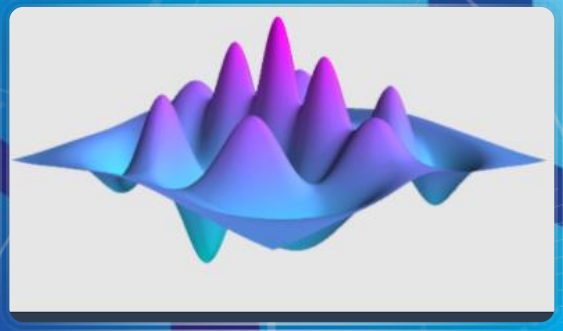

Qutip Introduction for Quantum Information Theory

Presented by:

Mohsen Izadyari

KOÇ UNIVERSITY

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Spring 2021

http://qutip.org/

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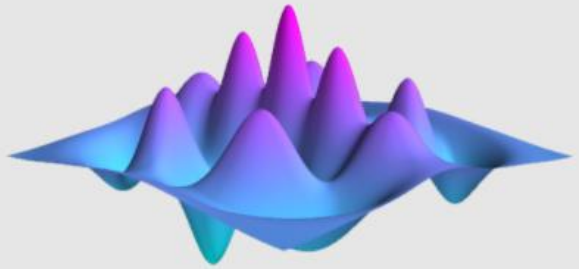

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We hope you enjoy using QuTiP. Please help us make QuTiP better by citing it in your publications.

A Technical Staff position for QuTiP is available, check it out under the jobs page.

QuTiP is open-source software for simulating the dynamics of open quantum systems. The QuTiP library depends on the excellent Numpy, Scipy, and Cython numerical packages. In addition, graphical output is provided by Matplotlib. QuTiP aims to provide user-friendly and efficient numerical simulations of a wide variety of Hamiltonians, including those with arbitrary time-dependence, commonly found in a wide range of physics applications such as quantum optics, trapped ions, superconducting circuits, and quantum nanomechanical resonators. QuTiP is freely available for use and/or modification on all major platforms such as Linux, Mac OSX, and Windows*. Being free of any licensing fees, QuTiP is ideal for exploring quantum mechanics and dynamics in the classroom.

*QuTiP is developed on Unix platforms only, and some features may not be available under Windows.

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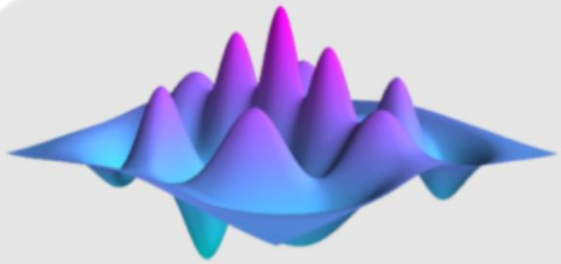

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Users guide

Latest release

Version 4.5

Online HTML documentation

PDF documentation

BASIC OPERATIONS ON QUANTUM OBJECTS

The key difference between classical and quantum mechanics lies in the use of operators instead of numbers as variables.

in computing the dynamics of quantum systems we need a data structure that is capable of encapsulating the properties of a quantum operator and ket/bra vectors. The quantum object class, qutip.Qobj, accomplishes this using matrix representation.

```
In [3]: Qobj()
Out[3]:
Quantum object: dims = [[1], [1]], shape = [1, 1], type = oper, isherm = True
Qobj data =
[[ 0.]]
```

```
In [4]: Qobj([[1],[2],[3],[4],[5]])
Out[4]:
Quantum object: dims = [[5], [1]], shape = [5, 1], type = ket
Qobj data =
[[ 1.]
 [ 2.]
 [ 3.]
 [ 4.]
 [ 5.]]

In [5]: x = np.array([[1, 2, 3, 4, 5]])

In [6]: Qobj(x)
Out[6]:
Quantum object: dims = [[1], [5]], shape = [1, 5], type = bra
Qobj data =
[[ 1.  2.  3.  4.  5.]]
```

