$$U_l\left(\frac{1}{\sqrt{2}}\Big[|\text{Dolaşıklık}\rangle + |\text{Uyumsuzluk}\rangle\Big]\right) \equiv \frac{1}{\sqrt{2}}\Big[|\text{Entanglement}\rangle + |\text{Discord}\rangle\Big]$$

Kuantum Dolaşıklık

States & Hilbert space & Dirac notation

States: A quantum state (or simply a state) is a complete description of a physical system. In quantum mechanics, a state is a ray in a Hilbert space. [What is a ray? It is an equivalence class of vectors that differ by multiplication by a nonzero complex scalar. We can choose a representative of this class (for any non-vanishing vector) to have unit norm.]

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$$\text{ket} : |x\rangle = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \ \text{bra} : \langle y| = \begin{bmatrix} y_1^* & y_2^* \end{bmatrix}, \ \langle y|x\rangle = x_1 y_1^* + x_2 y_2^*, \ |x\rangle\langle y| = \begin{bmatrix} x_1 y_1^* & x_1 y_2^* \\ x_2 y_1^* & x_2 y_2^* \end{bmatrix}.$$

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- It has an inner product $\langle\psi|\phi\rangle$ that maps an ordered pair of vectors to $\mathbb{C}$, defined by the properties: (i) Positivity: $\langle\psi|\psi\rangle > 0$ for $|\psi\rangle \neq 0$, (ii) Linearity: $\big(a\langle\psi_1| + b\langle\psi_2|\big)|\phi\rangle = a\langle\psi_1|\phi\rangle + b\langle\psi_2|\phi\rangle$, (iii) Skew symmetry: $\langle\psi|\phi\rangle = \langle\phi|\psi\rangle^*$.

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- It is complete in the norm, $||\psi|| = \sqrt{\langle\psi|\psi\rangle}$.

Qubit ($\boxed{\text{Qu}}$ antum $\boxed{\text{bit}}$)

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★ The corresponding unit of quantum information is called the "**quantum bit**" (or qubit): $\{|0\rangle$ or $|1\rangle\}$. The difference between bits and qubits is that a qubit can be in a state other than $|0\rangle$ or $|1\rangle$. It is also possible to form linear combinations of states:

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★ A geometrical representation of the pure state space of a two-dimensional quantum system can be illustrated as follows:

$$|\psi\rangle = \cos(\frac{\theta}{2})|0\rangle + e^{i\varphi}\sin(\frac{\theta}{2})|1\rangle, \ \left(\theta \in [0,\pi]\ ,\ \varphi \in [0,2\pi]\right)$$

where the numbers θ and φ define a point on the three-dimensional unit sphere: the **Bloch sphere**.

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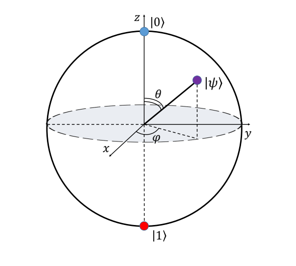

The probability to measure $|k\rangle$–Born rule–(for $k = 0, 1, 2, \ldots, d-1$) is given by

$$p(|k\rangle) = |\langle k|\psi\rangle|^2 = |c_k|^2 \left[p(|0\rangle) = |\cos(\frac{\theta}{2})|^2, \quad p(|1\rangle) = |e^{i\varphi}\sin(\frac{\theta}{2})|^2.\right]$$

Multipartite States & Tensor Product "$\otimes$"

- Two-qubit:

$$\left\{|00\rangle, |10\rangle, |01\rangle, |11\rangle\right\} \equiv \left\{|0\rangle \otimes |0\rangle, |1\rangle \otimes |0\rangle, \dots\right\} \equiv \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \dots \right\} \equiv \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

- Three-qubit: $\left\{|000\rangle, |100\rangle, |010\rangle, |001\rangle, |110\rangle, |101\rangle, |011\rangle, |111\rangle\right\}$

- n-qubit: $\left\{ \underbrace{|\overbrace{000...00}^{n}\rangle, |1000...00\rangle, |0100...00\rangle, |0010...00\rangle, ..., |111....11\rangle}_{2^n} \right\}$

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The general form of a two-qubit (often called bipartite) state is given by

$$|\psi\rangle = \sum_{i_1,i_2=0}^{1} c_{i_1 i_2} |i_1 i_2\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle,$$

where $c_{i_1 i_2} = e^{i\varphi_{i_1 i_2}} \lambda_{i_1 i_2}$ and $\sum_{i_1,i_2} |c_{i_1 i_2}|^2 = 1$.

