KUANTUM TEKNOLOJİLERİ & KAYNAK TEORİLERİ
⟨ QSB | KU ⟩

10 Nisan	*KUANTUM KAYNAK TEORİLERİNE GİRİŞ*
14.00-14.50	**Matematiksel Formalizm**
14.50-15.40	**Kuantum Üst Üste Binme** **Kuantum Eşevrelilik**
15.50-16.40	**Kuantum Dolaşıklık**
16.40-17.30	**Kuantum Uyumsuzluk**
17.30-18.15	**QuTiP'e Giriş**

11 Nisan	*KUANTUM TEKNOLOJİLERİNE ÖRNEKLER*
14.00-14.50	**Kuantum Enformasyon ve Hesaplama**
14.50-15.40	**Kuantum Termodinamik**
15.50-16.40	**Kuantum Metroloji**
16.40-17.30	**Kuantum Biyoloji**

Eğitimler Dr.Onur Pusuluk, Dr. Gökhan Torun ve Mohsen Izadyari tarafından verilecektir. ⟨Q|Turkey⟩

Dr. Gökhan Torun — Koç Üniversitesi — 11 Nisan 2021

1st round:

Kuantum Enformasyon ve Hesaplama

“Quantum computing is the use of quantum effects such as quantum coherence, superposition, and entanglement to perform computation.”

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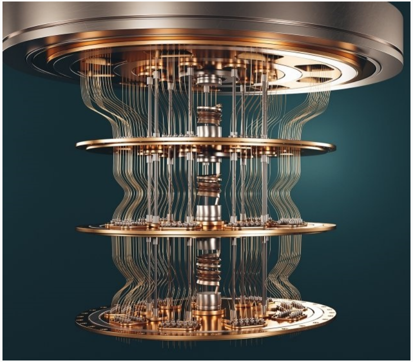


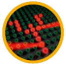
“The theory of computation has traditionally been studied almost entirely in the abstract, as a topic in pure mathematics. This is to miss the point of it. Computers are physical objects, and computations are physical processes. What computers can or cannot compute is determined by the laws of physics alone, and not by pure mathematics.” – David Deutsch

(taken from kuantumturkiye.org/kthack-2020/quantum-computer/)

Atomic quantum clocks can be synchronised with GPS to provide very high levels of timing stability and traceability, even in hostile environments where GPS is unavailable o denied. These timing solutions can be useful within future smart networks, for instance for the synchronization of energy grids, as well as in telecoms, broadcasting, energy and security.

Quantum sensors that exploit quantum superposition and/or entanglement to achieve higher sensitivity and resolution will be purchased and used by companies and public institutions for demanding construction projects; for instance, to measure voids under the ground and to detect mineral deposits or legacy infrastructure. They will also be use to provide non-invasive point-of-care diagnosis.

A secure **intercity quantum link** between a number of European capitals will allow transmission of highly sensitive data without any risk of interception. It may contain ground or satellite-based protected nodes derived from the development of trusted nodes and quantum repeaters.



Quantum simulators can be constructed for the special purpose of simulating materials or chemical reactions. Simulation allows new processes or properties to be explored before the material exists, as a tool to design new materials that are needed in multiple sectors, such as energy or transport.

A global **quantum-safe communication network** – a quantum internet combining quantum with classical information and encryption – offers security for internet transactions against the threat of a quantum computer breaking purely classical encryption schemes.

Universal quantum computers will be available with computational power at a level of performance that will exceed even the most powerful classical computers of the future. They will be reprogrammable machines used to solve demanding computational problems, such as optimisation tasks, database searches, machine learning and image recognition. They will contribute to Europe's smart industry, helping to make European manufacturing industries more efficient.

PROCEEDINGS OF THE ROYAL SOCIETY A | MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Rapid solution of problems by quantum computation

By David Deutsch[1] and Richard Jozsa[2]†

[1] *Wolfson College, Oxford OX2 6UD, U.K.*
[2] *St Edmund Hall, Oxford OX1 4AR, U.K.*

A class of problems is described which can be solved more efficiently by quantum computation than by any classical or stochastic method. The quantum computation solves the problem with certainty in exponentially less time than any classical deterministic computation.

