Semantics of Not Knowing

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1 Motivation for Instates

Booleans claim that it is rational for an agent to hold instates (indeterminate credal states) in addition to sharp credences, representing uncertainty. They are Bayesians in all other respects and defend Bayesian epistemology, proposing that it is better off without the requirement for sharp credences. Laplaceans, by contrast, require sharp credences for rational agents. We want to motivate indeterminacy as forcefully as possible so that the reader will see (a) the appeal of the Boolean approach, (b) the insufficiency of the critical response, and (c) the need for careful articulation of the Laplacean approach that can address the concerns which motivate some to resort to indeterminacy. Finally, (d) the undesirable semantics of the Boolean approach, documented both conceptually and by example, will lead to the conclusion that a Laplacean approach is the more promising alternative. Here is a list of reasons for the Boolean position found in the literature.

(A) The greatest emphasis motivating indeterminacy rests on lack of evidence or conflicting evidence and the assumption that single probability measures (sharp credences) do not represent such evidence as well as credal states composed by sets of probability measures (instates).

(B) The preference structure of a rational agent may be incomplete so that representation theorems do not yield single probability measures to represent such incomplete structures.

(C) There are more technical and paper-specific reasons, such as Thomas Augustin's attempt to mediate between the minimax pessimism of objectivists and the Bayesian optimism of subjectivists using interval probability (see Augustin, 2003, 35f); Alan Hájek and Michael Smithson's belief that there may be objectively indeterminate chances in the physical world (see Hájek and Smithson, 2012, 33); and Jake Chandler's claim that "the sharp model is at odds with a trio of plausible propositions regarding agnosticism" (Chandler, 2014, 4).

This paper mostly addresses (A), while taking (B) seriously as well and pointing towards solutions for it. I am leaving (C) to more specific responses to the issues presented in the cited articles, and for the remainder of this section I am adding a reason (D) that is poorly documented in the literature.

Motivation (D) is that the Boolean rational agent may do better accepting advantageous bets than the agent who on principle rejects instates. Walley conducted an experiment in which Boolean participants did significantly better than Laplacean participants, betting on soccer games played in the Soccer World Cup 1982 in Spain (see Walley, 1991, Appendix F).

I replicated the experiment using two computer players with rudimentary artificial intelligence and made them specify betting parameters (previsions) for games played in the Soccer World Cup 2014 in Brazil. I used the Poisson distribution (which is an excellent predictor for the outcome of soccer matches) and the FIFA ranking to simulate millions of counterfactual World Cup results and their associated bets, using Walley's evaluation method. The Boolean player had a slight but systematic advantage. In section 4, I will provide an explanation and show how it undermines any support the experiment might give to the Boolean position.

2 Dilation

Roger White introduces a problem for instates claiming that they lead to unacceptable doxastic scenarios involving dilation. In dilation, there is a widening of the instate upon updating instead of a narrowing. Sometimes this widening is extreme, from a maximally precise credal state to a vacuous instate, based on little information (see White, 2010 for examples). White's objection, even though I think it fails as it stands, triggers semantic concessions on the part of Booleans defending their position which will be important to my semantic criticism of indeterminacy.

In Joyce's response to White, two semantic concessions to White show why the dilated instates give us the right result. I agree with Joyce: dilation is what you would expect (1) if credences do not adequately represent evidence (the same instate can reflect different evidential situations); and (2) if instates do not reflect knowledge claims about objective chances (Joyce rejects White's Chance Grounding Thesis CGT, see Joyce, 2010, 289). Dilation from a sharp 0.5 to an indeterminate $[0.01, 0.99]$ or $\{0.01, 0.99\}$ (depending on whether convexity is required) is unproblematic in the following example, although the example already prefigures that there is something odd about the Boolean semantic approach. The example licences a 99:1 bet for one of the colours, but this is a problem that arises out of Boolean semantics without dilation, which we will address again in example 3.

> **Example 1: Dilating Urns.** You draw from an urn with 200 balls (100 red, 100 black) and receive the information that the urn actually had two chambers, one with 99 red balls and 1 black ball, the other with 1 red ball and 99 black balls.

