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Augustin's Concessions: A Problem for Indeterminate Credal States

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Abstract Many Bayesian epistemologists now accept that it is not necessary for a rational agent to hold sharp credences. There are various compelling formal theories how such a non-traditional view of credences can accommodate decision making and updating. They are motivated by a common complaint: that sharp credences can fail to represent incomplete evidence and exaggerate the information contained in it. Indeterminate credal states, the alternative to sharp credences, face challenges as well: they are vulnerable to dilation and under certain conditions do not permit learning. This paper focuses on two concessions that Thomas Augustin and James Joyce make to address these challenges. The concessions undermine the original case for indeterminate credal states. I use both conceptual arguments and hands-on examples to argue that rational agents always have sharp credences.

Keywords imprecise credences · indeterminate credal states · sharp credences · formal epistemology · dilation · partial beliefs · Bayesian epistemology

1 Introduction

The claim defended in this paper is that rational agents are subject to a norm requiring sharp credences. I defend this claim in spite of the initially promising features of indeterminate credal states to address problems of sharp credences reflecting a doxastic state.

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Traditionally, Bayesians have maintained that a rational agent, when holding a credence, holds a sharp credence. It has recently become popular to drop the requirement for credence functions to be sharp. There are now Bayesians who permit a rational agent to hold indeterminate credal states based on incomplete or ambiguous evidence. I will refer to Bayesians who continue to adhere to the classical theory of sharp credences for rational agents as 'Laplaceans' (e.g. Adam Elga and Roger White). I will refer to Bayesians who do not believe that a rational agent's credences need to be sharp as 'Booleans' (e.g. Richard Jeffrey, Peter Walley, Brian Weatherson, and James Joyce).

There is some terminological confusion around the adjectives 'imprecise,' 'indeterminate,' and 'mushy' credences. In the following, I will exclusively refer to indeterminate credences or credal states (abbreviated 'instates') and mean by them a set of sharp credence functions (which some Booleans require to be convex) which it may be rational for an agent to hold within an otherwise orthodox Bayesian framework (see Jeffrey, 1983).

More formally speaking, let Ω be a set of state descriptions or possible worlds and $\mathcal{X}$ a suitable algebra on Ω. Call $C_{\mathcal{X}}$, which is a set of probability functions on $\mathcal{X}$, a credal state with respect to $\mathcal{X}$. Sometimes the credal state is required to be convex so that $P_1 \in C_{\mathcal{X}}$ and $P_3 \in C_{\mathcal{X}}$ imply $P_2 \in C_{\mathcal{X}}$ if $P_2 = \vartheta P_1 + (1-\vartheta)P_3$. ϑ is a scalar between 0 and 1, which is multiplied by a probability function using conventional scalar multiplication.

The credal state is potentially different from an agent's doxastic state, which can be characterized in more detail than the credal state (examples will follow). The doxastic state of a rational agent contains all the information necessary to update, infer, and make decisions. Since updating, inference, and decision-making generally needs the quantitative information in a credal state, the credal state is a substate of the doxastic state. Credal states group together doxastic states which are indistinguishable on their formal representation by $C_{\mathcal{X}}$. Laplaceans require that the cardinality of $C_{\mathcal{X}}$ is 1. Booleans have a less rigid requirement of good behaviour for $C_{\mathcal{X}}$, for example they may require it to be a Borel set on the associated vector space if Ω is finite. $C_{\mathcal{X}}$ is a set restricted to probability functions for both Laplaceans and Booleans because both groups are Bayesian.

In the following, I will sometimes say that a credal state with respect to a proposition is a sub-interval of the unit interval, for example $(1/3, 2/3)$. This is a loose way of speaking, since credal states are sets of probability functions, not set-valued functions on a domain of propositions. What I mean, then, is that the credal state identifies $(1/3, 2/3)$ as the range of values that the probability functions in $C_{\mathcal{X}}$ take when they are applied to the proposition in question.

In this paper, I will introduce a *divide et impera* argument in favour of the Laplacean position. I assume that the appeal of the Boolean position is immediately obvious. Not only is it psychologically implausible that agents who strive for rationality should have all their credences worked out to crystal-clear precision; it seems epistemically doubtful to assign exact credences to propositions about which the agent has little or no information, incomplete or ambiguous evidence (Joyce calls the requirement for sharp credences "psychologically implausible and epistemologically calamitous," see Joyce, 2005, 156). From the perspective of information theory, it appears that an agent with sharp credences pretends to be in possession of information that she does not have.

The *divide et impera* argument runs like this: I show that the Boolean position is really divided into two mutually incompatible camps, *Bool-A* and *Bool-B*. *Bool-A* has the advantage of owning all the assets of appeal against the Laplacean position. The stock examples brought to bear against sharp credences have simple and compelling instate solutions given *Bool-A*. *Bool-A*, however, has deep conceptual problems which I will describe in detail below.

Bool-B's refinement of *Bool-A* is successful in so far as it resolves the conceptual problems. Their success depends on what I call Augustin's concessions, which undermine all the appeal that the Boolean position as a whole has over the Laplacean position. With a series of examples, I seek to demonstrate that in Simpson Paradox type fashion the Laplacean position looks genuinely inferior to the amalgamated Boolean position, but as soon as the mutually incompatible strands of the Boolean position have been identified, the Laplacean position is independently superior to both.

2 Partial Beliefs

When we first hear of the advantages of instates, three of them sound particularly persuasive.

- INTERN Instates represent the possibility range for objective chances (objective chances internal to the instate are not believed not to hold, objective chances external to the instate are believed not to hold).
- INCOMP Instates represent incompleteness or ambiguity of the evidence.
- INFORM Instates are responsive to the information content of evidence.

Here are some examples and explanations. Let a $coin_{\mathrm{x}}$ be a Bernoulli generator that produces successes and failures with probability p_{x} for success, labeled H_{x}, and $1 - p_{\mathrm{x}}$ for failure, labeled T_{x}. Physical coins may serve as Bernoulli generators, if we are willing to set aside that most of them are approximately fair.

