Augustin's Concessions: A Problem for Indeterminate Credal States

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1 Introduction

The claim is that rational agents are subject to a norm requiring sharp credences. I defend this claim in spite of the initially promising features of indeterminate credal states (from now on instates) to address problems which sharp credences have as they reflect the evidence at the foundation of a doxastic state. Traditionally, Bayesians have maintained that a rational agent, when holding a credence, holds a sharp credence. It has recently become popular to drop the requirement for credence functions to be sharp. There are now Bayesians who permit a rational agent to hold instates based on incomplete or ambiguous evidence. I will refer to Bayesians who continue to adhere to the classical theory of sharp credences for rational agents as 'Laplaceans' (e.g. Adam Elga and Roger White). I will refer to Bayesians who do not believe that a rational agent's credences are sharp as 'Booleans' (e.g. Peter Walley and James Joyce).

I will exclusively refer to indeterminate credal states (abbreviated 'instates', sometimes terminology such as 'imprecise' or 'mushy' credences is used as well) and mean by them a set of sharp credence functions (which some Booleans require to be convex) which it may be rational for an agent to hold within an otherwise orthodox Bayesian framework.

When we first hear of the advantages of instates, two of them sound particularly persuasive.

- RANGE Instates represent the possibility range for objective chances.
- INCOMPLETE Instates represent incompleteness or ambiguity of the evidence.

Here are some examples. Let a $coin_x$ be a Bernoulli generator that produces successes and failures with probability p_x for success, labeled H_x, and $1 - p_x$ for failure, labeled T_x. Physical coins may serve as Bernoulli generators, if we are willing to set aside that most of them are approximately fair.

> **Example 1: Range.** Bob has two Bernoulli Generators in his lab, $coin_{\mathrm{i}}$ and $coin_{\mathrm{ii}}$. Bob has a database of $coin_{\mathrm{i}}$ results and concludes on excellent evidence that $coin_{\mathrm{i}}$ is fair. Bob has no evidence about the bias of $coin_{\mathrm{ii}}$. As a Boolean, Bob assumes a sharp credence of $\{0.5\}$ for H_{i} and an indeterminate credal state of $[0, 1]$ for H_{ii}. He feels bad for Larry, his Laplacean colleague, who cannot distinguish between the two cases and who must assign a sharp credence of $\{0.5\}$ for both H_{i} and H_{ii}.

> **Example 2: Incomplete.** Bob has another Bernoulli Generator, $coin_{\mathrm{iii}}$, in his lab. His graduate student has submitted $coin_{\mathrm{iii}}$ to countless experiments and emails Bob the resulting bias, but fails to include whether the bias of $2/3$ is in favour of H_{iii} or in favour of T_{iii}. As a Boolean, Bob assumes an indeterminate credal state of $[1/3, 2/3]$ (or $\{1/3, 2/3\}$, depending on whether convexity is required) for H_{iii}. He feels bad for Larry who must assign a sharp credence of $\{0.5\}$ for H_{iii} when Larry concurrently knows that his credence gets the bias wrong.

Against the force of RANGE and INCOMPLETE, I maintain that the Laplacean approach of assigning subjective probabilities to partitions of the event space (e.g. objective chances) and then aggregating them by David Lewis' summation formula (see Lewis, 1981, 266f) into a single precise credence function is conceptually tidy and shares many of the formal virtues of Boolean theories. To put it provocatively, this paper defends a 0.5 sharp credence in heads in all three cases: for a coin of whose bias we are completely ignorant; for a coin whose fairness is supported by a lot of evidence; and even for a coin about whose bias we know that it is either 1/3 or 2/3 for heads.

2 Augustin's Concessions

Here are two potential problems for Booleans:

- DILATION Instates are vulnerable to dilation.
- OBTUSE Instates do not permit learning.

Again, these are best explained by examples. First, here is an example for DILATION (see White, 2010, 175f and Joyce, 2010, 296f).

> **Example 3: Dilation.** Larry has two Bernoulli Generators, $coin_{\text{iv}}$ and $coin_{\text{v}}$. He has excellent evidence that $coin_{\text{iv}}$ is fair and no evidence about the bias of $coin_{\text{v}}$. Larry's graduate student independently tosses both $coin_{\text{iv}}$ and $coin_{\text{v}}$. Then she tells Larry whether the two tosses are correlated or not ($H_{\text{iv}} \equiv H_{\text{v}}$ or $H_{\text{iv}} \equiv T_{\text{v}}$, where $X \equiv Y$ means $(X \wedge Y) \vee (\neg X \wedge \neg Y)$). Larry, who has a sharp credence for H_{v}, takes this information in stride, but he feels bad for Bob, whose credence in H_{iv} dilates to [0.1] even though Bob shares Larry's excellent evidence that $coin_{\text{iv}}$ is fair.

