1. Find solutions to **BVPs**.

   For part (i) we have that:

$$y'' + y = x \quad \text{for} \quad y(0) = y(\pi) = 0.$$

   We first find the homogeneous solution:

$$y'' + y = 0,$$

   and this gives: $y_H = \alpha \cos x + \beta \sin x$. The particular solution for the differential equation is $y_p = x$, and therefore our general solution is:

$$y = x + \alpha \cos x + \beta \sin x.$$

   However if we substitute our **boundary conditions** we find that:

$$y(0) = \alpha = 0$$

   yet

$$y(\pi) = \pi - \alpha = 0.$$

   This is clearly a contradiction and there are **no solutions** to this problem.

   For part (ii) we are asked:

$$y'' + 4y = \cos x \quad \text{for} \quad y'(0) = y'(\pi) = 0.$$

   The homogeneous equation is:

$$y'' + 4y = 0$$

   and has eigenvalues $\lambda = \pm 2i$, which corresponds to solution:

$$y_H = \alpha \cos 2x + \beta \sin 2x.$$

   We try for a particular solution of form $y_p = \gamma \cos x + \eta \sin x$ and after substituting our general solution takes the form:

$$y = \alpha \cos 2x + \beta \sin 2x + \frac{1}{3} \cos x.$$

   Taking derivatives and substituting boundary conditions we find $\beta = 0$, and so for this **BVP** there are infinitely many solutions of form:

$$y = \alpha \cos 2x + \frac{1}{3} \cos x,$$

   where $\alpha \in \mathbb{R}$.

2.