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Ex.3

Suppose that $1 < |z| < \infty$ and recall the Maclaurin series representation

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \qquad (|z| < 1).$$

This enables us to write

$$\frac{1}{1+z} = \frac{1}{z} \cdot \frac{1}{1+\frac{1}{z}} = \frac{1}{z}\sum_{n=0}^{\infty}\left(-\frac{1}{z}\right)^n = \sum_{n=0}^{\infty}\frac{(-1)^n}{z^{n+1}} \qquad (1 < |z| < \infty).$$

Replacing n by $n-1$ in this last series and then noting that

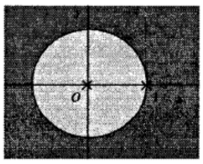
$$(-1)^{n-1} = (-1)^{n-1}(-1)^2 = (-1)^{n+1},$$

we arrive at the desired expansion:

$$\frac{1}{1+z} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{z^n} \qquad (1 < |z| < \infty).$$

Ex.4

The singularities of the function $f(z) = \dfrac{1}{z^2(1-z)}$ are at the points $z = 0$ and $z = 1$. Hence there are Laurent series in powers of z for the domains $0 < |z| < 1$ and $1 < |z| < \infty$ (see the figure below).



To find the series when $0 < |z| < 1$, recall that $\dfrac{1}{1-z} = \sum_{n=0}^{\infty} z^n$ $(|z| < 1)$ and write

$$f(z) = \frac{1}{z^2}\cdot\frac{1}{1-z} = \frac{1}{z^2}\sum_{n=0}^{\infty} z^n = \sum_{n=0}^{\infty} z^{n-2} = \frac{1}{z^2} + \frac{1}{z} + \sum_{n=2}^{\infty} z^{n-2} = \sum_{n=0}^{\infty} z^n + \frac{1}{z} + \frac{1}{z^2}.$$

As for the domain $1 < |z| < \infty$, note that $|1/z| < 1$ and write

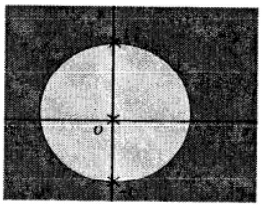
$$f(z) = -\frac{1}{z^3}\cdot\frac{1}{1-(1/z)} = -\frac{1}{z^3}\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)^n = -\sum_{n=0}^{\infty}\frac{1}{z^{n+3}} = -\sum_{n=3}^{\infty}\frac{1}{z^n}.$$

Ex.6

$$\begin{aligned}
\frac{z}{(z-1)(z-3)} &= \frac{z-1+1}{z-1}\frac{1}{z-3} \\
&= (1+\frac{1}{z-1})(-\frac{1}{2}\frac{1}{1-(z-1)/2}) \\
&= (1+\frac{1}{z-1})(-\frac{1}{2}\sum_{n=0}^{\infty}\frac{(z-1)^n}{2^n})(0<|(z-1)/2|<1) \\
&= -\frac{1}{2}\sum_{n=0}^{\infty}\frac{(z-1)^n}{2^n} - \frac{1}{2}\sum_{n=1}^{\infty}\frac{(z-1)^{n-1}}{2^n} - \frac{1}{2(z-1)}(0<|z-1|<2) \\
&= -2\sum_{n=0}^{\infty}\frac{(z-1)^n}{2^{n+2}} - \sum_{n=0}^{\infty}\frac{(z-1)^n}{2^{n+2}} - \frac{1}{2(z-1)} \\
&= -3\sum_{n=0}^{\infty}\frac{(z-1)^n}{2^{n+2}} - \frac{1}{2(z-1)}(0<|z-1|<2)
\end{aligned}$$

Ex.7

The function $f(z)=\dfrac{1}{z(1+z^2)}$ has isolated singularities at $z=0$ and $z=\pm i$, as indicated in the figure below. Hence there is a Laurent series representation for the domain $0<|z|<1$ and also one for the domain $1<|z|<\infty$, which is exterior to the circle $|z|=1$.



To find each of these Laurent series, we recall the Maclaurin series representation

$$\frac{1}{1-z}=\sum_{n=0}^{\infty}z^n \qquad (|z|<1).$$

For the domain $0<|z|<1$, we have

$$f(z)=\frac{1}{z}\cdot\frac{1}{1+z^2}=\frac{1}{z}\sum_{n=0}^{\infty}\left(-z^2\right)^n=\sum_{n=0}^{\infty}(-1)^n z^{2n-1}=\frac{1}{z}+\sum_{n=1}^{\infty}(-1)^n z^{2n-1}=\sum_{n=0}^{\infty}(-1)^{n+1}z^{2n+1}+\frac{1}{z}.$$

On the other hand, when $1<|z|<\infty$,

$$f(z)=\frac{1}{z^3}\cdot\frac{1}{1+\frac{1}{z^2}}=\frac{1}{z^3}\sum_{n=0}^{\infty}\left(-\frac{1}{z^2}\right)^n=\sum_{n=0}^{\infty}\frac{(-1)^n}{z^{2n+3}}=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{z^{2n+1}}.$$

In this second expansion, we have used the fact that $(-1)^{n-1}=(-1)^{n-1}(-1)^2=(-1)^{n+1}$.