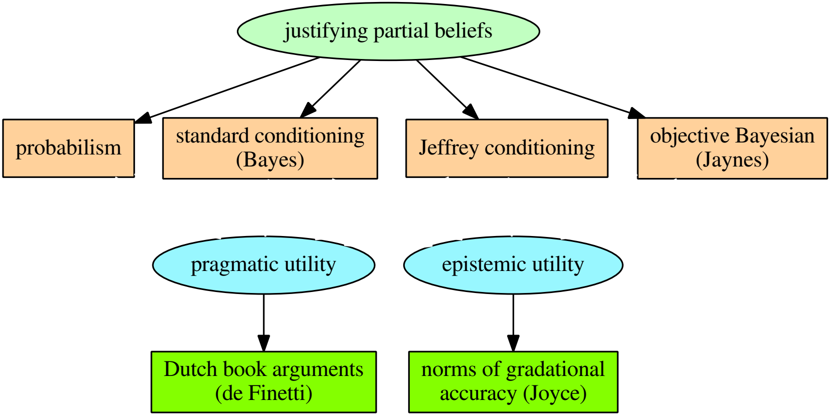
# Asymmetry and the Geometry of Reason

Causal and Probabilistic Reasoning Conference, LMU München

Stefan Lukits

June 19, 2015

# Epistemic Utility Approach to Justification



# Geometry of Reason vs. Information Theory I

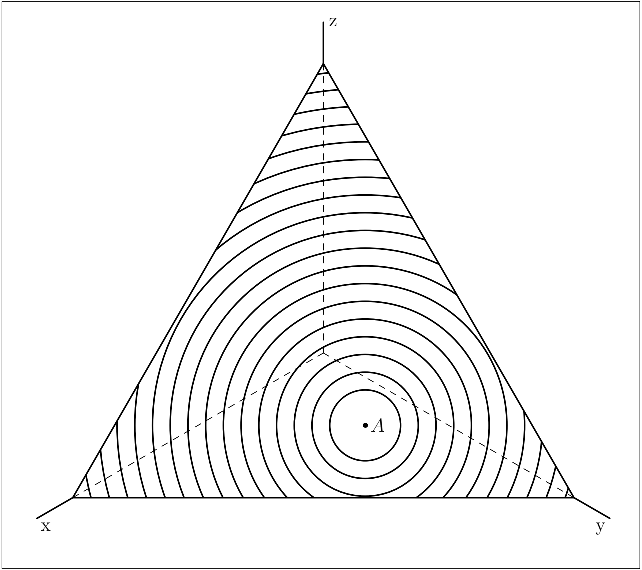
- *Geometry of Reason:* topology of the probability space is metric. A symmetric distance measure is used (Euclidean).

$$\|u - v\| = \sqrt{\sum_{i=1}^{n} (u_i - v_i)^2}. \tag{1}$$

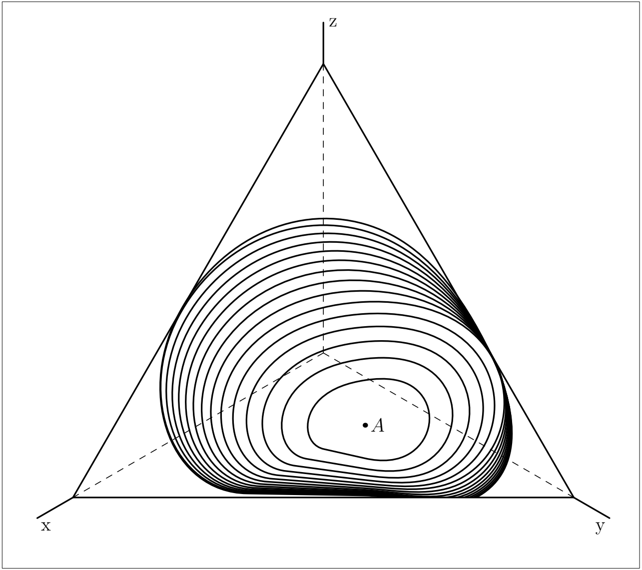
- *Information Theory:* topology of the probability space is not a metric. An asymmetric divergence measure is used (Kullback-Leibler).

