Faculty of Science, Technology, Engineering and Mathematics
M836 Coding theory

M836

TMA 01

Covers Units 1-4 **See module website for the cut-off date.**

TMA 01 is a formative assignment that does not count towards your final grade. However, in order to pass the module, you are required to submit at least three TMAs and score at least 30% on at least three of the TMAs submitted.

The substitution rule does not apply to this module.

To be sure of passing this module, you need to achieve a score of at least 50% in the examination and score at least 30% on three out of four TMAs. The final rank score will be completely determined by your overall exam score (OES).

You are strongly encouraged to submit your assignments online using the electronic TMA/EMA service. If you cannot submit an assignment electronically, then, with permission from your tutor, you may submit it by post. Please read the instructions under the 'Assessment' tab of the module website before starting your assignments. The assignment cut-off dates can be found on the module website.

Each TMA of this module examines the work contained in the block to which it relates, but may require knowledge of material in previous blocks. Within each TMA, there is not a one-to-one correspondence between questions and the units forming that block. The number of marks assigned to each part of a question is given in the right-hand margin. There are 100 marks available for each assignment. A high standard of presentation is required. Answers should be written in good English with appropriate explanations. There is no need to word-process your solutions; legible handwriting is perfectly acceptable.

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2.1

Question 1 – 25 marks

(a) Check whether the following are ISBN-10s

(i) 0303392611,

(ii) 0099417561,

(iii) 1584885086.

For the ones that are not ISBN-10s, and assuming that a single transposition error has been made in adjacent positions, determine if possible the correct ISBN-10s. *[8]*

(b) A transmission error that leaves a codeword with undecodeable symbols is called an *erasure error*. For example, if the codeword 130152 is received as ?301?2, where ? denotes an undecodeable symbol, then we say that two erasures have occurred. A code that can correct all instances of e erasures (assuming that no other errors are present) is called e*-erasure correcting*. Prove that the ISBN-10 code is 1-erasure correcting, but not 2-erasure correcting. Given the received vector 04862?263X, where ? denotes an undecodeable symbol, determine the correct ISBN-10. *[5]*

(c) The ISBN-10 code may be modified by appending to each codeword $\mathbf{x} = x_1 x_2 \cdots x_{10}$ an eleventh digit x_{11} given by $x_{11} = \sum_{i=1}^{10} x_i \pmod{11}$. Determine the minimum distance of the resulting code. If possible, correct the erasure errors in the following received vectors

(i) 297?2357099,

(ii) 2159?7?3011,

(iii) 42??325?897. *[9]*

(d) Determine the maximum value of e for which the modified ISBN-10 code is e-erasure correcting. *[3]*

Question 2 – 25 marks

(a) Determine which of the following codes are linear over the alphabet indicated

(i) $C_1 = \{00000, 11001, 10011, 01010\}$ over Z_2,

(ii) $C_2 = \{000, 001, 010, 100\}$ over Z_2,

(iii) $C_3 = \{00000, 11001, 10011, 01010\}$ over Z_3. [5]

(b) Prove that the binary linear codes with generator matrices

$$G_1 = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \quad \text{and} \quad G_2 = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

are equivalent as linear codes. [5]

(c) The code C is a ternary linear code with generator matrix

$$G = \begin{pmatrix} 1 & 2 & 0 & 1 & 2 \\ 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 2 & 2 \end{pmatrix}.$$

List the codewords of C. [5]

(d) Determine a parity-check matrix in standard form for the code C given in part (c). [5]

(e) (i) Suppose that D is a binary linear code having a generator matrix in which all the rows are of even weight. Prove that all the codewords of D have even weight.

(ii) Suppose that E is a binary linear code having a generator matrix in which at least one row has odd weight. Prove that precisely half of the codewords of E have odd weight. [5]

Question 3 – 25 marks

The binary linear code C has generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

(a) Write down the codewords of C. Construct a Slepian standard array for C and hence determine the number of coset leaders of weight i for $i = 0, 1, \ldots, 6$. If codewords are transmitted over a binary symmetric channel that has symbol error probability p, determine the word error probability $P_{err}(C)$, *assuming that decoding is by means of the standard array.* [*8*]

(b) Suppose now that codewords are transmitted over a binary *non-symmetric* channel, where the probability of 1 being received when 0 was transmitted is r, and the probability of 0 being received when 1 was transmitted is s. *Using the standard array decoding method*, show that the probability of a received vector being correctly decoded as 000000 is given by

$$(1 - r)^4(1 + 4r - 4r^2).$$

Similarly, show that the probability of a received vector being correctly decoded as each of the codewords of weight 3 is given by

$$(1 - r)^2(1 - s)^2(1 + 2r + 2s - 4rs).$$

For each of the remaining codewords, determine the corresponding probability of a received vector being correctly decoded. [There is no need to simplify your expressions.] [*12*]

(c) Applying the *incomplete decoding method* to C,determine how you would deal with each of the following received vectors

(i) 111111,

(ii) 110011,

(iii) 111101. [*5*]

Question 4 – 25 marks

The code D is a linear $[6,4]$-code over $GF(7) = \{0, 1, \ldots, 6\}$, the codewords $\mathbf{x} = x_1x_2x_3x_4x_5x_6$ of which are defined by the two parity-check equations

$$\sum_{i=1}^{6} x_i \equiv 0 \pmod 7 \quad \text{and} \quad \sum_{i=1}^{6} i x_i \equiv 0 \pmod 7.$$

(a) Describe an algorithm that will correct any single transmission error in a codeword of D and detect any transposition error involving two digits of a codeword of D. Apply your algorithm to the received vectors

(i) 113235,

(ii) 625152. [5]

(b) Explain carefully why the number of codewords of D that have the symbol "6" in any specified coordinate position is 7^3. Then explain why the number of codewords of D that have the symbol "6" in any specified pair of coordinate positions is 7^2, why the number of codewords of D that have the symbol "6" in any specified triple of coordinate positions is 7^1, and why the number of codewords of D that have the symbol "6" in any specified quadruple of coordinate positions is 7^0. [5]

(c) Use the two parity-check equations to prove that no codeword of D contains five or six "6"s. [You should produce a theoretical argument and not simply check all the codewords!] [5]

(d) Using your answers to parts (b) and (c), explain why the number of codewords of D that do not contain the symbol "6" is

$$7^4 - 6 \cdot 7^3 + 15 \cdot 7^2 - 20 \cdot 7^1 + 15 = 953.$$ [5]

(e) Apply a similar argument to that of part (d) to prove that the number of codewords of the $[10, 8]$ decimal code C defined on page 76 of **H** is 82 644 629. [5]