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In the case of systems composed of $m > 2$ subsystems:

$$|\psi\rangle = \sum_{i_1,i_2,\dots,i_m} c_{i_1 i_2 \dots i_m} |i_1 i_2 \dots i_m\rangle,$$

$$\left(\sum_{i_1,i_2,\dots,i_m} |c_{i_1 i_2 \dots i_m}|^2 = \sum_{i_1,i_2,\dots,i_m} p(|i_1 i_2 \dots i_m\rangle) = 1 \right)$$

Kuantum Dolaşıklık (Entanglement)

A bipartite pure state $|\psi\rangle_{AB}$ is called entangled if it cannot be written such that

$$|\psi\rangle_{AB} = |\varphi\rangle_A \otimes |\chi\rangle_B \quad \left[\rho_{AB} = |\psi\rangle_{AB}\langle\psi| = |\varphi\rangle_A\langle\varphi| \otimes |\chi\rangle_B\langle\chi|\right]$$

otherwise it is separable.

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In case of mixed states, a density matrix ρ_{AB} defined on a tensor product Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$ is entangled if it cannot be written in the form

$$\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i \quad \left(p_i \geq 0, \quad \sum_i p_i = 1\right).$$

The states that are not entangled in the light of the above definition are called separable.

$$
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|\psi\rangle_{AB} &= \frac{1}{\sqrt{2}}\Big(\alpha|00\rangle + \beta|10\rangle - \alpha|01\rangle - \beta|11\rangle\Big) = \big(\alpha|0\rangle + \beta|1\rangle\big) \otimes \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big) \\
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\rho = \left(\frac{1-z}{4}\right)\mathbb{I} + z|\psi\rangle\langle\psi| \quad ?
$$

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Quantifying Resource—Entanglement

A typical resource theory aims at quantifying the resource in hand.

Resource measure $\mathcal{M}$:

- $\mathcal{M}(\rho) = 0 \Leftrightarrow \rho \in \mathcal{F}$: faithfulness (or nonnegativity)
- $\mathcal{M}(\rho) \geq \mathcal{M}(\Lambda(\rho))$: monotonicity
- $\mathcal{M}(\sum_i p_i \rho_i) \leq \sum_i p_i \mathcal{M}(\rho_i)$: convexity
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✓ Majorization condition (a tool to compare the entanglement of bipartite pure states)

(R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, *Quantum Entanglement*, Rev. Mod. Phys. 81, 865 (2009))

Majorization condition:

Suppose $x \equiv (x_1, \ldots, x_d)^T$ and $y \equiv (y_1, \ldots, y_d)^T$ are real d-dimensional vectors whose components are in decreasing order. Then x is majorized by y, written $x \prec y$, if the inequalities $\sum_{i=1}^{k} x_i \leq \sum_{i=1}^{k} y_i$ are satisfied for any $k \in [1, d-1]$ with equality holding when $k = d$.

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$$|\psi\rangle_{AB} = \sqrt{\frac{1}{2}}|00\rangle + \sqrt{\frac{3}{8}}|11\rangle + \sqrt{\frac{1}{8}}|22\rangle, \quad |\varphi\rangle_{AB} = \sqrt{\frac{3}{8}}|00\rangle + \sqrt{\frac{3}{8}}|11\rangle + \frac{1}{2}|22\rangle$$

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$$\rho_A(\psi) \quad = \quad \mathrm{Tr}_B(\rho_{AB}(\psi)) = \mathrm{Tr}_B\left(\frac{1}{2}|00\rangle\langle 00| + \sqrt{\frac{3}{16}}|00\rangle\langle 11| + \frac{1}{4}|00\rangle\langle 22| + \sqrt{\frac{3}{16}}|11\rangle\langle 00| + \frac{3}{8}|11\rangle\langle 11| + \ldots\right)$$

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$$\begin{aligned} \rho_A(\psi) &= \mathrm{Tr}_B(\rho_{AB}(\psi)) = \mathrm{Tr}_B\left(\frac{1}{2}|00\rangle\langle 00| + \sqrt{\frac{3}{16}}|00\rangle\langle 11| + \frac{1}{4}|00\rangle\langle 22| + \sqrt{\frac{3}{16}}|11\rangle\langle 00| + \frac{3}{8}|11\rangle\langle 11| + \ldots\right) \\ &= \frac{1}{2}|0\rangle\langle 0| + \frac{3}{8}|1\rangle\langle 1| + \frac{1}{8}|2\rangle\langle 2| \end{aligned}$$

$$\rho(\varphi) : \{\frac{3}{8}, \frac{3}{8}, \frac{1}{4}\} \prec \rho(\psi) : \{\frac{1}{2}, \frac{3}{8}, \frac{1}{8}\}$$

