$$

$$1 = \mathrm{tr}[\rho_{\mathcal{S}}(t)] = \mathrm{tr}\Big[\underbrace{\sum_j K_j^\dagger(t,t_0)\,K_j(t,t_0)}_{\mathbb{I}}\;\rho_{\mathcal{S}}(t_0)\Big]$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{\mathcal{S}}(t) = \mathcal{L}_t\left[\rho_{\mathcal{S}}(t)\right]$$

$$= -\frac{i}{\hbar}[H_{\mathcal{S}} + H'_{\mathcal{S}}, \rho_{\mathcal{S}}(t)] + \sum_{j=0}^{N\times N} \gamma_j \left(A_j\, \rho_{\mathcal{S}}(t)\, A_j^{\dagger} - \frac{1}{2}\{A_j^{\dagger} A_j, \rho_{\mathcal{S}}(t)\}\right)$$

$$\frac{d\rho_{\mathcal{S}}}{dt} = -\frac{\mathrm{i}}{\hbar}[H_{\mathcal{S}} + \hbar H_{LS}, \rho_{\mathcal{S}}] + \mathcal{D}(\rho_{\mathcal{S}})$$

- $H_{LS} = \sum_{\omega} \sum_{j,j'} S_{jj'}(\omega)\, A_j^{\dagger}(\omega)\, A_{j'}(\omega)$,
- $\mathcal{D}(\rho_{\mathcal{S}}) = \sum_{\omega} \sum_{j,j'} \gamma_{jj'}(\omega) \left(A_{j'}(\omega)\rho_{\mathcal{S}} A_j^{\dagger}(\omega) - \frac{1}{2}\{A_j^{\dagger}(\omega)\, A_{j'}(\omega), \rho_{\mathcal{S}}\}\right)$,
- $A_j(\omega) = \sum_{\epsilon_m - \epsilon_{m'} = \omega} |\epsilon_{m'}\rangle\langle\epsilon_{m'}|A_{\alpha}|\epsilon_m\rangle\langle\epsilon_m|$.


Fin