$$D_{\mathrm{KL}}(u, v) = \sum_{i=1}^{3} u_i \ln \frac{u_i}{v_i} \tag{2}$$

# Geometry of Reason vs. Information Theory II



# Geometry of Reason vs. Information Theory III



# Weak Convexity and Symmetry

Here are two axioms for inaccuracy measures which use the language of the Geometry of Reason (Joyce):

**Weak Convexity** Let $m = (0.5b' + 0.5b'')$ be the midpoint of the line segment between $b'$ and $b''$. If $I(b', \omega) = I(b'', \omega)$, then it will always be the case that $I(b', \omega) \geq I(m, \omega)$ with identity only if $b' = b''$.

**Symmetry** If $I(b', \omega) = I(b'', \omega)$, then for any $\lambda \in [0, 1]$ one has $I(\lambda b' + (1 - \lambda)b'', \omega) = I((1 - \lambda)b' + \lambda b''), \omega)$.

# Local and Global Inaccuracy

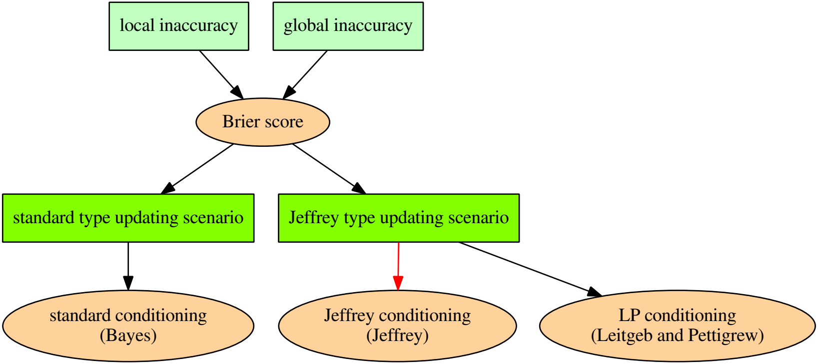
*Expected local inaccuracy* of degree of belief $x$ in proposition $A$ by the lights of belief function $b$ with respect to local inaccuracy measure $I$ and over the set $E$ of epistemically possible worlds:

$$\mathrm{LExp}_b(I, A, E, x) = \sum_{w \in E} b(\{w\}) I(A, w, x) = \sum_{w \in E} b(\{w\}) \lambda \left(\chi_A(w) - x\right)^2 . \tag{3}$$

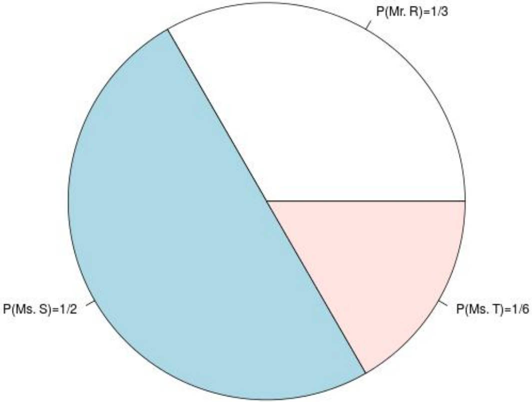
*Expected global inaccuracy* of belief function $b'$ by the lights of belief function $b$ with respect to global inaccuracy measure $G$ and over the set $E$ of epistemically possible worlds:

$$\mathrm{GExp}_b(G, E, b') = \sum_{w \in E} b(\{w\}) G(w, b') = \sum_{w \in E} b(\{w\}) \mu \|w - b\|^2 . \tag{4}$$