Bell states:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big), \quad |\phi^-\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle - |11\rangle\big),$$

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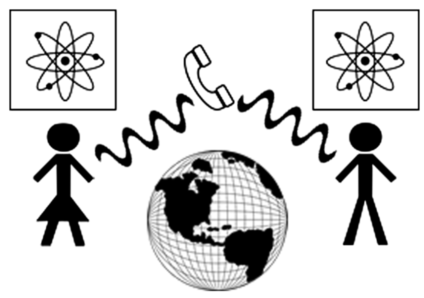
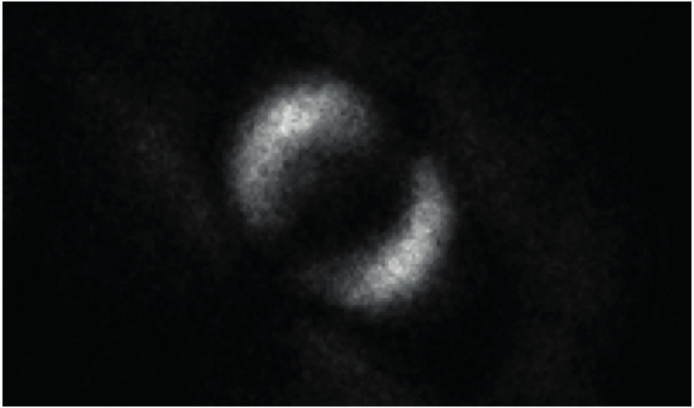


★ "Alice and Bob may be working in their own quantum laboratory while being separated from each other by some large distance. Because of current technological limitations, the only communication channel connecting their laboratories is classical, such as a telephone. Hence Alice cannot directly send quantum states to Bob and vice versa, and the free operations in this resource theory consist of LOCC. While the classical communication channel allows for the preparation of classically correlated states between the two laboratories, not every type of joint quantum state can be realized for Alice and Bob's systems using LOCC. A state is said to be entangled, and therefore a resource, precisely when it cannot be generated using the free operations of LOCC."

★(E.Chitambar and G.Gour, *Quantum Resource Theories*, Rev. Mod. Phys, 91, 025001 (2019))

(R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, *Quantum Entanglement*, Rev. Mod. Phys. 81, 865 (2009))



“For the first time ever, physicists have managed to take a photo of a strong form of quantum entanglement called Bell entanglement—capturing visual evidence of an elusive phenomenon which a baffled Albert Einstein once called ‘spooky action at a distance’.”

(https://phys.org/news/2019-07-scientists-unveil-first-ever-image-quantum.html)

Kuantum Uyumsuzluk

Shannon Entropi ve Klasik Enformasyon

The Shannon entropy associated with probability distribution, p_x, is defined by the following formula

$$H(X) \equiv H(p_1, ..., p_n) \equiv -\sum_x p_x \log p_x$$

where logarithms indicated by log are taken to base two. Then, it is conventional to say that entropies are measured in bits with this convention for the logarithm.

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$$H(X,Y) \equiv -\sum_{x,y} p(x,y) \log p(x,y)$$

$$H(A,B) \quad = \quad -\sum_{a,b} p_{a,b} \log(p_{a,b}) \qquad \left[p_{a|b} = \frac{p_{a,b}}{p_b}\right]$$

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H(A,B) &= -\sum_{a,b} p_{a,b} \log(p_{a,b}) \qquad \left[p_{a|b} = \frac{p_{a,b}}{p_b}\right] \\
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&= -\sum_{a,b} p_b\left(p_{a|b}\log(p_{a|b})\right) - \sum_{b} p_b\log(p_b) \\
&= H(A|B) + H(B)
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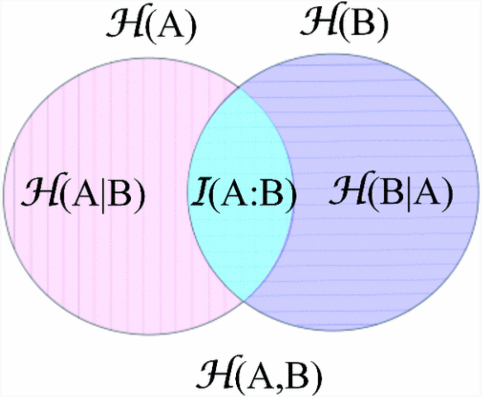
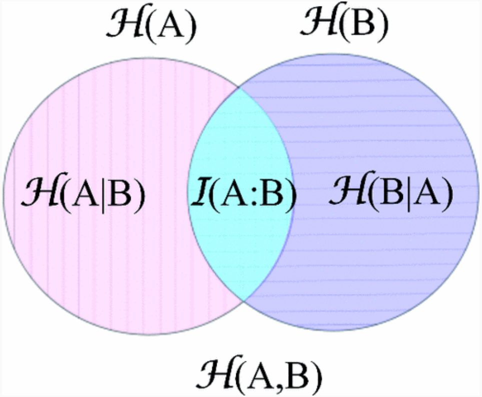
$$