> **Example 1: INTERN.** Blake has two Bernoulli generators in her lab, $coin_{\mathrm{i}}$ and $coin_{\mathrm{ii}}$. Blake has a database of $coin_{\mathrm{i}}$ results and concludes on excellent evidence that

> $coin_{\mathrm{i}}$ is fair. Blake has no evidence about the bias of $coin_{\mathrm{ii}}$. As a Boolean, Blake assumes a sharp credence of $\{0.5\}$ for H_{i} and an indeterminate credal state of $[0,1]$ for H_{ii}. She feels bad for Logan, her Laplacean colleague, who cannot distinguish between the two cases and who must assign a sharp credence to both H_{i} and H_{ii} (for example, $\{0.5\}$).
>
> **Example 2: INCOMP.** Blake has another Bernoulli generator, $coin_{\mathrm{iii}}$, in her lab. Her graduate student has submitted $coin_{\mathrm{iii}}$ to countless experiments and emails Blake the resulting bias, but fails to include whether the bias of 2/3 is in favour of H_{iii} or in favour of T_{iii}. As a Boolean, Blake assumes an indeterminate credal state of $[1/3, 2/3]$ (or $\{1/3, 2/3\}$, depending on the convexity requirement) for H_{iii}. She feels bad for Logan who must assign a sharp credence to H_{iii}. If Logan chooses $\{0.5\}$ as her sharp credence based not unreasonably on symmetry considerations, Logan concurrently knows that her credence gets the bias wrong.

Example 1 also serves as an example for INFORM: one way in which Blake feels bad for Logan is that Logan's $\{0.5\}$ credence for H_{ii} is based on very little information, a fact not reflected in Logan's credence. Walley notes that "the precision of probability models should match the amount of information on which they are based" (Walley, 1991, 34). Joyce explicitly criticizes the information overload for sharp credences in examples such as example 1. He says about sharp credences of this kind that, despite their maximal entropy compared to other sharp credences, they are "very informative" and "adopting [them] amounts to pretending that you have lots and lots of information that you simply don't have" (Joyce, 2010, 284).

Walley and Joyce appeal to intuition when they promote INFORM. It just feels as if there were more information in a sharp credence than in an instate. Neither of them ever makes this claim more explicit. Joyce admits:

> It is not clear how such an 'imprecise minimum information requirement' might be formulated, but it seems clear that C_1 encodes more information than C_2 whenever $C_1 \subset C_2$, or when C_2 arises from C_1 by conditioning. (Joyce, 2010, 288.)

Since for the rest of the paper the emphasis will be on INTERN and INCOMP, I will advance my argument against INFORM right away: not only is it not clear how Joyce's imprecise minimum information requirement might be formulated, I see no reason why it should give the results that Joyce envisions. To compare instates and sharp credences informationally, we would need a set function obeying Shannon's axioms for information (for an excellent summary see Klir, 2006). Attempts for such a generalized Shannon measure have been made, but they are all unsatisfactory. George Klir lists the requirements on page 235 (loc. cit.), and they are worth calling to mind here (a similar list is in Mork, 2013, 363):

(S1) **Probability Consistency** When $\mathcal{D}$ contains only one probability distribution, $\overline{S}$ assumes the form of the Shannon entropy.

(S2) **Set Consistency** When $\mathcal{D}$ consists of the set of all possible probability distributions on $A \subseteq X$, then $\overline{S}(\mathcal{D}) = \log_2 |A|$.
(S3) **Range** The range of $\overline{S}$ is $[0, \log_2 |X|]$ provided that uncertainty is measured in bits.
(S4) **Subadditivity** If $\mathcal{D}$ is an arbitrary convex set of probability distributions on $X \times Y$ and $\mathcal{D}_X$ and $\mathcal{D}_Y$ are the associated sets of marginal probability distributions on X and Y, respectively, then $\overline{S}(\mathcal{D}) \leq \overline{S}(\mathcal{D}_X) + \overline{S}(\mathcal{D}_Y)$.
(S5) **Additivity** If $\mathcal{D}$ is the set of joint probability distributions on $X \times Y$ that is associated with independent marginal sets of probability distributions, $\mathcal{D}_X$ and $\mathcal{D}_Y$, which means that $\mathcal{D}$ is the convex hull of the set $\mathcal{D}_X \otimes \mathcal{D}_Y$, then $\overline{S}(\mathcal{D}) = \overline{S}(\mathcal{D}_X) + \overline{S}(\mathcal{D}_Y)$.

Several things need an explanation here (I have retained Klir's nomenclature). $\mathcal{D}$ is the set of probability distributions constituting the instate. $\overline{S}$ is the proposed generalized Shannon measure defined on the set of possible instates. I will give an example below in (2). X is the event space. $\mathcal{D}_X \otimes \mathcal{D}_Y$ is defined as follows:

$$\mathcal{D}_X \otimes \mathcal{D}_Y = \{p(x,y) = p_X(x) \cdot p_Y(y) | x \in X, y \in Y, p_X \in \mathcal{D}_X, p_Y \in \mathcal{D}_Y\}. \quad (1)$$

One important requirement not listed is that indeterminateness should give us higher entropy (otherwise Joyce's and Walley's argument will fall flat). Klir's most hopeful contender for a generalized Shannon measure (see his equation 6.61) does not fulfill this requirement:

$$\overline{S}(\mathcal{D}) = \max_{p \in \mathcal{D}} \left\{ -\sum_{x \in X} p(x) \log_2 p(x) \right\}. \quad (2)$$

Notice that for any convex instate there is a sharp credence contained in the instate whose generalized Shannon measure according to (2) equals the generalized Shannon measure of the instate, but we would expect the entropy of the sharp credence to be lower than the entropy of the instate if it is indeterminate. Jonas Clausen Mork has noticed this problem as well and proposes a modified measure to reflect that more indeterminacy ought to mean higher entropy, all else being equal. He adds the following requirements to Klir's list above (the labels are mine; Mork calls them NC1 and NC2 in Mork, 2013, 363):

(S6) **Weak Monotonicity** If P is a superset of P', then $U(P) \geq U(P')$. A set containing another has at least as great uncertainty value.
(S7) **Strong Monotonicity** If (i) the lower envelope of P is dominated strongly by the lower envelope of P' and (ii) P is a strict superset of P', then $U(P) > U(P')$. When one set strictly contains another with a

strictly higher lower bound for at least one hypothesis, the greater set has strictly higher uncertainty value.

I have retained Mork's nomenclature and trust that the reader can see how it lines up with Klir's. Klir's generalized Shannon measure (2) fulfills weak monotonicity, but violates strong monotonicity. Mork's proposed alternative is the following (see Mork, 2013, 364):

$$\mathrm{CSU}(\mathit{\Pi}, P) = \max_{p^* \in P} \left\{ -\sum_{i=1}^{n} p^*(h_i) \min_{q^* \in P} \{\log_2 q^*(h_i)\} \right\}. \tag{3}$$

Mork fails to establish subadditivity, however, and it is more fundamentally unclear if Klir has not already shown with his disaggregation theory that fulfilling all desiderata (S1)–(S7) is impossible. Some of Klir's remarks seem to suggest this (see, for example, Klir, 2006, 218), but I was unable to discern a full-fledged impossibility theorem in his account. This would be an interesting avenue for further research.