Here is why Bob's credence in H_{iv} must dilate. His credence in H_{v} is $[0, 1]$, by stipulation. Let $c(X)$ be the set of sharp credences representing Bob's instate, for example $c(H_{\text{v}}) = [0, 1]$. Then

$$c(H_{\text{iv}} \equiv H_{\text{v}}) = c(H_{\text{iv}} \equiv T_{\text{v}}) = \{0.5\} \tag{1}$$

because the tosses are independent and $c(H_{\text{iv}}) = \{0.5\}$ by stipulation. Next,

$$c(H_{\text{iv}}|H_{\text{iv}} \equiv H_{\text{v}}) = c(H_{\text{v}}|H_{\text{iv}} \equiv H_{\text{v}}) \tag{2}$$

where $c(X|Y)$ is the updated instate after finding out Y. Booleans accept (2) because they are Bayesians and update by standard conditioning. Therefore,

$$\begin{aligned} &c(H_{\text{iv}}|H_{\text{iv}} \equiv H_{\text{v}}) = c(H_{\text{v}}|H_{\text{iv}} \equiv H_{\text{v}}) = \frac{c(H_{\text{iv}})c(H_{\text{v}})}{c(H_{\text{iv}})c(H_{\text{v}}) + c(T_{\text{iv}})c(T_{\text{v}})} \\ &= c(H_{\text{v}}) = [0, 1]. \end{aligned} \tag{3}$$

Bob's updated instate for H_{iv} has dilated from $\{0.5\}$ to $[0, 1]$.

This does not sound like a knock-down argument against Booleans (it is investigated in detail in Seidenfeld and Wasserman, 1993), but Roger White uses it to derive implications from instates which are worrisome (see especially his chocolate example in White, 2010, 183).

Second, here is an example for OBTUSE (see Susanna Rinard's objection cited in White, 2010, 84 and addressed in Joyce, 2010, 290f). It presumes Joyce's supervaluationist semantics of instates (see Joyce, 2010, 288; and Rinard, 2015), for which Joyce uses the helpful metaphor of committee members, each of whom holds a sharp credence. The instate consists then of the set of sharp credences from each committee member: for the purposes of updating, for example, each committee member updates as if she were holding a sharp credence. The aggregate of the committee members' updated sharp credences forms the updated instate. Supervaluationist semantics also permits comparisons, when for example a partial belief in X is stronger than a partial belief in Y because all committee members have sharp credences in X which exceed all the sharp credences held by committee members with respect to Y.

> **Example 4: Obtuse.** Bob has a Bernoulli Generator in his lab, $coin_{\mathrm{vi}}$, of whose bias he knows nothing and which he submits to experiments. At first, Bob's instate for H_{vi} is [0.1]. After a few experiments, it looks like $coin_{\mathrm{vi}}$ is fair. However, as committee members crowd into the centre and update their sharp credences to something closer to 0.5, they are replaced by extremists on the fringes. The instate remains at $[0, 1]$.

Joyce, an authoritative Boolean voice, has defended instates against DILATION and OBTUSE, making Augustin's concessions (AC1) and (AC2). I am naming them after Thomas Augustin, who has some priority over Joyce in the matter.

(AC1) Credences do not adequately represent a doxastic state. The same instate can reflect different doxastic states.

(AC2) Instates do not represent knowledge claims about objective chances. White's *Chance Grounding Thesis* is not an appropriate characterization of the Boolean position.

(AC1) and (AC2) are both necessary and sufficient to resolve DILATION and OBTUSE for instates. I will address this in more detail in a moment. Indeterminacy imposes a double task on credences (representing both uncertainty and available evidence) that they cannot coherently fulfill. I will present several examples where this double task stretches instates to the limits of plausibility. Joyce's idea that credences can represent balance, weight, and specificity of the evidence (in Joyce, 2005) is inconsistent with the use of indeterminacy. Joyce himself, in response to DILATION and OBTUSE, gives the argument why this is the case (see Joyce, 2010, 290ff for OBTUSE; and

Joyce, 2010, 296ff for DILATION). Let us look more closely at how (AC1) and (AC2) protect the Boolean position from DILATION and OBTUSE.

2.1 Augustin's Concession (AC1)

(AC1) says that credences do not adequately represent a doxastic state. The same instate can reflect different doxastic states.

