Sharp Credences and Prevision Betting

Stefan Lukits

Abstract

TBA.

1 Introduction

I am adding a reason (D) that is poorly documented in the literature: The Boolean rational agent may systematically do better accepting bets than the agent who on principle rejects instates. Walley conducted an experiment in which Boolean participants did significantly better than Laplacean participants, betting on soccer games played in the Soccer World Cup 1982 in Spain (see Walley, 1991, Appendix I). I replicated the experiment using two computer players with rudimentary artificial intelligence and made them specify betting parameters (previsions) for games played in the Soccer World Cup 2014 in Brazil. I used the Poisson distribution (which is an excellent predictor for the outcome of soccer matches) and the FIFA ranking to simulate millions of counterfactual World Cup results and their associated bets, using Walley's evaluation method. The Boolean player had a slight but systematic advantage. In section TBA, I will provide an explanation and show how it undermines any support the experiment might give to the Boolean position.

2 Evidence Differentials and Cushioning Credences

I want to proceed to the intriguing issue of who does better in betting situations: instates or sharp credences. I have given away the answer already in the introduction: instates do better. It is surprising that, except for a rudimentary allusion to this in Walley's book, no Boolean has caught on to this yet. After I found out that agents with instates do better betting on soccer games, I let Betsy and Linda play a more basic betting game.

An n-sided die is rolled (by the computer). The die is fair, unbeknownst to the players. Their bets are randomly and uniformly drawn from the simplex for which the probabilities attributed to the n results add up to 1. Betsy also surrounds her credences with an imprecision uniformly drawn from the interval $(0, y)$. I used Walley's pay off scheme (see Walley, 1991, 632) to settle the bets.

Here is an example: let $n = 2$, so the die is a fair *coin*. Betsy's and Linda's bets are randomly and uniformly drawn from the line segment from $(0, 1)$ to $(1, 0)$ (these are two-dimensional Cartesian coordinates), the two-dimensional simplex (for higher n, the simplex is a pentatope generalized for n dimensions with side length $2^{1/2}$). The previsions (limits at which bets are accepted) may be $(0.21, 0.79)$ for Linda and $(0.35 \pm 0.11, 0.65 \pm 0.11)$ for Betsy, where the indeterminacy ± 0.11 is also randomly and uniformly drawn from the imprecision interval $(0, y) \subseteq (0, 1)$. The first bet is on H, and Linda is willing to pay 22.5 cents for it, while Betsy is willing to pay 77.5 cents against it. The second bet is on T (if $n > 2$, there will not be the same symmetry as in the *coin* case between the two bets), for which Betsy is willing to pay 77.5 cents, and against which Linda is willing to pay 22.5 cents. Each bet pays \$1 if successful. Often, Linda's credal state will overlap with Betsy's sharp credence so that there will not be a bet.

The computer simulation clearly shows that Linda does better than Betsy in the long run. A defence of sharp credences for rational agents needs to have an explanation for this. We will call it partial belief cushioning, which is based on an evidence differential between the bettors.

In many decision-making contexts, we do not have the luxury of calling off the bet. We have to decide one way or another. This is a problem for instates, as Booleans have to find a way to decide without receiving instructions from the credal state. Booleans have addressed this point extensively (see for example Joyce, 2010, 311ff; for an opponent's view of this see Elga, 2010, 6ff). The problem for sharp credences arises when bets are noncompulsory, for then the data above suggest that agents holding instates systematically do better. Often, decision making happens as betting vis-à-vis uninformed nature or opponents which are at least as uninformed as the rational agent. Sometimes, however, bets are offered by better informed or potentially better informed bookies. In this case, even an agent with sharp credences must cushion her credences and is better off by rejecting bets that look attractive in terms of her partial beliefs.

If an agent does not cushion her partial beliefs (whether they are sharp or indeterminate), she will incur a loss in the long run. Since cushioning is permitted in Walley's experimental setup (the bets are noncompulsory), Laplacean agents should also have access to it and then no longer do worse than Boolean agents. One may ask what sharp credences do if they just end up being cushioned anyway and do not provide sufficient information to decide on rational bets. The answer is that sharp credences are sufficient where betting (or decision making more generally) is compulsory; the cushioning only supplies the information from the evidence inasmuch as betting is noncompulsory and so again properly distinguishes semantic categories. This task is much harder for Booleans, although I do not claim that it is insurmountable: instates can provide a coherent approach to compulsory betting. What they cannot do, once cushioning is introduced, is outperform sharp credences in noncompulsory betting situations.

Here are a few examples: even if I have little evidence on which to base my opinion, someone may force me to either buy Coca Cola shares or short them, and so I have to have a share price p in mind that I consider fair. I will buy Coca Cola shares for less than p, and short them for more than p, if forced to do one or the other. This does not mean that it is now reasonable for me to go (not forced by anyone) and buy Coca Cola shares for p. It may not even be reasonable to go (not forced by anyone) and buy Coca Cola share for $p - \delta$ with $\delta > 0$.

It may in fact be quite unreasonable, since there are many players who have much better evidence than I do and will exploit my ignorance. I suspect that most lay investors in the stock market make this mistake: even though they buy and sell stock at prices that seem reasonable to them, professional investors are much better and faster at exploiting arbitrage opportunities and more subtle regularities. If indices rise, lay investors will make a little less than their professional counterparts; and when they fall, lay investors lose a lot more. In sum, unless there is sustained growth and everybody wins, lay investors lose in the long term.

A case in point is the U.S. Commodity Futures Trading Commission's crackdown on the online prediction market Intrade. Intrade offered fair bets for or against events of public significance, such as election results or other events which had clear yes-or-no outcomes. Even though the bets were all fair and Intrade only received a small commission on all bets, and even though Intrade's predictions were remarkably accurate, the potential for professional arbitrageurs was too great and the CFTC shut Intrade down (see `https://www.intrade.com`).

Cushioning does not stand in the way of holding a sharp credence, even if the evidence is dim. The evidence determines for a rational agent the partial beliefs over possible states of the world operating in the background. The better the evidence, the more pointed the distributions of these partial beliefs will be and the more willing the rational agent will be to enter a bet, if betting is noncompulsory. The mathematical decision rule will be based on the underlying distribution of the partial beliefs, not only on the sharp credence. As we have stated before, a sharp credence is not a sufficient statistic for decision making, inference, or betting behaviour; and neither is an instate.

The rational agent with a sharp credence has resources at her disposal to use just as much differentiation with respect to accepting and rejecting bets as the agent with instates. Often (if she is able to and especially if the bets are offered to her by a better-informed agent), she will reject both of two complementary bets, even when they are fair. On the one hand, any advantage that the agent with an instate has over her can be counteracted based on her distribution over partial beliefs that she has with respect to all possibilities. On the other hand, the agent with instates suffers under both conceptual and practical problems that put her at a real disadvantage in terms of understanding the sources and consequences of her knowledge and her uncertainties.

Elga, Adam. "Subjective Probabilities Should Be Sharp." *Philosophers' Imprints* 10, 5: (2010) 1–11.

Joyce, James. "A Defense of Imprecise Credences in Inference and Decision Making." *Philosophical Perspectives* 24, 1: (2010) 281–323.

Walley, Peter. *Statistical Reasoning with Imprecise Probabilities.* London, UK: Chapman and Hall, 1991.