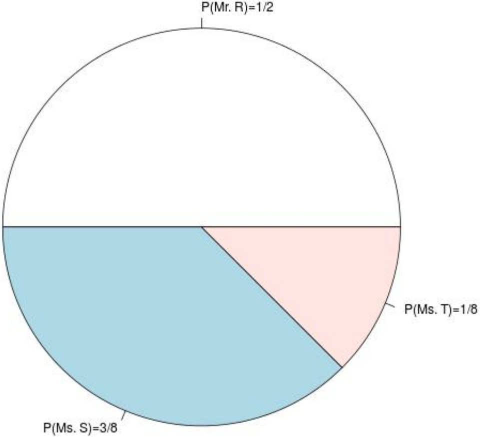
# Standard Conditioning, Jeffrey and LP Conditioning



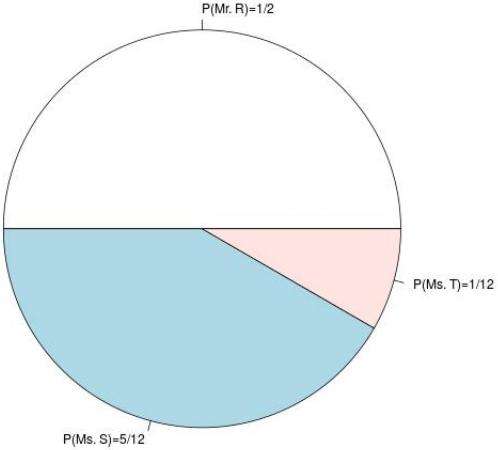
# Jeffrey Type Updating Scenario: Priors



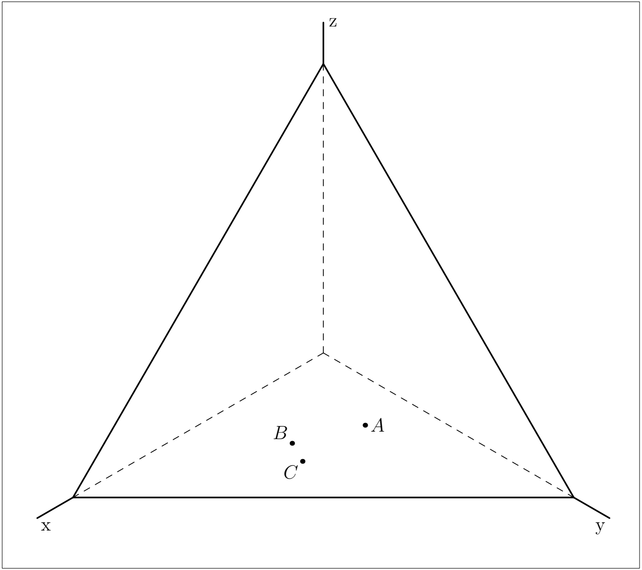
# Jeffrey Type Updating Scenario: Jeffrey Posteriors



# Jeffrey Type Updating Scenario: LP Posteriors



# Euclid and Kullback-Leibler



# Five Plausible Expectations for an Amujus

- CONTINUITY An amujus ought to be continuous with standard conditioning as a limiting case.
- INVARIANCE An amujus ought to be partition invariant.
- LEVINSTEIN An amujus ought not to give "extremely unattractive" results in a Levinstein scenario
- REGULARITY An amujus ought not to assign a posterior probability of 0 to an event which has a positive prior probability and about which the intervening evidence says nothing except that a strictly weaker event has a positive posterior probability.
- ASYMMETRY An amujus ought to reflect epistemic asymmetries.