Augustin recognizes the problem of inadequate representation before Joyce, with specific reference to instates: "The imprecise posterior does no longer contain all the relevant information to produce optimal decisions. Inference and decision do not coincide any more" (Augustin, 2003, 41) (see also an example for inadequate representation of evidence by instates in Bradley and Steele, 2013, 16). Joyce rejects the notion that identical instates encode identical beliefs by giving a simple example:

> **Example 5: Three-Sided Die.** Let $\mathcal{C}'$ and $\mathcal{C}''$ be sets of credence functions defined on a partition $\{X, Y, Z\}$ corresponding to the result of a roll of a three sided-die. $\mathcal{C}'$ contains all credence functions c for which $c(Z) \geq 1/2$. $\mathcal{C}''$ contains all credence functions c for which $c(X) = c(Y)$ (see Joyce, 2010, 294).

$\mathcal{C}'$ and $\mathcal{C}''$ represent the same instates, but they differ in the doxastic states that they encode. The doxastic state corresponding to $\mathcal{C}'$ regards X and Y as equiprobable, the doxastic state corresponding to $\mathcal{C}''$ does not. Joyce's contention is that Example 3 shares features with Example 5 in the sense that $H_{\text{iv}} \equiv H_{\text{v}}$ is inadmissible evidence so that the Principal Principle does not hold. To unpack this claim, note that the problem with DILATION in Example 3 is that on the surface we consider $H_{\text{iv}} \equiv H_{\text{v}}$ to be admissible so that Lewis' Principal Principle holds: (*) $H_{\text{iv}} \equiv H_{\text{v}}$ does not give anything away about H_{iv}, therefore (**) $c(H_{\text{iv}}|H_{\text{iv}} \equiv H_{\text{v}}) = c(H_{\text{iv}})$ by the Principal Principle and in contradiction to (3). The Principal Principle requires that my knowledge of objective chances is reflected in my credence, unless there is inadmissible evidence (such as knowing the outcome of a coin toss, in which case of course I do not need to have a credence for it corresponding to the bias of the coin).

Joyce attacks (*), but he cannot do so unless he makes concession (AC1). For $H_{\text{iv}} \equiv H_{\text{v}}$ is information that changes the doxastic state without changing the credence, just as in Example 5. As such $H_{\text{iv}} \equiv H_{\text{v}}$ is inadmissible information, and the argument for (**) fails. On this point, I agree with Joyce: given (AC1), $H_{\text{iv}} \equiv H_{\text{v}}$ is inadmissible and DILATION ceases to be a problem for the Boolean position. (AC1), however, undermines INCOMPLETE, an important argument which Joyce has used to reject the Laplacean position. If there is a lot more to a doxastic state than its reflection in a credal state, both for instates and for sharp credences, then the failure of sharp credences to report on incompleteness or ambiguity of the evidence is no longer a major obstacle for Laplaceans.

2.2 Augustin's Concession (AC2)

(AC2) says that instates do not reflect knowledge claims about objective chances. White's *Chance Grounding Thesis* is not an appropriate characterization of the Boolean position.

> **Chance Grounding Thesis:** Only on the basis of known chances can one legitimately have sharp credences. Otherwise one's spread of credence should cover the range of possible chance hypotheses left open by your evidence. (White, 2010, 174)

Joyce considers (AC2) to be as necessary for a coherent Boolean view of partial beliefs, blocking OBTUSE, as (AC1) is, blocking DILATION (see Joyce, 2010, 289f).

OBTUSE is related to VACUITY, another problem for Booleans:

- VACUITY If one were to be committed to the principle of regularity, that all states of the world considered possible have positive probability (for a defence see Savage et al., 1963); and to the solution of Henry Kyburg's lottery paradox, that what is rationally accepted should have probability 1 (for a defence of this principle see Douven and Williamson, 2006); and the CGT, that one's spread of credence should cover the range of possible chance hypotheses left open by the evidence (implied by much of Boolean literature); then one's instate would always be vacuous.

Booleans must deny at least one of the premises to avoid the conclusion. Joyce denies the CGT, giving us (AC2).

3 The Double Task

Sharp credences have one task: to represent epistemic uncertainty and serve as a tool for updating, inference, and decision making. They cannot fulfill this task without continued reference to the evidence which operates in the background. To use an analogy, credences are not sufficient statistics with respect to updating, inference, and decision making. What is remarkable about Joyce's response to DILATION and OBTUSE is that Joyce recognizes that instates are not sufficient statistics either. But this means that they fail at the double task which has been imposed on them: to represent both epistemic uncertainty and relevant features of the evidence.

In the following, I will provide a few examples where it becomes clear that instates have difficulty representing uncertainty because they are tangled in a double task which they cannot fulfill.

Example 6: Aggregating Expert Opinion. Bob has no information whether it will rain tomorrow (R) or not except the predictions of two weather forecasters. One of them forecasts 0.3 on channel GPY, the other 0.6 on channel QCT. Bob considers the QCT forecaster to be significantly more reliable, based on past experience.

