Professor Pap,

I am looking for a non-additive, continuous set function from a simplex Π_n of finite dimension $n-1$ into $[0, \infty]$. The motivation is as follows. Shannon defined the entropy of a probability measure on a finite event space $H(p_1, \ldots, p_n)$. He wanted the entropy to be (i) continuous in the p_i, (ii) if all the p_i are equal ($p_i = 1/n$) then H should be monotonic increasing in n (more equally probable events mean greater entropy, so for example $H_2(1/2, 1/2) < H_3(1/3, 1/3, 1/3)$), and (iii) if choices are subdivided the entropy should remain constant, such that, for example, $H_3(p_1, p_2, p_3) = H_2(p_1, q) + qH_2(p_2/q, p_3/q)$ for $q = p_2 + p_3$.

The only function fulfilling these requirements is the Shannon entropy

$$H(p) = -K \sum_{i=1}^{n} p_i \log p_i \tag{1}$$

I want to generalize the Shannon entropy from elements of Π_n to select subsets of Π_n (e.g. Borel subsets). Let π be a subset of Π_n. I guess here are some commonsense requirements for η, the generalization of H. (i) $\eta(\{p\}) = H(p)$. (ii) η is continuous in the sense that

$$\text{If } \bigcap_{k=1}^{\infty} \pi_k = \pi \text{ with } \pi_k \subseteq \pi_{k-1} \text{ then } \eta(\pi) = \lim_{k \to \infty} \eta(\pi_k)$$

and

$$\text{If } \bigcup_{k=1}^{\infty} \pi_k = \pi \text{ with } \pi_k \supseteq \pi_{k-1} \text{ then } \eta(\pi) = \lim_{k \to \infty} \eta(\pi_k)$$

One requirement that I can't quite articulate yet is that η won't be additive but more of an averageing function, such that $\eta(\{p, q\})$ is somewhere between $H(p)$ and $H(q)$, certainly not the sum of them. The intuition is that if a rational agent can permissibly hold these two probability functions, her information is roughly the average of the information where she is in the situation that she can hold only one of them.

My question for you: is it possible to give an explicit formula for η as in (1)? I imagine it would look very similar to (1) and involve some kind of integral over the points of the simplex.

I would love it if you could help me with this or point me to someone who could. Thank you,

Stefan