If one were to be committed to the principle of regularity, that all states of the world considered possible have positive probability (for a defence see Savage et al., 1963); and to the solution of Henry Kyburg's lottery paradox, that what is rationally accepted should have probability 1 (for a defence of this principle see Douven and Williamson, 2006); and the CGT, that one's spread of credence should cover the range of possible chance hypotheses left open by evidence (implied by much of Boolean literature); then one's instate is always vacuous. Booleans must deny at least one of the premises to avoid the conclusion. Joyce denies the CGT, but then he continues to make implicit use of it when he repeatedly complains that sharp credences "ignore a vast number of possibilities that are consistent with [the] evidence" (for example in (Joyce, 2005, 170)).

When updating dilates the credal state, it appears that the prior credal state was in some sense incorrect and did not properly reflect the state of the world, even though it properly reflected the epistemic state of the agent. Such a divergence between the proper reflections of epistemic state and state of the world undermines the subjective interpretation of probabilities at the heart of Bayesian epistemology. An instate semantically blurs the line between traditional full belief epistemology and formal partial belief epistemology (for semantically more intelligible attempts at reconciliation between the two see Moss, 2013, although Moss is committed to the Boolean approach, which may weaken her case; and Spohn, 2012, especially chapter 10). White's objection fails, however, because dilation for indeterminate credences is in principle not any more surprising than a piece of information that increases the Shannon entropy of a sharp credence. It is true for both sharp and indeterminate credences that information can make us less certain about things.

3 Semantics of Partial Belief

Central to my project is the proper semantic distinction between evidence, information, and partial belief. Both sharp credences and instates, within a Bayesian framework, try to represent the uncertainty of an agent with respect to the truth of a proposition. Instates have a greater ambition and therefore a double task: they also claim to represent properties of the evidence, such as its weight, conflict between its constituents, or its ambiguity.

The Laplacean approach uses partial beliefs about how a parameter is distributed and then applies Lewis' summation formula (see Lewis, 1981, 266f) to integrate over them and condense them to a sharp credence. Walley comments on this "reduction" in his section on Bayesian second order probabilities (see Walley, 1991, 258f), but he mistakenly represents the Laplacean approach as a second order approach, as if the probability distributions that are summarized by Lewis' formula are of the same kind as the resulting credences. They are not. They are objective chances or other partitions of the event space and the subjective probabilities that are associated with them. It is the Boolean approach which has elements of a second order approach and thus makes itself vulnerable to regress problems by adding another dimension of uncertainty to a parameter (the credence) which already represents uncertainty.

One of Joyce's complaints is that a sharp credence of 0.5 for a coin contains too much information if there is little or no evidence that the coin is fair. This complaint, of course, is only effective if we make a credence say something about the evidence. Joyce himself, however, admits that instates cannot adequately represent the evidence without violating the reflection principle due to White's dilation problem. He is quite clear that the same instate can represent different evidential scenarios (see, for example, Joyce, 2010, 302).

Sharp credences have one task: to represent epistemic uncertainty and serve as a tool for updating, inference, and decision-making. They cannot fulfill this task without continued reference to the evidence which operates in the background. To use an analogy, credences are not sufficient statistics with respect to updating, inference, and decision-making. What is remarkable about Joyce's response to White's dilation problem is that Joyce recognizes that instates are not sufficient statistics either. But this means that they fail at the double task which has been imposed on them. In the following, I will provide a few examples illustrating this failure.

> **Example 2: Aggregating Expert Opinion.** You have no information whether it will rain tomorrow (R) or not except the predictions of two weather forecasters. One of them forecasts 0.3 on channel GPY, the other 0.6 on channel QCT. You consider the QCT forecaster to be significantly more reliable, based on past experience.

An instate corresponding to this situation may be $[0.3, 0.6]$ (see Walley, 1991, 214), but it will have a difficult time representing the difference in reliability of the experts. A sharp credence of $P(R) = 0.55$, for example, does the right thing. Beliefs about objective chances make little sense in many situations where we have credences, since it is doubtful even in the case of rain tomorrow that there is an urn of nature from which balls are drawn (see White, 2010, 171). What is really at play is a complex interaction between epistemic states (for example, experts evaluating meteorological data) and the evidence which influences them.