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States	Command (# means optional)	Inputs
Fock state ket vector	`basis(N,#m)` / `fock(N,#m)`	N = number of levels in Hilbert space, m = level containing excitation (0 if no m given)
Fock density matrix (outer product of basis)	`fock_dm(N,#p)`	same as basis(N,m) / fock(N,m)
Coherent state	`coherent(N,alpha)`	alpha = complex number (eigenvalue) for requested coherent state
Coherent density matrix (outer product)	`coherent_dm(N,alpha)`	same as coherent(N,alpha)
Thermal density matrix (for n particles)	`thermal_dm(N,n)`	n = particle number expectation value

Operators	Command (# means optional)	Inputs
Identity	`qeye(N)`	N = number of levels in Hilbert space.
Lowering (destruction) operator	`destroy(N)`	same as above
Raising (creation) operator	`create(N)`	same as above
Number operator	`num(N)`	same as above
Single-mode displacement operator	`displace(N,alpha)`	N=number of levels in Hilbert space, alpha = complex displacement amplitude.
Single-mode squeezing operator	`squeeze(N,sp)`	N=number of levels in Hilbert space, sp = squeezing parameter.
Sigma-X	`sigmax()`	
Sigma-Y	`sigmay()`	
Sigma-Z	`sigmaz()`	
Sigma plus	`sigmap()`	
Sigma minus	`sigmam()`	

EXPECTATION VALUES

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$s_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\langle s_z \rangle_0 = (1 \quad 0) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$$

$$\langle s_z \rangle_1 = (0 \quad 1) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -1$$

```
In [72]: up = basis(2, 0)

In [73]: down = basis(2, 1)

In [74]: expect(sigmaz(), up)
Out[74]: 1.0

In [75]: expect(sigmaz(), down)
Out[75]: -1.0
```

TENSOR PRODUCTS

To describe the states of multipartite quantum systems - such as two coupled qubits, a qubit coupled to an oscillator, etc. - we need to expand the Hilbert space by taking the tensor product of the state vectors for each of the system components.

```
In [1]: tensor(basis(2, 0), basis(2, 0))
Out[1]:
Quantum object: dims = [[2, 2], [1, 1]], shape = (4, 1), type = ket
Qobj data =
[[1.]
 [0.]
 [0.]
 [0.]]
```

$$|0> \otimes |0>$$

For example, the state vector describing two qubits in their ground states is formed by taking the tensor product of the two single-qubit ground state vectors:

```
In [4]: tensor(sigmax(), sigmax())
Out[4]:
Quantum object: dims = [[2, 2], [2, 2]], shape = (4, 4), type = oper, isherm = True
Qobj data =
[[0. 0. 0. 1.]
 [0. 0. 1. 0.]
 [0. 1. 0. 0.]
 [1. 0. 0. 0.]]
```

$$\sigma_x \otimes \sigma_x$$

Partial trace

The partial trace is an operation that reduces the dimension of a Hilbert space by eliminating some degrees of freedom by averaging (tracing).

For example, the density matrix describing a single qubit obtained from a coupled two-qubit system is obtained via:

Example:

The two-qubit spin singlet $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ corresponds to the density matrix

$$|\psi^-\rangle\langle\psi^-| = \frac{1}{2}(|01\rangle\langle 01| - |01\rangle\langle 10| - |10\rangle\langle 01| + |10\rangle\langle 10|),$$

on which we act with the partial trace acording to the definition (1)

$$\begin{aligned}\mathrm{tr}_B|\psi^-\rangle\langle\psi^-| &= \frac{1}{2}(|0\rangle\langle 0|\,\langle 1\,|1\rangle - |0\rangle\langle 1|\,\langle 0\,|1\rangle - |1\rangle\langle 0|\,\langle 1\,|0\rangle + |1\rangle\langle 1|\,\langle 0\,|0\rangle)\\ &= \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|),\end{aligned}$$

```
In [70]: psi
Out[70]:
Quantum object: dims = [[2, 2], [1, 1]], shape = (4, 1), type = ket
Qobj data =
[[ 0.        ]
 [-0.70710678]
 [ 0.70710678]
 [ 0.        ]]

In [71]: psi.ptrace(0)
Out[71]:
Quantum object: dims = [[2], [2]], shape = (2, 2), type = oper, isherm = True
Qobj data =
[[0.5 0. ]
 [0.  0.5]]

In [72]: psi.ptrace(1)
Out[72]:
Quantum object: dims = [[2], [2]], shape = (2, 2), type = oper, isherm = True
Qobj data =
[[0.5 0. ]
 [0.  0.5]]
```

