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Ex.1

(a) Since

$$\arg\left(\frac{i}{-2-2i}\right) = \arg i - \arg(-2-2i),$$

one value of $\arg\left(\frac{i}{-2-2i}\right)$ is $\frac{\pi}{2} - \left(-\frac{3\pi}{4}\right)$, or $\frac{5\pi}{4}$. Consequently, the principal value is

$$\frac{5\pi}{4} - 2\pi, \text{ or } -\frac{3\pi}{4}.$$

(b) Since

$$\arg(\sqrt{3}-i)^6 = 6\arg(\sqrt{3}-i),$$

one value of $\arg(\sqrt{3}-i)^6$ is $6\left(-\frac{\pi}{6}\right)$, or $-\pi$. So the principal value is $-\pi + 2\pi$, or π.

Ex.6

Proof : Let $z_1 = r_1 e^{i\theta_1}$, $z_2 = r_2 e^{i\theta_2}$, $\theta_1 = \mathrm{Arg}(z_1)$, $\theta_2 = \mathrm{Arg}(z_2)$. For $\mathrm{Re}(z_1) > 0$, $\mathrm{Re}(z_2) > 0$, then $-\frac{\pi}{2} < \theta_1, \theta_2 < \frac{\pi}{2}$.

$\mathrm{Arg}(z_1 z_2) = \mathrm{Arg}(r_1 r_2 e^{i(\theta_1+\theta_2)})$,we have known that $-\pi < \theta_1 + \theta_2 < \pi$, then $\mathrm{Arg}(z_1 z_2) = \theta_1 + \theta_2 = \mathrm{Arg}(z_1) + \mathrm{Arg}(z_2)$.

Ex.8

First of all, given two nonzero complex numbers z_1 and z_2, suppose that there are complex numbers c_1 and c_2 such that $z_1 = c_1 c_2$ and $z_2 = c_1 \overline{c_2}$. Since

$$|z_1| = |c_1||c_2| \quad \text{and} \quad |z_2| = |c_1||\overline{c_2}| = |c_1||c_2|,$$

it follows that $|z_1| = |z_2|$.

Suppose, on the other hand, that we know only that $|z_1| = |z_2|$. We may write

$$z_1 = r_1 \exp(i\theta_1) \quad \text{and} \quad z_2 = r_1 \exp(i\theta_2).$$

If we introduce the numbers

$$c_1 = r_1 \exp\left(i\frac{\theta_1+\theta_2}{2}\right) \quad \text{and} \quad c_2 = \exp\left(i\frac{\theta_1-\theta_2}{2}\right),$$

we find that

$$c_1 c_2 = r_1 \exp\left(i\frac{\theta_1+\theta_2}{2}\right)\exp\left(i\frac{\theta_1-\theta_2}{2}\right) = r_1 \exp(i\theta_1) = z_1$$

and

$$c_1\bar{c}_2 = r_1 \exp\left(i\frac{\theta_1+\theta_2}{2}\right)\exp\left(-i\frac{\theta_1-\theta_2}{2}\right) = r_1 \exp\theta_2 = z_2.$$

That is,

$$z_1 = c_1 c_2 \quad \text{and} \quad z_2 = c_1\bar{c}_2.$$

Ex.10

We know from de Moivre's formula that

$$(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta,$$

or

$$\cos^3\theta + 3\cos^2\theta(i\sin\theta) + 3\cos\theta(i\sin\theta)^2 + (i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta.$$

That is,

$$(\cos^3\theta - 3\cos\theta\sin^2\theta) + i(3\cos^2\theta\sin\theta - \sin^3\theta) = \cos 3\theta + i\sin 3\theta.$$

By equating real parts and then imaginary parts here, we arrive at the desired trigonometric identities:

(a) $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$; (b) $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$.

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Ex.2

(a) Since $-16 = 16\exp[i(\pi + 2k\pi)]$ $(k = 0, \pm 1, \pm 2, \ldots)$, the needed roots are

$$(-16)^{1/4} = 2\exp\left[i\left(\frac{\pi}{4} + \frac{k\pi}{2}\right)\right] \qquad (k = 0, 1, 2, 3).$$

The principal root is

$$c_0 = 2e^{i\pi/4} = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 2\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = \sqrt{2}(1+i).$$

The other three roots are

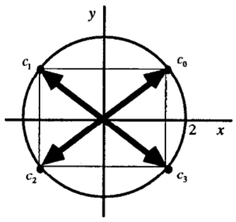
$$c_1 = (2e^{i\pi/4})e^{i\pi/2} = c_0 i = \sqrt{2}(1+i)i = -\sqrt{2}(1-i),$$

$$c_2 = (2e^{i\pi/4})e^{i\pi} = -c_0 = -\sqrt{2}(1+i),$$

and

$$c_3 = (2e^{i\pi/4})e^{i3\pi/2} = c_0(-i) = \sqrt{2}(1+i)(-i) = \sqrt{2}(1-i).$$

The four roots are shown below.



