Augustin's Concessions: A Problem for Indeterminate Credal States

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1 Introduction

The claim is that rational agents are subject to a norm requiring sharp credences. I defend this claim in spite of the initially promising features of indeterminate credal states (from now on instates) to address problems which sharp credences have as they reflect the evidence at the foundation of a doxastic state. Traditionally, Bayesians have maintained that a rational agent, when holding a credence, holds a sharp credence. It has recently become popular to drop the requirement for credence functions to be sharp. There are now Bayesians who permit a rational agent to hold instates based on incomplete or ambiguous evidence. I will refer to Bayesians who continue to adhere to the classical theory of sharp credences for rational agents as 'Laplaceans' (e.g. Adam Elga and Roger White). I will refer to Bayesians who do not believe that a rational agent's credences are sharp as 'Booleans' (e.g. Peter Walley and James Joyce).

There is some terminological confusion around the adjectives 'imprecise,' 'indeterminate,' and 'mushy' credences. In the following, I will exclusively refer to indeterminate credences or credal states (abbreviated 'instates') and mean by them a set of sharp credence functions (which some Booleans require to be convex) which it may be rational for an agent to hold within an otherwise orthodox Bayesian framework.

There is a sense in which, by linking knowledge of chances to its representation in credences, Booleans seek to reconcile traditional knowledge epistemology concerned with full belief and formal epistemology concerned with partial belief. There are other more recent reconciliation projects (see Spohn, 2012; and Moss, 2013). If my paper is correct then the Boolean approach will not contribute to this reconciliation because it mixes full belief and partial belief metaphors in ways that are problematic.

Instates represent in one credal state both degree of belief and properties of the evidence. Instates thus incorporate distinct features of the doxastic state. Representing multiple features of a state is not per se a bad thing when we fix the terms of a theory, in this case the terms of our theory of partial beliefs. In colour theory, the term ‘red’ represents both a phenomenological quality and a neighbourhood on the visible light spectrum. When we say one thing is more red than another thing, we effectively describe a relation based on both phenomenological and physical properties.

My examples and conceptual arguments show that instates fail to give us a superior terminology in our theory of partial beliefs, compared to sharp credences (much like scientific mineralogy prefers a terminology including ‘jadeite’ and ‘nephrite’ to one that lumps them together under the single concept of ‘jade.’). It turns out that in a credal state we do not want to represent everything that is relevant in a doxastic state to updating, inference, or decision making. Contrary to how it appears at first, it is the sharp credence which allows for greater latitude as to which possibilities are still being considered and assigned positive probability, compared to the instate. The main argument, however, adds to the conceptual comparison that what is most compelling about instates must be given up by Booleans as concessions to hands-on problems facing instates.

When we first hear of the advantages of instates, two of them sound particularly persuasive.

- RANGE Instates represent the possibility range for objective chances.
- INCOMPLETE Instates represent incompleteness or ambiguity of the evidence.

Let a $coin_{\mathrm{x}}$ be a Bernoulli generator that produces successes and failures with probability p_{x} for success, labeled H_{x}, and $1 - p_{\mathrm{x}}$ for failure, labeled T_{x}. Physical coins may serve as examples, if we are willing to set aside that most of them are approximately fair.

> **Example 1: Range.** Bob has two Bernoulli Generators in his lab, $coin_{\mathrm{i}}$ and $coin_{\mathrm{ii}}$. Bob has a database of $coin_{\mathrm{i}}$ results and concludes on excellent evidence that $coin_{\mathrm{i}}$ is fair. Bob has no evidence about the bias of $coin_{\mathrm{ii}}$. As a Boolean, Bob assumes a sharp credence of $\{0.5\}$ for H_{i} and an indeterminate

credal state of $[0,1]$ for H_{ii}. He feels bad for Larry, his Laplacean colleague, who cannot distinguish between the two cases and who must assign a sharp credence of $\{0.5\}$ for both H_{i} and H_{ii}.

Example 2: Incomp. Bob has another Bernoulli Generator, $coin_{\text{iii}}$, in his lab. His graduate student has submitted $coin_{\text{iii}}$ to countless experiments and emails Bob the resulting bias, but fails to include whether the bias of $2/3$ is in favour of H_{iii} or in favour of T_{iii}. As a Boolean, Bob assumes an indeterminate credal state of $[1/3, 2/3]$ (or $\{1/3, 2/3\}$, depending on whether convexity is required) for H_{iii}. He feels bad for Larry who must assign a sharp credence of $\{0.5\}$ for H_{iii} when Larry concurrently knows that his credence gets the bias wrong.

Against the force of RANGE and INCOMPLETE, I maintain that the Laplacean approach of assigning subjective probabilities to partitions of the event space (e.g. objective chances) and then aggregating them by David Lewis' summation formula into a single precise credence function is conceptually tidy and shares many of the formal virtues of Boolean theories. If the bad taste about numerical precision in a fuzzy and nebulous world lingers, I will point to philosophical projects in other domains where the concepts we use are sharply bounded, even though our ability to conceive of those sharp boundaries or know them is limited (in particular Timothy Williamson's accounts of vagueness and knowledge).

To put it provocatively, this paper defends a 0.5 sharp credence in heads in all three cases: for a coin of whose bias we are completely ignorant; for a coin whose fairness is supported by a lot of evidence; and even for a coin about whose bias we know that it is either 1/3 or 2/3 for heads. A few concise statements by proponents of indeterminacy (not all of whom are Booleans since they are not necessarily Bayesians) clarifies how their position is motivated:

- A refusal to make a determinate probability judgment does not derive from a lack of clarity about one's credal state. To the contrary, it may derive from a very clear and cool judgment that on the basis of the available evidence, making a numerically determinate judgment would be unwarranted and arbitrary. (Levi, 1985, 395.)
- If there is little evidence concerning [a claim,] then beliefs about [that claim] should be indeterminate, and probability models imprecise, to

reflect the lack of information. We regard this as the most important source of imprecision. (Walley, 1991, 212–213.)