The partial trace always results in a density matrix (mixed state), regardless of whether the composite system is a pure state (described by a state vector) or a mixed state (described by a density matrix)

QUANTUM INFORMATION TOOLS IN QUTIP

Name	Equation	Qutip Command
Von Neumann entropy	$H = -tr(\rho ln \rho)$ $= -\Sigma \lambda_i ln \lambda_i$	**entropy_vn**(rho)
Linear entropy of a density matrix	$S_L = 1 - tr(\rho^2)$	***entropy_linear**(rho)
Mutual information	$I(A:B) = H(A) + H(B) - H(AB)$	**entropy_mutual**(rho, selA, selB)
Concurrence entanglement	$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$	**concurrence**(rho)
Negativity	$N(\rho) = \frac{\left\|\left\|\rho^{\Gamma_A}\right\|\right\|_1 - 1}{2}$	**negativity**(rho, subsys)

*The linear entropy can range between zero, corresponding to a completely pure state, and $(1 - 1/d)$, corresponding to a completely mixed state. (Here, d is the dimension of the density matrix.)

TWO-QUBIT SYSTEM

$\psi_1 = |00>$

$\psi_2 = \frac{1}{\sqrt{2}}(|00> + |11>)$

$\rho_3 = \frac{1}{\sqrt{2}}(|0><0| \otimes |0><0| + |+><+| \otimes |1><1|)$

Quantum State	ψ_1	ψ_2	ρ_3
Von Neumann entropy	0	0	0
Linear entropy of a density matrix	0	0	0.5
Concurrence entanglement	0	0.999	0
	Pure separable state	Pure entangled state	Mixed state

QUANTUM DISCORD

In quantum information theory, **quantum discord** is a measure of nonclassical correlations between two subsystems of a quantum system. It includes correlations that are due to quantum physical effects but do not necessarily involve quantum entanglement.

The quantum discord, a measure of the quantumness of correlations, of a state ρ_{AB} under a von Neumann measurement $\{\Pi_a\}$ is defined as a difference between the quantum forms of mutual information,

$$\begin{aligned} \mathcal{D} &= I(A:B) - J(A:B) \\ &= S(\rho_A) - S(\rho_{AB}) + \min_{\{\Pi_a\}} \sum_a p_a S(\rho_{B|a}), \end{aligned} \tag{0.1}$$

where the minimization is over all local projectors.

Example: Consider the following state

$$\rho_{AB} = p\,|0\rangle\langle 0| \otimes |0\rangle\langle 0| + (1-p)\,|+\rangle\langle +| \otimes |1\rangle\langle 1|$$

where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.

$$\rho_{B|a} = \mathrm{tr}_A[(\Pi_a \otimes I)\rho_{AB}(\Pi_a^\dagger \otimes I)]/p_a, \quad \left(p_a = \mathrm{tr}[(\Pi_a \otimes I)\rho_{AB}(\Pi_a^\dagger \otimes I)]\right)$$

where von Neumann measurements are given by $\Pi_1 = |0\rangle\langle 0|$ and $\Pi_2 = |1\rangle\langle 1|$. Then we have

$$p_1 = \mathrm{tr}[(\Pi_1 \otimes I)\rho_{AB}(\Pi_1^\dagger \otimes I)] = \mathrm{tr}[p\,|0\rangle\langle 0| \otimes |0\rangle\langle 0| + \frac{(1-p)}{2}\,|0\rangle\langle 0| \otimes |1\rangle\langle 1|] = \frac{(1+p)}{2}$$

