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23214322	何昌烨
23214323	胡静静
23214324	黄项龙
23214326	刘润尧
23214329	宋珂
23214336	戴泳涛
23214338	杜冠男
23214339	段培明
23214345	黄瀚
23214346	黄樾
23214353	梁励
23214364	毛睿
23214369	钱甜奕
23214378	吴昊
23214383	徐博研
23214395	赵海洋
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23214446	何鸿荣
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23214452	洪桂航
23214460	黄泽林
23214466	赖柔成
23214474	李宏立
23214478	李茂锦
23214491	梁恒中
23214503	刘星宇
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23214542	王辉
23214564	熊泽华
23214565	徐浩耀
23214573	杨坤业
23214576	杨子逸
23214578	杨沅旭
23214590	易钰淇
23214594	曾家洋
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23214601	张晓逊
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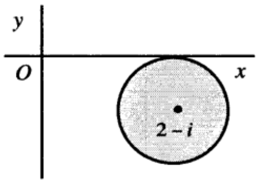
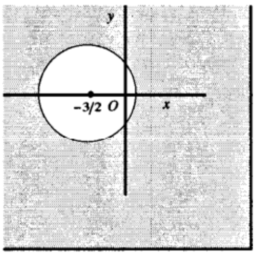
Ex.1

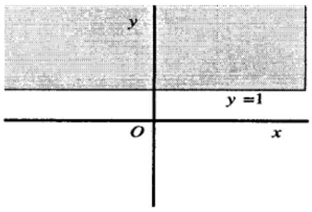
(a) Write $|z-2+i|\le 1$ as $|z-(2-i)|\le 1$ to see that this is the set of points inside and on the circle centered at the point $2-i$ with radius 1. It is *not* a domain.



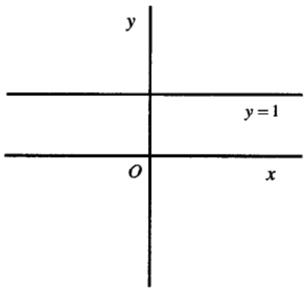
(b) Write $|2z+3|>4$ as $\left|z-\left(-\frac{3}{2}\right)\right|>2$ to see that the set in question consists of all points exterior to the circle with center at $-3/2$ and radius 2. It is a domain.



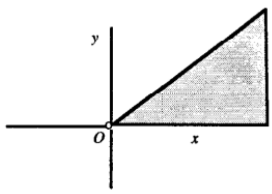
(c) Write $\operatorname{Im} z>1$ as $y>1$ to see that this is the half plane consisting of all points lying above the horizontal line $y=1$. It is a domain.



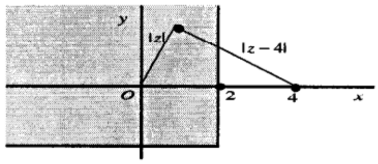
(d) The set $\operatorname{Im} z=1$ is simply the horizontal line $y=1$. It is *not* a domain.



(*e*) The set $0 \le \arg z \le \frac{\pi}{4}$ $(z \neq 0)$ is indicated below. It is *not* a domain.



(*f*) The set $|z-4| \ge |z|$ can be written in the form $(x-4)^2 + y^2 \ge x^2 + y^2$, which reduces to $x \le 2$. This set, which is indicated below, is *not* a domain. The set is also geometrically evident since it consists of all points z such that the distance between z and 4 is greater than or equal to the distance between z and the origin.

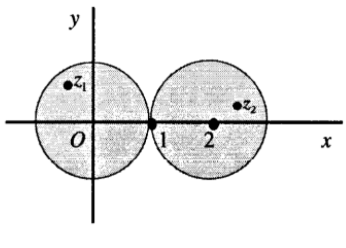


Ex.2

(e) is obviously not open for containing boundary points, and it's not closed because boundary point $z = 0$ is not in this set.

Ex.5

The set S consists of all points z such that $|z| < 1$ or $|z-2| < 1$, as shown below.



Since every polygonal line joining z_1 and z_2 must contain at least one point that is not in S, it is clear that S is not connected.

Ex.6

$Proof$: $(\Longrightarrow)$ If each point in S is an interior point, then S contains none of its boundary points. According to the definition, set S is open.

$(\Longleftarrow)$If set S is open, points in S are neither boundary points or exterior points.

Then each point in S is an interior point.

