AOS4 - Smartphone purchase

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Your parents have decided to buy you a new smartphone, and ask you to choose among a list they have prepared for you. Unfortunately for you, you tend to be **very thorough** when choosing your smartphone, but cannot actually change the list without your parents' approval.

Stuck with a fixed list, you resolve yourself to make the best decision according to your tastes. Fortunately, you have been studying multi-criteria decision-making (MCDM), what a coincidence!

After consulting with your parents, they give you the following guidelines:

1. 'The price cannot exceed 1000.00 €'

2. 'The camera or chipset should not influence your decision'

3. 'It is preferable if you give a ranking, rather than a choice'

After searching for the products' details online, you compile the information into table 1

> **Solution:** This exercise should be solved in class, as practice for MCDM. The familiar context enables students to confront their own reasoning to the studied decision theory, and facilitates reviewing taken decisions.
>
> The questions let students identify the types of criteria (order/type), apply Pareto Dominance, lexicographic/WS aggregation, normalization for aggregation, and further identify the different elicitation scenarios.
>
> The evaluation matrix does not include technical details to reduce the amount of assumed technical knowledge.

1. (a) According to your preferences, and only using the criteria contained in table 1, make a decision and list your top 3 choices.
   (b) For each criterion, answer the following questions, and compare your answers in group:
      (i) What objective have you chosen, *e.g.* max, min?
      (ii) If you cannot choose an objective, how did you order the values?
      (iii) Are there criteria that are 'not important', *i.e.* for which you have no preference?

> **Solution:** Introductory question, familiarize with the context and the data. Identify subjective aspects of the criteria, types of criteria, etc.
>
> For part (a), the idea is to introduce the data to the student, and make sure he has gone through the table. Any preferences the student has can serve for further reasoning. Why did they prefer certain criteria to others? Which objectives did they choose for each criterion?
>
> In part (b), criteria are studied more in depth and is directly following the previous line of thought, but explicitly.
>
> A suggested solution is the following:

(a) Google Pixel 7 Pro, Samsung Galaxy A52s 5G, Nothing Phone (1)

(b)   (a) (Brand) Google, Samsung, OnePlus, Nothing, ~~Apple/Xiaomi/Asus/Huawei/Motorola~~

   (b) (Release Date) No preference

   (c) (Dimensions) No preference

   (d) (5G) No preference

   (e) (Display Type) All the values indicated are identical

   (f) (Display Size) Maximum

   (g) (Operating System) Android, ~~iOS/EMUI~~

   (h) (Jack) No preference

   (i) (Battery capacity) Maximize

   (j) (Price) Minimize

2. Using the criteria that you identified as relevant, try to apply Pareto Dominance to make a decision.

Solution: Practice question for Pareto Dominance. Results might vary from individual to individual. The TA may instead choose a fixed list of interesting criteria, e.g. 5G, jack, display size, operating system, battery capacity, and price.

Almost certainly, students will showcase conflicting objectives, *e.g.* minimize price and maximize battery capacity.

Using brand, display size, operating system, battery capacity and price:

Name	Brand	Display Size	Operating System	Battery capacity	Price
Objective	Ordered	Maximize	Ordered	Maximize	Minimize
Google Pixel 7 Pro	Google	6.7 "	Android	5000 mA h	812.00 €
Samsung Galaxy A52s 5G	Samsung	6.5 "	Android	4500 mA h	349.99 €
Samsung Galaxy S22 Ultra 5G	Samsung	6.8 "	Android	5000 mA h	928.00 €
OnePlus 10 Pro	OnePlus	6.7 "	Android	5000 mA h	724.99 €
Nothing Phone (1)	Nothing	6.55 "	Android	4500 mA h	399.00 €
Asus Zenfone 9	~~Asus~~	5.9 "	Android	4300 mA h	743.89 €
Asus ROG Phone 6D Ultimate	~~Asus~~	6.78 "	Android	6000 mA h	1399.00 €
Huawei Mate 50 Pro	~~Huawei~~	6.74 "	~~EMUI~~	4700 mA h	1154.99 €
Xiaomi 11T Pro	~~Xiaomi~~	6.67 "	Android	5000 mA h	412.99 €
Xiaomi 12 Pro	~~Xiaomi~~	6.73 "	Android	4600 mA h	758.00 €
Motorola Moto X40	~~Motorola~~	6.7 "	Android	4600 mA h	465.79 €
Apple iPhone 13 Pro Max	~~Apple~~	6.7 "	~~iOS~~	4352 mA h	1379.00 €

We sorted the table by brand, meaning that a phone cannot be Pareto-dominated by a lower one of a different brand. Brands that are not considered are eliminated from the alternatives.

As such, we begin from eligible products from the top to eliminate dominated alternatives:

- The Google Pixel 7 Pro is at least beaten by lower alternatives on price, display size, price, and price respectively.