(b) First write $-8-8\sqrt{3}i = 16\exp\left[i\left(-\frac{2\pi}{3}+2k\pi\right)\right]$ $(k = 0,\pm1,\pm2,\ldots)$. Then

$$(-8-8\sqrt{3}i)^{1/4} = 2\exp\left[i\left(-\frac{\pi}{6}+\frac{k\pi}{2}\right)\right] \qquad (k = 0,1,2,3).$$

The principal root is

$$c_0 = 2e^{-i\pi/6} = 2\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right) = 2\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right) = \sqrt{3} - i.$$

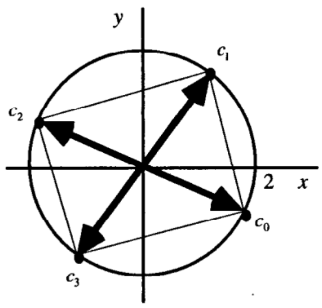
The others are

$$c_1 = (2e^{-i\pi/6})e^{i\pi/2} = c_0 i = 1 + \sqrt{3}i,$$

$$c_2 = (2e^{-i\pi/6})e^{i\pi} = -c_0 = -(\sqrt{3} - i),$$

$$c_3 = (2e^{-i\pi/6})e^{i3\pi/2} = c_0(-i) = -(1 + \sqrt{3}i).$$

These roots are all shown below.



Ex.4

The three cube roots of the number $z_0 = -4\sqrt{2} + 4\sqrt{2}i = 8\exp\left(i\frac{3\pi}{4}\right)$ are evidently

$$(z_0)^{1/3} = 2\exp\left[i\left(\frac{\pi}{4} + \frac{2k\pi}{3}\right)\right] \qquad (k = 0,1,2).$$

In particular,

$$c_0 = 2\exp\left(i\frac{\pi}{4}\right) = \sqrt{2}(1+i).$$

With the aid of the number $\omega_3 = \dfrac{-1+\sqrt{3}i}{2}$, we obtain the other two roots:

$$c_1 = c_0\omega_3 = \sqrt{2}(1+i)\left(\frac{-1+\sqrt{3}i}{2}\right) = \frac{-(\sqrt{3}+1)+(\sqrt{3}-1)i}{\sqrt{2}},$$

$$c_2 = c_0\omega_3^2 = (c_0\omega_3)\omega_3 = \left[\frac{-(\sqrt{3}+1)+(\sqrt{3}-1)i}{\sqrt{2}}\right]\left(\frac{-1+\sqrt{3}i}{2}\right) = \frac{(\sqrt{3}-1)-(\sqrt{3}+1)i}{\sqrt{2}}.$$

Ex.5

(a) Let a denote any fixed real number. In order to find the two square roots of $a+i$ in exponential form, we write

$$A = |a+i| = \sqrt{a^2+1} \quad \text{and} \quad \alpha = \operatorname{Arg}(a+i).$$

Since

$$a + i = A\exp[i(\alpha + 2k\pi)] \qquad (k = 0,\pm1,\pm2,\ldots),$$

we see that

$$(a+i)^{1/2} = \sqrt{A}\exp\left[i\left(\frac{\alpha}{2} + k\pi\right)\right] \qquad (k = 0,1).$$

That is, the desired square roots are

$$\sqrt{A}e^{i\alpha/2} \quad \text{and} \quad \sqrt{A}e^{i\alpha/2}e^{i\pi} = -\sqrt{A}e^{i\alpha/2}.$$

(b) Since $a+i$ lies above the real axis, we know that $0 < \alpha < \pi$. Thus $0 < \frac{\alpha}{2} < \frac{\pi}{2}$, and this tells us that $\cos\left(\frac{\alpha}{2}\right) > 0$ and $\sin\left(\frac{\alpha}{2}\right) > 0$. Since $\cos\alpha = \frac{a}{A}$, it follows that

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \frac{1}{\sqrt{2}}\sqrt{1+\frac{a}{A}} = \frac{\sqrt{A+a}}{\sqrt{2}\sqrt{A}}$$

and

$$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \frac{1}{\sqrt{2}}\sqrt{1-\frac{a}{A}} = \frac{\sqrt{A-a}}{\sqrt{2}\sqrt{A}}.$$

Consequently,

$$\pm\sqrt{A}e^{i\alpha/2} = \pm\sqrt{A}\left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\right) = \pm\sqrt{A}\left(\frac{\sqrt{A+a}}{\sqrt{2}\sqrt{A}} + i\frac{\sqrt{A-a}}{\sqrt{2}\sqrt{A}}\right)$$
$$= \pm\frac{1}{\sqrt{2}}(\sqrt{A+a} + i\sqrt{A-a}).$$

Ex.7

Let c be any nth root of unity other than unity itself. With the aid of the identity (see Exercise 9, Sec. 8),

$$1+z+z^2+\cdots+z^{n-1}=\frac{1-z^n}{1-z} \qquad (z\neq 1),$$

we find that

$$1+c+c^2+\cdots+c^{n-1}=\frac{1-c^n}{1-c}=\frac{1-1}{1-c}=0.$$

Ex.9

Observe first that

$$(z^{1/m})^{-1}=\left[\sqrt[m]{r}\exp\frac{i(\theta+2k\pi)}{m}\right]^{-1}=\frac{1}{\sqrt[m]{r}}\exp\frac{i(-\theta-2k\pi)}{m}=\frac{1}{\sqrt[m]{r}}\exp\frac{i(-\theta)}{m}\exp\frac{i(-2k\pi)}{m}$$

and

$$(z^{-1})^{1/m}=\sqrt[m]{\frac{1}{r}}\exp\frac{i(-\theta+2k\pi)}{m}=\frac{1}{\sqrt[m]{r}}\exp\frac{i(-\theta)}{m}\exp\frac{i(2k\pi)}{m},$$

where $k=0,1,2,\ldots,m-1$. Since the set

$$\exp\frac{i(-2k\pi)}{m} \qquad (k=0,1,2,\ldots,m-1)$$

is the same as the set

$$\exp\frac{i(2k\pi)}{m} \qquad (k=0,1,2,\ldots,m-1),$$

but in reverse order, we find that $(z^{1/m})^{-1}=(z^{-1})^{1/m}$.