- Imprecise probabilities and related concepts [...] provide a powerful language which is able to reflect the partial nature of the knowledge suitably and to express the amount of ambiguity adequately. (Augustin, 2003, 34.)
- As sophisticated Bayesians like Isaac Levi (1980), Richard Jeffrey (1983), Mark Kaplan (1996), have long recognized, the proper response to symmetrically ambiguous or incomplete evidence is not to assign probabilities symmetrically, but to refrain from assigning precise probabilities at all. Indefiniteness in the evidence is reflected not in the values of any single credence function, but in the spread of values across the family of all credence functions that the evidence does not exclude. This is why modern Bayesians represent credal states using sets of credence functions. It is not just that sharp degrees of belief are psychologically unrealistic (though they are). Imprecise credences have a clear epistemological motivation: they are the proper response to unspecific evidence. (Joyce, 2005, 170f.)

Consider therefore the following reasons that incline Booleans to permit instates for rational agents:

(A) The greatest emphasis motivating indeterminacy rests on RANGE and INCOMPLETE.

(B) The preference structure of a rational agent may be incomplete so that representation theorems do not yield single probability measures to represent such incomplete structures.

(C) There are more technical and paper-specific reasons, such as Thomas Augustin's attempt to mediate between the minimax pessimism of objectivists and the Bayesian optimism of subjectivists using interval probability (see Augustin, 2003, 35f); Alan Hájek and Michael Smithson's belief that there may be objectively indeterminate chances in the physical world (see Hájek and Smithson, 2012, 33); and Jake Chandler's claim that "the sharp model is at odds with a trio of plausible propositions regarding agnosticism" (Chandler, 2014, 4).

This paper mostly addresses (A), while taking (B) seriously as well and pointing towards solutions for it. I am leaving (C) to more specific responses to the issues presented in the cited articles. I am adding a reason (D) that is poorly documented in the literature: The Boolean rational agent may systematically do better accepting bets than the agent who on principle rejects instates. Walley conducted an experiment in which Boolean participants did significantly better than Laplacean participants, betting on soccer games played in the Soccer World Cup 1982 in Spain (see Walley, 1991, Appendix I). I replicated the experiment using two computer players with rudimentary artificial intelligence and made them specify betting parameters (previsions) for games played in the Soccer World Cup 2014 in Brazil. I used the Poisson distribution (which is an excellent predictor for the outcome of soccer matches) and the FIFA ranking to simulate millions of counterfactual World Cup results and their associated bets, using Walley's evaluation method. The Boolean player had a slight but systematic advantage. In section 4, I will provide an explanation and show how it undermines any support the experiment might give to the Boolean position.

There are four sections to come.[TBD]

2 Augustin's Concessions

Here are two potential problems for Booleans:

- DILATION Instates are vulnerable to dilation.
- OBTUSE Instates do not permit learning.

Again, these are best explained by examples. First, here is an example for DILATION (see White, 2010, 175f and Joyce, 2010, 296f).

> **Example 3: Dilation.** Larry has two Bernoulli Generators, $coin_{\text{iv}}$ and $coin_{\text{v}}$. He has excellent evidence that $coin_{\text{iv}}$ is fair and no evidence about the bias of $coin_{\text{v}}$. Larry's graduate student independently tosses both $coin_{\text{iv}}$ and $coin_{\text{v}}$. Then she tells Larry whether the two tosses are correlated or not ($H_{\text{iv}} \equiv H_{\text{v}}$ or $H_{\text{iv}} \equiv T_{\text{v}}$, where $X \equiv Y$ means $(X \wedge Y) \vee (\neg X \wedge \neg Y)$). Larry, who has a sharp credence for H_{v}, takes this information in stride, but he feels bad for Bob, whose credence in H_{iv} dilates to [0.1] even though Bob shares Larry's excellent evidence that $coin_{\text{iv}}$ is fair.

Here is why Bob's credence in H_{iv} must dilate. His credence in H_{v} is $[0,1]$, by stipulation. Let $c(X)$ be the set of sharp credences representing Bob's instate, for example $c(H_{\mathrm{v}}) = [0,1]$. Then

$$c(H_{\mathrm{iv}} \equiv H_{\mathrm{v}}) = c(H_{\mathrm{iv}} \equiv T_{\mathrm{v}}) = \{0.5\} \tag{1}$$

because the tosses are independent and $c(H_{\mathrm{iv}}) = \{0.5\}$ by stipulation. Next,

$$c(H_{\mathrm{iv}}|H_{\mathrm{iv}} \equiv H_{\mathrm{v}}) = c(H_{\mathrm{v}}|H_{\mathrm{iv}} \equiv H_{\mathrm{v}}) \tag{2}$$

where $c(X|Y)$ is the updated instate after finding out Y. Booleans accept (2) because they are Bayesians and update by standard conditioning. Therefore,

$$\begin{aligned} &c(H_{\mathrm{iv}}|H_{\mathrm{iv}} \equiv H_{\mathrm{v}}) = c(H_{\mathrm{v}}|H_{\mathrm{iv}} \equiv H_{\mathrm{v}}) = \frac{c(H_{\mathrm{iv}})c(H_{\mathrm{v}})}{c(H_{\mathrm{iv}})c(H_{\mathrm{v}}) + c(T_{\mathrm{iv}})c(T_{\mathrm{v}})} \\ &= c(H_{\mathrm{v}}) = [0,1]. \end{aligned} \tag{3}$$

Bob's updated instate for H_{iv} has dilated from $\{0.5\}$ to $[0,1]$.