- The Samsung Galaxy A52s 5G is beaten by lower alternatives on display size.

- The Samsung Galaxy S22 Ultra 5G is beaten by lower alternatives on price.

- The OnePlus 10 Pro is beaten by lower alternatives on price.

There are no Pareto dominance relationships.

3. You have been probably unable to apply Pareto dominance relationships to eliminate alternatives.

   (a) Try to think of reasons why, and write them down. *Hint: are your criteria completely independent from each other?*

   (b) Which method would be suitable to solve this decision problem, if you were making a choice?

Solution:

Some objectives might be conflicting, meaning that you cannot at the same time maximize display size, battery capacity and minimize the price.

One alternative method to solve this problem could be even swaps, as you resolve the trade-offs by trading criteria for others, *e.g.* $500\,\mathrm{mA\,h}$ might be traded for $100.00\,€$.

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4. As you are unable to rank the products using Pareto optimization techniques, you decide to merge the criteria together. For that, you believe both lexicographic and weighted sum aggregation are appropriate techniques, and you wish to compare the results.

   (a) Use lexicographic aggregation. What issues does this method have?

   (b) Use weighted sum aggregation. What issues does this method have?

Solution:

The question introduces aggregation techniques for practice purposes. Optionally, the Choquet Integral can be introduced.

For the lexicographic aggregation, the students must first rank the criteria, then solve the problem. As for issues, serial **dictactorship** prioritizes certain criteria over others. Given the limited list, if the student prefers a Google phone over all other brands, then they only have a single choice, regardless of all other criteria.

For the weighted sum aggregation, the students must assign numeric values to each criterion, then also convert the symbolic criteria to numeric ones, *e.g.* by ranking, and finally solve the problem. The critical issues here are to appropriately convert symbolic criteria to numeric ones, and objectives to the same one (either minimize or maximize). A further issue is the difference in numeric range for different criteria. Preferably, the student should **normalize** values first.

An example would be the following:

For part (a), brand is prioritized first, thus the only choice is the Google Pixel 7 Pro.

For part (b), the weights for brand, display size, battery capacity and price are respectively: 0.2, 0.2, 0.3, 0.3

Unnormalized scores are:

$$\begin{aligned}
\text{Score(Google Pixel 7 Pro)} &:= 0.2 \times 4 + 0.2 \times 6.7 + 0.3 \times 5000 - 0.3 \times 812.00 = 1258.54 \\
\text{Score(Samsung Galaxy A52s 5G)} &:= 0.2 \times 3 + 0.2 \times 6.5 + 0.3 \times 4500 - 0.3 \times 349.99 = 1246.90 \\
\text{Score(Samsung Galaxy S22 Ultra 5G)} &:= 0.2 \times 3 + 0.2 \times 6.8 + 0.3 \times 5000 - 0.3 \times 928.00 = 1223.56 \\
\text{Score(OnePlus 20 Pro)} &:= 0.2 \times 3 + 0.2 \times 6.7 + 0.3 \times 5000 - 0.3 \times 724.99 = 1284.44 \\
\text{Score(Nothing Phone (1))} &:= 0.2 \times 1 + 0.2 \times 6.55 + 0.3 \times 4500 - 0.3 \times 399.00 = 1231.155
\end{aligned}$$

The ranking is thus: OnePlus 20 Pro, Google Pixel 7 Pro, Samsung Galaxy A52s 5G, Nothing Phone (1), Samsung Galaxy S22 Ultra 5G.

If we normalize (mean/variance) the scores using all data from table 1, then:

$$\text{Score*(Google Pixel 7 Pro)} := 0.5951$$
$$\text{Score*(Samsung Galaxy A52s 5G)} := 0.3392$$
$$\text{Score*(Samsung Galaxy S22 Ultra 5G)} := 0.4428.$$
$$\text{Score*(OnePlus 20 Pro)} := 0.3920$$
$$\text{Score*(Nothing Phone (1))} := 0.0625$$

The final ranking is Google Pixel 7 Pro, Samsung Galaxy S22 Ultra 5G, OnePlus 20 Pro, Samsung Galaxy A52s 5G, and Nothing Phone (1).

The results are remarkably different between unnormalized and normalized data. Although, the Google Pixel 7 Pro is still chosen by both lexicographic and weighted sum models.

5. Do you agree with the decision made using aggregation? Why?
   (a) Are the ranking coherent with your preferences?
   (b) How did you rank the criteria? How did you estimate the weights?
   (c) Suggest a better aggregation procedure.

Solution:

The question aims at providing more insight into how exactly the aggregation chose to rank the alternatives, and which criteria took the most importance. The students must realize that there exists a lot of subjectivity into how the criteria are aggregated, and also difficulties in the aggregation itself.

