

probabilities? This problem is best understood by example (see Wagner's *Linguist* problem in Section 4). Wagner solves it using a natural generalization of Jeffrey conditioning, which I will call Wagner conditioning. It is not based on PME, but on what I call Jeffrey's updating principle, or JUP for short:

[JUP] In a diachronic updating process, keep the ratio of probabilities constant as long as they are unaffected by the constraints that the evidence poses.

As is the case for PME, there is a debate whether updating on evidence by rational agents is bound by JUP (for a defence see [19]; for detractors see [20]). Our interest in this paper is the relationship between PME and JUP, both of which are updating principles. Wagner contends that his natural generalization of Jeffrey conditioning, based on JUP, contradicts PME. Among formal epistemologists, there is a widespread view that, while PME is a generalization of Jeffrey conditioning, it is an inappropriate updating method in certain cases and does not enjoy the generality of Jeffrey conditioning. Wagner's claims support this view inasmuch as Wagner conditioning is based on the relatively plausible JUP and naturally generalizes Jeffrey conditioning, but according to Wagner it contradicts PME, which gives wrong results in these cases.

This paper resists Wagner's conclusions and shows that PME generalizes both Jeffrey conditioning and Wagner conditioning, providing a much more integrated approach to probability updating. This integrated approach also gives a coherent answer to the obverse Majerník problem posed above.

3. Jeffrey Conditioning

Richard Jeffrey proposes an updating method for cases in which the evidence is uncertain, generalizing standard probabilistic conditioning. I will present this method in unusual notation, anticipating using my notation to solve Wagner's *Linguist* problem and to give a general solution for the obverse Majerník problem. Let Ω be a finite event space and $\{\theta_j\}_{j=1,\dots,n}$ a partition of Ω . Let κ be an $m \times n$ matrix for which each column contains exactly one 1, otherwise 0. Let $P = P_{\text{prior}}$ and $\hat{P} = P_{\text{posterior}}$. Then $\{\omega_i\}_{i=1,\dots,m}$, for which

$$\omega_i = \bigcup_{j=1,\dots,n} \theta_{ij}^*, \quad (1)$$

is likewise a partition of Ω (the ω are basically a more coarsely grained partition than the θ). $\theta_{ij}^* = \emptyset$ if $\kappa_{ij} = 0$, $\theta_{ij}^* = \theta_j$ otherwise. Let β be the vector of prior probabilities for $\{\theta_j\}_{j=1,\dots,n}$ ($P(\theta_j) = \beta_j$) and $\hat{\beta}$ the vector of posterior probabilities ($\hat{P}(\theta_j) = \hat{\beta}_j$); likewise for α and $\hat{\alpha}$ corresponding to the prior and posterior probabilities for $\{\omega_i\}_{i=1,\dots,m}$, respectively.

A Jeffrey-type problem is when β and $\hat{\alpha}$ are given and we are looking for $\hat{\beta}$. A mathematically more concise characterization of a Jeffrey-type problem is the triple $(\kappa, \beta, \hat{\alpha})$. The solution, using Jeffrey conditioning, is

$$\hat{\beta}_j = \beta_j \sum_{i=1}^n \frac{\kappa_{ij} \hat{\alpha}_i}{\sum_{l=1}^m \kappa_{il} \beta_l} \text{ for all } j = 1, \dots, n. \quad (2)$$

The notation is more complicated than it needs to be for Jeffrey conditioning. In Section 5, however, I will take full advantage of it to present a generalization where the ω_i do not range over the θ_j . In the meantime, here is an example to illustrate (2).