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Ex.2

(a) $\forall \varepsilon > 0, \exists \delta = \frac{\varepsilon}{|a|} > 0$ such that

$$|(az+b)-(az_0+b)| = |a||z-z_0| < \varepsilon \quad whenever \quad 0 < |z-z_0| < \delta$$

then

$$\lim_{z\to z_0}(az+b) = az_0+b$$

(b)

$$|(z^2+c)-(z_0^2+c)| = |z^2-z_0^2| = |z+z_0||z-z_0| \le (|z|+|z_0|)|z-z_0| \le (|z-z_0|+2|z_0|)|z-z_0|$$

若 $|z_0| = 0$，则 $\forall \varepsilon > 0, \exists \delta = \sqrt{\varepsilon} > 0$ 使得

$$|(z^2+c)-(z_0^2+c)| = |z^2| = |z|^2 < \varepsilon \quad whenever \quad 0 < |z| < \delta$$

若 $|z_0| \neq 0$，则 $\forall \varepsilon > 0, \exists \delta = \min\{\sqrt{\frac{\varepsilon}{2}}, \frac{\varepsilon}{4|z_0|}\}$ 使得

$$|(z^2+c)-(z_0^2+c)| \le |z-z_0|^2 + 2|z_0||z-z_0| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} < \varepsilon \quad whenever \quad 0 < |z-z_0| < \delta$$

因此

$$\lim_{z\to z_0}(z^2+c) = z_0^2+c$$

(c) $\forall \varepsilon > 0, \exists \delta = \frac{\varepsilon}{3} > 0$ such that

$$|x+i(2x+y)-(1+i)| = |x-1+i(y+1)+2i(x-1)| \le 3|x-1+i(y+1)| < \varepsilon$$

$$whenever \quad 0 < |x-1+i(y+1)| < \delta$$

then

$$\lim_{z\to 1-i}[x+i(2x+y)] = 1+i$$

Ex.3

(a) For $\lim_{z\to z_0} 1 = 1, \lim_{z\to z_0} z^n = z_0^n$, then $\lim_{z\to z_0} 1/z^n = 1/z_0^n$.

(b) For $\lim_{z\to i}(iz^3-1) = -i^2-1 = 0, \lim_{z\to i}(z+i) = 2i$, then $\lim_{z\to i} \frac{iz^3-1}{z+i} = \frac{0}{2i} = 0$.

(c) For $\lim_{z\to z_0} P(z) = P(z_0), \lim_{z\to z_0} Q(z) = Q(z_0)$, then $\lim_{z\to z_0} \frac{P(z)}{Q(z)} = \frac{P(z_0)}{Q(z_0)}$.

Ex.6

(a) 令 $f(z) = u(x,y)+iv(x,y), F(z) = U(x,y)+iV(x,y)$, 则 $g(z) = f(z)+F(z) = u(x,y)+U(x,y)+i[v(x,y)+V(x,y)]$.

已知 $\lim_{z\to z_0} f(z) = w_0 = u_0+iv_0, \lim_{(x,y)\to(x_0,y_0)} u = u_0, \lim_{(x,y)\to(x_0,y_0)} v = v_0$ 以及 $\lim_{z\to z_0} F(z) = W_0 = U_0+iV_0, \lim_{(x,y)\to(x_0,y_0)} U = U_0, \lim_{(x,y)\to(x_0,y_0)} V = V_0$，根据二元实函数极限的性质，有 $\lim_{(x,y)\to(x_0,y_0)}(u+U) = u_0+U_0, \lim_{(x,y)\to(x_0,y_0)}(v+V) = v_0+V_0$，则 $\lim_{z\to z_0} g(z) = \lim_{(x,y)\to(x_0,y_0)}(u+U)+i\lim_{(x,y)\to(x_0,y_0)}(v+V) = u_0+U_0+i(v_0+V_0) = w_0+W_0$.

(b) 已知 $\lim_{z\to z_0} f(z) = w_0, \lim_{z\to z_0} F(z) = W_0$，则 $\forall\varepsilon>0, \exists\delta_1,\delta_2>0$，满足

$$|f(z)-w_0| < \frac{\varepsilon}{2} \quad whenever \quad 0<|z-z_0|<\delta_1$$

$$|F(z)-W_0| < \frac{\varepsilon}{2} \quad whenever \quad 0<|z-z_0|<\delta_2$$

则 $\forall\varepsilon>0, \exists\delta = \min\{\delta_1,\delta_2\}$, 满足

$$|f(z)+F(z)-(w_0+W_0)| \le |f(z)-w_0|+|F(z)-W_0| < \varepsilon \quad whenever \quad 0<|z-z_0|<\delta$$

因此

$$\lim_{z\to z_0}[f(z)+F(z)] = w_0+W_0$$

Ex.9

已知 $\exists\delta_1 > 0$, 满足

$$|g(z)| \leq M \quad whenever \quad |z - z_0| < \delta_1$$

由 $\lim_{z\to z_0} f(z) = 0$, 可得 $\forall \frac{\varepsilon}{M} > 0, \exists\delta_2 > 0$ 满足

$$|f(z)| < \frac{\varepsilon}{M} \quad whenener \quad 0 < |z - z_0| < \delta_2$$

则 $\forall\varepsilon > 0, \exists\delta = \min\{\delta_1, \delta_2\}$，满足

$$|f(z)g(z)| = |f(z)||g(z)| < \frac{\varepsilon}{M}M = \varepsilon \quad whenever \quad 0 < |z - z_0| < \delta$$

因此

$$\lim_{z\to z_0} f(z)g(z) = 0$$

Ex.11

In this problem, we consider the function

$$T(z) = \frac{az+b}{cz+d} \qquad (ad - bc \neq 0).$$

(a) Suppose that $c = 0$. Statement (3), Sec. 17, tells us that $\lim_{z\to\infty} T(z) = \infty$ since

$$\lim_{z\to 0} \frac{1}{T(1/z)} = \lim_{z\to 0} \frac{c+dz}{a+bz} = \frac{c}{a} = 0.$$

(b) Suppose that $c \neq 0$. Statement (2), Sec. 17, reveals that $\lim_{z\to\infty} T(z) = \frac{a}{c}$ since

$$\lim_{z\to 0} T\left(\frac{1}{z}\right) = \lim_{z\to 0} \frac{a+bz}{c+dz} = \frac{a}{c}.$$

Also, we know from statement (1), Sec. 16, that $\lim_{z\to -d/c} T(z) = \infty$ since

$$\lim_{z\to -d/c} \frac{1}{T(z)} = \lim_{z\to -d/c} \frac{cz+d}{az+b} = 0.$$

Ex.12

假设 $\lim_{z\to\infty} f(z) = w_1$ 以及 $\lim_{z\to\infty} f(z) = w_2 \neq w1$，则有 $\lim_{z\to 0} f(1/z) = w_1, \lim_{z\to 0} f(1/z) = w_2 \neq w1$，与函数在 $z_0 = 0$ 的极限具有唯一性矛盾，因此函数在无穷点处的极限也具有唯一性。