Against the force of INTERN, INCOMP, and INFORM, I maintain that the Laplacean approach of assigning subjective probabilities to partitions of the event space (e.g. objective chances) and then aggregating them by David Lewis' summation formula (see Lewis, 1981, 266f) into a single precise credence function is conceptually tidy and shares many of the formal virtues of Boolean theories. If the bad taste about numerical precision lingers, I will point to philosophical projects in other domains where the concepts we use are sharply bounded, even though our ability to conceive of those sharp boundaries or know them is limited (in particular Timothy Williamson's accounts of vagueness and knowledge). To put it provocatively, this paper defends a 0.5 sharp credence in heads in all three cases: for a coin of whose bias we are completely ignorant; for a coin whose fairness is supported by a lot of evidence; and even for a coin about whose bias we know that it is either 1/3 or 2/3 for heads.

Consider the following reasons that incline Booleans to permit instates for rational agents:

(A) The greatest emphasis motivating indeterminacy rests on INTERN, INCOMP, and INFORM.
(B) The preference structure of a rational agent may be incomplete so that representation theorems do not yield single probability measures to represent such incomplete structures.
(C) There are more technical and paper-specific reasons, such as Thomas Augustin's attempt to mediate between the minimax pessimism of objectivists and the Bayesian optimism of subjectivists using interval probability (see Augustin, 2003, 35f); Alan Hájek and Michael Smithson's belief that there may be objectively indeterminate chances in the physical world

(see Hájek and Smithson, 2012, 33, but also Hájek, 2003, 278, 307); Jake Chandler's claim that "the sharp model is at odds with a trio of plausible propositions regarding agnosticism" (Chandler, 2014, 4); and Brian Weatherson's claim that for the Boolean position, open-mindedness and modesty may be consistent when for the Laplacean they are not (see Weatherson, 2015, using a result by Gordon Belot, see Belot, 2013).

This paper mostly addresses (A), while taking (B) seriously as well and pointing towards solutions for it. I am leaving (C) for more specific responses to the issues presented in the cited articles. I will address in section 6 Weatherson's more general claim that it is a distinctive and problematic feature of the Laplacean position "that it doesn't really have a good way of representing a state of indecisiveness or open-mindedness" (Weatherson, 2015, 9), i.e. that sharp credences cannot fulfill what I will call the double task. Weatherson's more particular claim about open-mindedness and modesty is a different story and shall be told elsewhere.

3 Two Camps: *Bool-A* and *Bool-B*

My *divide et impera* argument rests on the distinction between two Boolean positions. The difference is best captured by a simple example to show how epistemologists advocate for *Bool-A* or relapse into it, even when they have just advocated the more refined *Bool-B*.

Example 3: Skittles. Every skittles bag contains 42 pieces of candy. It is filled by robots from a giant randomized pile of candies in a warehouse, where the ratio of five colours is 8:8:8:9:9, orange being the last of the five colours. Logan picks one skittle from a bag and tries to guess what colour it is before she looks at it. She has a sharp credence of 9/42 that the skittle is orange.

Bool-A Booleans reject Logan's sharp credence on the basis that she does not know that there are 9 orange skittles in her particular bag. A 9/42 credence suggests to them a knowledge claim on Logan's part, based on very thin evidence, that her bag contains 9 orange skittles. Logan's doxastic state, however, is much more complicated than her credal state. She knows about the robots and the warehouse. Therefore, her credences that there are k orange skittles in the bag conform to the Bernoulli distribution:

$$C(k) = \binom{42}{k} \left(\frac{9}{42}\right)^{k} \left(\frac{33}{42}\right)^{42-k} \tag{4}$$

For instance, her sharp credence that there are in fact 9 orange skittles in the bag is approximately 14.9%. One of Augustin's concessions, the refinements that *Bool-B* makes to *Bool-A*, clarifies that a coherent Boolean position must agree with the Laplacean position that doxastic states are not fully captured by credal states. We will see in the next section why this is the case.

It is a characteristic of *Bool-A*, however, to require that the credal state be sufficient for inference, updating, and decision making. Susanna Rinard, for example, considers it the goal of instates to provide "a complete characterization of one's doxastic attitude" (Rinard, 2015, 5) and reiterates a few pages later that it is "a primary purpose of the set of functions model to represent the totality of the agent's actual doxastic state" (Rinard, 2015, 12).

I will give a few examples of *Bool-A* in the literature, where the authors usually consider themselves to be defending an amalgamated Boolean position which is *pro toto* superior to the Laplacean position. Here is an illustration in Hájek and Smithson.

> **Example 4: Lung Cancer.** Your doctor is your sole source of information about medical matters, and she assigns a credence of $[0.4, 0.6]$ to your getting lung cancer.

Hájek and Smithson go on to say that

> it would be odd, and arguably irrational, for you to assign this proposition a sharper credence—say, 0.5381. How would you defend that assignment? You could say, I don't have to defend it, it just happens to be my credence. But that seems about as unprincipled as looking at your sole source of information about the time, your digital clock, which tells that the time rounded off to the nearest minute is 4:03—and yet believing that the time is in fact 4:03 and 36 seconds. Granted, you may just happen to believe that; the point is that you have no business doing so. (Hájek and Smithson, 2012, 38f.)

This is an argument against Laplaceans by *Bool-A* because it conflates partial belief and full belief. The precise credences in Hájek and Smithson's example, on any reasonable Laplacean interpretation, do not represent full beliefs that the objective chance of getting lung cancer is 0.5381 or that the time of the day is 4:03:36. A sharp credence rejects no hypothesis about objective chances (unlike an instate for *Bool-A*). It often has a subjective probability distribution operating in the background, over which it integrates to yield the sharp credence (it would do likewise in Hájek and Smithson's example for the prognosis of the doctor or the time of the day). The integration proceeds by Lewis' summation formula (see Lewis, 1981, 266f),

$$C(R) = \int_0^1 \zeta P\left(\pi(R) = \zeta\right)\, d\zeta. \tag{5}$$

If, for example, S is the proposition that Logan's randomly drawn skittle in example 3 is orange, then

$$C(S) = \sum_{k=0}^{42} \frac{k}{42} \binom{42}{k} \left(\frac{9}{42}\right)^k \left(\frac{33}{42}\right)^{42-k} = 9/42. \tag{6}$$

No objective chance $\pi(S)$ needs to be excluded by it. Any updating will merely change the partial beliefs, but no full beliefs. Instates, on the other hand, by giving ranges of acceptable objective chances suggest that there is a full belief that the objective chance does not lie outside what is indicated by the instate (corresponding to INTERN). When a *Bool-A* advocate learns that an objective chance lies outside her instate, she needs to resort to belief revision rather than updating her partial beliefs. A *Bool-B* advocate can avoid this by accepting one of Augustin's concessions that I will introduce in section 5.