# Continuity Violation I

To illustrate a CONTINUITY violation, consider the case where Sherlock Holmes reduces his credence that the culprit was male to $\varepsilon_n = 1/n$ for $n = 4, 5, \ldots$

# Continuity Violation II

Straightforward conditionalization on the evidence that "the culprit is female" gives us

$$\begin{aligned} P'_{\text{SC}}(E_1) &= 0 \\ P'_{\text{SC}}(E_2) &= 3/4 \\ P'_{\text{SC}}(E_3) &= 1/4. \end{aligned} \quad (5)$$

# Continuity Violation III

Letting $n \to \infty$ for Jeffrey conditioning yields

$$\begin{array}{rcccl} P'_{\mathrm{JC}}(E_1) & = & 1/n & \to & 0 \\ P'_{\mathrm{JC}}(E_2) & = & 3(n-1)/4n & \to & 3/4 \\ P'_{\mathrm{JC}}(E_3) & = & (n-1)/4n & \to & 1/4, \end{array} \tag{6}$$

# Continuity Violation IV

Letting $n \to \infty$ for LP conditioning yields

$$\begin{array}{rcccl} P'_{\text{LP}}(E_1) & = & 1/n & \to & 0 \\ P'_{\text{LP}}(E_2) & = & (4n-1)/6n & \to & 2/3 \\ P'_{\text{LP}}(E_3) & = & (2n-1)/6n & \to & 1/3. \end{array} \tag{7}$$

LP conditioning violates CONTINUITY.

# Invariance Violation I

Consider the Sherlock Holmes scenario. Jane Marple is on the same case and arrives at the same relatively prior probability distribution as Sherlock Holmes. Jane Marple, however, has a more fine-grained probability assignment than Sherlock Holmes and distinguishes between the case where Ms. S went to boarding school with her, of which she has a vague memory, and the case where Ms. S did not and the vague memory is only about a fleeting resemblance of Ms. S with another boarding school mate.

# Invariance Violation II

$$\begin{aligned} Q(E_1) &= 1/3 \\ Q(E_2^*) &= 1/4 \\ Q(E_2^{**}) &= 1/4 \\ Q(E_3) &= 1/6. \end{aligned} \tag{8}$$

# Invariance Violation III

Now note that while Sherlock Holmes and Jane Marple agree on the relevant facts of the criminal case (who is the culprit?) in their posterior probabilities if they use Jeffrey conditioning,

$$\begin{aligned} P'_{\text{JC}}(E_1) &= 1/2 \\ P'_{\text{JC}}(E_2) &= 3/8 \\ P'_{\text{JC}}(E_3) &= 1/8 \end{aligned} \tag{9}$$

# Invariance Violation IV

$$\begin{aligned}
Q'_{\mathrm{JC}}(E_1) &= 1/2 \\
Q'_{\mathrm{JC}}(E_2^*) &= 3/16 \\
Q'_{\mathrm{JC}}(E_2^{**}) &= 3/16 \\
Q'_{\mathrm{JC}}(E_3) &= 1/8
\end{aligned} \tag{10}$$

# Invariance Violation V

They do not agree if they use LP conditioning,

$$\begin{aligned} P'_{\mathrm{LP}}(E_1) &= 1/2 \\ P'_{\mathrm{LP}}(E_2) &= 5/12 \\ P'_{\mathrm{LP}}(E_3) &= 1/12 \end{aligned} \tag{11}$$

# Invariance Violation VI

$$\begin{aligned} Q'_{\mathrm{LP}}(E_1) &= 1/2 \\ Q'_{\mathrm{LP}}(E_2^*) &= 7/36 \\ Q'_{\mathrm{LP}}(E_2^{**}) &= 7/36 \\ Q'_{\mathrm{LP}}(E_3) &= 1/9. \end{aligned} \tag{12}$$

LP conditioning violates INVARIANCE.

# Levinstein Violation I

There is a car behind an opaque door, which you are almost sure is blue but which you know might be red. You are almost certain of materialism, but you admit that there's some minute possibility that ghosts exist. Now the opaque door is opened, and the lighting is fairly good. You are quite surprised at your sensory input: your new credence that the car is red is very high.