"In the Deutsch–Jozsa problem, we are given a black box quantum computer known as an oracle that implements some function $f : \{0, 1\}^n \to \{0, 1\}$. The function takes n-digit binary values as input and produces either a 0 or a 1 as output for each such value. We are promised that the function is either constant (0 on all outputs or 1 on all outputs) or balanced (returns 1 for half of the input domain and 0 for the other half) The task then is to determine if f is constant or balanced by using the oracle." (en.wikipedia.org/wiki/Deutsch-Jozsa-algorithm)

PROCEEDINGS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

The Royal Society

Quantum algorithms revisited

By R. Cleve[1], A. Ekert[2], C. Macchiavello[2,3] and M. Mosca[2,4]

[1] *Department of Computer Science, University of Calgary, Calgary, Alberta, Canada T2N 1N4*

[2] *Clarendon Laboratory, Department of Physics, University of Oxford, Parks Road, Oxford OX1 3PU, UK*

[3] *I.S.I. Foundation, Villa Gualino, Viale Settimio Severo 65, 1033 Torino, Italy*

[4] *Mathematical Institute, University of Oxford, 24–29 St. Giles', Oxford OX1 3LB, UK*

Quantum computers use the quantum interference of different computational paths to enhance correct outcomes and suppress erroneous outcomes of computations. A common pattern underpinning quantum algorithms can be identified when quantum computation is viewed as multiparticle interference. We use this approach to review (and improve) some of the existing quantum algorithms and to show how they are related to different instances of quantum phase estimation. We provide an explicit algorithm for generating any prescribed interference pattern with an arbitrary precision.

Keywords: quantum computation; quantum factoring; quantum networks; quantum algorithms; quantum phase estimation

Hadamard	$\boxed{H}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$
Phase	$\boxed{S}$	$\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$
$\pi/8$	$\boxed{T}$	$\begin{bmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{bmatrix}$
Pauli-X	$\boxed{X}$	$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$
Pauli-Y	$\boxed{Y}$	$\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$
Pauli-Z	$\boxed{Z}$	$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$

measurement		Projection onto $\|0\rangle$ and $\|1\rangle$
qubit	———	wire carrying a single qubit (time goes left to right)
classical bit	$=\!=\!=$	wire carrying a single classical bit
n qubits	$—/^{n}—$	wire carrying n qubits

Hadamard	$-\boxed{H}-$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
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Pauli-X	$-\boxed{X}-$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y	$-\boxed{Y}-$	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z	$-\boxed{Z}-$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$\alpha|0\rangle + \beta|1\rangle \;-\!\!\boxed{X}\!\!-\; \beta|0\rangle + \alpha|1\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \;-\!\!\boxed{Z}\!\!-\; \alpha|0\rangle - \beta|1\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \;-\!\!\boxed{H}\!\!-\; \alpha\frac{|0\rangle+|1\rangle}{\sqrt{2}} + \beta\frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

measu… …nto $|0\rangle$ and $|1\rangle$

qu… …a single qubit …t to right)

classic… …single classical bit

n qubits — $\not{}^{\,n}$ — wire carrying n qubits

Pauli-X	$-\boxed{X}-$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	Pauli-Y	$-\boxed{Y}-$	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

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$$\alpha|0\rangle + \beta|1\rangle$$

Pauli-X	$-\boxed{X}-$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	Pauli-Y	$-\boxed{Y}-$	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$$\alpha|0\rangle + \beta|1\rangle \to \sigma_x\Big[\alpha|0\rangle + \beta|1\rangle\Big]$$

Pauli-X	$-\boxed{X}-$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	Pauli-Y	$-\boxed{Y}-$	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$$\alpha|0\rangle + \beta|1\rangle \to \sigma_x\Big[\alpha|0\rangle + \beta|1\rangle\Big] \to \Big[|1\rangle\langle 0| + |0\rangle\langle 1|\Big]\Big[\alpha|0\rangle + \beta|1\rangle\Big]$$

Pauli-X	$-\boxed{X}-$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	Pauli-Y	$-\boxed{Y}-$	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$$\alpha|0\rangle + \beta|1\rangle \to \sigma_x\Big[\alpha|0\rangle + \beta|1\rangle\Big] \to \Big[|1\rangle\langle 0| + |0\rangle\langle 1|\Big]\Big[\alpha|0\rangle + \beta|1\rangle\Big] = \alpha|1\rangle + \beta|0\rangle$$

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$$|00\rangle = |0\rangle \otimes |0\rangle$$