While most of these examples have been examples of presenting *Bool-A* as an amalgamated Boolean position, without heed to the refinements of Augustin and Joyce, it is also the case that refined Booleans belonging to *Bool-B* relapse into *Bool-A* patterns when they argue against the Laplacean position. This is not surprising, because, as we will see, the refined Boolean position *Bool-B* is more coherent than the more simple Boolean position *Bool-A*, but also left without resources to address the problems that have made the Laplacean position vulnerable in the first place.

Joyce, for instance, refers to an example that is again in all relevant respects like example 3 and states that Logan is "committing herself to a definite view about the relative proportions of skittles in the bag" (see Joyce, 2010, 287, pronouns and example-specific nouns changed to fit example 3). Augustin defends the Boolean position with another example of relapse:

> Imprecise probabilities and related concepts ... provide a powerful language which is able to reflect the partial nature of the knowledge suitably and to express the amount of ambiguity adequately. (Augustin, 2003, 34.)

Augustin himself (see section 5 on Augustin's concessions) details the demise of the idea that indeterminate credal states can "express the amount of ambiguity adequately." Before I go into these details, however, I need to make the case that Augustin's concessions are necessary in order to refine *Bool-A* and make it more coherent. It is two problems for instates that make this case for us: dilation and the impossibility of learning. Note that these problems are not sufficient to reject instates—they only compel us to refine the more simple Boolean position via Augustin's concessions. The final game is between *Bool-B* and Laplaceans, where I will argue that the *Bool-B* position has lost the intuitive appeal of the amalgamated Boolean position to present solutions to prima facie problems facing the Laplacean position.

4 Dilation and Learning

Here are two potential problems for Booleans:

- DILATION Instates are vulnerable to dilation.
- OBTUSE Instates do not permit learning.

Both of these can be resolved by making Augustin's concessions. I will introduce these problems in the present section, then Augustin's concessions in the next section, and the implications for the more general disagreement between Booleans and Laplaceans in the final section.

4.1 Dilation

Consider the following example for DILATION (see White, 2010, 175f and Joyce, 2010, 296f).

> **Example 5: Dilation.** Logan has two Bernoulli generators, $coin_{\text{iv}}$ and $coin_{\text{v}}$. She has excellent evidence that $coin_{\text{iv}}$ is fair and no evidence about the bias of $coin_{\text{v}}$. Logan's graduate student independently tosses both $coin_{\text{iv}}$ and $coin_{\text{v}}$. Then she tells Logan whether the results of the two tosses correspond or not ($H_{\text{iv}} \equiv H_{\text{v}}$ or $H_{\text{iv}} \equiv T_{\text{v}}$, where $X \equiv Y$ means $(X \wedge Y) \vee (\neg X \wedge \neg Y)$). Logan, who has a sharp credence for H_{v}, takes this information in stride, but she feels bad for Blake, whose credence in H_{iv} dilates to $[0,1]$ even though Blake shares Logan's excellent evidence that $coin_{\text{iv}}$ is fair.

Here is why Blake's credence in H_{iv} must dilate. Her credence in H_{v} is $[0,1]$, by stipulation. Let $c(X)$ be the range of probabilities represented by Blake's instate with respect to the proposition X, for example $c(H_{\text{v}}) = [0,1]$. Then

$$c(H_{\text{iv}} \equiv H_{\text{v}}) = c(H_{\text{iv}} \equiv T_{\text{v}}) = \{0.5\} \tag{7}$$

because the tosses are independent and $c(H_{\text{iv}}) = \{0.5\}$ by stipulation. Next,

$$c(H_{\text{iv}} | H_{\text{iv}} \equiv H_{\text{v}}) = c(H_{\text{v}} | H_{\text{iv}} \equiv H_{\text{v}}) \tag{8}$$

where $c(X|Y)$ is the updated instate after finding out Y. Booleans accept (8) because they are Bayesians and update by standard conditioning. Therefore,

$$c(H_{\text{iv}} | H_{\text{iv}} \equiv H_{\text{v}}) = c(H_{\text{v}} | H_{\text{iv}} \equiv H_{\text{v}}) = c(H_{\text{v}}) = [0,1]. \tag{9}$$

To see that (9) is true, note that in the rigorous definition of a credal state as a set of probability functions, each probability function P in Blake's instate for the proposition $H_{\mathrm{v}}|H_{\mathrm{iv}} \equiv H_{\mathrm{v}}$ has the following property by Bayes' theorem:

$$P(H_{\mathrm{v}}|H_{\mathrm{iv}} \equiv H_{\mathrm{v}}) = \frac{P(H_{\mathrm{iv}})P(H_{\mathrm{v}})}{P(H_{\mathrm{iv}})P(H_{\mathrm{v}}) + P(T_{\mathrm{iv}})P(T_{\mathrm{v}})} \tag{10}$$

In our loose way of speaking of credal states as set-valued functions of propositions, Blake's updated instate for H_{iv} has dilated from $\{0.5\}$ to $[0, 1]$.

This does not sound like a knock-down argument against Booleans (it is investigated in detail in Seidenfeld and Wasserman, 1993), but Roger White uses it to derive implications from instates which are worrisome.

> **Example 6: Chocolates.** Four out of five chocolates in the box have cherry fillings, while the rest have caramel. Picking one at random, what should my credence be that it is cherry-filled? Everyone, including the staunchest [Booleans], seems to agree on the answer 4/5. Now of course the chocolate I've chosen has many other features, for example this one is circular with a swirl on top. Noticing such features could hardly make a difference to my reasonable credence that it is cherry filled (unless of course I have some information regarding the relation between chocolate shapes and fillings). Often chocolate fillings do correlate with their shapes, but I haven't the faintest clue how they do in this case or any reason to suppose they correlate one way rather than another ... the further result is that while my credence that the chosen chocolate is cherry-filled should be 4/5 prior to viewing it, once I see its shape (whatever shape it happens to be) my credence that it is cherry-filled should dilate to become [indeterminate]. But this is just not the way we think about such matters. (White, 2010, 183.)