This does not sound like a knock-down argument against Booleans (it is investigated in detail in Seidenfeld and Wasserman, 1993), but Roger White uses it to derive implications from instates which are worrisome.

> **Example 4: Chocolates.** Four out of five chocolates in the box have cherry fillings, while the rest have caramel. Picking one at random, what should my credence be that it is cherry-filled? Everyone, including the staunchest [Booleans], seems to agree on the answer 4/5. Now of course the chocolate I've chosen has many other features, for example this one is circular with a swirl on top. Noticing such features could hardly make a difference to my reasonable credence that it is cherry filled (unless of course I have some information regarding the relation between chocolate shapes and fillings). Often chocolate fillings do correlate with their shapes, but I haven't the faintest clue how they do in this case or any reason to suppose they correlate one way rather than another ... the further result is that while my credence that the chosen chocolate is cherry-filled should be 4/5 prior to viewing it,

> once I see its shape (whatever shape it happens to be) my credence that it is cherry-filled should dilate to become [indeterminate]. But this is just not the way we think about such matters. (Quoted verbatim from White, 2010, 183.)

Second, here is an example for VACUITY (see Susanna Rinard's objection cited in White, 2010, 84 and addressed in Joyce, 2010, 290f). It presumes Joyce's supervaluationist semantics of instates (see Joyce, 2010, 288 and Rinard, 2015), for which Joyce uses the helpful metaphor of committee members, each of whom holds a sharp credence. The instate consists then of the set of sharp credences from each committee member: for the purposes of updating, for example, each committee member updates as if she were holding a sharp credence. The aggregate of the committee members' updated sharp credences forms the updated instate. Supervaluationist semantics also permits comparisons, when for example a partial belief in X is stronger than a partial belief in Y because all committee members have sharp credences in X which exceed all the sharp credences held by committee members with respect to Y.

> **Example 5: learning.** Bob has a Bernoulli Generator in his lab, $coin_{\text{vi}}$, of whose bias he knows nothing and which he submits to experiments. At first, Bob's instate for H_{vi} is [0.1]. After a few experiments, it looks like $coin_{\text{vi}}$ is fair. However, as committee members crowd into the centre and update their sharp credences to something closer to 0.5, they are replaced by extremists on the fringes. The instate remains at $[0, 1]$.

Joyce, an authoritative Boolean voice, has defended instates against DILATION and OBTUSE, making Augustin's concessions (AC1) and (AC2). I named them after Thomas Augustin, who has some priority over Joyce in the matter, and the name James Joyce does not lend itself to literary double entendre.

(AC1) Credences do not adequately represent a doxastic state. The same instate can reflect different doxastic states.

(AC2) Instates do not represent knowledge claims about objective chances. White's *Chance Grounding Thesis* is not an appropriate characterization of the Boolean position.

I agree with Joyce that (AC1) and (AC2) are both necessary and sufficient to resolve DILATION and OBTUSE for instates. I will address this in more detail in a moment. I disagree with Joyce what this means for an overall recommendation to accept the Boolean rather than the Laplacean position. I will show that (AC1) and (AC2) neutralize both RANGE and INCOMPLETE, the two major impulses for rejecting the Laplacean position. Laplaceans, more modestly, consider a credence to reflect the doxastic state while filtering out some evidential features, which need to be independently represented. Overall, a sharp credence reflects the doxastic state and does not represent it (cashing out the difference in the sense that a credence which reflects the doxastic state is not sufficient for inference, updating, and decision making; whereas a credence which represents the doxastic state would be sufficient). If instates could successfully integrate all relevant features of the doxastic state, they would represent it. (AC1) and (AC2) manifest that they cannot.

Indeterminacy imposes a double task on credences (representing both uncertainty and available evidence) that they cannot coherently fulfill. I will present several examples where this double task stretches instates to the limits of plausibility. Joyce's idea that credences can represent balance, weight, and specificity of the evidence (in Joyce, 2005) is inconsistent with the use of indeterminacy. Joyce himself, in response to DILATION and OBTUSE, gives the argument why this is the case (see Joyce, 2010, 290ff for OBTUSE and Joyce, 2010, 296ff for DILATION). Let us look at how (AC1) and (AC2) protect the Boolean position from DILATION and OBTUSE more closely.

2.1 Augustin's Concession (AC1)

(AC1) says that credences do not adequately represent a doxastic state. The same instate can reflect different doxastic states.

Augustin recognizes the problem of inadequate representation before Joyce, with specific reference to instates: "The imprecise posterior does no longer contain all the relevant information to produce optimal decisions. Inference and decision do not coincide any more" (Augustin, 2003, 41) (see also an example for inadequate representation of evidence by instates in Bradley and Steele, 2013, 16). Joyce rejects the notion that identical instates encode identical beliefs by giving a simple example:

> **Example 6: Three-Sided Die.** Let $\mathcal{C}'$ and $\mathcal{C}''$ be sets of credence functions defined on a partition $\{X, Y, Z\}$ corresponding to the result of a roll of a

three sided-die. $\mathcal{C}'$ contains all credence functions c for which $c(Z) \geq 1/2$. $\mathcal{C}''$ contains all credence functions c for which $c(X) = c(Y)$ (see Joyce, 2010, 294).