Ex.10

Proof :Let $S := \{z_1, z_2, ..., z_n\}$ and $\epsilon = \min\{|z_i - z_j||1 \le i, j \le n\}$, obviously $\epsilon > 0$. Assume that point z_i is an accumulation point in S, then each deleted neighborhood of z_i contains at least one point in S, which contradicts with the fact that $0 < |z - z_i| < \epsilon$ contains none of points in S. Then S cannot have any accumulation points.

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Ex.1

(a) The function $f(z)=\dfrac{1}{z^2+1}$ is defined everywhere in the finite plane except at the points $z=\pm i$, where $z^2+1=0$.

(b) The function $f(z)=\text{Arg}\left(\dfrac{1}{z}\right)$ is defined throughout the entire finite plane except for the point $z=0$.

(c) The function $f(z)=\dfrac{z}{z+\overline{z}}$ is defined everywhere in the finite plane except for the imaginary axis. This is because the equation $z+\overline{z}=0$ is the same as $x=0$.

(d) The function $f(z)=\dfrac{1}{1-|z|^2}$ is defined everywhere in the finite plane except on the circle $|z|=1$, where $1-|z|^2=0$.

Ex.2

$$
\begin{aligned}
f(z) &= z^3 + z + 1 \\
&= (x+iy)^3 + (x+iy) + 1 \\
&= (x^3 - xy^2 - 2xy^2) + i(2x^2y + x^2y - y^3) + x + iy + 1 \\
&= (x^3 - 3xy^2 + x + 1) + i(3x^2y - y^3 + y)
\end{aligned}
$$

Ex.4

$$\begin{aligned}
f(z) &= z + \frac{1}{z}\ (z \neq 0) \\
&= re^{i\theta} + \frac{1}{r}e^{-i\theta} \\
&= r(\cos\theta + i\sin\theta) + \frac{1}{r}(\cos\theta - i\sin\theta) \\
&= (r + \frac{1}{r})\cos\theta + i(r - \frac{1}{r})\sin\theta
\end{aligned}$$