I will characterize the problems that dilation causes for the Boolean position by three aspects (all of which originate in White, 2010):

- RETENTION This is the problem in example 6. When we tie the outcome of a random process whose objective chance we know to the outcome of a random process whose chance we do not know, White maintains that we should be entitled to *retain* the credence that is warranted by the known objective chance. One worry about the Boolean position in this context is that credences become excessively dependent on the mode of representation of a problem.
- REPETITION Consider again example 5, although this time Blake runs the experiment 10,000 times. Each time, her graduate student tells her whether $H^n_{\mathrm{iv}} \equiv H^n_{\mathrm{v}}$ or $H^n_{\mathrm{iv}} \equiv T^n_{\mathrm{v}}$, n signifying the n-th experiment. After running the experiment that many times, approximately half of the outcomes of $coin_{\mathrm{iv}}$ are heads. Now Blake runs the experiment one more time. Again, the Boolean position mandates dilation, but should Blake not just on inductive grounds persist in a sharp credence of 0.5 for H_{iv}, given that about

half of the coin flips so far have come up heads? Attentive readers will notice a sleight of hand here: as many times as Blake performs the experiment, it must not have any evidential impact on the assumptions of example 5, especially that the two coin flips remain independent and that the credal state for $coin_v$ is still, even after all these experiments, maximally indeterminate.
- REFLECTION Consider again example 5. Blake's graduate student will tell Blake either $H_{iv} \equiv H_v$ or $H_{iv} \equiv T_v$. No matter what the graduate student tells Blake, Blake's credence in H_{iv} dilates to an instate of $[0, 1]$. Blake therefore is subject to Bas van Fraassen's reflection principle stating that

$$P_t^a(A|p_{t+x}^a(A) = r) = r \tag{11}$$

where P_t^a is the agent a's credence function at time t, x is any non-negative number, and $p_{t+x}^a(A) = r$ is the proposition that at time $t + x$, the agent a will bestow degree r of credence on the proposition A (see van Fraassen, 1984, 244). Van Fraassen had sharp credences in mind, but it is not immediately obvious why the reflection principle should not also hold for instates.

To address the force of RETENTION, REPETITION, and REFLECTION, Joyce hammers home what I have called Augustin's concessions. According to Joyce, none of White's attacks succeed if a refinement of *Bool-A*'s position takes place and instates are not required either to reflect knowledge of objective chances or doxastic states. I will address this in detail in section 5, concluding that Augustin's concessions are only necessary based on REFLECTION, whereas solutions for RETENTION and REPETITION have less far-reaching consequences and do not impugn the Boolean position of *Bool-A*.

4.2 Learning

Here is an example for OBTUSE (see Rinard's objection cited in White, 2010, 84 and addressed in Joyce, 2010, 290f). It presumes Joyce's supervaluationist semantics of instates (see Hájek, 2003; Joyce, 2010, 288; and Rinard, 2015; for a problem with supervaluationist semantics see Lewis, 1993; and an alternative which may solve the problem see Weatherson, 2015, 7), for which Joyce uses the helpful metaphor of committee members, each of whom holds a sharp credence. The instate consists then of the set of sharp credences from each committee member: for the purposes of updating, for example, each committee member updates as if she were holding a sharp credence. The aggregate of the committee members' updated sharp credences forms the updated instate. Supervaluationist semantics also permits comparisons, when for example a partial belief in X is stronger than a partial belief in Y because all committee members have sharp credences in X which exceed all the sharp credences held by committee members with respect to Y.

Example 7: Learning. Blake has a Bernoulli generator in her lab, $coin_{\mathrm{vi}}$, of whose bias she knows nothing and which she submits to experiments. At first, Blake's instate for H_{vi} is $(0, 1)$. After a few experiments, it looks like $coin_{\mathrm{vi}}$ is fair. However, as committee members crowd into the centre and update their sharp credences to something closer to 0.5, they are replaced by extremists on the fringes. The instate remains at $(0, 1)$.

It is time now to examine refinements of the Boolean position to address these problems.

5 Augustin's Concessions

Joyce has defended instates against DILATION and OBTUSE, making Augustin's concessions (AC1) and (AC2). I am naming them after Thomas Augustin, who has some priority over Joyce in the matter. Augustin's concessions distinguish *Bool-A* and *Bool-B*, the former of which does not make the concessions and identifies partial beliefs with full beliefs about objective chances. A sophisticated view of partial beliefs recognizes that they are sui generis, which necessitates a substantial reconciliation project between full belief epistemology and partial belief epistemology. My task at hand is to agree with the refined position of *Bool-B* in their argument against *Bool-A* that this reconciliation project is indeed substantial and that partial beliefs are not full beliefs about objective chances; but also that *Bool-B* fails to summon arguments against the Laplacean position without relapsing into an unrefined version of indeterminacy.

Here, then, are Augustin's concessions:

(AC1) Credal states do not adequately represent doxastic states. The same instate can reflect different doxastic states, even when the difference in the doxastic states matters for updating, inference, and decision making.

(AC2) Instates do not represent full belief claims about objective chances. White's *Chance Grounding Thesis* is not an appropriate characterization of the Boolean position.

I agree with Joyce that (AC1) and (AC2) are both necessary and sufficient to resolve DILATION and OBTUSE for instates. I disagree with Joyce about what this means for an overall recommendation to accept the Boolean rather than the Laplacean position. After I have already cast doubt on INFORM, I will show that (AC1) and (AC2) neutralize INTERN and INCOMP, the major impulses for rejecting the Laplacean position.

Indeterminacy imposes a double task on credences (representing both uncertainty and available evidence) that they cannot coherently fulfill. I will present several examples where this double task stretches instates to the limits of plausibility. Joyce's idea that credences can represent balance, weight, and specificity of the evidence (in Joyce, 2005) is inconsistent with the use of indeterminacy. Joyce himself, in response to DILATION and OBTUSE, gives the argument why this is the case (see Joyce, 2010, 290ff, for OBTUSE; and Joyce, 2010, 296ff, for DILATION). Let us begin by looking more closely at how (AC1) and (AC2) protect *Bool-B* from DILATION and OBTUSE.

5.1 Augustin's Concession (AC1)

(AC1) says that credences do not adequately represent a doxastic state. The same instate can reflect different doxastic states, where the difference is relevant to updating, inference, and decision making.

