D1.1.1: Complex numbers

Let $z = x + iy$ and $w = a + ib$ where $x, y, a, b \in \mathbb{R}$. Then z and w are complex numbers. Furthermore:

1. $z = w$ **iff** $x = a$ and $y = b$.
2. $\mathrm{Re}(z) := x$ and $\mathrm{Im}(z) := y$.
3. $|z| := \sqrt{x^2 + y^2}$
4. The **complex conjugate** of z is:
$$\overline{z} := x - iy.$$
5. Addition and multiplication:
$$(x+iy)+(a+ib) := (x+a)+i(y+b)$$
$$(x+iy)(a+ib) := .$$

L1.1.3

properties of complex numbers

Remark

$\mathbb{C}$ is a field.

D1.1.5 and D1.1.7

polar and exp forms

L1.1.6

de moivre

L1.1.9

conjugate properties

L1.1.10 – 11: Triangle inequalities

D1.1.12: Argument of z

P1.1.14

properties of arg z

Remark

set addition

D1.2.1: Open and closed ϵ-discs

Let $z_0 \in \mathbb{C}$ and $\epsilon > 0$.

1. An **open** ϵ-disc centred at z_0 is:
$$D_\epsilon(z_0) := \{z \in \mathbb{C} : |z - z_0| < \epsilon\}.$$
2. A **closed** ϵ-disc centred at z_0 is:
$$\overline{D}_\epsilon(z_0) := \{z \in \mathbb{C} : |z - z_0| \leq \epsilon\}.$$

A **punctured** ϵ-disc centred at z_0 is:
$$D'_\epsilon(z_0) := \{z \in \mathbb{C} : 0 < |z - z_0| < \epsilon\}.$$

D1.2.2: Open sets

Let $U \subset \mathbb{C}$. Set U is **open** if:
$$\forall z_0 \in U; \exists \epsilon > 0 : D_\epsilon(z_0) \subseteq U.$$

Subset F is **closed** if $\mathbb{C} \setminus F$ is open.

A **neighbourhood** of point $z_0 \in \mathbb{C}$ is an open set that contains z_0.

L1.2.3

Punctured disc $D'_\epsilon(z_0)$ is open.

D1.2.4: Limit points

Let $S \subseteq \mathbb{C}$. z_0 is a **limit point** of S if:
$$\forall \epsilon > 0; D'_\epsilon(z_0) \cap S \neq \emptyset.$$

The **closure** of S is set $\overline{S}$ and contains S and **all** its limit points.

L1.2.6

Let $S \subseteq \mathbb{C}$. S is closed **iff** $S = \overline{S}$.

D1.2.7: Bounded sets

Let $S \subseteq \mathbb{C}$. Set S is **bounded** if:
$$\forall z \in S; \exists M > 0 : |z| \leq S.$$

D1.2.8: ϵ-N convergence

Let $\mathbb{N} = \{0, 1, 2, \dots\}$.

Let $(z_n)_{n\in\mathbb{N}} \subseteq \mathbb{C}$ be a sequence and $z \in \mathbb{C}$. Then $\lim\limits_{n\to\infty} z_n = z$ if:
$$\forall \epsilon > 0; \exists N \in \mathbb{N} : \forall n \geq N \implies |z_n - z| < \epsilon.$$

L1.2.9

Let $z_n, z \in \mathbb{C}$ where $z_n = a_n + ib_n$.

Then $\lim\limits_{n\to\infty} z_n = z$ **iff**:
$$\mathrm{Re}(z) = \lim_{n\to\infty} a_n \text{ and } \mathrm{Im}(z) = \lim_{n\to\infty} b_n.$$

L1.2.10

Let $S \subseteq \mathbb{C}$ and $z \in \mathbb{C}$. Then $z \in \overline{S}$ **iff**:
$$\exists z_n \in S : z = \lim_{n\to\infty} z_n.$$

D1.2.11: Cauchy sequences

z_n is a Cauchy sequence if:
$$\forall \epsilon > 0; \exists N \in \mathbb{N} : \forall n, m \geq N \implies |z_n - z_m| < \epsilon.$$

L1.2.12

z_n is convergent **iff** z_n is Cauchy.

D1.2.14: Bounded sequences

z_n is bounded if:
$$\forall n \in \mathbb{N}; \exists M > 0 : |z_n| \leq M.$$

L1.2.15: Bolzano-Weierstrass

Let z_n be a bounded sequence. Then:
$$\exists (z_{n_k})_{k, n_k \in \mathbb{N}} : \lim_{k\to\infty} z_{n_k} = z \in \mathbb{C}$$

or that z_n has a convergent subsequence.

Remark

define a complex valued function

D1.3.1: Bounded functions

D1.3.2: ϵ-δ convergence

L1.3.3 ?

L1.3.4

results on function limits

L1.3.5

limit algebra

D1.3.6: ϵ-δ continuity

L1.3.7

L1.3.8

composition of functions are also continuous

L1.3.9

L1.3.10