> **Example 3: Precise Credences.** Your sharp credence for rain tomorrow, based on the expert opinion of channel GPY and channel QCT (you have no other information) is 0.55. Is it reasonable, considering how little evidence you have, to reject the belief that the chance of rain tomorrow is 0.54 or 0.56; or to prefer a 54.9 cent bet on rain to a 45.1 cent bet on no rain?

The first question is confused in instructive ways. A sharp credence rejects no hypothesis about objective chances (unlike an instate). It often has a subjective probability distribution operating in the background, over which it integrates to yield the sharp credence. The subjective probability distribution is condensed by Lewis' summation formula to a sharp credence, without being reduced to it. Lewis' 1981 paper "A Subjectivist's Guide to Objective Chance" addresses the question of the relationship between credence, subjective probability, and objective chance. The Laplacean properly separates the semantic dimensions, without using second-order probabilities. Formal partial belief epistemology deals with sharp credences and how they represent uncertainty and serve as a tool in inference, updating, and decision making; Bayesian epistemology provides the interpretation of probabilities;

and Lewis' Humean speculations and his interpretation of the principal principle cover the relationship between subjective probabilities and objective chance.

The second question is also instructive: why would we prefer a 54.9 cent bet on rain to a 45.1 cent bet on no rain, given that we do not possess the power of descrimination between these two bets? The answer to this question ties in with the issue of incomplete preference structure referenced above as motiviation (B) for instates. Via representation theorems, preferences may conceptually precede an agent's probability and utility functions, but that does not mean that we cannot inform the axioms we use for a rational agent's preferences by undesirable consequences downstream. Completeness may sound like an unreasonable imposition at the outset, but if incompleteness has unwelcome semantic consequences for credences, it is not illegitimate to revisit the issue. Timothy Williamson goes through this exercise with vague concepts, showing that all upstream logical solutions to the problem fail and that it has to be solved downstream with an epistemic solution (see Williamson, 1996).

Vague concepts, like sharp credences, are sharply bounded, but not in a way that is luminous to the agent (for anti-luminosity see chapter 4 in Williamson, 2000). Anti-luminosity answers the original question: the rational agent prefers the 54.9 cent bet on rain to a 45.1 cent bet on no rain based on her sharp credence without being in a position to have this preference necessarily or have it based on physical or psychological ability (for the analogous claim about knowledge see Williamson, 2000, 95).

In a way, advocates of indeterminacy have solved this problem for us. There is strong agreement among most of them that the issue of indeterminacy for credences is not an issue of elicitation. The appeal of preferences is that we can elicit them more easily than assessments of probability and utility functions. The indeterminacy issue has been raised to the probability level (or moved downstream) by indeterminacy advocates themselves who feel justifiably uncomfortable with an interpretation of their theory in behaviourist terms. So it shall be solved there, and this paper makes an appeal to reject indeterminacy on this level. Isaac Levi seems to agree with me on this point: when he talks about indeterminacy, it proceeds from the level of probability judgment to preferences, not the other way around (see Levi, 1981, 533).

Example 4: Jaynes' Monkeys. Let urn A contain 4 balls, two red and two black. A monkey randomly fills urn B from urn A with two balls. We draw from urn B. (See Jaynes and Bretthorst, 2003, 160.)

The sharp credence of drawing a red ball is 0.5, following Lewis' summation formula for the different combinations of balls in urn B. This solution is more intuitive in terms of further inference, decision making, and betting behaviour than an instate of $\{0, 1/2, 1\}$ or $[0, 1]$ (depending on the convexity requirement), since the instate would licence an exorbitant bet in favour of one colour.

Example 5: Three Prisoners. The Three Prisoners problem is well-documented (see Mosteller, 1987, 28). Let the three prisoners be X_1, X_2, X_3 and the warden tell X_1 that X_3 will be executed.