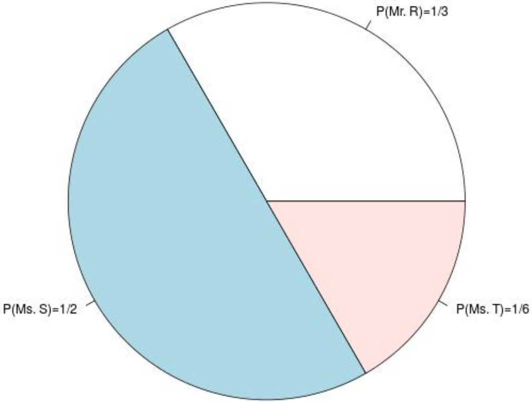
# Levinstein Violation II

Jeffrey conditioning leads to no change in opinion about ghosts. Under LP conditioning, however, seeing the car raises the probability that there are ghosts to an astonishing 47%, given Levinstein's reasonable priors.

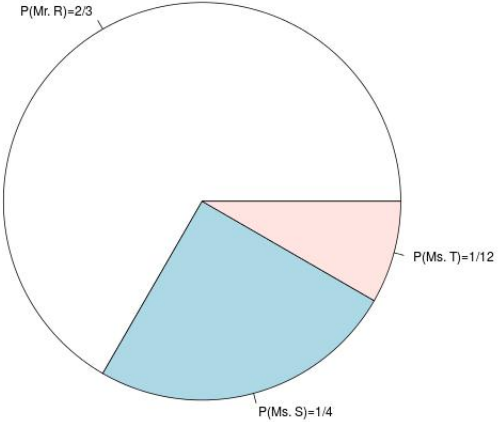
# Regularity Violation I

LP conditioning violates REGULARITY because formerly positive probabilities can be reduced to 0 even though the new information in the Jeffrey-type updating scenario makes no such requirements (as is usually the case for standard conditioning).

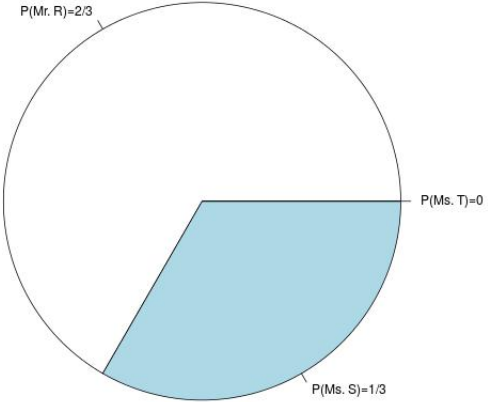
# Regularity Violation Priors



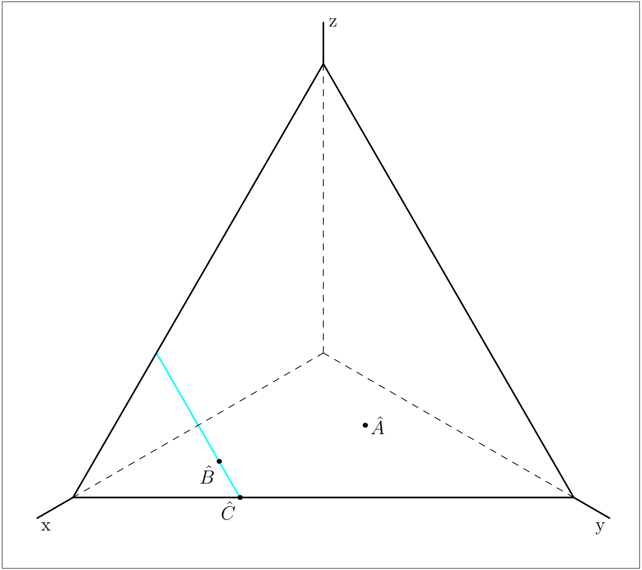
# Regularity Violation Jeffrey Posteriors