Conditional entropy:

$$
\boxed{H(A|B)} \equiv H(A,B) - H(B) = -\sum_{a,b} p_b \left(p_{a|b} \log(p_{a|b})\right) = \boxed{\sum_{a,b} p_b H(A|b)}
$$

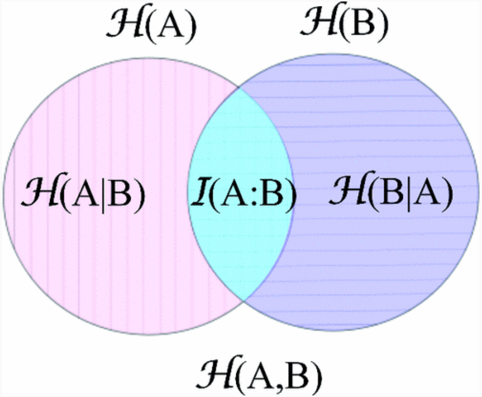

$\mathcal{H}(A)$
$\mathcal{H}(B)$
$\mathcal{H}(A|B)$
$\mathcal{I}(A:B)$
$\mathcal{H}(B|A)$
$\mathcal{H}(A,B)$



$$H(A|B) \equiv H(A,B) - H(B)$$

$$H(B|A) \equiv H(A,B) - H(A)$$

. . . .



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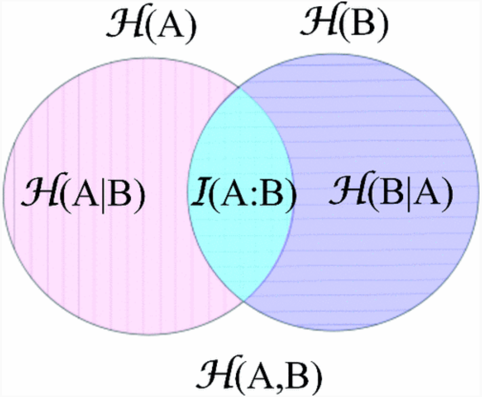
. . . .

Mutual information:

$$I(A:B) = H(A) + H(B) - H(A,B)$$



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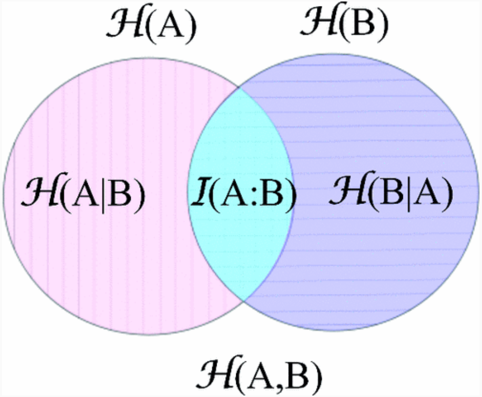

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Mutual information:

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The relative entropy:

$$H(p(x)||q(x)) \equiv \sum_x p(x) \log \frac{p(x)}{q(x)} == -H(X) - \sum_x p(x) \log q(x)$$

von Neumann Entropi ve Kuantum Enformasyon

$$S(\rho) \equiv -tr(\rho \log \rho)$$

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In this formula logarithms are taken to base two, as usual. If λ_x are the eigenvalues of ρ then von Neumann's definition can be re-expressed as

$$S(\rho) = -\sum_x \lambda_x \log \lambda_x$$

similar to the Shannon entropy.

$$J(A:B) = S(\rho_B) - S(B|A) \quad , \quad I(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

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The state of B, after the outcome corresponding to M_a has been obtained, is

$$\rho_{B|a} = \mathrm{tr}_A[(M_a \otimes I)\rho_{AB}(M_a^\dagger \otimes I)]/p_a$$