$\mathcal{C}'$ and $\mathcal{C}''$ represent the same instates, but they surely differ in the doxastic states that they encode. The doxastic state corresponding to $\mathcal{C}'$ regards X and Y as equiprobable, the doxastic state corresponding to $\mathcal{C}''$ does not. Joyce's contention is that Example 3 shares features with Example 6 in the sense that $H_{\text{iv}} \equiv H_{\text{v}}$ is inadmissible evidence so that the Principal Principle does not hold. To unpack this claim, note that the problem with DILATION in Example 3 is that on the surface we consider $H_{\text{iv}} \equiv H_{\text{v}}$ to be admissible so that Lewis' Principal Principle holds: (*) $H_{\text{iv}} \equiv H_{\text{v}}$ does not give anything away about H_{iv}, therefore (**) $c(H_{\text{iv}}|H_{\text{iv}} \equiv H_{\text{v}}) = c(H_{\text{iv}})$ by the Principal Principle and in contradiction to (3). The Principal Principle requires that my knowledge of objective chances is reflected in my credence, unless there is inadmissible evidence (such as knowing the outcome of a coin toss, in which case of course I do not need to have a credence for it corresponding to the bias of the coin).

Joyce attacks (*), but he cannot do so unless he makes concession (AC1). For $H_{\text{iv}} \equiv H_{\text{v}}$ is information that changes the doxastic state without changing the credence, just as in Example 6. As such $H_{\text{iv}} \equiv H_{\text{v}}$ is inadmissible information, and the argument for (B) fails. On this point, I agree with Joyce: given (AC1), $H_{\text{iv}} \equiv H_{\text{v}}$ is inadmissible and DILATION ceases to be a problem for the Boolean position. (AC1), however, undermines INCOMPLETE, an important argument which Joyce has used to reject the Laplacean position.

One of Joyce's complaints is that a sharp credence of 0.5 for a *coin* contains too much information if there is little or no evidence that the *coin* is fair (see, for example, Joyce, 2010, 284). This complaint, of course, is only effective if the indeterminacy of the credence is anticorrelated to the amount of information in the evidence. Walley's and Joyce's claim that instates are less informative than sharp credences (see Walley, 1991, 34; and Joyce, 2010, 311 for examples, but this attitude is passim) has no foundation in information theory. To compare instates and sharp credences informationally, we would need a non-additive set function obeying Shannon's axioms for information. This is a non-trivial task. I have not succeeded solving it (nor do I need to carry the Boolean's water), but I am not convinced that it will result in an information measure which assigns, for instance, more information to a sharp credence such as $\{0.5\}$ than to an instate such as $\{x|1/3 \leq x \leq 2/3\}$.

We would usually expect more information to sharpen our credal states (see Walley's "the more information the more precision" principle and his response to this problem in 1991, 207 and 299), an intuition violated by both DILATION and OBTUSE. As far as DILATION is concerned, however, the loss of precision is in principle not any more surprising than information that increases the Shannon entropy of a sharp credence.

> **Example 7: Rumour.** A rumour that the Canadian prime minister has been assassinated raises your initially very low probability that this event is taking place today to approximately 50%.

It is true for both sharp and indeterminate credences that information can make us less certain about things, and it is true for both sharp and indeterminate credences that they do not encode the doxastic state. It is therefore not surprising that Joyce has found an argument against DILATION. What is surprising is that he must admit (AC1), which undermines his general argument for the Boolean position and against the Laplacean position.

Here is how dilation is as unproblematic as a gain in entropy after more information in Example 7:

> **Example 8: Dilating Urns.** You draw one ball from an urn with 200 balls (100 red, 100 black) and receive the information that the urn actually had two chambers, one with 99 red balls and 1 black ball, the other with 1 red ball and 99 black balls.

Dilation from a sharp credence of $\{0.5\}$ to an instate of $[0.01, 0.99]$ (or $\{0.01, 0.99\}$, depending on whether convexity is required) is unproblematic, although the example prefigures that there is something odd about the Boolean conceptual approach. The example licences a 99:1 bet for one of the colours (if the instate is interpreted as upper and lower previsions), but this is a problem that arises out of the Boolean position quite apart from DILATION, which we will address see again in Example 12.

2.2 Augustin's Concession (AC2)

(AC2) says that instates do not reflect knowledge claims about objective chances. White's *Chance Grounding Thesis* is not an appropriate characterization of the Boolean position.

> **Chance Grounding Thesis:** Only on the basis of known chances can one legitimately have sharp credences. Otherwise one's spread of credence should cover the range of possible chance hypotheses left open by your evidence. (White, 2010, 174)

Joyce considers (AC2) to be as necessary for a coherent Boolean view of partial beliefs, blocking OBTUSE, as (AC1) is, blocking DILATION (see Joyce, 2010, 289f).

OBTUSE is related to VACUITY, another problem for Booleans:

- VACUITY If one were to be committed to the principle of regularity, that all states of the world considered possible have positive probability (for a defence see Savage et al., 1963); and to the solution of Henry Kyburg's lottery paradox, that what is rationally accepted should have probability 1 (for a defence of this principle see Douven and Williamson, 2006); and the CGT, that one's spread of credence should cover the range of possible chance hypotheses left open by the evidence (implied by much of Boolean literature); then one's instate would always be vacuous.

Booleans must deny at least one of the premises to avoid the conclusion. Joyce denies the CGT, giving us (AC2).

