

of the half-life? If so, you can claim to have verified the code for this problem.

## Decay series

For heavy nuclei, a daughter nucleus resulting from radioactive decay, may itself be unstable and undergo decay. The process may continue through several "generations" until the decay results in a stable nucleus. This is called a decay series or decay chain. There are four radioactive decay series found in nature: the decay of Uranium-235, Uranium-238, Thorium-232 and Neptunium-237.

← NATURAL DECAY SERIES

Modify your code to simulate a radioactive decay series which decays through two or more generations. Each element in the series should have a different half-life. You can either choose your own values for a hypothetical series, or select values from a section of one of the natural decay series; for example, the half-lives for the last four decays in the Uranium-235 series are:

Lead-211  $\xrightarrow{T_{1/2}=36 \text{ mins}}$  Bismuth-211  $\xrightarrow{T_{1/2}=2.1 \text{ mins}}$  Thallium-207  $\xrightarrow{T_{1/2}=4.8 \text{ mins}}$  Lead-207 (stable)

## 2.4 Relevant course sections

Studying the following course sections will help you complete this checkpoint:

- Classes, Objects and Methods
- Object Oriented Design

Additional material that you may find useful:

- Errors and Exceptions

## 2.5 Marking Scheme

- Radioactive Decay Checkpoint Marking Scheme

## 3 Mandelbrot Set

*"Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line."*

Benoit Mandelbrot

### 3.1 Aim

A fractal is a geometric shape that contains self-similar images within itself - you can zoom in on a section and it will have just as much detail as the whole fractal. Many objects in the natural world exhibit fractal behaviour. For example, the human circulatory system is a fractal. If you look at the blood vessels in your hand, they resemble the overall shape that the complete system takes on. The Mandelbrot set is also an example of a fractal - it is recursively defined and infinitely detailed.

The Mandelbrot Set is a set of complex numbers  $C$  resulting from repeated iterations of the following function:

$$z_{n+1} = z_n^2 + C$$

with the initial condition  $z_0 = 0$ .

A given complex number  $C$  belongs to the Mandelbrot set if  $|z_n|$ , the magnitude of  $z_n$ , remains bounded, i.e. does not diverge. If  $|z_n|$  diverges then  $C$  does not belong to the Mandelbrot set.

In fact, it can be shown that if  $|z_n| > 2$  for some value of  $n$ , it will subsequently rapidly tend to infinity, i.e. diverge, meaning that  $C$  is not in the Mandelbrot set.

You can also assume that if  $|z_n|$  has not diverged after 255 iterations, it will not diverge at larger values of  $n$ , meaning that  $C$  is in the Mandelbrot set.

### 3.2 Checkpoint task

Write a PYTHON program that will give a visual representation of the Mandelbrot set.

To obtain a visual plot of the Mandelbrot set, the complex plane can be represented as a 2D grid and the value of  $N$  (the number of iterations needed to reach the threshold  $|z_n| > 2$ ) calculated for complex numbers  $C$  corresponding to points on the grid. As explained above, you should set an upper iteration limit of  $N = 255$ .

The value of  $N$  can then be converted to a colour and plotted on the grid.

You should explore values of  $C$  in the range  $x = \{-2.025 \rightarrow 0.6\}$  and  $y = \{-1.125 \rightarrow 1.125\}$ . Note that as you increase the number of points on the grid you will not only increase the resolution of the display but will also significantly increase the computation time.

### 3.3 Optional extra

Write a program to display a Julia set. Julia Sets are produced from the same formula as the Mandelbrot set but used in a different way. When making a picture of a Julia set,  $C$  remains fixed during the whole generation process, while the value of  $Z_0$  varies. The value of  $C$  determines the shape of the Julia set: in other words, each point of the complex plane is associated with a particular Julia set.

Some of the more famous Julia sets are:

← NOTE

- $C = -1, 0$
- $C = 0, -1$  – the Dendrite
- $C = 0.5, 0$
- $C = -0.1, 0.8$  – the Rabbit
- $C = 0.36, 0.1$  – the Dragon

### 3.4 Relevant course sections

Studying the following course sections will help you complete this checkpoint:

- [NumPy arrays](#)
- [Plotting using Matplotlib](#)

### 3.5 Marking Scheme

- [Mandelbrot Set Checkpoint Marking Scheme](#)

## 4 Traffic

### 4.1 Aim

This checkpoint requires you to code a simple cellular automaton which attempts to model traffic flow. This is an example where the effective theory is not known, so we invent a theory that has the basic properties that we believe are important. By running simulations we can look for emergent phenomena, in this case the onset of congestion (traffic jams).

### 4.2 The Model

The simulation box is a straight line of  $N$  cells (the road) which can each only have two values: 1 if a car is present on that section of road, 0 otherwise. The [update rules](#) for each iteration are very simple:

- If the space in front of a car is empty then it moves forward one cell;
- Otherwise it stays where it is.

If we use  $c(j)$  to indicate the state of the  $j$ th cell, and use a subscript  $n$  to represent the iteration, we can write down rules that determine  $c_{n+1}(j)$  from values at the previous iteration  $c_n(j-1)$ ,  $c_n(j)$  and  $c_n(j+1)$ .