Augustin recognizes the problem of inadequate representation before Joyce, with specific reference to instates: "The imprecise posterior does no longer contain all the relevant information to produce optimal decisions. Inference and decision do not coincide any more" (Augustin, 2003, 41) (see also an example for inadequate representation of evidence by instates in Bradley and Steele, 2014, 1300). Joyce rejects the notion that identical instates encode identical beliefs by giving two examples. The first one is problematic. The second one, which is example 5 given earlier, addresses the issue of DILATION more directly. Here is the first example.

> **Example 8: Three-Sided Die.** Suppose $\mathcal{C}'$ and $\mathcal{C}''$ are defined on a partition $\{X, Y, Z\}$ corresponding to the result of a roll of a three sided-die. Let $\mathcal{C}'$ contain all credence functions defined on $\{X, Y, Z\}$ such that $c(Z) \geq 1/2$, and let $\mathcal{C}''$ be the subset of $\mathcal{C}''$ whose members also satisfy $c(X) = c(Y)$ (see Joyce, 2010, 294).

Joyce then goes on to say,

> It is easy to show that $\mathcal{C}'$ and $\mathcal{C}''$ generate the same range of probabilities for all Boolean combinations of $\{X, Y, Z\}$, and so LP and PSET deem them equivalent. But they are surely different: the $\mathcal{C}''$-person believes everything the $\mathcal{C}'$-person believes, but she also regards X and Y as equiprobable.

example 8 is problematic because $\mathcal{C}'$ and $\mathcal{C}''$ do not generate the same range of probabilities: if, as Joyce says, $c(Z) \geq 1/2$, then $c(X) = c(Y)$ implies $c(X) \leq 1/4$ for $\mathcal{C}''$, but not for $\mathcal{C}'$. What Joyce wants to say is that the same instate can encode doxastic states which are relevantly different when it comes to updating probabilities, and the best example for this is example 5 itself.

To explain this in more detail, we need to review for a moment what Lewis means by the Principal Principle and by inadmissibility. The Principal Principle requires that my knowledge of objective chances is reflected in my credence, unless there is inadmissible evidence. Inadmissible evidence would for instance be knowledge of a coin toss outcome, in which case of course I do not need to have a credence for it corresponding to the bias of the coin. In example 5, I could use the Principal Principle in the spirit of RETENTION to derive a contradiction to the Boolean formalism.

$$H_{\text{iv}} \equiv H_{\text{v}} \text{ does not give anything away about } H_{\text{iv}}, \tag{12}$$

therefore

$$c(H_{\text{iv}}|H_{\text{iv}} \equiv H_{\text{v}}) = c(H_{\text{iv}}) \tag{13}$$

by the Principal Principle and in contradiction to (9).

Joyce explains how (12) is false and blocks the conclusion (13), which would undermine the Boolean position. $H_{\text{iv}} \equiv H_{\text{v}}$ is clearly inadmissible, even without (AC1), since it is information that not only changes Blake's doxastic state, but also her credal state.

We would usually expect more information to sharpen our credal states (see Walley's anti-dilation principle and his response to this problem in 1991, 207 and 299), an intuition violated by both DILATION and OBTUSE. As far as DILATION is concerned, however, the loss of precision is in principle not any more surprising than information that increases the Shannon entropy of a sharp credence.

Example 9: Rumour. A rumour that the Canadian prime minister has been assassinated raises your initially very low probability that this event is taking place today to approximately 50%.

It is true for both sharp and indeterminate credences that information can make us less certain about things. This is the simple solution for RETENTION. If one of Joyce's committee members has a sharp credence of 1 in H_{v} and learns $H_{\text{iv}} \equiv H_{\text{v}}$, then her sharp credence for H_{iv} should obviously be 1 as well; ditto and mutatis mutandis for the committee member who has a sharp credence of 0 in H_{v}. (AC1) is unnecessary.

Here is how dilation is as unproblematic as a gain in entropy after more information in example 9:

Example 10: Dilating Urns. You are about to draw a ball from an urn with 200 balls (100 red, 100 yellow). Just before you draw, you receive the information that the urn has two chambers which are obscured to you as you draw the ball, one with 99 red balls and 1 yellow ball, the other with 1 red ball and 99 yellow balls.

Dilation from a sharp credence of $\{0.5\}$ to an instate of $[0.01, 0.99]$ (or $\{0.01, 0.99\}$, depending on whether convexity is required) is unproblematic, although the example prefigures that there is something odd about the Boolean conceptual approach. The example licences a 99:1 bet for one of the colours (if the instate is interpreted as upper and lower previsions), which inductively would be a foolish move. This is a problem that arises out of the Boolean position quite apart from DILATION and in conjunction with betting licences, which we will address in example 13.

So far we have not found convincing reasons to accept (AC1). Neither dilation as a phenomenon nor Lewis' inadmissibility criterion need to compel a *Bool-A* advocate to admit (AC1), despite Joyce's claims in the abstract of his paper that reactions to dilation "are based on an overly narrow conception of imprecise belief states which assumes that we know everything there is to know about a person's doxastic attitudes once we have identified the spreads of values for her imprecise credences." Not only does example 8 not give us the desired results, we can also resolve RETENTION and REPETITION without recourse to (AC1). It is, in the final analysis, REFLECTION which will make the case for (AC1).

It is odd that Joyce explicitly says that the argument REPETITION "goes awry by assuming that credal states which assign the same range of credences for an event reflect the same opinions about that event" (Joyce, 2010, 304), when in the following he makes an air-tight case against the anti-Boolean force of REPETITION without ever referring to (AC1). Joyce's case against REPETITION is based on the sleight of hand to which I made reference when I introduced REPETITION. There is no need to repeat Joyce's argument here.

Let me add that despite my agreement with Joyce on how to handle REPETITION, although we disagree that it has anything to do with (AC1), a worry about the Boolean position with respect to REPETITION lingers. Joyce requires that Blake, in so far as Blake wants to be rational, has a maximally indeterminate credal state for H_{iv}^{10000} after hearing from her graduate student. The oddity of this, when we have just had about 5000 heads and 5000 tails, remains. Blake's maximally indeterminate credal state rests on her stubborn conviction that her credal state must be inclusive of the objective chance of $coin_{\mathrm{iv}}^{10000}$ landing heads.

Logan, if she were to do this experiment, would relax, reason inductively as well as based on her prior probability, and give H_{iv} a credence of 0.5, since her credal state is not in the same straight-jacket of underlying objective chances—just as in example 3 Logan is able to have a sharp credence of $9/42$ for picking an orange skittle, when her credence that there are 9 orange skittles in the bag is only 14.9%. This worry supports Augustin's second concession (AC2) that the relationship between instates and objective chances is much looser than for *Bool-A*, but more about this in the next subsection.

