D: Functions

injections, surjections and bijections

D: Groups

D: Abelian groups

D1.2.1(i): Fields

A field F is a set defined with:

1. Addition function $(+)$:

   $$(+) : F \times F \to F; (\lambda, \mu) \mapsto \lambda + \mu$$

2. Multiplication function $(\cdot)$:

   $$(\cdot) : F \times F \to F; (\lambda, \mu) \mapsto \lambda \cdot \mu$$

3. $\exists 0_F, 1_F \in F$ where $0_F \neq 1_F$ such that $(F, +)$ and $(F \backslash \{0_F\}, \cdot)$ form Abelian groups.
4. $\exists (-\lambda) \in F : \lambda + (-\lambda) = 0_F$
5. $\exists (\lambda^{-1}) \in F : \lambda \cdot (\lambda^{-1}) = 1_F$
6. $\lambda(\mu + \nu) = \lambda\mu + \lambda\nu \in F$

D1.2.1(ii): Vector spaces

A vector space V over a field F is an Abelian group $V := (V, +)$ with mapping:

$$F \times V \to V : (\lambda, \boldsymbol{v} \mapsto \lambda \boldsymbol{v})$$

where for $\forall \lambda, \mu \in F$ and $\forall \boldsymbol{v}, \boldsymbol{w} \in V$:

1. $\lambda(\boldsymbol{v} + \boldsymbol{w}) = (\lambda \boldsymbol{v}) + (\mu \boldsymbol{w})$
2. $(\lambda + \mu)\boldsymbol{v} = (\lambda \boldsymbol{v}) + (\mu \boldsymbol{w})$
3. $\lambda(\mu \boldsymbol{v}) = (\lambda\mu)\boldsymbol{v}$
4. $1_F \boldsymbol{v} = \boldsymbol{v}$

and is an F-vector space.

L1.2.2

L1.2.3

L1.2.4

really obvious stuff

D: Cartesian products

D1.4.1: Vector subspaces

P1.4.5: ???

? subset subspace

D1.4.7: Generating set

D1.4.9: Power sets

D1.5.1: Linearly independent

D1.5.8: Basis

T1.5.11: ???

T1.5.12: ???

C1.5.13