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Ex.2

(a) P.V. $i^i = \exp(i\text{Log}\, i) = \exp\left[i\left(\ln 1 + i\frac{\pi}{2}\right)\right] = \exp\left(-\frac{\pi}{2}\right).$

(b) P.V. $\left[\frac{e}{2}\left(-1-\sqrt{3}i\right)\right]^{3\pi i} = \exp\left\{3\pi i \text{Log}\left[\frac{e}{2}\left(-1-\sqrt{3}i\right)\right]\right\} = \exp\left[3\pi i\left(\ln e - i\frac{2\pi}{3}\right)\right]$

$$= \exp(2\pi^2)\exp(i3\pi) = -\exp(2\pi^2).$$

(c) P.V. $(1-i)^{4i} = \exp[4i\text{Log}(1-i)] = \exp\left[4i\left(\ln\sqrt{2} - i\frac{\pi}{4}\right)\right] = e^{\pi}e^{i4\ln\sqrt{2}}$

$$= e^{\pi}[\cos(4\ln\sqrt{2}) + i\sin(4\ln\sqrt{2})] = e^{\pi}[\cos(2\ln 2) + i\sin(2\ln 2)].$$

Ex.4

(a)

$$\begin{aligned}
(-1+\sqrt{3}i)^{3/2} &= [(-1+\sqrt{3}i)^{1/2}]^3 \\
&= [\sqrt{2}e^{i(2\pi/3+2n\pi)/2}]^3 \\
&= 2\sqrt{2}e^{i(\pi+3n\pi)} \\
&= \pm 2\sqrt{2}
\end{aligned}$$

(b)

$$\begin{aligned}
(-1+\sqrt{3}i)^{3/2} &= [(-1+\sqrt{3}i)^3]^{1/2} \\
&= [2^3 e^{i(2\pi+6n\pi)}]^{1/2} \\
&= (8e^{i2n\pi})^{1/2} \\
&= 2\sqrt{2}e^{in\pi} \\
&= \pm 2\sqrt{2}
\end{aligned}$$

Ex.8

(a) $z^{c_1}z^{c_2} = e^{c_1\text{Log}z}e^{c_2\text{Log}z} = e^{(c_1+c_2)\text{Log}z} = z^{c_1+c_2}$

(b) $z^{c_1}/z^{c_2} = e^{c_1\text{Log}z}/e^{c_2\text{Log}z} = e^{(c_1-c_2)\text{Log}z} = z^{c_1-c_2}$

(c) $(z^c)^n = (e^{c\text{Log}z})^n = e^{nc\text{Log}z} = z^{nc},\ (n = 1, 2, ...)$

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Ex.2

(a)

$$\begin{aligned} e^{iz_1}e^{iz_2} &= (\cos z_1 + i\sin z_1)(\cos z_2 + i\sin z_2) \\ &= \cos z_1 \cos z_2 - \sin z_1 \sin z_2 + i(\sin z_1 \cos z_2 + \cos z_1 \sin z_2) \end{aligned}$$

$$\begin{aligned} e^{-iz_1}e^{-iz_2} &= \cos(-z_1)\cos(-z_2) - \sin(-z_1)\sin(-z_2) + i[\sin(-z_1)\cos(-z_2) + \cos(-z_1)\sin(-z_2)] \\ &= \cos z_1 \cos z_2 - \sin z_1 \sin z_2 - i(\sin z_1 \cos z_2 + \cos z_1 \sin z_2) \end{aligned}$$

(b)

$$\begin{aligned} \sin(z_1 + z_2) &= \frac{1}{2i}(e^{iz_1}e^{iz_2} - e^{-iz_1}e^{-iz_2}) \\ &= \frac{1}{2i}2i(\sin z_1 \cos z_2 + \cos z_1 \sin z_2) \\ &= \sin z_1 \cos z_2 + \cos z_1 \sin z_2 \end{aligned}$$

Ex.3

We know from Exercise 2*(b)* that

$$\sin(z+z_2)=\sin z\cos z_2+\cos z\sin z_2.$$

Differentiating each side yields

$$\cos(z+z_2)=\cos z\cos z_2-\sin z\sin z_2.$$

Then, by setting $z=z_1$, we have

$$\cos(z_1+z_2)=\cos z_1\cos z_2-\sin z_1\sin z_2.$$

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Ex.3

Identity (9), Sec. 34, is $\sin^2 z+\cos^2 z=1$. Replacing z by iz here and using the identities

$$\sin(iz)=i\sinh z \quad\text{and}\quad \cos(iz)=\cosh z,$$

we find that $i^2\sinh^2 z+\cosh^2 z=1$, or

$$\cosh^2 z-\sinh^2 z=1.$$

Identity (6), Sec. 34, is $\cos(z_1+z_2)=\cos z_1\cos z_2-\sin z_1\sin z_2$. Replacing z_1 by iz_1 and z_2 by iz_2 here, we have $\cos[i(z_1+z_2)]=\cos(iz_1)\cos(iz_2)-\sin(iz_1)\sin(iz_2)$. The same identities that were used just above then lead to