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This allows us to define a classical-quantum version of the conditional entropy

$$S(B|A) = \sum_a p_a S(\rho_{B|a}) = -\sum_a p_a \left(p_{b|a} \log(p_{b|a})\right)$$

Kuantum Uyumsuzluk (Discord)

The "**quantum discord**" of a state ρ_{AB} under a measurement $\{\Pi_a\}$ is defined as a difference between the quantum forms of mutual information,

$$\begin{aligned}\mathcal{D} &= I(A:B) - J(A:B) \\ &= \Big[S(\rho_A) + S(\rho_B) - S(\rho_{AB})\Big] - \Big[S(\rho_B) - S(B|A)\Big] \\ &= S(\rho_A) - S(\rho_{AB}) + \min_{\{\Pi_a\}} \sum_a p_a S(\rho_{B|a}),\end{aligned}$$

where the minimization is over all local projectors, that is $\{\Pi_a\}$.

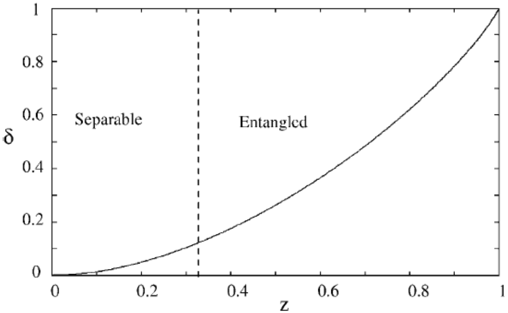

FIG. Value of the discord for Werner states $\frac{1-z}{4}\mathbf{1} + z|\psi\rangle\langle\psi|$, with $|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. Discord does not depend on the basis of measurement in this case because both $\mathbf{1}$ and $|\psi\rangle$ are invariant under local rotations.

(H. Ollivier and W. H. Zurek, *Quantum Discord: A Measure of the Quantumness of Correlations*, PRL 88, 017901 (2001).)

$$\rho_{cc} = \sum_{i=0}^{n} p_i |i\rangle\langle i|_A \otimes |i\rangle\langle i|_B.$$

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If a local operation is performed on Alice's subsystem, then the subsystem of Bob remains unchanged. In this case, the Kraus operators have the form $M_i = M_i^A \otimes I^B$. That is, $\Lambda^a(\rho_{cc}) = \sum_i (M_i^A \otimes I^B)\rho_{cc}((M_i^A)^\dagger \otimes I^B)$, where $\sum_i (M_i^A)^\dagger M_i^A = I$. For $M_i^A = |\psi_i\rangle\langle i|_A$, we obtain

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$$\Lambda^b(\rho_{cc}) = \sum_i p_i |i\rangle\langle i|_A \otimes |\phi_i\rangle\langle\phi_i|_B = \rho_{cq}.$$

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$$\rho_{AB} = p|0\rangle\langle 0| \otimes |0\rangle\langle 0| + (1-p)|+\rangle\langle +| \otimes |1\rangle\langle 1|$$

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\end{aligned}$$

$$\begin{aligned}
\rho_A &= \mathrm{tr}_B(\rho_{AB}) \\
&= \mathrm{tr}_B\Big[p|0\rangle\langle 0| \otimes |0\rangle\langle 0| \\
&+ \frac{(1-p)}{2}((|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|) \otimes |1\rangle\langle 1|)\Big] \\
&= p|0\rangle\langle 0| + \frac{(1-p)}{2}(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|) \\
&= \frac{(1+p)}{2}|0\rangle\langle 0| + \frac{(1-p)}{2}(|0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|)
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$$\begin{aligned} p_1 &= \mathrm{tr}[(\Pi_1 \otimes I)\rho_{AB}(\Pi_1^\dagger \otimes I)] \\ &= \mathrm{tr}[p|0\rangle\langle 0| \otimes |0\rangle\langle 0| + \frac{(1-p)}{2}|0\rangle\langle 0| \otimes |1\rangle\langle 1|] \\ &= \frac{(1+p)}{2}, \end{aligned}$$

$$\begin{aligned} p_2 &= \mathrm{tr}[(\Pi_2 \otimes I)\rho_{AB}(\Pi_2^\dagger \otimes I)] \\ &= \mathrm{tr}[\frac{(1-p)}{2}|1\rangle\langle 1| \otimes |1\rangle\langle 1|] \\ &= \frac{(1-p)}{2}. \end{aligned}$$

$$\begin{aligned} \rho_A &= \mathrm{tr}_B(\rho_{AB}) \\ &= \mathrm{tr}_B\Big[p|0\rangle\langle 0| \otimes |0\rangle\langle 0| \\ &+ \frac{(1-p)}{2}((|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|) \otimes |1\rangle\langle 1|)\Big] \\ &= p|0\rangle\langle 0| + \frac{(1-p)}{2}(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|) \\ &= \frac{(1+p)}{2}|0\rangle\langle 0| + \frac{(1-p)}{2}(|0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|) \end{aligned}$$