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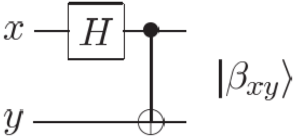
$$\begin{aligned} |00\rangle = |0\rangle \otimes |0\rangle \quad &\to \quad \big(\sigma_x \otimes \sigma_y\big)|00\rangle \\ &\to \quad \Big(\big[|1\rangle\langle 0| + |0\rangle\langle 1|\big] \otimes \big[i|1\rangle\langle 0| - i|0\rangle\langle 1|\big]\Big)|00\rangle = i|11\rangle \end{aligned}$$

In	Out
$\lvert 00\rangle$	$(\lvert 00\rangle + \lvert 11\rangle)/\sqrt{2} \equiv \lvert\beta_{00}\rangle$
$\lvert 01\rangle$	$(\lvert 01\rangle + \lvert 10\rangle)/\sqrt{2} \equiv \lvert\beta_{01}\rangle$
$\lvert 10\rangle$	$(\lvert 00\rangle - \lvert 11\rangle)/\sqrt{2} \equiv \lvert\beta_{10}\rangle$
$\lvert 11\rangle$	$(\lvert 01\rangle - \lvert 10\rangle)/\sqrt{2} \equiv \lvert\beta_{11}\rangle$



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$\lvert 00\rangle$	$(\lvert 00\rangle + \lvert 11\rangle)/\sqrt{2} \equiv \lvert\beta_{00}\rangle$
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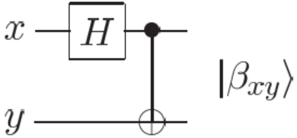
$\lvert 11\rangle$	$(\lvert 01\rangle - \lvert 10\rangle)/\sqrt{2} \equiv \lvert\beta_{11}\rangle$



$$|01\rangle = |0\rangle \otimes |1\rangle$$

In	Out
$\lvert 00\rangle$	$(\lvert 00\rangle + \lvert 11\rangle)/\sqrt{2} \equiv \lvert\beta_{00}\rangle$
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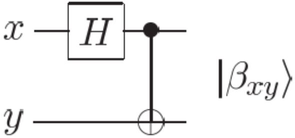
$\lvert 11\rangle$	$(\lvert 01\rangle - \lvert 10\rangle)/\sqrt{2} \equiv \lvert\beta_{11}\rangle$



$$|01\rangle = |0\rangle \otimes |1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |1\rangle$$

In	Out
$\|00\rangle$	$(\|00\rangle + \|11\rangle)/\sqrt{2} \equiv \|\beta_{00}\rangle$
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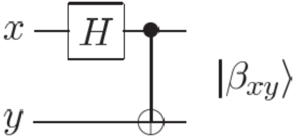

$$|01\rangle = |0\rangle \otimes |1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |1\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |11\rangle)$$

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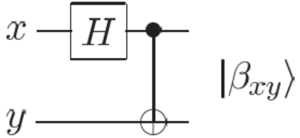

$$|01\rangle = |0\rangle \otimes |1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |1\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |11\rangle) \to \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

Quantum Teleportation (Kuantum Işınlanma)

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1st step:

$$|\phi\rangle = (\alpha|0\rangle + \beta|1\rangle)_C, \quad |\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB}$$

$$\left(|\psi\rangle = a|00\rangle + b|11\rangle \;,\; a^2 \geq b^2 > 0 \;,\; \boxed{2b^2} \right)$$

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$$\left(|\psi\rangle = a|00\rangle + b|11\rangle \ , \ a^2 \geq b^2 > 0 \ , \ \boxed{2b^2} \right)$$

2nd step:

$$|\phi\rangle \otimes |\psi\rangle = (\alpha|0\rangle + \beta|1\rangle)_C \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB}.$$

3rd and 4th steps:

$$
\begin{aligned}
|\phi\rangle \otimes |\psi\rangle &= \frac{1}{\sqrt{2}}\Big(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\Big)_{CAB} \\
&= \frac{1}{\sqrt{2}}\Big(\alpha|00\rangle \otimes |0\rangle + \alpha|01\rangle \otimes |1\rangle + \beta|10\rangle \otimes |0\rangle + \beta|11\rangle \otimes |1\rangle\Big)_{AB}
\end{aligned}
$$

3rd and 4th steps:

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&= \frac{1}{\sqrt{2}}\Big(\alpha|00\rangle \otimes |0\rangle + \alpha|01\rangle \otimes |1\rangle + \beta|10\rangle \otimes |0\rangle + \beta|11\rangle \otimes |1\rangle\Big)_{AB}
\end{aligned}
$$

$$
|00\rangle_A = \frac{1}{\sqrt{2}}(|\phi^+\rangle + |\phi^-\rangle), \quad |11\rangle_A = \frac{1}{\sqrt{2}}(|\phi^+\rangle - |\phi^-\rangle)
$$

$$
|01\rangle_A = \frac{1}{\sqrt{2}}(|\psi^+\rangle + |\psi^-\rangle), \quad |10\rangle_A = \frac{1}{\sqrt{2}}(|\psi^+\rangle - |\psi^-\rangle)
$$

5th step:

$$|\phi\rangle \otimes |\psi\rangle = \frac{1}{2}|\phi^+\rangle_A(\alpha|0\rangle + \beta|1\rangle)_B + \frac{1}{2}|\phi^-\rangle_A(\alpha|0\rangle - \beta|1\rangle)_B + \frac{1}{2}|\psi^+\rangle_A(\alpha|1\rangle + \beta|0\rangle)_B + \frac{1}{2}|\psi^-\rangle_A(\alpha|1\rangle - \beta|0\rangle)_B$$

$$P_1 = |\phi^+\rangle\langle\phi^+|, \quad P_2 = |\phi^-\rangle\langle\phi^-|, \quad P_3 = |\psi^+\rangle\langle\psi^+|, \quad P_4 = |\psi^-\rangle\langle\psi^-|, \quad p(|\phi^+\rangle) = p(|\phi^-\rangle) = p(|\psi^+\rangle) = p(|\psi^-\rangle) = \frac{1}{4}.$$

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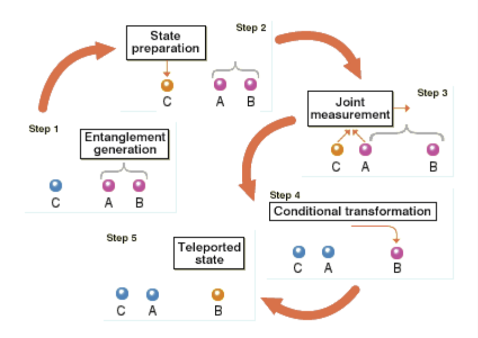
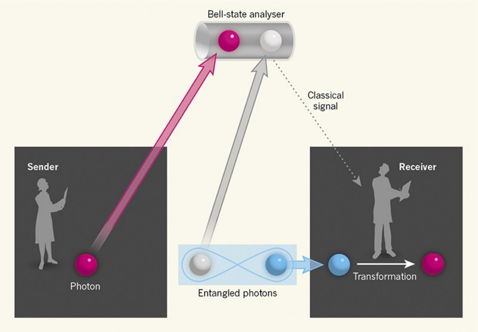
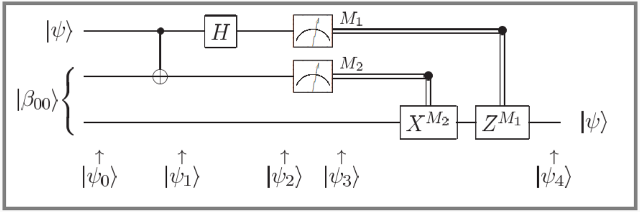
$$P_1 = |\phi^+\rangle\langle\phi^+|, \quad P_2 = |\phi^-\rangle\langle\phi^-|, \quad P_3 = |\psi^+\rangle\langle\psi^+|, \quad P_4 = |\psi^-\rangle\langle\psi^-|, \quad p(|\phi^+\rangle) = p(|\phi^-\rangle) = p(|\psi^+\rangle) = p(|\psi^-\rangle) = \frac{1}{4}.$$



(taken from physicsworld.com/a/teleportation-breaks-new-ground/)



(taken from nature.com/articles/d41586-017-07689-5)


|ψ⟩
|β00⟩
H
M1
M2
X^M2
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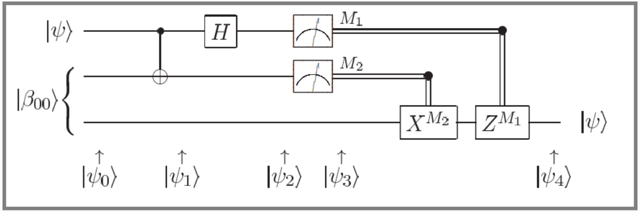
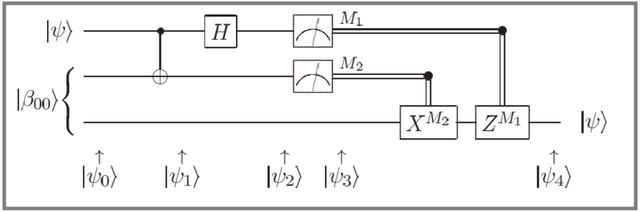
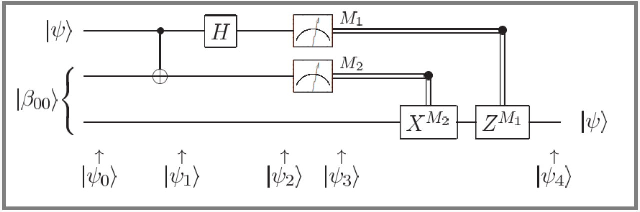
|ψ3⟩
|ψ4⟩



