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Ex.2

Inequalities (3), Sec. 4, are

$$\operatorname{Re} z \leq |\operatorname{Re} z| \leq |z| \quad \text{and} \quad \operatorname{Im} z \leq |\operatorname{Im} z| \leq |z|.$$

These are obvious if we write them as

$$x \leq |x| \leq \sqrt{x^2 + y^2} \quad \text{and} \quad y \leq |y| \leq \sqrt{x^2 + y^2}.$$

Ex.4

In order to verify the inequality $\sqrt{2}|z| \geq |\operatorname{Re} z| + |\operatorname{Im} z|$, we rewrite it in the following ways:

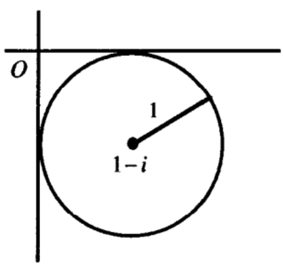
$$\sqrt{2}\sqrt{x^2 + y^2} \geq |x| + |y|,$$
$$2(x^2 + y^2) \geq |x|^2 + 2|x||y| + |y|^2,$$
$$|x|^2 - 2|x||y| + |y|^2 \geq 0,$$
$$(|x| - |y|)^2 \geq 0.$$

This last form of the inequality to be verified is obviously true since the left-hand side is a perfect square.

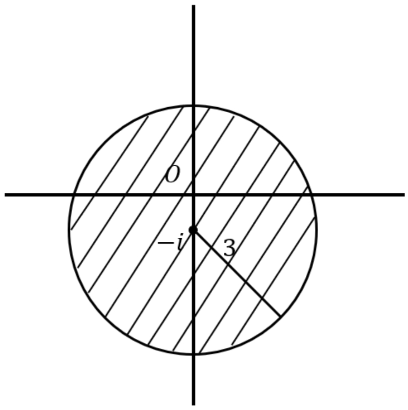
Ex.5

(a)

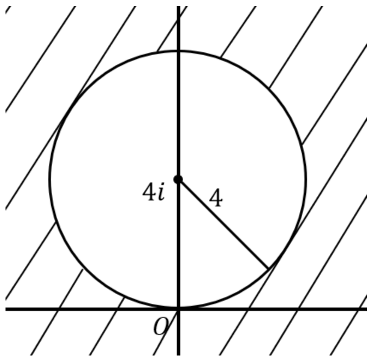
Rewrite $|z - 1 + i| = 1$ as $|z - (1 - i)| = 1$. This is the circle centered at $1 - i$ with radius 1. It is shown below.



(b) Rewrite $|z + i| \leq 3$ as $|z - (-i)| \leq 3$. This is a disk centered at $-i$ with radius 3.



(c) $|z - 4i| \geq 4$ is outside the disk centered at $4i$ with radius 4.



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Ex.1

(a) $\overline{\overline{z}+3i} = \overline{\overline{z}} + \overline{3i} = z - 3i$;

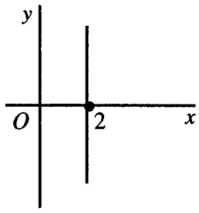
(b) $\overline{iz} = \overline{i}\,\overline{z} = -i\overline{z}$;

(c) $\overline{(2+i)^2} = \left(\overline{2+i}\right)^2 = (2-i)^2 = 4-4i+i^2 = 4-4i-1 = 3-4i$;

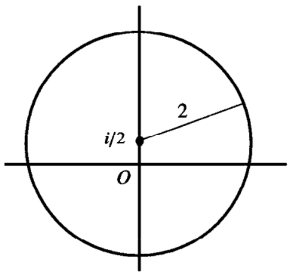
(d) $|(2\overline{z}+5)(\sqrt{2}-i)| = |2\overline{z}+5||\sqrt{2}-i| = |\overline{2z+5}|\sqrt{2+1} = \sqrt{3}\;|2z+5|$.

Ex.2

(a) Rewrite $\mathrm{Re}(\bar{z}-i)=2$ as $\mathrm{Re}[x+i(-y-1)]=2$, or $x=2$. This is the vertical line through the point $z=2$, shown below.



(b) Rewrite $|2\bar{z}+i|=4$ as $2\left|\bar{z}+\frac{i}{2}\right|=4$, or $\left|z-\frac{i}{2}\right|=2$. This is the circle centered at $\frac{i}{2}$ with radius 2, shown below.



Ex.7

In this problem, we shall use the inequalities (see Sec. 4)

$$|\mathrm{Re}\, z|\le|z| \quad \text{and} \quad |z_1+z_2+z_3|\le|z_1|+|z_2|+|z_3|.$$

Specifically, when $|z|\le 1$,

$$|\mathrm{Re}(2+\bar{z}+z^3)|\le|2+\bar{z}+z^3|\le 2+|\bar{z}|+|z^3|=2+|z|+|z|^3\le 2+1+1=4.$$

Ex.14

Since $x=\dfrac{z+\bar{z}}{2}$ and $y=\dfrac{z-\bar{z}}{2i}$, the hyperbola $x^2-y^2=1$ can be written in the following ways:

$$\left(\frac{z+\bar{z}}{2}\right)^2-\left(\frac{z-\bar{z}}{2i}\right)^2=1,$$
$$\frac{z^2+2z\bar{z}+\bar{z}^2}{4}+\frac{z^2-2z\bar{z}+\bar{z}^2}{4}=1,$$
$$\frac{2z^2+2\bar{z}^2}{4}=1,$$
$$z^2+\bar{z}^2=2.$$