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page 160-163

Ex.1

(a) $f(z)=\frac{z^2}{z-3}$在$z=3$以外的地方解析，即在简单闭曲线C及其包围的区域上解析，由Cauchy-Goursat定理得到$\int_C f(z)dz=0$。

(b) $f(z)=ze^{-z}$在复平面上解析，由Cauchy-Goursat定理得到$\int_C f(z)dz=0$。

(c) $f(z)=\frac{1}{z^2+2z+2}$在$z=-1\pm i$以外的地方解析，处于C包围的区域外，由Cauchy-Goursat定理有$\int_C f(z)dz=0$。

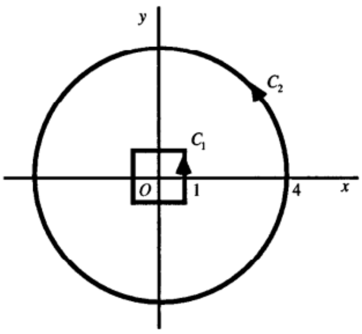
(d) $f(z)=\mathrm{sech}z=\frac{2}{e^z+e^{-z}}$在$z=i\pi(\frac{1}{2}+n),(n=0,\pm1,\pm2,...)$以外解析，处于$C$包围的区域以外，由Cauchy-Goursat定理有$\int_C f(z)dz=0$。

(e) $f(z)=\tan z$在$z=\pi(\frac{1}{2}+n),(n=0,\pm1,\pm2,...)$以外解析，处于$C$包围的区域以外，由Cauchy-Goursat定理有$\int_C f(z)dz=0$。

(f) $f(z)=\mathrm{Log}(z+2)$在实轴上以$z=-2$分割的左半轴外的区域解析，处于C包围的区域以外，由Cauchy-Goursat定理有$\int_C f(z)dz=0$。

Ex.2

The contours C_1 and C_2 are as shown in the figure below.



In each of the cases below, the singularities of the integrand lie inside C_1 or outside of C_2; and so the integrand is analytic on the contours and between them. Consequently,

$$\int_{C_1} f(z)dz=\int_{C_2} f(z)dz.$$

(a) When $f(z)=\dfrac{1}{3z^2+1}$, the singularities are the points $z=\pm\dfrac{1}{\sqrt{3}}i$.

(b) When $f(z)=\dfrac{z+2}{\sin(z/2)}$, the singularities are at $z=2n\pi\ (n=0,\pm1,\pm2,\ldots)$.

(c) When $f(z)=\dfrac{z}{1-e^z}$, the singularities are at $z=2n\pi i\ (n=0,\pm1,\pm2,\ldots)$.

Ex.3

令C_0为以$2+i$为圆心，半径$0<R<1$的圆，方向为逆时针。$f(z)=(z-2-i)^{n-1}, n=0,\pm 1,\pm 2,...$在$z=2+i$以外的区域可以确定为解析的，即在$C$和$C_0$之间的区域解析，由推论可以得到

$$\int_C (z-2-i)^{n-1}dz=\int_{C_0}(z-2-i)^{n-1}dz=\begin{cases}0 & \text{when n}=\pm 1,\pm 2,... \\ 2\pi i & \text{when n }=0\end{cases}$$

Ex.7

令

$$f(z)=\bar{z}=u(x,y)+iv(x,y)=x-iy$$

由格林公式可得

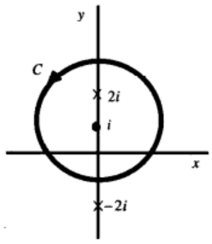
$$\begin{aligned}\int_C f(z)dz &= \iint_R(-v_x-u_y)dA+i\iint_R(u_x-v_y)dA \\ &= 2i\iint_R dA\end{aligned}$$

故C所围区域的面积$\iint_R dA=\frac{1}{2i}\int_C \bar{z}dz$。

page 170-172

Ex.2

Let C denote the positively oriented circle $|z-i|=2$, shown below.