. . . .

$$\begin{aligned} \rho_{B|a=1} &= \mathrm{tr}_A[(\Pi_1 \otimes I)\rho_{AB}(\Pi_1^\dagger \otimes I)]/p_1 \\ &= \mathrm{tr}_A[p|0\rangle\langle 0| \otimes |0\rangle\langle 0| + \frac{(1-p)}{2}|0\rangle\langle 0| \otimes |1\rangle\langle 1|]/p_1 \\ &= [p|0\rangle\langle 0| + \frac{(1-p)}{2}|1\rangle\langle 1|]/(\frac{1+p}{2}) \end{aligned}$$

. . . .

Consider the following state

$$\rho_{AB} = p|0\rangle\langle 0| \otimes |0\rangle\langle 0| + (1-p)|+\rangle\langle +| \otimes |1\rangle\langle 1|$$

$$\left[|\mp\rangle = \frac{1}{\sqrt{2}}(|0\rangle \mp |1\rangle)\right], \left[\Pi_1 = |0\rangle\langle 0|, \Pi_2 = |1\rangle\langle 1|\right].$$

$$\rho_{B|a} = \mathrm{tr}_A[(\Pi_a \otimes I)\rho_{AB}(\Pi_a^\dagger \otimes I)]/p_a.$$

$$p_a = \mathrm{tr}[(\Pi_a \otimes I)\rho_{AB}(\Pi_a^\dagger \otimes I)]$$

$$\begin{aligned} p_1 &= \mathrm{tr}[(\Pi_1 \otimes I)\rho_{AB}(\Pi_1^\dagger \otimes I)] \\ &= \mathrm{tr}[p|0\rangle\langle 0| \otimes |0\rangle\langle 0| + \frac{(1-p)}{2}|0\rangle\langle 0| \otimes |1\rangle\langle 1|] \\ &= \frac{(1+p)}{2}, \end{aligned}$$

$$\begin{aligned} p_2 &= \mathrm{tr}[(\Pi_2 \otimes I)\rho_{AB}(\Pi_2^\dagger \otimes I)] \\ &= \mathrm{tr}[\frac{(1-p)}{2}|1\rangle\langle 1| \otimes |1\rangle\langle 1|] \\ &= \frac{(1-p)}{2}. \end{aligned}$$

$$\begin{aligned} \rho_A &= \mathrm{tr}_B(\rho_{AB}) \\ &= \mathrm{tr}_B\Big[p|0\rangle\langle 0| \otimes |0\rangle\langle 0| \\ &+ \frac{(1-p)}{2}((|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|) \otimes |1\rangle\langle 1|)\Big] \\ &= p|0\rangle\langle 0| + \frac{(1-p)}{2}(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|) \\ &= \frac{(1+p)}{2}|0\rangle\langle 0| + \frac{(1-p)}{2}(|0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|) \end{aligned}$$

$$\begin{aligned} \rho_{B|a=1} &= \mathrm{tr}_A[(\Pi_1 \otimes I)\rho_{AB}(\Pi_1^\dagger \otimes I)]/p_1 \\ &= \mathrm{tr}_A[p|0\rangle\langle 0| \otimes |0\rangle\langle 0| + \frac{(1-p)}{2}|0\rangle\langle 0| \otimes |1\rangle\langle 1|]/p_1 \\ &= [p|0\rangle\langle 0| + \frac{(1-p)}{2}|1\rangle\langle 1|]/(\frac{1+p}{2}) \end{aligned}$$

$$\begin{aligned} \rho_{B|a=2} &= \mathrm{tr}_A[(\Pi_2 \otimes I)\rho_{AB}(\Pi_2^\dagger \otimes I)]/p_2 \\ &= \mathrm{tr}_A[\frac{(1-p)}{2}|1\rangle\langle 1| \otimes |1\rangle\langle 1|]/p_2 \\ &= |1\rangle\langle 1| \end{aligned}$$

Kaynaklar

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