Review for manuscript (PHIL-D-16-00054):

Augustin's Concessions: A Problem for Indeterminate Credal States

Summary:

The main hypothesis of the paper under review is that rational agents (e.g. decision makers) should always hold sharp credences about their beliefs, i.e. should always base their description of the uncertainty underlying a (decision) situation on one *single* probability distribution. As a consequence, the paper strongly disagrees with, and argues against, the approach followed by 'Booleans', relying on the theory of *imprecise probabilities* ('instates'). After giving an example-based overview about the use of imprecise probabilities, the author points out that there is no unanimously accepted measure for the informativeness of an instate that is comparable to Shannon's entropy in the case of classical probability theory. In the second part, the author develops the distinction between two mutually incompatible camps of Booleans (*Bool-A*, and *Bool-B*, the later relying on what is called Augustin's concessions here.) Discussing them a *divide et impera* argument is developed that favors the use of sharp credences ('Laplacean position'). This position advocates a precise uncertainty description that is computed as the expectation with respect to some second order probability according to the summation formula of Lewis.

Overall Evaluation

Fundamentally questioning and challenging the appropriateness of the theory of imprecise probabilities for describing the doxastic state of a rational agent, the paper makes a substantial contribution to a current topic of general interest.
Although we do not fully agree to all the conclusions drawn in the paper, we strongly welcome the paper. It is one of the very few contributions that try to built up a sound and detailed argumentation defending precise over imprecise probabilities. It enriches an urgently needed discussion, overcoming naive pragmatism and dogmatism on both sides (the dogma of ideal precision, mainly present in statistics, as well an recently appearing unreflected dogma of imprecision.)
Nevertheless, there are quite some issues that (at least in parts) remain unclear to us, and urgently need clarification to improve the argumentation.

- The distinction between the camps *Bool-A* and *Bool-B* is described on a rather vague level and doesn't become perfectly clear: On page 3, paragraph 2 the two camps are said to be mutually incompatible, whereas in the next paragraph B is said to be a refinement of A. Since this distinction is essential for the understanding of the main argument, a clear definition would be helpful.

- By the way, it would also be helpful for the reader to briefly explain why the imprecisionists are called Booleans. Do you refer to Boole as one of the first authors using set of probabilities? If so, does this really fit? If we read Boole correctly, he is not motivated by ideas of indeterminacy or imprecision as fundamental concepts, but uses set of probabilities just to describe situations where the assumed true probability can not be described exactly.

- In Chapter 2, the author recalls different axiomatic approaches for measuring the informativeness of instates, similar to Shannon's entropy value in the case of a classical probability distribution. In this context, (s)he recalls different candidates for concrete measures from literature and lists certain disadvantages of those. More principally, it is even conjectured that a measure satisfying all desiderata is impossible to exist.

  The discussion is quite interesting, but it is rather isolated from the rest of the paper and should be better connected to it. It may also be worth considering to make a section of its own from this.

  It would also be interesting to discuss approaches that take the multidimensional of uncertainty addressed by instates into account, by constructing a so-to-say multivariate version of an information measure. (See, for instance, the work by Abellan and co-authors on this topic.)

- In some of the hands-on examples (particularly Examples 3 and 10) and the follow-up discussions it remains unclear why a Boolean agent necessarily has to reject a sharp credence. One has a situation of ideal stochasticity with completely known chances. Most probably, an imprecionist would just adopt the principle of direct inference to transfer objective chances into epistemic probabilities (cf., e.g., Walley, 1991, Section 7.2.4).

  We think it is important to be more precise here. Moreover, again, the difference between Bool-A and Bool-B is not quite clear in this respect.

- In equation (5) the author refers to the Lewis' summation formula, which is used to argue that also sharp credences can reflect the ambiguity underlying a situation if they are computed as the expected values with respect to some (subjective) second order probability. This may indeed be possibly extended to a convincing argument in favour of the Laplacean position. However, mindful of its importance for the whole paper, in the form it is currently presented it seems unmotivated and it is not fully understandable.

- Example 8 in Chapter 5 is adapted from an article by Joyce. The author criticizes (with good reason, in our eyes) that the example is incorrect. Although correct, it is not at all clear why this should be of any relevance for the paper. Thus, the author should either drop the example or give some motivation why it is relevant here.

- In the end, the often-announced *divide et impera* argumentation could not fully convince us: Throughout the whole paper, the authors demonstrate (on basis of toy examples) what problems appear when using instates for representing doxastic states, namely dilation, learning,... In contrast, very few effort is made for arguing why the Laplacean position is superior in this aspects.
  A similar problem arises in the last chapter: It is argued that instates wouldn't be able to address the double task of modelling epistemic uncertainty and amount of available information simultaneously. Although this might be a fair point, what makes the argument for sharp credences: Not trying it at all?

- In this context in addition note that the mentioned problems with respect to learning are not necessarily general problems of instates, but of instates relying on the *Generalized Bayes Rule* (GBR). Also among Booleans the point-wise update of the credal set arising from GBR is not at all uncontroversial, and there are alternative update rules that promise to avoid the problems discussed in this paper (see, for instance, the discussion at the end of the Generalized Bayesian Inference section in the recent *Introduction to Imprecise Probabilities* (Wiley 2014, edited by Augustin, Coolen, de Cooman and Troffaes)). It may also be worth mentioning here that Walley (1991), who is generally understood as a strong supporter of the GBR, himself argued against focusing on the GBR as the only updating rule. He explicitly states that "[...] there is a role for other updating strategies, not because the updated beliefs constructed through the GBR are unjustified, but because they are often indeterminate." (Walley, 1991, p. 334), see also more generally the discussion in Walley (1991, Chapter 6.11), and, as one prototypical example, Held et al. (2008, International Journal of Approximate Reasoning), for suggesting an alternative updating rule.

Minor comments and notational issues:

In the following, we list up some minor comments that mainly concern notation and small typos:

- End of page 4: It would be much more naturally, if the relevant notation was introduced *before* citing the axioms (S1) to (S5). Also it simplifies the understanding if the notations within axioms (S1) to (S5) and axioms (S6) and (S7) would be unified (first the measure is called $\bar{S}$ and then U,...).

- Equations with undefined components are truly problematic to follow in detail. Please check in particular:
  Equation (3): Why does the measure CSU suddenly depend on two arguments (that's strange, since it is a candidate for U)? How is q^* defined? ((A bracket is missing after h_i.))
  Equation (5): Why is the event suddenly called R? How is P defined? Is it a second order probability?

- Page 15, end of the paragraph after example 9: Could you add some more words explaining why the given argumentation makes (AC1) unnecessary?

- It would be helpful to add the identity

  $$P(H_v|H_{iv} \equiv H_v) = \frac{P(H_{iv})P(H_v)}{P(H_{iv} \equiv H_v)}$$

  to equation (10), since this makes the conclusion of (9) from (7) and (8) more directly.

- Please give some more explanation on example 7: What exactly is the point with instates and learning? The paragraph ends pretty abruptly. Also some short words on what is meant by OBTUSE would be helpful.