The lexicographic is too drastic, the WS aggregation is linear, the student must suggest a better aggregation method.

It is not asked that students define a fuzzy measure for this problem, but they should mention the Choquet, or the Sugeno integral.

6. After a very long and arduous decision process, you finally have a ranking of potential smartphones, albeit with some faults!

   However, you cannot afford to spend more time on this, and decide to examine the final criterion: the design!

   You group together all the visuals in fig. 1.
   (a) Which products matches your tastes the best? Rank them.
   (b) Compare your tastes with your class, by counting who likes which products. Can you identify clusters of similar products?
   (c) If you had to assign a cause to these clusters, what would they be? Discuss.

Solution:

Introduce a very subjective visual criteria.

One example answer:

   • g, h, k, l, j, f, a, d, c, b, i, e

- In practice, just fill in a matrix students/products. Hopefully, there should be some groups of products easily identifiable.

- This is a discussion on the cause of natural clustering in smartphones. Some causes might be: design features, *e.g.* notches, back cameras, etc., product personality, brand, etc.

The discussion should naturally evolve into listing a number of criteria for choosing the design, resulting in making the final criterion a MCDM problem in itself.

Collecting users' preferences is a very difficult task.

(a) Samsung Galaxy A52s 5G

(b) Apple iPhone 13 Pro Max
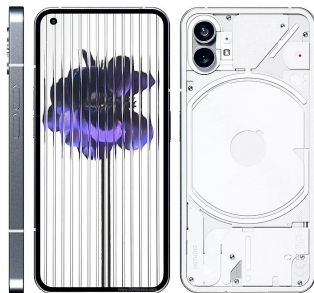

(c) Xiaomi 11T Pro

(d) Xiaomi 12 Pro

(e) Asus Zenfone 9

(f) OnePlus 10 Pro

(g) Nothing Phone (1)

(h) Google Pixel 7 Pro

(i) Asus ROG Phone 6D Ultimate

(j) Huawei Mate 50 Pro

(k) Samsung Galaxy S22 Ultra 5G

(l) Motorola Moto X40

Figure 1: Smartphones Designs

Table 1: Smartphones details

Name	Brand	Release Date	Dimensions	Weight	5G	Display Type	Display Size	Operating System	3.5 mm Jack	Battery capacity	Price
Samsung Galaxy A52s 5G	Samsung	1st September 2021	159.9 mm × 75.1 mm × 8.4 mm	189 g	Yes	OLED	6.5 "	Android	Yes	4500 mA h	349.99 €
Apple iPhone 13 Pro Max	Apple	24th September 2021	160.8 mm × 78.1 mm × 7.7 mm	240 g	Yes	OLED	6.7 "	iOS	No	4352 mA h	1379.00 €
Xiaomi 11T Pro	Xiaomi	5th October 2021	164.1 mm × 76.9 mm × 8.8 mm	204 g	Yes	OLED	6.67 "	Android	No	5000 mA h	412.99 €
Xiaomi 12 Pro	Xiaomi	31st December 2021	163.6 mm × 74.6 mm × 8.2 mm	204 g	Yes	OLED	6.73 "	Android	No	4600 mA h	758.00 €
Asus Zenfone 9	Asus	15th September 2022	146.5 mm × 68.1 mm × 9.1 mm	169 g	Yes	OLED	5.9 "	Android	Yes	4300 mA h	743.89 €
OnePlus 10 Pro	OnePlus	13th January 2022	163 mm × 73.9 mm × 8.6 mm	201 g	Yes	OLED	6.7 "	Android	No	5000 mA h	724.99 €
Nothing Phone (1)	Nothing	16th June 2022	159.2 mm × 75.8 mm × 8.3 mm	193.5 g	Yes	OLED	6.55 "	Android	No	4500 mA h	399.00 €
Google Pixel 7 Pro	Google	13th October 2022	162.9 mm × 76.6 mm × 8.9 mm	212 g	Yes	OLED	6.7 "	Android	No	5000 mA h	812.00 €
Asus ROG Phone 6D Ultimate	Asus	7th October 2022	173 mm × 77 mm × 10.4 mm	247 g	Yes	OLED	6.78 "	Android	Yes	6000 mA h	1399.00 €
Huawei Mate 50 Pro	Huawei	28th September 2022	162.1 mm × 75.5 mm × 8.5 mm	205 g	No	OLED	6.74 "	EMUI	No	4700 mA h	1154.99 €
Samsung Galaxy S22 Ultra 5G	Samsung	25th February 2022	163.3 mm × 77.9 mm × 8.9 mm	228 g	Yes	OLED	6.8 "	Android	No	5000 mA h	928.00 €
Motorola Moto X40	Motorola	22nd December 2022	161.2 mm × 74 mm × 8.6 mm	199 g	Yes	OLED	6.7 "	Android	No	4600 mA h	465.79 €