Peter Walley maintains that for the Monty Hall problem and the Three Prisoners problem, the probabilities of a rational agent should dilate rather than settle on the commonly accepted solutions. For the Three Prisoners problem, there is a compelling case for standard conditioning and the result that the chances of pardon for prisoner X_1 are unchanged after the update (see Lukits, 2014, 1421f). Walley's dilated solution would give prisoner X_1 hope on the doubtful possibility (and unfounded assumption) that the warden might prefer to provide X_3's name in case prisoner X_1 was pardoned.

Booleans charge that sharp credences often reflect independence of variables where such independence is unwarranted. Booleans prefer to dilate over the possible dependence relationships (independence included). White's dilation problem is an instance of this. The fallacy in the argument for instates, illustrated by the Three Prisoners problem, is that the probabilistic independence of sharp credences does not imply independence of variables (the converse is correct), but only that it is unknown whether there is dependence, and if yes, whether it is correlation or inverse correlation.

Example 6: Wagner's Linguist. A linguist hears the utterance of a native and concludes that the native cannot be part of certain population groups, depending on what the utterance means. The linguist is uncertain between some options about the meaning of the utterance. (For full details see Wagner, 1992, 252; and Spohn, 2012, 197.)

The mathematician Carl Wagner proposed a natural generalization of Jeffrey Conditioning for his Linguist example (see Wagner, 1992). Since the principle of maximum entropy is already a generalization of Jeffrey Conditioning, the question naturally arises whether the two generalizations agree. Wagner makes the case that they do not agree and deduces that the principle of maximum entropy is sometimes an inappropriate updating mechanism, in line with many earlier criticisms of the principle of maximum entropy (see van Fraassen, 1981; Shimony, 1985; Skyrms, 1987; and, later on, Grove and Halpern, 1997).

What is interesting about this case is that Wagner uses instates for his deduction, so that even if you agree with his natural generalization of Jeffrey Conditioning (which I find plausible), the inconsistency with the principle of maximum entropy can only be inferred assuming instates. On the assumption of sharp credences Wagner's generalization of Jeffrey conditioning perfectly accords with the principle of maximum entropy. Wagner's argument, instead of undermining the principle of maximum entropy, just shows that instates are as wedded to rejecting the claims of the principle of maximum entropy as the principle of maximum entropy is wedded to sharp credences.

4 Evidence Differentials and Cushioning Credences

After I found out that agents with instates do better betting on soccer games (see section 1), I let player X (who uses sharp credences) and player Y (who uses instates) play a more basic betting game and used Walley's pay off scheme (see Walley, 1991, 632) to settle the bets. The simulation results show that player Y does better for $n > 2$ while player X does better for $n = 2$. A defence of sharp credences for rational agents needs to have an explanation why, for $n > 2$, player Y does better. We will call it partial belief cushioning, which is based on an evidence differential between the bettors.

Compulsory betting is a problem for instates, as Booleans have to find a way to decide without receiving instructions from the credal state. Booleans have addressed this point extensively (see for example Joyce, 2010, 311ff; for an opponent's view of this see Elga, 2010, 6ff). The problem for sharp credences arises when bets are noncompulsory, for then the data above suggests that agents holding instates sometimes do better. Bets are sometimes offered by better informed or potentially better informed bookies. In this case, even an agent with sharp credences must cushion her credences and is better off by rejecting bets that look attractive in terms of her partial beliefs. If an agent does not cushion her partial beliefs (whether they are sharp or indeterminate), she will incur a loss in the long run. Since cushioning is permitted in Walley's experimental setup (the bets are noncompulsory), Laplacean agents should also have access to it and then no longer do worse than Boolean agents.

The rational agent with a sharp credence has resources at her disposal to use just as much differentiation with respect to accepting and rejecting bets as the agent with instates. Often (if she is able to and especially if the bets are offered to her by a better-informed agent), she will reject both of two complementary bets, even when they are fair. In 2013, the U.S. Commodity Futures Trading Commission shut down the online prediction market Intrade, even though it was offering fair bets, because of expert arbitrageurs.

Any advantage that the agent with an instate has can be counteracted based on distribution over partial beliefs with respect to all possibilities. The agent with instates suffers from a semantic mixing of metaphors between evidential and epistemic dimensions that puts her at a real disadvantage in terms of understanding the sources and consequences of her knowledge and her uncertainties.