3 The Double Task

Sharp credences have one task: to represent epistemic uncertainty and serve as a tool for updating, inference, and decision making. They cannot fulfill this task without continued reference to the evidence which operates in the background. To use an analogy, credences are not sufficient statistics with respect to updating, inference, and decision making. What is remarkable about Joyce's response to DILATION and OBTUSE is that Joyce recognizes that instates are not sufficient statistics either. But this means that they fail at the double task which has been imposed on them: to represent both epistemic uncertainty and relevant features of the evidence.

In the following, I will provide a few examples where it becomes clear that instates have difficulty representing uncertainty because they are tangled in a double task which they cannot fulfill.

Example 9: Aggregating Expert Opinion. You have no information whether it will rain tomorrow (R) or not except the predictions of two weather forecasters. One of them forecasts 0.3 on channel GPY, the other 0.6 on channel QCT. You consider the QCT forecaster to be significantly more reliable, based on past experience.

An instate corresponding to this situation may be $[0.3, 0.6]$ (see Walley, 1991, 214), but it will have a difficult time representing the difference in reliability of the experts. We could try $[0.2, 0.8]$ (since the greater reliability of QCT suggests that the chance of rain tomorrow is higher rather than lower) or $[0.1, 0.7]$ (since the greater reliability of QCT suggests that its estimate is more precise), but it remains obscure what the criteria might be.

A sharp credence of $P(R) = 0.53$, for example, does the right thing. Such a credence says nothing about any beliefs that the objective chance is restricted to a subset of the unit interval, but it accurately reflects the degree of uncertainty that the rational agent has over the various possibilities. Beliefs about objective chances make little sense in many situations where we have credences, since it is doubtful even in the case of rain tomorrow that there is an urn of nature from which balls are drawn. What is really at play is a complex interaction between epistemic states (for example, experts evaluating meteorological data) and the evidence which influences them.

As we will see in the next example, it is an advantage of sharp credences that they do not exclude objective chances, even extreme ones, because they are fully committed to partial belief and do not suggest, as indeterminate credences do, that there is full belief knowledge that the objective chance is a member of a proper subset of the possibilities.

Example 10: Precise Credences. Your sharp credence for rain tomorrow, based on the expert opinion of channel GPY and channel QCT (you have no other information) is 0.53. Is it reasonable, considering how little evidence you have, to reject the belief that the chance of rain tomorrow is 0.52 or 0.54; or to prefer a 52.9 cent bet on rain to a 47.1 cent bet on no rain?

The first question in Example 10 is confused, but in instructive ways (a display of this confusion is found in Hájek and Smithson, 2012, 38f, and their doctor and their time of the day analogy). A sharp credence rejects no hypothesis about objective chances (unlike an instate, unless (AC2) is firmly in place). It often has a subjective probability distribution operating in the background, over which it integrates to yield the sharp credence (it would do likewise in Hájek and Smithson's example for the prognosis of the doctor or the time of the day, without any problems). This subjective probability distribution may look like this:

$P(\pi(R) = 0.00)$	$=$	0.0001
$P(\pi(R) = 0.01)$	$=$	0.0003
$P(\pi(R) = 0.02)$	$=$	0.0007
...		...
$P(\pi(R) = 0.30)$	$=$	0.0015
$P(\pi(R) = 0.31)$	$=$	0.0016
...		...
$P(\pi(R) = 0.52)$	$=$	0.031
$P(\pi(R) = 0.53)$	$=$	0.032
$P(\pi(R) = 0.54)$	$=$	0.030
...		...

It is condensed by Lewis' summation formula to a sharp credence, without being reduced to it:

$$C(R) = \int_0^1 \zeta P\left(\pi(R) = \zeta\right) \, d\zeta \tag{4}$$

Lewis' 1981 paper "A Subjectivist's Guide to Objective Chance" addresses the question what the relationship between π (objective chance), P (subjective probability), and C (credence) is. The point is that we have properly separated the conceptual dimensions and that the Laplacean approach is not a second order probability approach. Partial belief epistemology deals with sharp credences and how they represent uncertainty and serve as a tool in inference, updating, and decision making; while Lewis' Humean speculations and his interpretation of the Principal Principle cover the relationship between subjective probabilities and objective chance.

A sharp credence constrains partial beliefs in objective chances by Lewis' summation formula (which we will provide in the next section). No objective chance is excluded by it (principle of regularity) and any updating will merely change the partial beliefs, but no full beliefs. Instates, on the other hand, by giving ranges of acceptable objective chances suggest that there is a full belief that the objective chance does not lie outside what is indicated by the instate. A Boolean can avoid this situation by accepting (AC2).

Here is a brief example to illustrate the difference between the Laplacean theory of partial beliefs based on the principle of regularity and the Boolean position which introduces an obscure grey zone between partial beliefs and full belief, update and revision, traditional epistemology and formal epistemology.

> **Example 11: Bavarian King.** Matthias Perth, an Austrian civil servant, observes the Bavarian king at the Congress of Vienna in 1815 and writes in his diary that the king "appears to be a man between 45 and 47 years old" (see `http://www.das-perth-projekt.at`).

If Perth learns that the king was 49 years old, he must revise, not just update, his earlier judgment. The appropriate formal instrument is belief revision, not probability update, requiring a substantial reconciliation project between formal and traditional epistemology operating in the background. I do not see this project articulated in the Boolean literature (for an example of such a project see Spohn, 2012, especially chapter 10). Sarah Moss also undertakes it and assumes the Boolean approach (see Moss, 2013), but I fail to see how the Boolean approach is essential to her reconciliation or how her reconciliation gives independent arguments for the Boolean approach. If Perth had wanted to express a sharp credence, he would have said, "my best guess is that the king is 46 years old," and the information that the king was 49 would have triggered the appropriate update, without any revision of full beliefs.

