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page 92-93

Ex.1

(a) $\exp(2\pm 3\pi i) = e^2 \exp(\pm 3\pi i) = -e^2$, since $\exp(\pm 3\pi i) = -1$.

(b) $$\exp\frac{2+\pi i}{4} = \left(\exp\frac{1}{2}\right)\left(\exp\frac{\pi i}{4}\right) = \sqrt{e}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$= \sqrt{e}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = \sqrt{\frac{e}{2}}(1+i).$$

(c) $\exp(z+\pi i) = (\exp z)(\exp \pi i) = -\exp z$, since $\exp \pi i = -1$.

Ex.6

First write

$$\left|\exp(z^2)\right| = \left|\exp[(x+iy)^2]\right| = \left|\exp(x^2-y^2) + i2xy\right| = \exp(x^2-y^2)$$

and

$$\exp(|z|^2) = \exp(x^2+y^2).$$

Since $x^2 - y^2 \le x^2 + y^2$, it is clear that $\exp(x^2-y^2) \le \exp(x^2+y^2)$. Hence it follows from the above that

$$\left|\exp(z^2)\right| \le \exp(|z|^2).$$

Ex.8

(a) Write $e^z = -2$ as $e^x e^{iy} = 2e^{i\pi}$. This tells us that

$$e^x = 2 \text{ and } y = \pi + 2n\pi \qquad (n = 0, \pm 1, \pm 2, \ldots).$$

That is,

$$x = \ln 2 \text{ and } y = (2n+1)\pi \qquad (n = 0, \pm 1, \pm 2, \ldots).$$

Hence

$$z = \ln 2 + (2n+1)\pi i \qquad (n = 0, \pm 1, \pm 2, \ldots).$$

(b) Write $e^z = 1 + \sqrt{3}i$ as $e^x e^{iy} = 2e^{i(\pi/3)}$, from which we see that

$$e^x = 2 \text{ and } y = \frac{\pi}{3} + 2n\pi \qquad (n = 0, \pm 1, \pm 2, \ldots).$$

That is,

$$x = \ln 2 \text{ and } y = \left(2n + \frac{1}{3}\right)\pi \qquad (n = 0, \pm 1, \pm 2, \ldots).$$

Consequently,

$$z = \ln 2 + \left(2n + \frac{1}{3}\right)\pi i \qquad (n = 0, \pm 1, \pm 2, \ldots).$$

(c) Write $\exp(2z-1) = 1$ as $e^{2x-1}e^{i2y} = 1e^{i0}$ and note how it follows that

$$e^{2x-1} = 1 \quad \text{and} \quad 2y = 0 + 2n\pi \qquad (n = 0, \pm 1, \pm 2, \ldots).$$

Evidently, then,

$$x = \frac{1}{2} \quad \text{and} \quad y = n\pi \qquad (n = 0, \pm 1, \pm 2, \ldots);$$

and this means that

$$z = \frac{1}{2} + n\pi i \qquad (n = 0, \pm 1, \pm 2, \ldots).$$

page 97-98

Ex.1

(a) $\mathrm{Log}(-ei) = \ln|-ei| + i\mathrm{Arg}(-ei) = \ln e - \frac{\pi}{2}i = 1 - \frac{\pi}{2}i.$

(b) $\mathrm{Log}(1-i) = \ln|1-i| + i\mathrm{Arg}(1-i) = \ln\sqrt{2} - \frac{\pi}{4}i = \frac{1}{2}\ln 2 - \frac{\pi}{4}i.$

Ex.3

(a) Observe that

$$\mathrm{Log}(1+i)^2 = \mathrm{Log}(2i) = \ln 2 + \frac{\pi}{2}i$$

and

$$2\mathrm{Log}(1+i) = 2\left(\ln\sqrt{2} + i\frac{\pi}{4}\right) = \ln 2 + \frac{\pi}{2}i.$$

Thus

$$\mathrm{Log}(1+i)^2 = 2\mathrm{Log}(1+i).$$

(b) On the other hand,

$$\mathrm{Log}(-1+i)^2 = \mathrm{Log}(-2i) = \ln 2 - \frac{\pi}{2} i$$

and

$$2\mathrm{Log}(-1+i) = 2\left(\ln\sqrt{2} + i\frac{3\pi}{4}\right) = \ln 2 + \frac{3\pi}{2} i.$$

Hence

$$\mathrm{Log}(-1+i)^2 \neq 2\mathrm{Log}(-1+i).$$

Ex.4

(a) Consider the branch

$$\log z = \ln r + i\theta \qquad \left(r > 0, \frac{\pi}{4} < \theta < \frac{9\pi}{4}\right).$$

Since

$$\log(i^2) = \log(-1) = \ln 1 + i\pi = \pi i \quad \text{and} \quad 2\log i = 2\left(\ln 1 + i\frac{\pi}{2}\right) = \pi i,$$

we find that $\log(i^2) = 2\log i$ when this branch of $\log z$ is taken.

(b) Now consider the branch

$$\log z = \ln r + i\theta \qquad \left(r > 0, \frac{3\pi}{4} < \theta < \frac{11\pi}{4}\right).$$

Here

$$\log(i^2) = \log(-1) = \ln 1 + i\pi = \pi i \quad \text{and} \quad 2\log i = 2\left(\ln 1 + i\frac{5\pi}{2}\right) = 5\pi i.$$

Hence, for this particular branch, $\log(i^2) \neq 2\log i$.