# Regularity Violation LP Posteriors



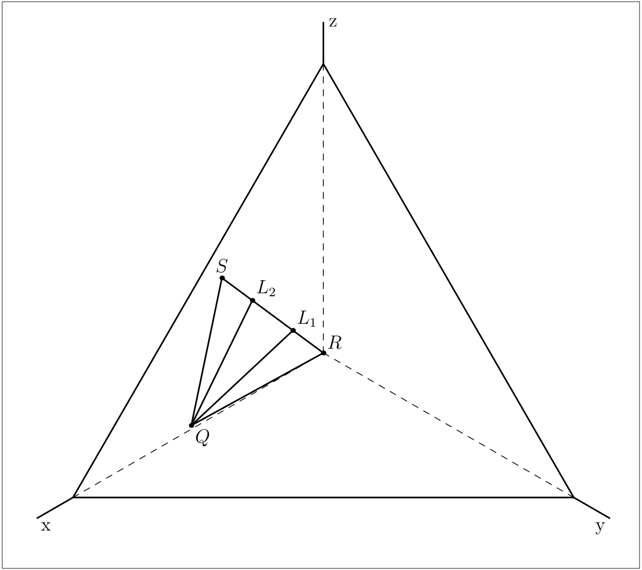
# Asymmetry Violation I



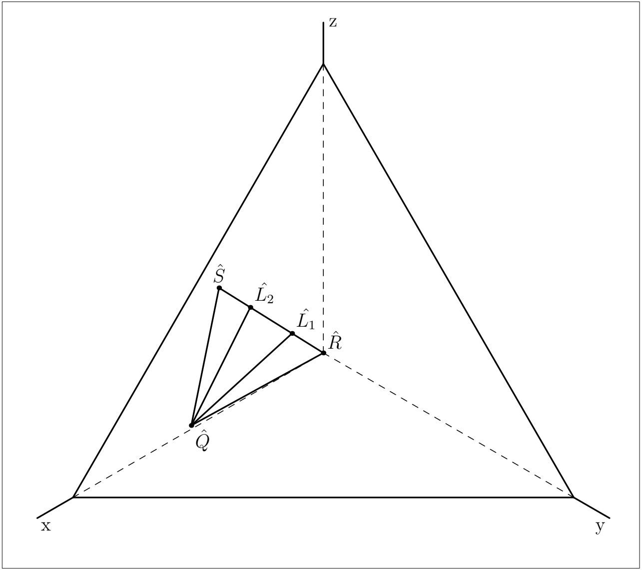
# Asymmetry Violation II

If $I(b', \omega) = I(b'', \omega)$, then for any $\lambda \in [0, 1]$ one has $I(\lambda b' + (1 - \lambda)b'', \omega) = I((1 - \lambda)b' + \lambda b''), \omega)$.

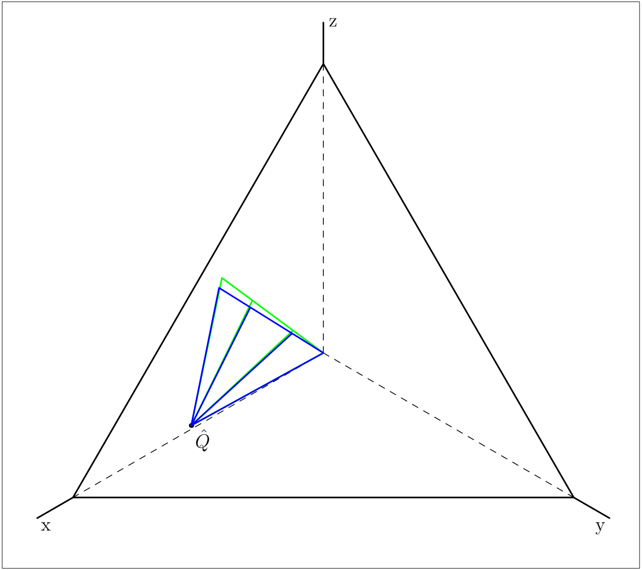
# Asymmetry Violation III



# Asymmetry Violation IV



## Asymmetry Violation V



# End of Presentation

Thank you for your attention.