$$p_2 = \mathrm{tr}[(\Pi_2 \otimes I)\rho_{AB}(\Pi_2^\dagger \otimes I)] = \mathrm{tr}[\frac{(1-p)}{2}\,|1\rangle\langle 1| \otimes |1\rangle\langle 1|] = \frac{(1-p)}{2}$$

$$\begin{aligned}
\rho_{B|a=1} &= \mathrm{tr}_A[(\Pi_1 \otimes I)\rho_{AB}(\Pi_1^\dagger \otimes I)]/p_1 \\
&= \mathrm{tr}_A[p\,|0\rangle\langle 0| \otimes |0\rangle\langle 0| + \frac{(1-p)}{2}\,|0\rangle\langle 0| \otimes |1\rangle\langle 1|]/p_1 \\
&= [p\,|0\rangle\langle 0| + \frac{(1-p)}{2} \otimes |1\rangle\langle 1|]/(\frac{1+p}{2}) \qquad (0.2)
\end{aligned}$$

$$\begin{aligned}
\rho_{B|a=2} &= \mathrm{tr}_A[(\Pi_2 \otimes I)\rho_{AB}(\Pi_2^\dagger \otimes I)]/p_2 \\
&= \mathrm{tr}_A[\frac{(1-p)}{2}\,|1\rangle\langle 1| \otimes |1\rangle\langle 1|]/p_2 \\
&= |1\rangle\langle 1| \qquad (0.3)
\end{aligned}$$

$$
\begin{aligned}
\rho_A &= \mathrm{tr}_B(\rho_{AB}) \\
&= \mathrm{tr}_B(p\,|0\rangle\langle 0| \otimes |0\rangle\langle 0| + \frac{(1-p)}{2}\Big((|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|) \otimes |1\rangle\langle 1|\Big)) \\
&= p\,|0\rangle\langle 0| + \frac{(1-p)}{2}(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|) \\
&= \frac{(1+p)}{2}\,|0\rangle\langle 0| + \frac{(1-p)}{2}(|0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|) \qquad (0.5)
\end{aligned}
$$

Then we have

$$\text{Eigenvalues of } \rho_A : \left\{\frac{1}{2}\left(1-\sqrt{2p^2-2p+1}\right), \frac{1}{2}\left(\sqrt{2p^2-2p+1}+1\right)\right\}$$

$$\text{Eigenvalues of } \rho_{AB} : \{0, 0, 1-p, p\}$$

$$\text{Eigenvalues of } \rho_{B|a=1} : \left\{\frac{1-p}{p+1}, \frac{2p}{p+1}\right\}$$