Ex.9

(a) $g(z) = z - i$ 为整函数，$f[g(z)] = \mathrm{Log}[g(z)]$ 在除 $\mathrm{Arg}[g(z)] = \pi$ 之外的区域解析，

$$\mathrm{Arg}[g(z)] = \pi \Rightarrow \frac{x}{\sqrt{x^2 + (y-1)^2}} = -1, \ \frac{y-1}{\sqrt{x^2 + (y-1)^2}} = 0 \Rightarrow x \leq 0, y = 1$$

(b) $f(z) = \mathrm{Log}(z+4)/(z^2+i)$ 在除 $z^2 + i = 0$，$\mathrm{Arg}(z+4) = \pi$ 之外的区域解析，

$$z^2 + i = 0 \Rightarrow z = e^{-i\pi/4}, e^{i3\pi/4} = \pm\frac{1-i}{\sqrt{2}}$$

$$\operatorname{Arg}(z+4)=\pi \Rightarrow \frac{x+4}{\sqrt{(x+4)^2+y^2}}=-1,\ \frac{y}{\sqrt{(x+4)^2+y^2}}=0 \Rightarrow x\le -4, y=0$$

Ex.10

Since $\ln(x^2+y^2)$ is the real component of any (analytic) branch of $2\log z$, it is harmonic in every domain that does not contain the origin. This can be verified directly by writing $u(x,y)=\ln(x^2+y^2)$ and showing that $u_{xx}(x,y)+u_{yy}(x,y)=0$.

page 100

Ex.1

Suppose that $\operatorname{Re} z_1>0$ and $\operatorname{Re} z_2>0$. Then

$$z_1=r_1\exp i\Theta_1 \quad\text{and}\quad z_2=r_2\exp i\Theta_2,$$

where

$$-\frac{\pi}{2}<\Theta_1<\frac{\pi}{2} \quad\text{and}\quad -\frac{\pi}{2}<\Theta_2<\frac{\pi}{2}.$$

The fact that $-\pi<\Theta_1+\Theta_2<\pi$ enables us to write

$$\begin{aligned}\operatorname{Log}(z_1z_2)&=\operatorname{Log}[(r_1r_2)\exp i(\Theta_1+\Theta_2)]=\ln(r_1r_2)+i(\Theta_1+\Theta_2)\\&=(\ln r_1+i\Theta_1)+(\ln r_2+i\Theta_2)=\operatorname{Log}(r_1\exp i\Theta_1)+\operatorname{Log}(r_2\exp i\Theta_2)\\&=\operatorname{Log} z_1+\operatorname{Log} z_2.\end{aligned}$$

Ex.2

$$\operatorname{Log}(z_1z_2)=\operatorname{Log}(r_1r_2e^{i(\theta_1+\theta_2)})=\ln(r_1r_2)+i(\Theta_1+\Theta_2+2N\pi)$$

$\Theta_1,\Theta_2\in(-\pi,\pi],\ \Theta_1+\Theta_2\in(-2\pi,2\pi]$。

当 $\Theta_1+\Theta_2\in(-2\pi,-\pi]$， $N=1,\Theta_1+\Theta_2+2\pi\in(0,\pi]\subset(-\pi,\pi]$。

当 $\Theta_1+\Theta_2\in(-\pi,\pi]$， $N=0$。

当 $\Theta_1+\Theta_2\in(\pi,2\pi]$， $N=-1,\Theta_1+\Theta_2-2\pi\in(-\pi,0]\subset(-\pi,\pi]$。

故 $\mathrm{Log}(z_1z_2)=\ln r_1+i\Theta_1+\ln r_2+i\Theta_2+i2N\pi=\mathrm{Log}z_1+\log z_2+i2N\pi$,$N$ 取 $0,\pm1$。

Ex.3

We are asked to show in two different ways that

$$\log\left(\frac{z_1}{z_2}\right)=\log z_1-\log z_2 \qquad (z_1\neq 0, z_2\neq 0).$$

(a) One way is to refer to the relation $\arg\left(\frac{z_1}{z_2}\right)=\arg z_1-\arg z_2$ in Sec. 7 and write

$$\log\left(\frac{z_1}{z_2}\right)=\ln\left|\frac{z_1}{z_2}\right|+i\arg\left(\frac{z_1}{z_2}\right)=(\ln|z_1|+i\arg z_1)-(\ln|z_2|+i\arg z_2)=\log z_1-\log z_2.$$

(b) Another way is to first show that $\log\left(\frac{1}{z}\right)=-\log z\,(z\neq 0)$. To do this, we write $z=re^{i\theta}$

and then

$$\log\left(\frac{1}{z}\right)=\log\left(\frac{1}{r}e^{-i\theta}\right)=\ln\left(\frac{1}{r}\right)+i(-\theta+2n\pi)=-[\ln r+i(\theta-2n\pi)]=-\log z,$$

where $n=0,\pm1,\pm2,\ldots$ This enables us to use the relation

$$\log(z_1z_2)=\log z_1+\log z_2$$

and write

$$\log\left(\frac{z_1}{z_2}\right)=\log\left(z_1\frac{1}{z_2}\right)=\log z_1+\log\left(\frac{1}{z_2}\right)=\log z_1-\log z_2.$$