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Ex.2

$u = x^2 - y^2 = c_1 < 0$ 在 xy 平面有上下两分支，

$$u = c_1,\ v = 2x\sqrt{x^2 - c_1}\ (-\infty < x < \infty)$$

$$u = c_1,\ v = -2x\sqrt{x^2 - c_1}\ (-\infty < x < \infty)$$

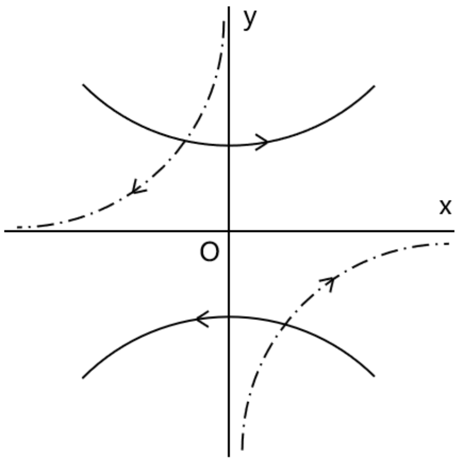
$u = c_1$ 方向朝上时，对应上下分支的方向分别为从左到右和从右到左。

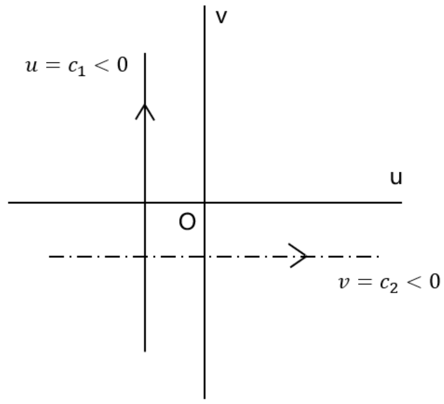
$v = 2xy = c_2 < 0$ 在 xy 平面同样存在两分支，

$$u = x^2 - \frac{c_2^2}{4x^2},\ v = c_2,\ (-\infty < x < 0)$$

$$u = \frac{c_2^2}{4y^2} - y^2,\ v = c_2,\ (-\infty < y < 0)$$

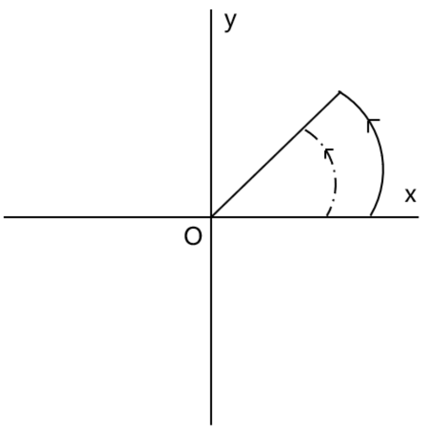
$v = c_2$ 方向朝右时，对应左右分支的方向分别为从上向下和从下向上。

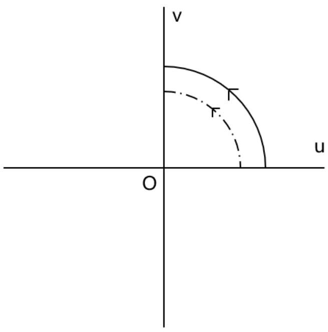




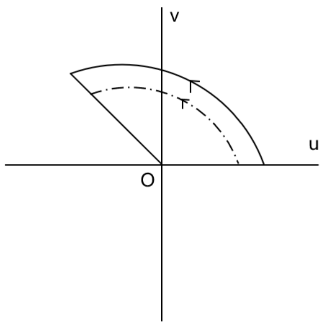
Ex.3

原区域

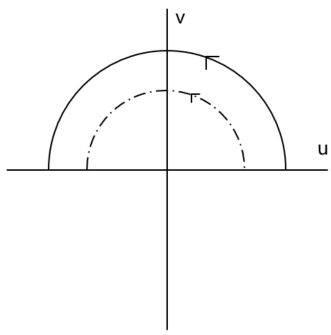




(a)



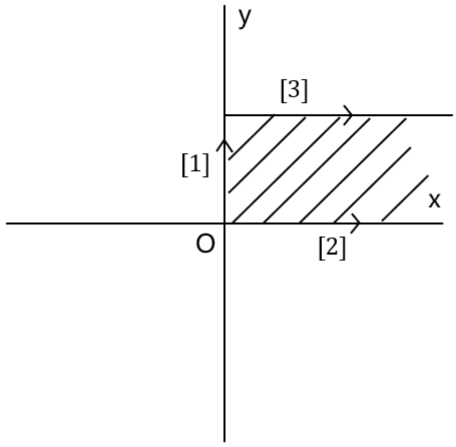
(b)



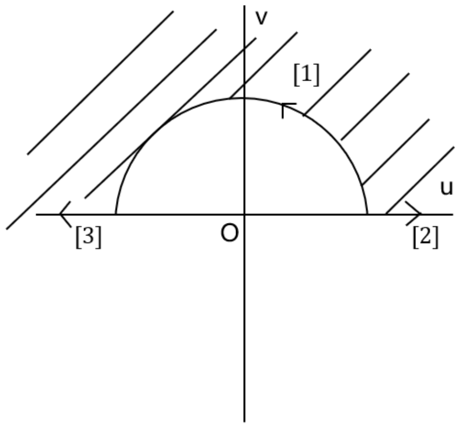
(c)

Ex.7

原区域

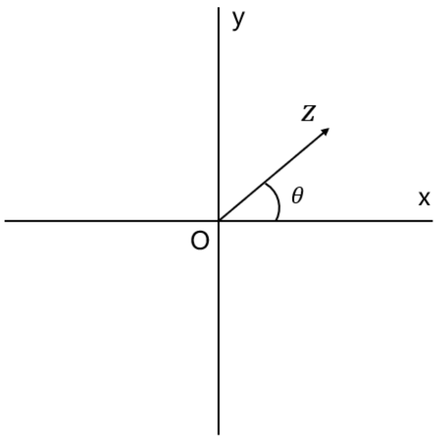


映射区域

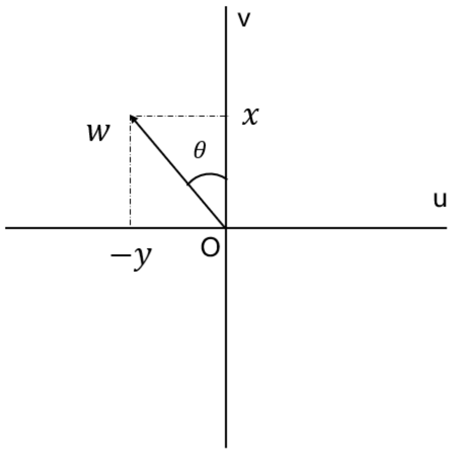


Ex.8

设 $w = f(z)$ 的定义域为 xy 平面，对平面的某点 $z = x + iy$，



(a) $w = iz = -y + ix$，w 对应向量为



(b) $w = \frac{z}{|z|} = \frac{x}{\sqrt{x^2+y^2}} + i\frac{y}{\sqrt{x^2+y^2}}$，$w$ 对应向量为