

Definitions

Isolated system: No exchanges

Closed system: Only energy exchange

Open system: Energy & mass exchange

Intensive state variables:

Independent of mass

Extensive state variables:

Proportional to mass

Reservoirs: Infinite/very large system that remains unchanged when in contact with finite system.

Mechanical equilibrium:

No unbalanced forces

Thermal equilibrium:

No temperature differences

Thermodynamic equilibrium:

Intensive state variables of system are constant. Alternatively our system is in mechanical and thermal equilibrium.

Reversible processes:

Every intermediate is an equilibrium state.

Quasi-static processes:

Process sufficiently slow such that only infinitesimal temperature or pressure gradients exist.

Frictionless quasi-static processes are reversible.

Cyclic processes:

$$\Delta U = 0 \text{ and } W = Q$$

For conservative forces:

$$\oint dX = 0$$

where X is a state variable.

Adiabatic processes: $\Delta Q = 0$

Isothermal processes: $\Delta T = 0$

Zeroth law

If A is in thermal equilibrium with B and C separately then B and C are also in thermal equilibrium.

Ideal gas state equation

Given n moles of gas at temperature T :

$$\begin{aligned} PV &= nRT \\ &= Nk_B T \end{aligned}$$

where $R = N_A k_B = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$ and N the number of molecules.

Calculus identities

$$\begin{aligned} 1. \left(\frac{\partial Z}{\partial Y} \right)_X &= \left[\left(\frac{\partial Y}{\partial Z} \right)_X \right]^{-1} \\ 2. \left(\frac{\partial X}{\partial Y} \right)_Z \left(\frac{\partial Y}{\partial Z} \right)_X \left(\frac{\partial Z}{\partial X} \right)_Y &= -1 \end{aligned}$$

First law

Total energy E is conserved and:

$$\Delta U = Q - W$$

$$dU = dQ - dW$$

$$\dot{U} = \dot{Q} - \dot{W}.$$

Note $Q > 0$ represents energy transferred into system. When system does work on surroundings we have $W > 0$.

Work done by a fluid in reversible adiabatic processes:

$$dW = PdV$$

and is in Joules (J).

Isochoric heat capacity

$$C_V(T) = \left(\frac{dQ}{dT} \right)_V = \left(\frac{\partial U}{\partial T} \right)_V$$

Isoobaric heat capacity

$$\begin{aligned} C_P &= \left(\frac{dQ}{dT} \right)_P \\ &= C_V + \left[P + \left(\frac{\partial U}{\partial V} \right)_T \right] \left(\frac{\partial V}{\partial T} \right)_P \end{aligned}$$

Heat capacity has units JK^{-1} .

For ideal gases we have that:

$$C_P - C_V = nR.$$

Under reversible adiabatic processes:

$$TV^{\gamma-1} = \text{constant}$$

$$PV^{\gamma} = \text{constant}$$

$$T^{\frac{1}{\gamma-1}} V = \text{constant}$$

where γ is the adiabatic exponent:

$$U = \frac{f}{2} nRT$$

$$\gamma = \frac{C_P}{C_V} = \frac{f+2}{f}$$

regardless of degrees of freedom f .

State function enthalpy

$$H = U + PV$$

$$dH = dU + VdP + PdV$$

$$= dQ + VdP$$

$$\therefore C_P = \left(\frac{\partial H}{\partial T} \right)_P$$

Enthalpy has units J .

Latent heats

Latent heat is heat needed for sample to undergo a phase transition:

$$\Delta U_{m,V} = Q_{m,V} - P\Delta V_{m,V}$$

$$Q_{m,V} = \Delta H_{m,V}.$$

Chemical reactions

$$Q = \Delta U + P_0 \Delta V = \Delta H$$

Here P_0 is constant.

- $Q < 0$: exothermic (heat is released)
- $Q > 0$: endothermic (heat is absorbed)

Carnot's theorem

Peak efficiency of a cyclic heat engine:

$$\eta = \frac{\dot{W}}{\dot{Q}_H} = 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H}$$

and is either in terms of rate or value, with units J or Js^{-1} .

State function entropy

$$S = \frac{Q}{T}$$

$$\therefore dU = TdS - PdV$$

Entropy of mixing

$$\Delta S = n_A R \ln \frac{V_A + V_B}{V_A} + n_B R \ln \frac{V_A + V_B}{V_B}$$

$$\Delta s_{mix} = -R(x_A \ln x_A + x_B \ln x_B)$$

$$x_A = \frac{n_A}{n_A + n_B} \text{ and } x_B = \frac{n_B}{n_A + n_B}$$

Second law

$$\Delta S_{total} = \Delta s_{system} + \Delta s_{reservoir} \geq 0$$

$$dS \geq \frac{dQ}{T}$$

Helmholtz free energy

$$F = U - TS$$

$$dF = -SdT - PdV$$

Gibbs free energy

$$G = H - TS$$

$$dG = -SdT + VdP$$

Chemical reactions are spontaneous if:

$$\Delta G = \Delta H - T\Delta S < 0.$$

Maxwell relations

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$$-\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{\partial V}{\partial T}\right)_P$$

The isobaric expansivity is defined as:

$$\beta = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_P$$

and the isothermal compressibility:

$$\kappa_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P}\right)_T.$$

Clausius-Clapeyron equation

The slope of any phase boundaries is:

$$\frac{dP}{dT} = \frac{\Delta S}{\Delta V}.$$

Van der Waals state equation

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

Chemical potentials

The Euler equation for a 1-component **open** system with N particles is:

$$U = TS - PV + \mu N$$

with modified first law statement:

$$dU = TdS - PdV + \mu dN.$$

This gives the Gibbs-Duhem relation:

$$SdT - VdP + Nd\mu = 0$$

where μ is the chemical potential:

$$\mu = \frac{G}{N}$$

since G is extensive. At constant T with ideal gas assumptions:

$$\mu(P, T) = RT \ln \frac{P}{P_0} + \mu_0.$$

Third law

$S = 0$ at $T = 0\text{K}$.