(a) The Cauchy integral formula enables us to write

$$\int_C \frac{dz}{z^2+4}=\int_C \frac{dz}{(z-2i)(z+2i)}=\int_C \frac{1/(z+2i)}{z-2i}dz=2\pi i\left(\frac{1}{z+2i}\right)_{z=2i}=2\pi i\left(\frac{1}{4i}\right)=\frac{\pi}{2}.$$

(b) Applying the extended form of the Cauchy integral formula, we have

$$\int_C \frac{dz}{(z^2+4)^2}=\int_C \frac{dz}{(z-2i)^2(z+2i)^2}=\int_C \frac{1/(z+2i)^2}{(z-2i)^{1+1}}\,dz=\frac{2\pi i}{1!}\left[\frac{d}{dz}\frac{1}{(z+2i)^2}\right]_{z=2i}$$

$$=2\pi i\left[\frac{-2}{(z+2i)^3}\right]_{z=2i}=\frac{-4\pi i}{(4i)^3}=\frac{-4\pi i}{-(16)(4)i}=\frac{\pi}{16}.$$

Ex.4

令$f(s)=s^3+2s$,则

$$g(z)=\int_C \frac{f(s)}{(s-z)^3}ds$$

因为$f(s)$在整个复平面解析，若z在C里面，根据柯西积分公式有

$$g(z)=\frac{2\pi i}{2!}f^{(2)}(z)=6\pi iz$$

若z在C外面，则根据Cauchy-Goursat定理

$$g(z)=\int_C \frac{s^3+2s}{(s-z)^3}ds=0$$

Ex.5

Suppose that a function f is analytic inside and on a simple closed contour C and that z_0 is not on C. If z_0 is inside C, then

$$\int_C \frac{f'(z)dz}{z-z_0} = 2\pi i f'(z_0) \quad \text{and} \quad \int_C \frac{f(z)dz}{(z-z_0)^2} = \int_C \frac{f(z)dz}{(z-z_0)^{1+1}} = \frac{2\pi i}{1!} f'(z_0).$$

Thus

$$\int_C \frac{f'(z)dz}{z-z_0} = \int_C \frac{f(z)dz}{(z-z_0)^2}.$$

The Cauchy-Goursat theorem tells us that this last equation is also valid when z_0 is exterior to C, each side of the equation being 0.

Ex.7

Let C be the unit circle $z = e^{i\theta}$ $(-\pi \le \theta \le \pi)$, and let a denote any real constant. The Cauchy integral formula reveals that

$$\int_C \frac{e^{az}}{z} dz = \int_C \frac{e^{az}}{z-0} dz = 2\pi i \left[e^{az}\right]_{z=0} = 2\pi i.$$

On the other hand, the stated parametric representation for C gives us

$$\int_C \frac{e^{az}}{z} dz = \int_{-\pi}^{\pi} \frac{\exp(ae^{i\theta})}{e^{i\theta}} ie^{i\theta} d\theta = i \int_{-\pi}^{\pi} \exp[a(\cos\theta + i\sin\theta)] d\theta$$

$$= i \int_{-\pi}^{\pi} e^{a\cos\theta} e^{ia\sin\theta} d\theta = i \int_{-\pi}^{\pi} e^{a\cos\theta} [\cos(a\sin\theta) + i\sin(a\sin\theta)] d\theta$$

$$= -\int_{-\pi}^{\pi} e^{a\cos\theta} \sin(a\sin\theta) d\theta + i \int_{-\pi}^{\pi} e^{a\cos\theta} \cos(a\sin\theta) d\theta.$$

Equating these two different expressions for the integral $\int_C \frac{e^{az}}{z} dz$, we have

$$-\int_{-\pi}^{\pi} e^{a\cos\theta} \sin(a\sin\theta) d\theta + i \int_{-\pi}^{\pi} e^{a\cos\theta} \cos(a\sin\theta) d\theta = 2\pi i.$$

Then, by equating the imaginary parts on each side of this last equation, we see that

$$\int_{-\pi}^{\pi} e^{a\cos\theta} \cos(a\sin\theta) d\theta = 2\pi;$$

and, since the integrand here is even,

$$\int_{0}^{\pi} e^{a\cos\theta} \cos(a\sin\theta) d\theta = \pi.$$