The burden for the Boolean is to show what kind of coherence there is in defending indeterminacy when it neither fulfills the promise of adequately representing evidence nor the promise of reconciling traditional full belief 'knowledge' epistemology and Bayesian partial belief epistemology as outlined in the CGT, but only adds another hierarchical layer of uncertainty to a numerical quantity (a sharp credence) whose job it already is to represent uncertainty.

It is important not to confuse the claim that it is reasonable to hold both X and Y with the claim that it is reasonable to hold either X (without Y) or Y (without X). It is the reasonableness of holding X and Y concurrently that is controversial, not the reasonableness of holding Y (without holding X) when it is reasonable to hold X. It is a fallacy to think that $R(S,X,t)$ and $R(S,Y,t)$ imply $R(S,X \wedge Y,t)$, where $R(S,Z,t)$ means "it is rational for S to believe Z at time t." Beliefs somehow grounded in subjectivity (such as beliefs about etiquette or colour perception) serve as counter-examples.

In a moment, I will talk about anti-luminosity, the fact that a rational agent may not be able to distinguish psychologically between a 54.9 cent bet on an event and a 45.1 bet on its negation, when her sharp credence is 0.55. She must reject one of them not to incur sure loss, so proponents of indeterminacy suggest that she choose one of them freely without being constrained by her credal state or reject both of them. I claim that a sharp credence will make a recommendation between the two so that only one of the bets is rational given her particular credence, but that does not mean that another sharp credence which would give a different recommendation may not also be rational for her to have.

Instates, by contrast, mix features of a doxastic state so that in the end we get a muddle where a superficial reading of indeterminacy suddenly follows a converse Principal Principle of sorts, namely that objective chances are constrained by the factivity of a rational agent's credence when this credence is knowledge (Lewis actually talks about such a converse, but in completely different and epistemologically more intelligible terms, see Lewis, 1981, 289). Sharp credences are more, not less, permissive with respect to objective chances operating externally (compared to the internal belief state of the agent, which the credence reflects). By the principle of regularity and in keeping with statistical practice, all objective chances as possible states of the world are given positive subjective probabilities, even though they may be very small. Instates, on the other hand, mix partial belief epistemology with full belief epistemology and presumably exclude objective chances which lie outside the credal state from consideration because they are fully known not to hold (see Levi, 1981, 540, "inference derives credal probability from knowledge of the chances of possible outcomes").

The second question in Example 10 is also instructive: why would we prefer a 52.9 cent bet on rain to a 47.1 cent bet on no rain, given that we do not possess the power of descrimination between these two bets? The answer to this question ties in with the issue of incomplete preference structure referred to above as motiviation (B) for instates.

> It hardly seems a requirement of rationality that belief be precise (and preferences complete); surely imprecise belief (and corresponding incomplete preferences) are at least rationally permissible. (Bradley and Steele, 2013, 2.)

The development of representation theorems beginning with Frank Ramsey (followed by increasingly more compelling representation theorems in Savage, 1954; and Jeffrey, 1965; and numerous other variants in contemporary literature) puts the horse before the cart and bases probability and utility functions of an agent on her preferences, not the other way around. Once completeness as an axiom for the preferences of an agent is jettisoned, indeterminacy follows automatically. Indeterminacy may thus be a natural consequence of the proper way to think about credences in terms of the preferences that they represent.

In response, preferences may very well logically and psychologically precede an agent's probability and utility functions, but that does not mean that we cannot inform the axioms we use for a rational agent's preferences by undesirable consequences downstream. Completeness may sound like an unreasonable imposition at the outset, but if incompleteness has unwelcome consequences for credences downstream, it is not illegitimate to revisit the issue. Timothy Williamson goes through this exercise with vague concepts, showing that all upstream logical solutions to the problem fail and that it has to be solved downstream with an epistemic solution (see Williamson, 1996). Vague concepts, like sharp credences, are sharply bounded, but not in a way that is luminous to the agent (for anti-luminosity see chapter 4 in Williamson, 2000). Anti-luminosity answers the original question: the rational agent prefers the 52.9 cent bet on rain to a 47.1 cent bet on no rain based on her sharp credence without being in a position to have this preference necessarily or have it based on physical or psychological ability (for the analogous claim about knowledge see Williamson, 2000, 95).

In a way, advocates of indeterminacy have solved this problem for us. There is strong agreement among most of them that the issue of determinacy for credences is not an issue of elicitation (sometimes the term 'indeterminacy' is used instead of 'imprecision' to underline this difference; see Levi, 1985, 395). The appeal of preferences is that we can elicit them more easily than assessments of probability and utility functions. The indeterminacy issue has been raised to the probability level (or moved downstream) by indeterminacy advocates themselves who feel justifiably uncomfortable with an interpretation of their theory in behaviourist terms. So it shall be solved there, and this paper makes an appeal to reject indeterminacy on this level. The solution then has to be carried upstream (or lowered to the logically more basic level of preferences), where we recognize that completeness for preferences is after all a desirable axiom for rationality. Isaac Levi agrees with me on this point: when he talks about indeterminacy, it proceeds from the level of probability judgment to preferences, not the other way around (see Levi, 1981, 533).

> **Example 12: Monkey-Filled Urns.** Let urn A contain 4 balls, two red and two black. A monkey randomly fills urn B from urn A with two balls. We draw from urn B (a precursor to this example is in Jaynes and Bretthorst, 2003, 160).