$$\text{Eigenvalues of } \rho_{B|a=2} : \{1, 0\}$$

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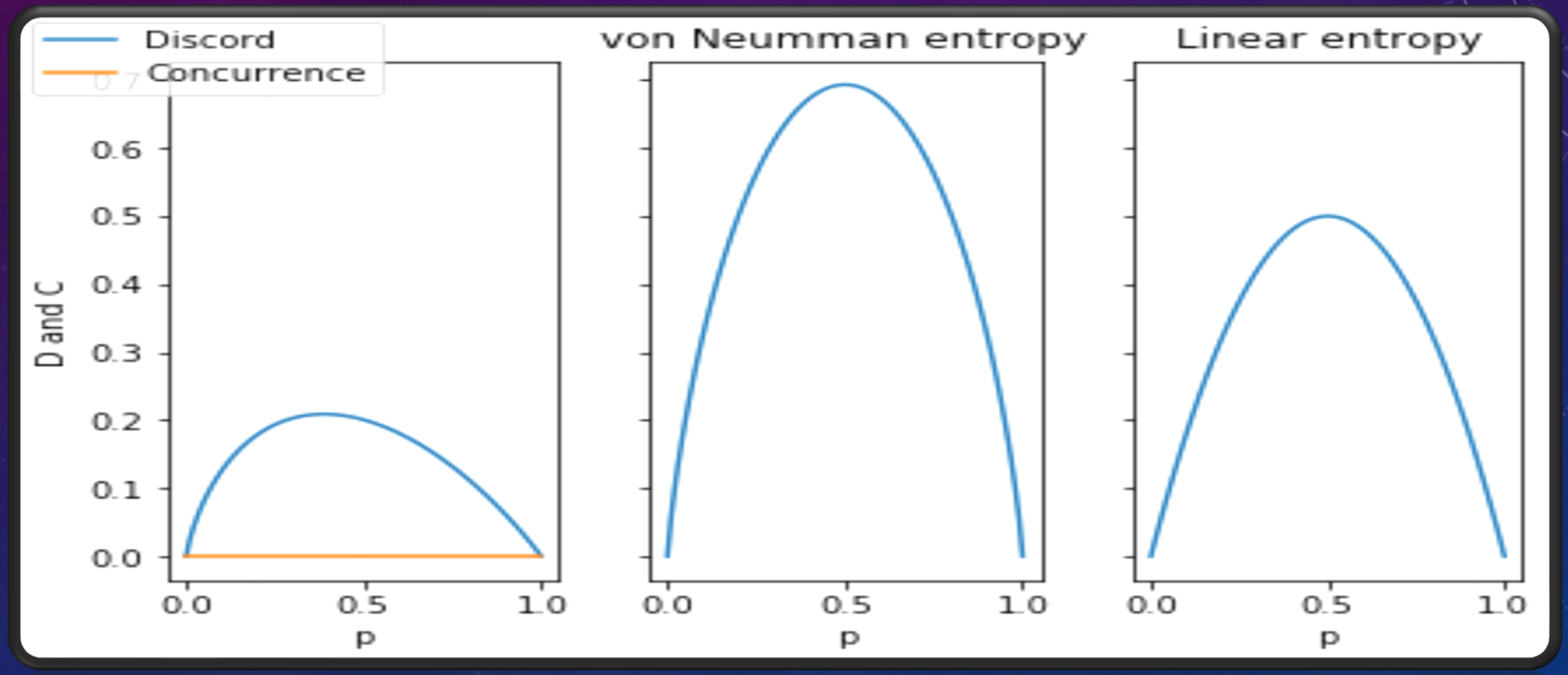
$$
\begin{aligned}
\mathcal{D} &= I(A:B) - J(A:B) \\
&= S(\rho_A) - S(\rho_{AB}) + \min_{\{\Pi_a\}} \sum_a p_a S(\rho_{B|a}) \\
&= -\Big[\frac{1}{2}\left(1-\sqrt{2p^2-2p+1}\right)\log\frac{1}{2}\left(1-\sqrt{2p^2-2p+1}\right) \\
&\quad +\frac{1}{2}\left(1+\sqrt{2p^2-2p+1}\right)\log\frac{1}{2}\left(1+\sqrt{2p^2-2p+1}\right)\Big] \\
&\quad +\Big[(1-p)\log(1-p)+p\log p\Big] \\
&\quad +p_1\left(-\frac{1-p}{p+1}\log(\frac{1-p}{p+1})-\frac{2p}{p+1}\log(\frac{2p}{p+1})\right)+p_2\big(-1\log 1\big) \qquad (0.6)
\end{aligned}
$$

$$\rho_{AB} = \mathrm{p}|0\rangle\langle 0|\otimes|0\rangle\langle 0| + (1-p)|+\rangle\langle +|\otimes|1\rangle\langle 1|$$



