

## Probabilities

The probability of an event in a trial is:

$$\mathbb{P}(\text{event}) := \lim_{N \rightarrow \infty} \frac{n}{N}$$

for  $n$  is number of occurrences in  $N$  trials.

For discrete probabilities:

$$\sum_{i=1}^q \mathbb{P}(i) = 1$$

$$\mathbb{P}(i \text{ or } j) = \mathbb{P}(i) + \mathbb{P}(j)$$

$$\mathbb{P}(i \text{ and } j) = \mathbb{P}(i)\mathbb{P}(j).$$

Given continuous random variables:

$$\mathbb{P}([x, x + dx]) = P(x)dx$$

for  $P$  is the probability density function:

$$\int_{-\infty}^{\infty} P(x)dx = 1.$$

We define the **mean** and **variance** as:

$$\bar{x} = \int_{-\infty}^{\infty} xP(x)dx$$

$$\overline{\Delta x^2} = \int_{-\infty}^{\infty} (x - \bar{x})^2 P(x)dx.$$

The weight of a function is given by:

$$\overline{f(x)} = \int_{-\infty}^{\infty} f(x)P(x)dx.$$

## Binomial distribution

The probability of observing  $n$  events each with probability  $p$  in  $N$  trials is:

$$\mathbb{P}_n = \binom{N}{n} p^n (1-p)^{N-n}$$

where  $\binom{N}{n} = \frac{N!}{n!(N-n)!}$  with:

$$\bar{n} = Np \text{ and } \overline{\Delta n^2} = Np(1-p).$$

For large  $N$ :

$$\ln(N!) \approx N \ln(N) - N$$

and is **Stirling's approximation**.

## Poisson distribution

## Gaussian distribution