The sharp credence of drawing a red ball is 0.5, following Lewis' summation formula for the different combinations of balls in urn B. This solution is more intuitive in terms of further inference, decision making, and betting behaviour than a credal state of $\{0, 1/2, 1\}$ or $[0, 1]$ (depending on the convexity requirement), since this instate would licence an exorbitant bet in favour of one colour, for example one that costs \$9,999 and pays \$10,000 if red is drawn and nothing if black is drawn.

To make this example more vivid consider a Hand Urn, where you draw by hand from an urn with 100 balls, 50 red balls and 50 black balls. When your hand retreats from the urn, does it not contain either a red ball or a black ball and so serve itself as an urn, from which in a sense you draw a ball? Your hand contains one ball, either red or black, and the indeterminate credal state that it is one or the other should be $[0, 1]$. This contradicts our intuition that our credence should be a sharp 0.5. As is the case in Example 4, instates appear to be highly contingent on a problem's mode of representation, more so than intuition allows.

> **Example 13: Three Prisoners.** Prisoner X_1 knows that two out of three prisoners (X_1, X_2, X_3) will be executed and one of them pardoned. He asks the warden of the prison to tell him the name of another prisoner who will be executed, hoping to gain knowledge about his own fate. When the warden tells him that X_3 will be executed, X_1 erroneously updates his probability of pardon from 1/3 to 1/2, since either X_1 or X_2 will be spared.

Walley maintains that for the Monty Hall problem and the Three Prisoners problem, the probabilities of a rational agent should dilate rather than settle on the commonly accepted solutions. For the Three Prisoners problem, there is a compelling case for standard conditioning and the result that the credence for prisoner X_1 to have been pardoned ought to be unchanged after the update (see Lukits, 2014, 1421f). Walley's dilated solution would give prisoner X_1 hope on the doubtful possibility (and unfounded assumption) that the warden might prefer to provide X_3's (rather than X_2's) name in case prisoner X_1 was pardoned.

This example brings an interesting issue to the forefront. Sharp credences often reflect independence of variables where such independence is unwarranted. Booleans (more specifically, detractors of the principle of indifference or the principle of maximum entropy, principles which are used to generate sharp credences for rational agents) tend to point this out gleefully. They prefer to dilate over the possible dependence relationships (independence included). DILATION is an instance of this. The fallacy in the argument for instates, illustrated by the Three Prisoners problem, is that the probabilistic independence of sharp credences does not imply independence of variables. Only the converse is correct.

In the Three Prisoners problem, there is no evidence about the degree or the direction of the dependence, and so prisoner X_1 should take no comfort in the information that she receives. The prisoner's probabilities will reflect probabilistic independence, but make no claims about causal independence. Walley has unkind things to say about sharp credences and their ability to respond to evidence (for example that their "inferences rarely conform to evidence", see Walley, 1991, 396), but in this case it appears to me that they outperform the Boolean approach.

> **Example 14: Wagner's Linguist.** A linguist hears the utterance of a native and concludes that the native cannot be part of certain population groups, depending on what the utterance means. The linguist is uncertain between some options about the meaning of the utterance. (For full details see Wagner, 1992, 252; and Spohn, 2012, 197.)

The mathematician Carl Wagner proposes a natural generalization of Jeffrey Conditioning for his Linguist example (see Wagner, 1992). Since the principle of maximum entropy is already a generalization of Jeffrey Conditioning, the question naturally arises whether the two generalizations agree. Wagner makes the case that they do not agree and deduces that the principle of maximum entropy is sometimes an inappropriate updating mechanism, in line with many earlier criticisms of the principle of maximum entropy (see van Fraassen, 1981; Shimony, 1985; Skyrms, 1987; and, later on, Grove and Halpern, 1997). What is interesting about this case is that Wagner uses instates for his deduction, so that even if you agree with his natural generalization of Jeffrey Conditioning (which I find plausible), the inconsistency with the principle of maximum entropy can only be inferred assuming instates. Wagner is unaware of this, and it can be shown that on the assumption of sharp credences Wagner's generalization of Jeffrey conditioning accords with the principle of maximum entropy (see Lukits, 2015).

This will not convince Booleans, since they are already unlikely to believe in the general applicability of the principle of maximum entropy (just as Wagner's argument is unlikely to convince a proponent of the principle of maximum entropy, since they have a tendency to reject instates). The battle lines are clearly drawn. Wagner's argument, instead of undermining the principle of maximum entropy, shows that instates are as wedded to rejecting the claims of the principle of maximum entropy as the principle of maximum entropy is wedded to sharp credences (these marriages are only unilaterally monogamous, however, as it is perfectly coherent to reject both the principle of maximum entropy and the Boolean position; or to reject both the Laplacean position and instates).

Endorsement of instates, however, implies that there are situations of probability update in which the posterior probability distribution is more informative than it might be in terms of information theory. Indeterminate credences violate the relatively natural intuition that we should not gain information from evidence when a less informative updated probability will do the job of responding to the evidence. This is not a strong argument in favour of sharp credences. I consider it to be much easier to convince someone to reject instates on independent conceptual grounds than to convince them to reconsider the principle of maximum entropy after its extensive criticism.

4 Evidence Differentials and Cushioning Credences

I want to proceed to the intriguing issue of who does better in betting situations: instates or sharp credences. I have given away the answer already in the introduction: instates do better. It is surprising that, except for a rudimentary allusion to this in Walley's book, no Boolean has caught on to this yet. After I found out that agents with instates do better betting on soccer games, I let Betsy and Linda play a more basic betting game. An n-sided die is rolled (by the computer). The die is fair, unbeknownst to the players. Their bets are randomly and uniformly drawn from the simplex for which the probabilities attributed to the n results add up to 1. Betsy also surrounds her credences with an imprecision uniformly drawn from the interval $(0, y)$. I used Walley's pay off scheme (see Walley, 1991, 632) to settle the bets.

