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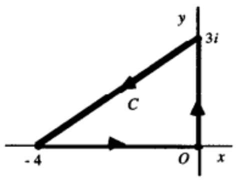
Ex.3

The contour C is the closed triangular path shown below.



To find an upper bound for $\left|\int_C (e^z - \bar{z})dz\right|$, we let z be a point on C and observe that

$$|e^z - \bar{z}| \le |e^z| + |\bar{z}| = e^x + \sqrt{x^2 + y^2}.$$

But $e^x \le 1$ since $x \le 0$, and the distance $\sqrt{x^2+y^2}$ of the point z from the origin is always less than or equal to 4. Thus $|e^z - \bar{z}| \le 5$ when z is on C. The length of C is evidently 12. Hence, by writing $M = 5$ and $L = 12$, we have

$$\left|\int_C (e^z - \bar{z})dz\right| \le ML = 60.$$

Ex.4

Note that if $|z| = R$ $(R > 2)$, then

$$|2z^2 - 1| \le 2|z|^2 + 1 = 2R^2 + 1$$

and

$$|z^4 + 5z^2 + 4| = |z^2 + 1||z^2 + 4| \ge \left||z|^2 - 1\right|\left||z|^2 - 4\right| = (R^2 - 1)(R^2 - 4).$$

Thus

$$\left|\frac{2z^2-1}{z^4+5z^2+4}\right| = \frac{|2z^2-1|}{|z^4+5z^2+4|} \le \frac{2R^2+1}{(R^2-1)(R^2-4)}$$

when $|z|=R$ $(R>2)$. Since the length of C_R is πR, then,

$$\left|\int_{C_R} \frac{2z^2-1}{z^4+5z^2+4}dz\right| \le \frac{\pi R(2R^2+1)}{(R^2-1)(R^2-4)} = \frac{\dfrac{\pi}{R}\left(2+\dfrac{1}{R^2}\right)}{\left(1-\dfrac{1}{R^2}\right)\left(1-\dfrac{4}{R^2}\right)};$$

and it is clear that the value of the integral tends to zero as R tends to infinity.

Ex.5

Here C_R is the positively oriented circle $|z|=R(R>1)$. If z is a point on C_R, then

$$\left|\frac{\mathrm{Log}\,z}{z^2}\right| = \frac{|\ln R + i\Theta|}{R^2} \le \frac{\ln R + |\Theta|}{R^2} \le \frac{\pi + \ln R}{R^2},$$

since $-\pi < \Theta \le \pi$. The length of C_R is, of course, $2\pi R$. Consequently, by taking

$$M = \frac{\pi + \ln R}{R^2} \quad \text{and} \quad L = 2\pi R,$$

we see that

$$\left|\int_{C_R} \frac{\mathrm{Log}\,z}{z^2}dz\right| \le ML = 2\pi\left(\frac{\pi+\ln R}{R}\right).$$

Since

$$\lim_{R\to\infty} \frac{\pi+\ln R}{R} = \lim_{R\to\infty} \frac{1/R}{1} = 0,$$

it follows that

$$\lim_{R\to\infty} \int_{C_R} \frac{\mathrm{Log}\,z}{z^2}dz = 0.$$

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Ex.2

(a) $\int_i^{i/2} e^{\pi z}\,dz = \left[\frac{e^{\pi z}}{\pi}\right]_i^{i/2} = \frac{e^{i\pi/2}-e^{i\pi}}{\pi} = \frac{i+1}{\pi} = \frac{1+i}{\pi}.$

(b) $\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right)dz = 2\left[\sin\left(\frac{z}{2}\right)\right]_0^{\pi+2i} = 2\sin\left(\frac{\pi}{2}+i\right) = 2\frac{e^{i\left(\frac{\pi}{2}+i\right)}-e^{-i\left(\frac{\pi}{2}+i\right)}}{2i} = -i\left(e^{i\pi/2}e^{-1}-e^{-i\pi/2}e\right)$

$= -i\left(\frac{i}{e}+ie\right) = \frac{1}{e}+e = e+\frac{1}{e}.$

(c) $\int_1^3 (z-2)^3\,dz = \left[\frac{(z-2)^4}{4}\right]_1^3 = \frac{1}{4}-\frac{1}{4} = 0.$

Ex.3

Note the function $(z-z_0)^{n-1}$ $(n=\pm1,\pm2,\ldots)$ always has an antiderivative in any domain that does not contain the point $z=z_0$. So, by the theorem in Sec. 44,

$$\int_{C_0}(z-z_0)^{n-1}dz = 0$$

for any closed contour C_0 that does not pass through z_0.

Ex.4

$f_2(z)=\sqrt{r}e^{i\theta/2}(r>0,\frac{\pi}{2}<\theta<\frac{5\pi}{2})$ 原函数为 $F_2(z)=\frac{2}{3}z^{\frac{3}{2}}=\frac{2}{3}r\sqrt{r}e^{i3\theta/2}$，$f_2(z)$ 在 C_2 上除了 $z=3$ 以外的值与 $z^{1/2}$ 相等，因此有

$$\int_{C_2} z^{1/2}dz = \int_{-3}^{3} f_2(z)dz = F_2(z)\Big|_{-3}^{3} = 2\sqrt{3}(e^{i3\pi}-e^{i3\pi/2}) = 2\sqrt{3}(-1+i)$$

$$\int_{C_2-C_1} z^{1/2}dz = \int_{C_2} z^{1/2}dz - \int_{C_1} z^{1/2}dz = 2\sqrt{3}(-1+i-1-i) = -4\sqrt{3}$$