> A token is pulled from a bag containing 3 yellow tokens, 2 blue tokens, and 1 purple token. You are colour blind and cannot distinguish between the blue and the purple token when you see it. When the token is pulled, it is shown to you in poor lighting and then obscured again. You come to the conclusion based on your observation that the probability that the pulled token is yellow is $1/3$ and that the probability that the pulled token is blue or purple is $2/3$. What is your updated probability that the pulled token is blue?

Let $P(\text{blue})$ be the prior subjective probability that the pulled token is blue and $\hat{P}(\text{blue})$ the respective posterior subjective probability. Jeffrey conditioning, based on JUP (which mandates, for example, that $\hat{P}(\text{blue}|\text{blue or purple}) = P(\text{blue}|\text{blue or purple})$) recommends

$$\begin{aligned} & \hat{P}(\text{blue}) \\ = \; & \hat{P}(\text{blue}|\text{blue or purple})\hat{P}(\text{blue or purple}) + \hat{P}(\text{blue}|\text{neither blue nor purple})\hat{P}(\text{neither blue nor purple}) \\ = \; & P(\text{blue}|\text{blue or purple})\hat{P}(\text{blue or purple}) \\ = \; & 4/9 \end{aligned} \tag{3}$$

In the notation of (2), the example is calculated with $\beta = (1/2, 1/3, 1/6)^{\top}, \hat{\alpha} = (1/3, 2/3)^{\top}$,

$$\kappa = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \tag{4}$$

and yields the same result as (3) with $\hat{\beta}_2 = 4/9$.

4. Wagner Conditioning

Carl Wagner uses JUP (explained in more detail in [21]) to solve a problem which cannot be solved by Jeffrey conditioning. Here is the narrative (call this the *Linguist* problem):

> You encounter the native of a certain foreign country and wonder whether he is a Catholic northerner (θ_1), a Catholic southerner (θ_2), a Protestant northerner (θ_3), or a Protestant southerner (θ_4). Your prior probability p over these possibilities (based, say, on population statistics and the judgment that it is reasonable to regard this individual as a random representative of his country) is given by $p(\theta_1) = 0.2, p(\theta_2) = 0.3, p(\theta_3) = 0.4$, and $p(\theta_4) = 0.1$. The individual now utters a phrase in his native tongue which, due to the aural similarity of the phrases in question, might be a traditional Catholic piety (ω_1), an epithet uncomplimentary to Protestants (ω_2), an innocuous southern regionalism (ω_3), or a slang expression used throughout the country in question (ω_4). After reflecting on the matter you assign subjective probabilities $u(\omega_1) = 0.4, u(\omega_2) = 0.3, u(\omega_3) = 0.2$, and $u(\omega_4) = 0.1$ to these alternatives. In the light of this new evidence how should you revise p? (See [18] (p.252) and [22] (p197).)

Let us call a problem of this type a Wagner-type problem. It is an instance of the more general obverse Majerník problem where partitions are given with logical relationships between them as well as some marginal probabilities. Wagner-type problems seek as a solution missing marginals, while obverse Majerník problems seek the conditional probabilities as well, both of which I will eventually provide using PME.

Wagner's solution for such problems (from now on Wagner conditioning) rests on JUP and a formal apparatus established by Arthur Dempster in [23], which is quite different from our notational approach.