Here is an example: let $n = 2$, so the die is a fair *coin*. Betsy's and Linda's bets are randomly and uniformly drawn from the line segment from $(0, 1)$ to $(1, 0)$ (these are two-dimensional Cartesian coordinates), the two-dimensional simplex (for higher n, the simplex is a pentatope generalized for n dimensions with side length $2^{1/2}$). The previsions (limits at which bets are accepted) may be $(0.21, 0.79)$ for Linda and $(0.35 \pm 0.11, 0.65 \pm 0.11)$ for Betsy, where the indeterminacy ± 0.11 is also randomly and uniformly drawn from the imprecision interval $(0, y) \subseteq (0, 1)$. The first bet is on H, and Linda is willing to pay 22.5 cents for it, while Betsy is willing to pay 77.5 cents against it. The second bet is on T (if $n > 2$, there will not be the same symmetry as in the *coin* case between the two bets), for which Betsy is willing to pay 77.5 cents, and against which Linda is willing to pay 22.5 cents. Each bet pays \$1 if successful. Often, Linda's credal state will overlap with Betsy's sharp credence so that there will not be a bet.

The computer simulation clearly shows that Linda does better than Betsy in the long run. A defence of sharp credences for rational agents needs to have an explanation for this. We will call it partial belief cushioning, which is based on an evidence differential between the bettors.

In many decision-making contexts, we do not have the luxury of calling off the bet. We have to decide one way or another. This is a problem for instates, as Booleans have to find a way to decide without receiving instructions from the credal state. Booleans have addressed this point extensively (see for example Joyce, 2010, 311ff; for an opponent's view of this see Elga, 2010, 6ff). The problem for sharp credences arises when bets are noncompulsory, for then the data above suggest that agents holding instates systematically do better. Often, decision making happens as betting vis-à-vis uninformed nature or opponents which are at least as uninformed as the rational agent. Sometimes, however, bets are offered by better informed or potentially better informed bookies. In this case, even an agent with sharp credences must cushion her credences and is better off by rejecting bets that look attractive in terms of her partial beliefs.

If an agent does not cushion her partial beliefs (whether they are sharp or indeterminate), she will incur a loss in the long run. Since cushioning is permitted in Walley's experimental setup (the bets are noncompulsory), Laplacean agents should also have access to it and then no longer do worse than Boolean agents. One may ask what sharp credences do if they just end up being cushioned anyway and do not provide sufficient information to decide on rational bets. The answer is that sharp credences are sufficient where betting (or decision making more generally) is compulsory; the cushioning only supplies the information from the evidence inasmuch as betting is noncompulsory and so again properly distinguishes semantic categories. This task is much harder for Booleans, although I do not claim that it is insurmountable: instates can provide a coherent approach to compulsory betting. What they cannot do, once cushioning is introduced, is outperform sharp credences in noncompulsory betting situations.

Here are a few examples: even if I have little evidence on which to base my opinion, someone may force me to either buy Coca Cola shares or short them, and so I have to have a share price p in mind that I consider fair. I will buy Coca Cola shares for less than p, and short them for more than p, if forced to do one or the other. This does not mean that it is now reasonable for me to go (not forced by anyone) and buy Coca Cola shares for p. It may not even be reasonable to go (not forced by anyone) and buy Coca Cola share for $p - \delta$ with $\delta > 0$.

It may in fact be quite unreasonable, since there are many players who have much better evidence than I do and will exploit my ignorance. I suspect that most lay investors in the stock market make this mistake: even though they buy and sell stock at prices that seem reasonable to them, professional investors are much better and faster at exploiting arbitrage opportunities and more subtle regularities. If indices rise, lay investors will make a little less than their professional counterparts; and when they fall, lay investors lose a lot more. In sum, unless there is sustained growth and everybody wins, lay investors lose in the long term.

A case in point is the U.S. Commodity Futures Trading Commission's crackdown on the online prediction market Intrade. Intrade offered fair bets for or against events of public significance, such as election results or other events which had clear yes-or-no outcomes. Even though the bets were all fair and Intrade only received a small commission on all bets, and even though Intrade's predictions were remarkably accurate, the potential for professional arbitrageurs was too great and the CFTC shut Intrade down (see `https://www.intrade.com`).

Cushioning does not stand in the way of holding a sharp credence, even if the evidence is dim. The evidence determines for a rational agent the partial beliefs over possible states of the world operating in the background. The better the evidence, the more pointed the distributions of these partial beliefs will be and the more willing the rational agent will be to enter a bet, if betting is noncompulsory. The mathematical decision rule will be based on the underlying distribution of the partial beliefs, not only on the sharp credence. As we have stated before, a sharp credence is not a sufficient statistic for decision making, inference, or betting behaviour; and neither is an instate.

The rational agent with a sharp credence has resources at her disposal to use just as much differentiation with respect to accepting and rejecting bets as the agent with instates. Often (if she is able to and especially if the bets are offered to her by a better-informed agent), she will reject both of two complementary bets, even when they are fair. On the one hand, any advantage that the agent with an instate has over her can be counteracted based on her distribution over partial beliefs that she has with respect to all possibilities. On the other hand, the agent with instates suffers under both conceptual and practical problems that put her at a real disadvantage in terms of understanding the sources and consequences of her knowledge and her uncertainties.

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