Probabilities

The probablity of an event in a trial is:

$$\mathbb{P}(\text{event}) := \lim_{N\to\infty} \frac{n}{N}$$

for n is number of occurrences in N trials.
For discrete probabilities:

$$\sum_{i=1}^{q} \mathbb{P}(i) = 1$$

$$\mathbb{P}(i \text{ or } j) = \mathbb{P}(i) + \mathbb{P}(j)$$

$$\mathbb{P}(i \text{ and } j) = \mathbb{P}(i)\mathbb{P}(j).$$

Given continuous random variables:

$$\mathbb{P}([x, x + \mathrm{d}x]) = P(x)\mathrm{d}x$$

for P is the probability density function:

$$\int_{-\infty}^{\infty} P(x)\mathrm{d}x = 1.$$

We define the **mean** and **variance** as:

$$\overline{x} = \int_{-\infty}^{\infty} xP(x)\mathrm{d}x$$

$$\overline{\Delta x^2} = \int_{-\infty}^{\infty} (x - \overline{x})^2 P(x)\mathrm{d}x.$$

The weight of a function is given by:

$$\overline{f(x)} = \int_{-\infty}^{\infty} f(x)P(x)\mathrm{d}x.$$

Binomial distribution

The probability of observing n events each with probability p in N trials is:

$$P_n = \binom{N}{n} p^n (1 - p)^{N-n}$$

where $\binom{N}{n} = \dfrac{N!}{n!(N-n)!}$ with:

$$\overline{n} = Np \text{ and } \overline{\Delta n^2} = Np(1 - p).$$

For large N:

$$\ln(N!) \approx N \ln(N) - N$$

known as **Stirling's approximation**.

Poisson distribution

Let $N \gg n$ and let p be the probability of an event in a trial. Assume that as $N \to \infty$, $p \to 0$. Under such conditions the binomial probability of observing n events in N trials is:

$$P_n \approx (\overline{n})^n \frac{\exp(-\overline{n})}{n!}$$

with mean and variance Np.

Gaussian distribution