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Ex.2

Note that if $z_n = 2 + i\frac{(-1)^n}{n^2}$ $(n = 1,2,\ldots)$, then

$$\Theta_{2n} = \operatorname{Arg} z_{2n} \to 0 \quad \text{and} \quad \Theta_{2n-1} = \operatorname{Arg} z_{2n-1} \to 0 \qquad (n = 1,2,\ldots)$$

Hence the sequence Θ_n $(n = 1,2,\ldots)$ does converge.

Ex.3

Suppose that $\lim_{n\to\infty} z_n = z$. That is, for each $\varepsilon > 0$, there is a positive integer n_0 such that $|z_n - z| < \varepsilon$ whenever $n > n_0$. In view of the inequality (see Sec. 4)

$$|z_n - z| \geq ||z_n| - |z||,$$

it follows that $||z_n| - |z|| < \varepsilon$ whenever $n > n_0$. That is, $\lim_{n\to\infty} |z_n| = |z|$.

Ex.5

假设一个收敛的复数数列$z_n = x_n + iy_n$有两个不同极限值$L_1 = X_1 + iY_1$和$L_2 = X_2 + iY_2$，根据实数数列极限的唯一性有

$$\lim_{n\to\infty} x_n = X_1 = X_2, \lim_{n\to\infty} y_n = Y_1 = Y_2$$

得到$L_1 = L_2$，与假设矛盾，故复数数列的极限也具有唯一性。

Ex.9

(a) 由于z_n的极限是z，根据定义有，对任意$\epsilon > 0$，存在$n_0 > 0$，当$n > n_0$，有$|z_n - z| < \epsilon$，则$|z_n| = |z + (z_n - z)| \leq |z| + |z_n - z| < |z| + \epsilon$，取$\epsilon = 1$，有$|z_n| < |z| + 1$。令$M = \max\{|z_1|, |z_2|, ..., |z_{n_0}|, |z| + 1\}$，有$|z_n| \leq M$。

(b) $z_n = x_n + iy_n$收敛，故x_n与y_n也收敛，根据实数数列的性质，存在$M_1, M_2 > 0$，使得$\forall n$有$|x_n| \leq M_1, |y_n| \leq M_2$。$|z_n| = |x_n + iy_n| \leq |x_n| + |iy_n| \leq M_1 + M_2$。令$M = M_1 + M_2$，则$\forall n$有$|z_n| \leq M$。

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Ex.2

(a) $f(z) = e^z$是完全函数，对于整个复平面都有麦克劳林级数展开，$f^{(n)}(z) = e^z(n = 0, 1, 2, ...)$，因此$f^{(n)}(1) = e(n = 0, 1, 2, ...)$,有

$$e^z = \sum_{n=0}^{\infty} e\frac{(z-1)^n}{n!} = e\sum_{n=0}^{\infty}\frac{(z-1)^n}{n!}(|z-1| < \infty)$$

(b) Replacing z by $z-1$ in the known expansion

$$e^z = \sum_{n=0}^{\infty}\frac{z^n}{n!} \qquad (|z| < \infty),$$

we have

$$e^{z-1} = \sum_{n=0}^{\infty}\frac{(z-1)^n}{n!} \qquad (|z| < \infty).$$

So

$$e^z = e^{z-1}e = e\sum_{n=0}^{\infty}\frac{(z-1)^n}{n!} \qquad (|z| < \infty).$$

Ex.3

We want to find the Maclaurin series for the function

$$f(z) = \frac{z}{z^4+9} = \frac{z}{9}\cdot\frac{1}{1+(z^4/9)}.$$

To do this, we first replace z by $-(z^4/9)$ in the known expansion

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \qquad (|z| < 1),$$

as well as its condition of validity, to get

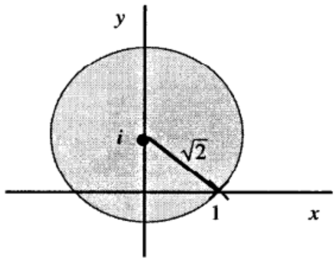
$$\frac{1}{1+(z^4/9)} = \sum_{n=0}^{\infty}\frac{(-1)^n}{3^{2n}}z^{4n} \qquad (|z| < \sqrt{3}).$$

Then, if we multiply through this last equation by $\frac{z}{9}$, we have the desired expansion:

$$f(z) = \sum_{n=0}^{\infty}\frac{(-1)^n}{3^{2n+2}}z^{4n+1} \qquad (|z| < \sqrt{3}).$$

Ex.7

The function $\frac{1}{1-z}$ has a singularity at $z=1$. So the Taylor series about $z=i$ is valid when $|z-i|<\sqrt{2}$, as indicated in the figure below.



To find the series, we start by writing

$$\frac{1}{1-z}=\frac{1}{(1-i)-(z-i)}=\frac{1}{1-i}\cdot\frac{1}{1-(z-i)/(1-i)}.$$

This suggests that we replace z by $(z-i)/(1-i)$ in the known expansion

$$\frac{1}{1-z}=\sum_{n=0}^{\infty}z^n \qquad (|z|<1)$$

and then multiply through by $\frac{1}{1-i}$. The desired Taylor series is then obtained:

$$\frac{1}{1-z}=\sum_{n=0}^{\infty}\frac{(z-i)^n}{(1-i)^{n+1}} \qquad (|z-i|<\sqrt{2}).$$

Ex.11

(a)

$$\frac{e^z}{z^2}=\frac{1}{z^2}\sum_{n=0}^{\infty}\frac{z^n}{n!}=\frac{1}{z^2}+\frac{1}{z}+\frac{1}{2!}+\frac{z}{3!}+\frac{z^2}{4!}+\ldots$$

(b)

$$\begin{aligned}\frac{\sin(z^2)}{z^4}&=\frac{1}{z^4}\frac{e^{iz^2}-e^{-iz^2}}{2i}\\&=\frac{1}{z^4}\sum_{n=0}^{\infty}(-1)^n\frac{(z^2)^{2n+1}}{(2n+1)!}\\&=\sum_{n=0}^{\infty}(-1)^n\frac{z^{4n-2}}{(2n+1)!}\\&=\frac{1}{z^2}-\frac{z^2}{3!}+\frac{z^6}{5!}-\frac{z^{10}}{7!}+\ldots\end{aligned}$$