We are left with REFLECTION, which bears most of the burden in Joyce's argument for (AC1). It is indeed odd that a rational agent should have two strictly distinct credal states C_1 and C_2, when C_2 follows upon C_1 from a piece of information that says either X or $\neg X$—but it does not matter which of the two. Why does the rational agent not assume C_2 straight away? Joyce introduces an important distinction for van Fraassen's reflection principle: It is not the degree of credence that is decisive as in van Fraassen's original formulation of the principle, but the doxastic state. X and $\neg X$ (in example 5, they refer to $H_{\text{iv}} \equiv H_{\text{v}}$ and $H_{\text{iv}} \equiv T_{\text{v}}$) both lead from a sharp credence of 0.5 to an instate of $[0, 1]$ for H_{iv}, but this updated instate reflects different doxastic states. In Joyce's words, "the beliefs about H_{iv} you will come to have upon learning $H_{\text{iv}} \equiv H_{\text{v}}$ are complementary to the beliefs you will have upon learning $H_{\text{iv}} \equiv T_{\text{v}}$" (Joyce, 2010, 304). Committee members representing complementary beliefs agree on the instate, but single committee members take opposite views depending on the information, X or $\neg X$. Credal states keep track only of the committee's aggregate credal state, whereas doxastic states keep track of each committee member's individual sharp credences.

This resolves the anti-Boolean force of REFLECTION by making the concession (AC1). Ironically, of course, not being able to represent a doxastic state, but only to reflect it inadequately, was just the problem that Laplacean sharp credences had which Boolean instates were supposed to fix. One way *Bool-B* could respond is like this:

> **Riposte:** If you are going to make a difference between *representing* a doxastic state and *reflecting* a doxastic state, where the former poses an identity relationship between credal states and doxastic states and the latter a supervenience relationship, then all you have succeeded in making me concede is that both instates and sharp credences reflect doxastic states. Instates may still be more successful in reflecting doxastic states than sharp credences are and so solve the abductive task of explaining partial beliefs better.

There are several reasons why I doubt this line of argument is successful. The Laplacean position has formal advantages, for example that it is able to cooperate with information theory, an ability which *Bool-B* lacks. There is no coherent theory of how relations between instates can be evaluated on the basis of information and entropy, which are powerful tools when it comes to justifying fundamental Bayesian tenets (see Shore and Johnson, 1980; Giffin, 2008; and Lukits, 2015). Furthermore, the Laplacean position is conceptually tidy. It distinguishes between the quantifiable aspects of a doxastic state, which it integrates to yield the sharp credence, and other aspects of the evidence, such as incompleteness or ambiguity. Instates dabble in what I will call the double task: trying to reflect both aspects without convincing success in either.

5.2 Augustin's Concession (AC2)

(AC2) says that instates do not reflect knowledge claims about objective chances. White's *Chance Grounding Thesis* (which White does not endorse, being a Laplacean) is not an appropriate characterization of the Boolean position.

> **Chance Grounding Thesis (CGT):** Only on the basis of known chances can one legitimately have sharp credences. Otherwise one's spread of credence should cover the range of possible chance hypotheses left open by your evidence. (White, 2010, 174)

Joyce considers (AC2) to be as necessary for a coherent Boolean view of partial beliefs, blocking OBTUSE, as (AC1) is, blocking DILATION (see Joyce, 2010, 289f).

OBTUSE is related to VACUITY, another problem for Booleans:

- VACUITY If one were to be committed to the principle of regularity, that all states of the world considered possible have positive probability (for a defence see Savage et al, 1963, 211); and to the solution of Henry Kyburg's lottery paradox, that what is rationally accepted should have probability 1 (for a defence of this principle see Douven and Williamson, 2006); and the CGT, that one's spread of credence should cover the range of possible chance hypotheses left open by the evidence (implied by much of Boolean literature); then one's instate would always be vacuous.

Booleans must deny at least one of the premises to avoid the conclusion. Joyce denies the CGT, giving us (AC2). It is by no means necessary to sign on to regularity and to the above-mentioned solution of Henry Kyburg's lottery paradox in order to see how (AC2) is a necessary refinement of *Bool-A*. The link between objective chances and credal states expressed in the CGT is suspect for many other reasons. I have referred to them passim, but will not go into more detail here.

6 The Double Task

Sharp credences have a single task: to reflect epistemic uncertainty as a tool for updating, inference, and decision making. They cannot fulfill this task without continued reference to the evidence which operates in the background. To use an analogy, credences are not sufficient statistics with respect to updating, inference, and decision making. What is remarkable about Joyce's response to DILATION and OBTUSE is that Joyce recognizes that instates are not sufficient statistics either. But this means that they fail at the double task which has been imposed on them: to represent both epistemic uncertainty and relevant features of the evidence.

In the following, I will provide a few examples where it becomes clear that instates have difficulty representing uncertainty because they are tangled in a double task which they cannot fulfill.

Example 11: Aggregating Expert Opinion. Blake has no information whether it will rain tomorrow (R) or not except the predictions of two weather forecasters. One of them forecasts 0.3 on channel GPY, the other 0.6 on channel QCT. Blake considers the QCT forecaster to be significantly more reliable, based on past experience.

An instate corresponding to this situation may be $[0.3, 0.6]$ (see Walley, 1991, 214), but it will have a difficult time representing the difference in reliability of the experts. We could try $[0.2, 0.8]$ (since the greater reliability of QCT suggests that the chance of rain tomorrow is higher rather than lower) or $[0.1, 0.7]$ (since the greater reliability of QCT suggests that its estimate is more precise), but it remains obscure what the criteria are.

A sharp credence of $P(R) = 0.53$, for example, does the right thing. Such a credence says nothing about any beliefs that the objective chance is restricted to a subset of the unit interval, but it accurately reflects the degree of uncertainty that the rational agent has over the various possibilities. Beliefs about objective chances make little sense in many situations where we have credences, since it is doubtful even in the case of rain tomorrow that there is an urn of nature from which balls are drawn. What is really at play is a complex interaction between epistemic states (for example, experts evaluating meteorological data) and the evidence which influences them.

As we will see in the next example, it is an advantage of sharp credences that they do not exclude objective chances, even extreme ones, because they express partial belief and do not suggest, as instates do for *Bool-A*, that there is full belief knowledge that the objective chance is a member of a proper subset of the possibilities (for an example of a crude version of indeterminacy that reduces partial beliefs to full beliefs see Levi, 1981, 540, "inference derives credal probability from knowledge of the chances of possible outcomes"; or Kaplan, 2010, 45, "you should rule out all and only the assignments the evidence warrants your regarding as too high or too low, and you should remain in suspense between those degree of confidence assignments that are left").

