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Ex.1

(a) Start by writing

$$I=\int_{-b}^{-a} w(-t)dt=\int_{-b}^{-a} u(-t)dt+i\int_{-b}^{-a} v(-t)dt.$$

The substitution $\tau=-t$ in each of these two integrals on the right then yields

$$I=-\int_{b}^{a} u(\tau)d\tau-i\int_{b}^{a} v(\tau)d\tau=\int_{a}^{b} u(\tau)d\tau+i\int_{a}^{b} v(\tau)d\tau=\int_{a}^{b} w(\tau)d\tau.$$

That is,

$$\int_{-b}^{-a} w(-t)dt=\int_{a}^{b} w(\tau)d\tau.$$

(b) Start with

$$I=\int_{a}^{b} w(t)dt=\int_{a}^{b} u(t)dt+i\int_{a}^{b} v(t)dt$$

and then make the substitution $t=\varphi(\tau)$ in each of the integrals on the right. The result is

$$I=\int_{\alpha}^{\beta} u[\phi(\tau)]\phi'(\tau)d\tau+i\int_{\alpha}^{\beta} v[\phi(\tau)]\phi'(\tau)d\tau=\int_{\alpha}^{\beta} w[\phi(\tau)]\phi'(\tau)d\tau.$$

That is,

$$\int_{a}^{b} w(t)dt=\int_{\alpha}^{\beta} w[\phi(\tau)]\phi'(\tau)d\tau.$$

Ex.3

The slope of the line through the points (α, a) and (β, b) in the τt plane is

$$m=\frac{b-a}{\beta-\alpha}.$$

So the equation of that line is

$$t-a=\frac{b-a}{\beta-\alpha}(\tau-\alpha).$$

Solving this equation for t, one can rewrite it as

$$t=\frac{b-a}{\beta-\alpha}\tau+\frac{a\beta-b\alpha}{\beta-\alpha}.$$

Since $t=\phi(\tau)$, then,

$$\phi(\tau)=\frac{b-a}{\beta-\alpha}\tau+\frac{a\beta-b\alpha}{\beta-\alpha}.$$

Ex.4

If $Z(\tau)=z[\phi(\tau)]$, where $z(t)=x(t)+iy(t)$ and $t=\phi(\tau)$, then

$$Z(\tau)=x[\phi(\tau)]+iy[\phi(\tau)].$$

Hence

$$Z'(\tau)=\frac{d}{d\tau}x[\phi(\tau)]+i\frac{d}{d\tau}y[\phi(\tau)]=x'[\phi(\tau)]\phi'(\tau)+iy'[\phi(\tau)]\phi'(\tau)$$

$$=\{x'[\phi(\tau)]+iy'[\phi(\tau)]\}\phi'(\tau)=z'[\phi(\tau)]\phi'(\tau).$$

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Ex.2

(a) The arc is $C: z=1+e^{i\theta}\ (\pi \le \theta \le 2\pi)$. Then

$$\int_C (z-1)dz = \int_{\pi}^{2\pi} (1+e^{i\theta}-1)ie^{i\theta}d\theta = i\int_{\pi}^{2\pi} e^{i2\theta}d\theta = i\left[\frac{e^{i2\theta}}{2i}\right]_{\pi}^{2\pi}$$
$$=\frac{1}{2}\left(e^{i4\pi}-e^{i2\pi}\right)=\frac{1}{2}(1-1)=0.$$

(b) Here $C: z=x\ (0 \le x \le 2)$. Then

$$\int_C (z-1)dz = \int_0^2 (x-1)dx = \left[\frac{x^2}{2}-x\right]_0^2 = 0.$$

Ex.5

The contour C has some parametric representation $z=z(t)\ (a \le t \le b)$, where $z(a)=z_1$ and $z(b)=z_2$. Then

$$\int_C dz = \int_a^b z'(t)dt = \left[z(t)\right]_a^b = z(b)-z(a) = z_2 - z_1.$$

Ex.7

$z(\theta)=e^{i\theta}(0\le\theta\le\pi)$，则

$$f[z(\theta)]z'(\theta)=ie^{(i-1)\theta}(0\le\theta<\pi)$$

定义 $f[z(\theta)]z'(\theta)$ 在 $\theta=\pi$ 取值为 $ie^{(i-1)\pi}$,

$$\begin{aligned}\int_C f(z)dz &= i\int_0^{\pi} e^{(i-1)\theta}d\theta \\ &= \frac{i}{i-1}e^{(i-1)\theta}\Big|_0^{\pi} \\ &= -\frac{1+e^{-\pi}}{2}(1-i)\end{aligned}$$

Ex.8

Let C be the positively oriented circle $|z|=1$, with parametric representation $z=e^{i\theta}\,(0\le\theta\le 2\pi)$, and let m and n be integers. Then

$$\int_C z^m \overline{z}^n dz = \int_0^{2\pi} \left(e^{i\theta}\right)^m \left(e^{-i\theta}\right)^n ie^{i\theta} d\theta = i\int_0^{2\pi} e^{i(m+1)\theta} e^{-in\theta} d\theta.$$

But we know from Exercise 3, Sec. 38, that

$$\int_0^{2\pi} e^{im\theta} e^{-in\theta} d\theta = \begin{cases} 0 & \text{when} \quad m \ne n, \\ 2\pi & \text{when} \quad m = n. \end{cases}$$

Consequently,

$$\int_C z^m \overline{z}^n dz = \begin{cases} 0 & \text{when} \quad m+1 \ne n, \\ 2\pi i & \text{when} \quad m+1 = n. \end{cases}$$

Ex.10

(a) Since

$$\int_{C_0} f(z-z_0)dz = \int_{-\pi}^{\pi} f(Re^{i\theta})Rie^{i\theta} d\theta$$

and

$$\int_C f(z)dz = \int_{-\pi}^{\pi} f(Re^{i\theta})Rie^{i\theta} d\theta,$$

we have

$$\int_{C_0} f(z-z_0)dz = \int_C f(z)dz.$$

(b) The results

$$\int_{C_0} (z-z_0)^{n-1} dz = 0\,(n=\pm 1, \pm 2, \ldots) \quad \text{and} \quad \int_{C_0} \frac{dz}{z-z_0} = 2\pi i$$

are immediate consequences of part *(a)* and integrals (5) and (6) in Sec. 42.