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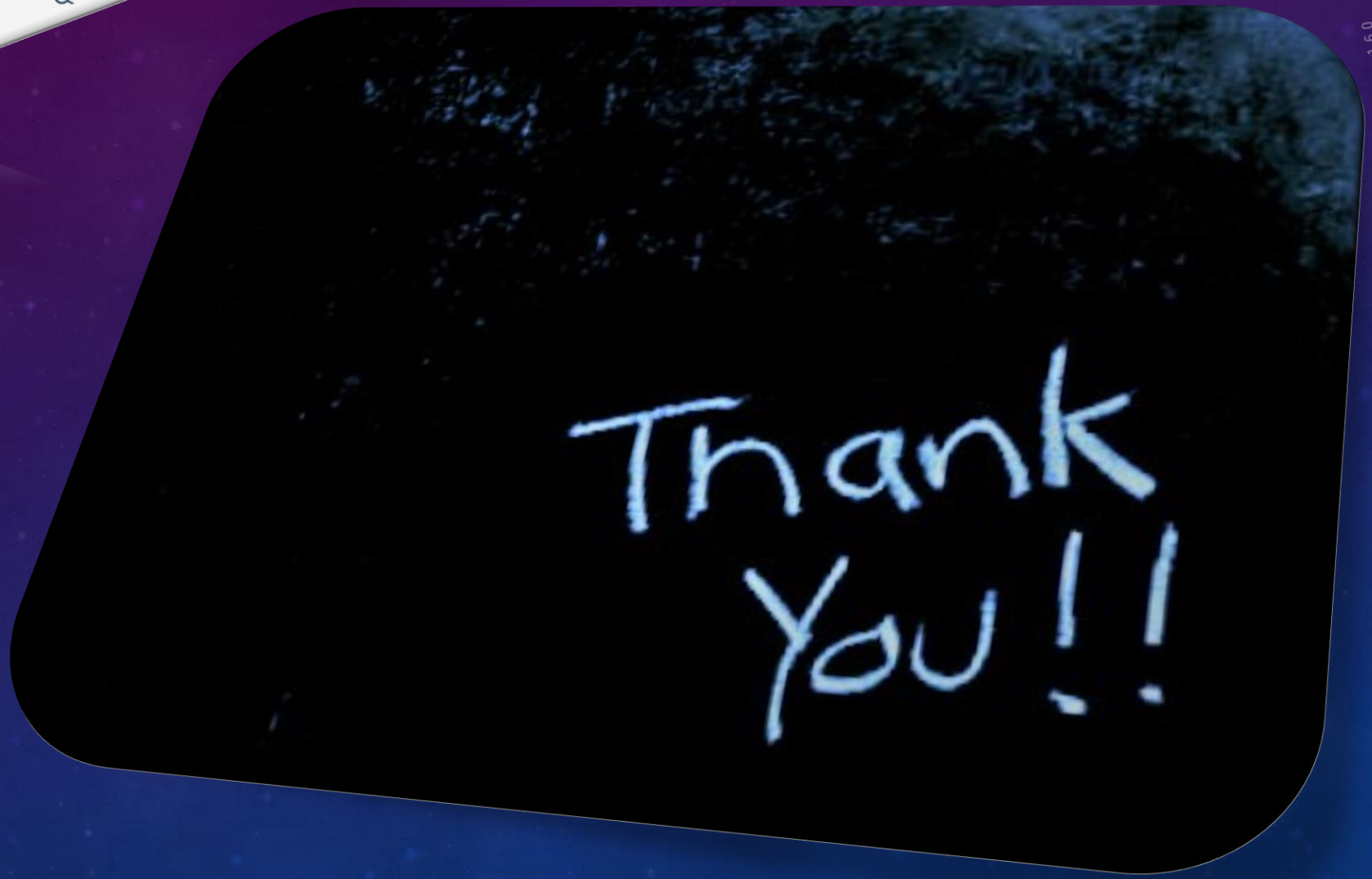

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10 Nisan	*KUANTUM KAYNAK TEORİLERİNE GİRİŞ*
14.00-14.50	**Matematiksel Formalizm**
14.50-15.40	**Kuantum Üst Üste Binme Kuantum Eşevrelilik**
15.50-16.40	**Kuantum Dolaşıklık**
16.40-17.30	**Kuantum Uyumsuzluk**
17.30-18.15	**QuTiP'e Giriş**

11 Nisan	*KUANTUM TEKNOLOJİLERİNE ÖRNEKLER*
14.00-14.50	**Kuantum Enformasyon ve Hesaplama**
14.50-15.40	**Kuantum Termodinamik**
15.50-16.40	**Kuantum Metroloji**
16.40-17.30	**Kuantum Biyoloji**

Eğitimler Dr.Onur Pusuluk, Dr. Gökhan Torun ve Mohsen Izadyari tarafından verilecektir.

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