Example 12: Precise Credences. Logan's credence for rain tomorrow, based on the expert opinion of channel GPY and channel QCT (she has no other information) is 0.53. Is it reasonable for Logan, considering how little evidence she has, to reject the belief that the chance of rain tomorrow is 0.52 or 0.54; or to prefer a 52.9 cent bet on rain to a 47.1 cent bet on no rain?

The first question in example 12 is as confused as the *Bool-A* confusion found in example 4 discussed earlier. As for the second question in example 12: why would we prefer a 52.9 cent bet on rain to a 47.1 cent bet on no rain, given that we do not possess the power of discrimination between these two bets? It is important not to confuse the claim that it is reasonable to hold both X and Y with the claim that it is reasonable to hold either X (without Y) or Y (without X). It is the reasonableness of holding X and Y concurrently that is controversial, not the reasonableness of holding Y (without holding X) when it is reasonable to hold X.

Let $U(S, Z, t)$ mean "it is rational for S to believe Z at time t." Then U is exportable (see Rinard, 2015, 6) if and only if $U(S, X, t)$ and $U(S, Y, t)$ imply $U(S, X \wedge Y, t)$. Beliefs somehow grounded in subjectivity, such as beliefs about etiquette or colour perception are counter-examples for the exportability of U. Vagueness also gives us cases of non-exportability. Rinard considers the connection between vagueness and indeterminacy to be an argument in favour of indeterminacy.

My argument is that non-exportability blunts an argument against the Laplacean position. In a moment, I will talk about anti-luminosity, the fact that a rational agent may not be able to distinguish psychologically between a 54.9 cent bet on an event and a 45.1 bet on its negation, when her sharp credence is 0.55. She must reject one of them not to incur sure loss, so proponents of indeterminacy suggest that she choose one of them freely without being constrained by her credal state or reject both of them. I claim that a sharp credence will make a recommendation between the two so that only one of the bets is rational given her particular credence, but that does not mean that another sharp credence which would give a different recommendation may not also be rational for her to have. Partial beliefs are non-exportable.

The answer to the second question in example 12 ties in with the issue of incomplete preference structure referred to above as motivation (B) for instates (see page 6).

> It hardly seems a requirement of rationality that belief be precise (and preferences complete); surely imprecise belief (and corresponding incomplete preferences) are at least rationally permissible. (Bradley and Steele, 2014, 1288, for a similar sentiment see Kaplan, 2010, 44.)

The development of representation theorems beginning with Frank Ramsey (followed by increasingly more compelling representation theorems in Savage, 1954; and Jeffrey, 1965; and numerous other variants in contemporary literature) bases probability and utility functions of an agent on her preferences, not the other way around. Once completeness as an axiom for the preferences of an agent is jettisoned, indeterminacy follows automatically. Indeterminacy may thus be a natural consequence of the proper way to think about credences in terms of the preferences that they represent.

In response, preferences may very well logically and psychologically precede an agent's probability and utility functions, but that does not mean that we cannot inform the axioms we use for a rational agent's preferences by undesirable consequences downstream. Completeness may sound like an unreasonable imposition at the outset, but if incompleteness has unwelcome consequences for credences downstream, it is not illegitimate to revisit the issue. Timothy Williamson goes through this exercise with vague concepts, showing that all upstream logical solutions to the problem fail and that it has to be solved downstream with an epistemic solution (see Williamson, 1996). Vague concepts, like sharp credences, are sharply bounded, but not in a way that is luminous to the agent (for anti-luminosity see chapter 4 in Williamson, 2000). Anti-luminosity answers the second question in example 12: the rational agent prefers the 52.9 cent bet on rain to a 47.1 cent bet on no rain based on her sharp credence without being in a position to have this preference necessarily or have it based on physical or psychological ability (for the analogous claim about knowledge see Williamson, 2000, 95).

In a way, advocates of indeterminacy have solved this problem for us. There is strong agreement among most of them that the issue of determinacy for credences is not an issue of elicitation (sometimes the term 'indeterminacy' is used instead of 'imprecision' to underline this difference; see Levi, 1985, 395). The appeal of preferences is that we can elicit them more easily than assessments of probability and utility functions. The indeterminacy issue has been raised to the probability level (or moved downstream) by indeterminacy advocates themselves who feel justifiably uncomfortable with an interpretation of their theory in behaviourist terms. So it shall be solved there, and this paper makes an appeal to reject indeterminacy on this level. The solution then has to be carried upstream (or lowered to the logically more basic level of preferences), where we recognize that completeness for preferences is after all a desirable axiom for rationality and "perfectly rational agents always have perfectly sharp probabilities" (Elga, 2010, 1). When Levi talks about indeterminacy, it also proceeds from the level of probability judgment to preferences, not the other way around (see Levi, 1981, 533).

Example 13: Monkey-Filled Urns. Let urn A contain 4 balls, two red and two yellow. A monkey randomly fills urn B from urn A with two balls. We draw from urn B (a precursor to this example is in Jaynes and Bretthorst, 2003, 160).

One reasonable sharp credence of drawing a red ball is 0.5, following Lewis' summation formula for the different combinations of balls in urn B and symmetry considerations. This solution is more intuitive in terms of further inference, decision making, and betting behaviour than a credal state of $\{0, 1/2, 1\}$ or $[0, 1]$ (depending on the convexity requirement), since this instate would licence an exorbitant bet in favour of one colour, for example one that costs \$9,999 and pays \$10,000 if red is drawn and nothing if yellow is drawn.

How a bet is licenced is different on various Boolean accounts. Rinard, for example, contrasts a moderate account with a liberal account (see Rinard, 2015, 7). According to the liberal account, the \$9,999 bet is licenced, whereas according to the moderate account, it is only indeterminate whether the bet is licenced. The moderate account does not take away from the force of example 13, where it should be determinate that a \$9,999 bet is not licenced.

To conclude, a Boolean in the light of Joyce's two Augustinian concessions has three alternatives, of which I favour the third: (a) to find fault with Joyce's reasoning as he makes those concessions; (b) to think (as Joyce presumably does) that the concessions are compatible with the promises of Booleans, such as INTERN, INCOMP, and INFORM, to solve prima facie problems of sharp credences; or (c) to abandon the Boolean position because (AC1), (AC2), and an array of examples in which sharp credences are conceptually and pragmatically more appealing show that the initial promise of the Boolean position is not fulfilled.

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