An instate corresponding to this situation may be $[0.3, 0.6]$ (see Walley, 1991, 214), but it will have a difficult time representing the difference in reliability of the experts. We could try $[0.2, 0.8]$ (since the greater reliability of QCT suggests that the chance of rain tomorrow is higher rather than lower) or $[0.1, 0.7]$ (since the greater reliability of QCT suggests that its estimate is more precise), but it remains obscure what the criteria might be.

A sharp credence of $P(R) = 0.53$, for example, does the right thing. Such a credence says nothing about any beliefs that the objective chance is restricted to a subset of the unit interval, but it accurately reflects the degree of uncertainty that the rational agent has over the various possibilities. As we will see in the next example, it is an advantage of sharp credences that they do not exclude objective chances, even extreme ones, because they are fully committed to partial belief and do not suggest, as indeterminate credences do, that there is full belief knowledge that the objective chance is a member of a proper subset of the possibilities.

> **Example 7: Precise Credences.** Larry's credence for rain tomorrow, based on the expert opinion of channel GPY and channel QCT (he has no other information) is 0.53. Is it reasonable for Larry, considering how little evidence he has, to reject the belief that the chance of rain tomorrow is 0.52 or 0.54; or to prefer a 52.9 cent bet on rain to a 47.1 cent bet on no rain?

The first question in Example 7 is confused, but in instructive ways. A sharp credence rejects no hypothesis about objective chances (unlike an instate, unless (AC2) is firmly in place). It constrains partial beliefs in objective chances by Lewis' summation formula. No objective chance is excluded by it (principle of regularity) and any updating will merely change the partial beliefs, but no full beliefs. Instates, on the other hand, by giving ranges of acceptable objective chances suggest that there is a full belief that the objective chance does not lie outside what is indicated by the instate. A Boolean can avoid this situation by accepting (AC2).

The second question in Example 7 is also instructive: why would we prefer a 52.9 cent bet on rain to a 47.1 cent bet on no rain, given that we do not possess the power of descrimination between these two bets? The answer to this question ties in with the issue of incomplete preference structure referred to above as motiviation (B) for instates.

The development of representation theorems beginning with Frank Ramsey (followed by increasingly more compelling representation theorems in Savage, 1954; and Jeffrey, 1965; and numerous other variants in contemporary literature) bases probability and utility functions of an agent on her preferences, not the other way around. Once completeness as an axiom for the preferences of an agent is jettisoned, indeterminacy follows automatically. Indeterminacy may thus be a natural consequence of the proper way to think about credences in terms of the preferences that they represent.

In response, preferences may very well logically and psychologically precede an agent's probability and utility functions, but that does not mean that we cannot inform the axioms we use for a rational agent's preferences by undesirable consequences downstream. Completeness may sound like an unreasonable imposition at the outset, but if incompleteness has unwelcome consequences for credences downstream, it is not illegitimate to revisit the issue. Timothy Williamson goes through this exercise with vague concepts, showing that all upstream logical solutions to the problem fail and that it has to be solved downstream with an epistemic solution (see Williamson, 1996). Vague concepts, like sharp credences, are sharply bounded, but not in a way that is luminous to the agent (for anti-luminosity see chapter 4 in Williamson, 2000). Anti-luminosity answers the original question: the rational agent prefers the 52.9 cent bet on rain to a 47.1 cent bet on no rain based on her sharp credence without being in a position to have this preference necessarily or have it based on physical or psychological ability (for the analogous claim about knowledge see Williamson, 2000, 95).

> **Example 8: Monkey-Filled Urns.** Let urn A contain 4 balls, two red and two black. A monkey randomly fills urn B from urn A with two balls. We

draw from urn B (a precursor to this example is in Jaynes and Bretthorst, 2003, 160).

The sharp credence of drawing a red ball is 0.5, following Lewis' summation formula for the different combinations of balls in urn B. This solution is more intuitive in terms of further inference, decision making, and betting behaviour than a credal state of $\{0, 1/2, 1\}$ or $[0, 1]$ (depending on the convexity requirement), since this instate would licence an exorbitant bet in favour of one colour, for example one that costs \$9,999 and pays \$10,000 if red is drawn and nothing if black is drawn.

To conclude, a Boolean in the light of Joyce's two Augustinian concessions has three alternatives, of which I favour the third: (a) to find fault with Joyce's reasoning as he makes those concessions; (b) to think (as Joyce presumably does) that the concessions are compatible with the promises of Booleans, such as RANGE and INCOMPLETE, to solve prima facie problems of sharp credences; or (c) to abandon the Boolean position because (AC1), (AC2), and an array of examples in which sharp credences are conceptually and pragmatically more appealing show that the initial promise of the Boolean position is not fulfilled.

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