$$\cosh(z_1+z_2)=\cosh z_1\cosh z_2+\sinh z_1\sinh z_2.$$

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Ex.1

(a)

$$\begin{aligned}
\tan^{-1}(2i) &= \frac{i}{2}\log\frac{i+2i}{i-2i} \\
&= \frac{i}{2}\log\frac{3i}{-i} \\
&= \frac{i}{2}\log(-3) \\
&= \frac{i}{2}[\ln 3 + i(\pi + 2n\pi)] \\
&= -(n+\frac{1}{2})\pi + \frac{i}{2}\ln 3,\ \ n = 0, \pm 1, \pm 2, .. \\
&= (n+\frac{1}{2})\pi + \frac{i}{2}\ln 3,\ \ n = 0, \pm 1, \pm 2, ..
\end{aligned}$$

(b)

$$\begin{aligned}
\tan^{-1}(1+i) &= \frac{i}{2}\log\frac{i+1+i}{i-(1+i)} \\
&= \frac{i}{2}\log(-1-2i) \\
&= \frac{i}{2}[\ln 5 + i\arg(-1-2i)] \\
&= \frac{i}{2}\ln 5 - \frac{1}{2}\arg(-1-2i)
\end{aligned}$$

(c)

$$\begin{aligned}
\cosh^{-1}(-1) &= \log[-1 + (1-1)^{1/2}] \\
&= \log(-1) \\
&= \ln 1 + i(\pi + 2n\pi) \\
&= i\pi(1+2n),\ \ n = 0, \pm 1, \pm 2, ..
\end{aligned}$$

(d)

$$\begin{aligned}\tanh^{-1} 0 &= \frac{1}{2}\log\frac{1+0}{1-0}\\ &= \frac{1}{2}\log 1\\ &= \frac{1}{2}i2n\pi\\ &= in\pi,\ \ n = 0, \pm1, \pm2, ..\end{aligned}$$

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Ex.1

(a)

$$\begin{aligned}\frac{d}{dt}w(-t) &= \frac{d}{dt}u(-t) + i\frac{d}{dt}v(-t)\\ &= -u'(-t) - iv'(-t)\\ &= -[u'(-t) + iv'(-t)]\\ &= -w'(-t)\end{aligned}$$

(b)

$$\begin{aligned}\frac{d}{dt}[w(t)]^2 &= \frac{d}{dt}[u^2(t) - v^2(t) + 2iu(t)v(t)]\\ &= 2u(t)u'(t) - 2v(t)v'(t) + 2iu'(t)v(t) + 2iu(t)v'(t)\\ &= 2[u(t) + iv(t)][u'(t) + iv'(t)]\\ &= 2w(t)w'(t)\end{aligned}$$

Ex.2

(a) $\int_1^2 \left(\frac{1}{t}-i\right)^2 dt = \int_1^2 \left(\frac{1}{t^2}-1\right)dt - 2i\int_1^2 \frac{dt}{t} = -\frac{1}{2} - 2i\ln 2 = -\frac{1}{2} - i\ln 4;$

(b) $\int_0^{\pi/6} e^{i2t}dt = \left[\frac{e^{i2t}}{2i}\right]_0^{\pi/6} = \frac{1}{2i}\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} - 1\right] = \frac{\sqrt{3}}{4} + \frac{i}{4};$

(c) Since $|e^{-bz}| = e^{-bx}$, we find that

$$\int_0^{\infty} e^{-zt}dt = \lim_{b\to\infty}\int_0^b e^{-zt}dt = \lim_{b\to\infty}\left[\frac{e^{-zt}}{-z}\right]_{t=0}^{t=b} = \frac{1}{z}\lim_{b\to\infty}\left(1-e^{-bz}\right) = \frac{1}{z} \quad \text{when } \operatorname{Re} z > 0.$$

Ex.4

First of all,

$$\int_0^{\pi} e^{(1+i)x}dx = \int_0^{\pi} e^x \cos x\, dx + i\int_0^{\pi} e^x \sin x\, dx.$$

But also,

$$\int_0^{\pi} e^{(1+i)x}dx = \left[\frac{e^{(1+i)x}}{1+i}\right]_0^{\pi} = \frac{e^{\pi}e^{i\pi}-1}{1+i} = \frac{-e^{\pi}-1}{1+i}\cdot\frac{1-i}{1-i} = -\frac{1+e^{\pi}}{2} + i\frac{1+e^{\pi}}{2}.$$

Equating the real parts and then the imaginary parts of these two expressions, we find that

$$\int_0^{\pi} e^x \cos x\, dx = -\frac{1+e^{\pi}}{2} \quad \text{and} \quad \int_0^{\pi} e^x \sin x\, dx = \frac{1+e^{\pi}}{2}.$$