$$\psi_0 = \frac{1}{\sqrt{2}}\Big(\alpha|0\rangle[|00\rangle + |11\rangle] + \beta|1\rangle[|00\rangle + |11\rangle]\Big),$$

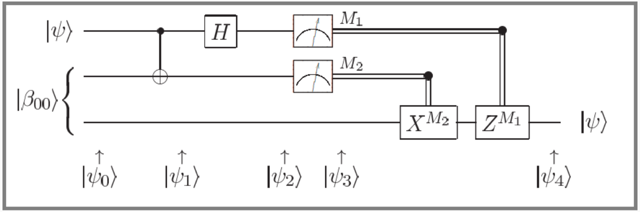


$$\psi_0 = \frac{1}{\sqrt{2}}\Big(\alpha|0\rangle[|00\rangle + |11\rangle] + \beta|1\rangle[|00\rangle + |11\rangle]\Big), \quad \psi_1 = \frac{1}{\sqrt{2}}\Big(\alpha|0\rangle[|00\rangle + |11\rangle] + \beta|1\rangle[|10\rangle + |01\rangle]\Big)$$



$$\psi_0 = \frac{1}{\sqrt{2}}\Big(\alpha|0\rangle[|00\rangle + |11\rangle] + \beta|1\rangle[|00\rangle + |11\rangle]\Big), \quad \psi_1 = \frac{1}{\sqrt{2}}\Big(\alpha|0\rangle[|00\rangle + |11\rangle] + \beta|1\rangle[|10\rangle + |01\rangle]\Big)$$

$$\begin{aligned}
\psi_2 \quad &= \quad \frac{1}{2}\Big(\alpha(|0\rangle + |1\rangle)[|00\rangle + |11\rangle] + \beta(|0\rangle - |1\rangle)[|10\rangle + |01\rangle]\Big) \\
&= \quad \frac{1}{2}\Big(|00\rangle[\alpha|0\rangle + \beta|1\rangle] + |01\rangle[\alpha|1\rangle + \beta|0\rangle] + |10\rangle[\alpha|0\rangle - \beta|1\rangle] + |11\rangle[\alpha|1\rangle - \beta|0\rangle]\Big)
\end{aligned}$$



$$\psi_0 = \frac{1}{\sqrt{2}}\Big(\alpha|0\rangle[|00\rangle + |11\rangle] + \beta|1\rangle[|00\rangle + |11\rangle]\Big), \quad \psi_1 = \frac{1}{\sqrt{2}}\Big(\alpha|0\rangle[|00\rangle + |11\rangle] + \beta|1\rangle[|10\rangle + |01\rangle]\Big)$$

$$\begin{aligned} \psi_2 &= \frac{1}{2}\Big(\alpha(|0\rangle + |1\rangle)[|00\rangle + |11\rangle] + \beta(|0\rangle - |1\rangle)[|10\rangle + |01\rangle]\Big) \\ &= \frac{1}{2}\Big(|00\rangle[\alpha|0\rangle + \beta|1\rangle] + |01\rangle[\alpha|1\rangle + \beta|0\rangle] + |10\rangle[\alpha|0\rangle - \beta|1\rangle] + |11\rangle[\alpha|1\rangle - \beta|0\rangle]\Big) \end{aligned}$$

"What can we learn from quantum teleportation? Quite a lot! It's much more than just a neat trick one can do with quantum states. Quantum teleportation emphasizes the interchangeability of different resources in quantum mechanics, showing that one shared EPR pair together with two classical bits of communication is a resource at least the equal of one qubit of communication. Quantum computation and quantum information has revealed a plethora of methods for interchanging resources, many built upon quantum teleportation." (M. Nielsen, and I. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, (2000).)

BREAK..