probabilities $P(\theta_j) = \beta_j, \hat{P}(\theta_j) = \hat{\beta}_j, P(\omega_i) = \alpha_i, \hat{P}(\omega_i) = \hat{\alpha}_i$, we therefore also have joint probabilities $\mu_{ij} = P(\omega_i \cap \theta_j)$ and $\hat{\mu}_{ij} = \hat{P}(\omega_i \cap \theta_j)$.

Given the specific nature of Wagner-type problems, there are a few constraints on the triple $(\kappa, \beta, \hat{\alpha})$. The last row $(\mu_{mj})_{j=1,\ldots,n}$ is special because it represents the probability of ω_m, which is the negation of the events deemed possible after the observation. In the *Linguist* problem, for example, ω_5 is the event (initially highly likely, but impossible after the observation of the native's utterance) that the native does not make any of the four utterances. The native may have, after all, uttered a typical Buddhist phrase, asked where the nearest bathroom was, complimented your fedora, or chosen to be silent. κ will have all 1s in the last row. Let $\hat{\kappa}_{ij} = \kappa_{ij}$ for $i = 1, \ldots, m-1$ and $j = 1, \ldots, n$; and $\hat{\kappa}_{mj} = 0$ for $j = 1, \ldots, n$. $\hat{\kappa}$ equals κ except that its last row are all 0s, and $\hat{\alpha}_m = 0$. Otherwise the 0s are distributed over κ (and equally over $\hat{\kappa}$) so that no row and no column has all 0s, representing the logical relationships between the ω_is and the θ_js ($\kappa_{ij} = 0$ if and only if $\hat{P}(\omega_i \cap \theta_j) = \mu_{ij} = 0$). We set $P(\omega_m) = x$ ($\hat{P}(\omega_m) = 0$), where x depends on the specific prior knowledge. Fortunately, the value of x cancels out nicely and will play no further role. For convenience, we define

$$\zeta = (0, \ldots, 0, 1)^\top \tag{5}$$

with $\zeta_m = 1$ and $\zeta_i = 0$ for $i \neq m$.

The best way to visualize such a problem is by providing the joint probability matrix $M = (\mu_{ij})$ together with the marginals α and β in the last column/row, here for example as for the *Linguist* problem with $m = 5$ and $n = 4$ (note that this is not the matrix M, which is $m \times n$, but M expanded with the marginals in improper matrix notation):

$$\begin{bmatrix} \mu_{11} & \mu_{12} & 0 & 0 & \alpha_1 \\ \mu_{21} & \mu_{22} & 0 & 0 & \alpha_2 \\ 0 & \mu_{32} & 0 & \mu_{34} & \alpha_3 \\ \mu_{41} & \mu_{42} & \mu_{43} & \mu_{44} & \alpha_4 \\ \mu_{51} & \mu_{52} & \mu_{53} & \mu_{54} & x \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 & 1.00 \end{bmatrix}. \tag{6}$$

The $\mu_{ij} \neq 0$ where $\kappa_{ij} = 1$. Ditto, mutatis mutandis, for $\hat{M}, \hat{\alpha}, \hat{\beta}$. To make this a little less abstract, Wagner's *Linguist* problem is characterized by the triple $(\kappa, \beta, \hat{\alpha})$,

$$\kappa = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \text{ and } \hat{\kappa} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

$$\beta = (0.2, 0.3, 0.4, 0.1)^\top \text{ and } \hat{\alpha} = (0.4, 0.3, 0.2, 0.1, 0)^\top. \tag{8}$$

Wagner's solution, based on JUP, is

$$\hat{\beta}_j = \beta_j \sum_{i=1}^{m-1} \frac{\hat{\kappa}_{ij} \hat{\alpha}_i}{\sum_{\hat{\kappa}_{il}=1} \beta_l} \text{ for all } j = 1, \ldots, n. \tag{9}$$