Q 1.

(a)

(b)

(c)

Q 2.

(a)

(b)

(c)

Q 3.

(a)

(b)

Q 4.

Q 5.

Q 6.

(a) The model of tumour growth under radiotherapy from time $t = 0$ to $t = t_1 > 0$ is given as

$$\frac{\mathrm{d}C}{\mathrm{d}t} = -C \log\left(\frac{C}{C_{max}}\right) - \frac{DC}{1+D}, \tag{6.1}$$

where, at time t, $C(t)$ is the size of the tumour, $C_{max} > 0$ is the maximum size of the tumour, a constant, so that $0 < C(t) \leq C_{max}$, and $D(t) \geq 0$ is the rate at which the drug is administered.

Now, making a change of variable

$$x = \log\left(\frac{C}{C_{max}}\right),$$

then,

$$\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= \frac{\mathrm{d}C}{\mathrm{d}t} \cdot \frac{\mathrm{d}x}{\mathrm{d}C}, \\
\frac{\mathrm{d}x}{\mathrm{d}t} &= \frac{\mathrm{d}C}{\mathrm{d}t} \cdot \frac{1}{C}, \\
\frac{\mathrm{d}x}{\mathrm{d}t} &= \left[\cancelto{-1}{-C} \log\left(\frac{C}{C_{max}}\right) - \frac{D\cancelto{1}{C}}{1+D}\right] \cdot \frac{1}{\cancelto{1}{C}}, \\
\frac{\mathrm{d}x}{\mathrm{d}t} &= -\log\left(\frac{C}{C_{max}}\right) - \frac{D}{1+D}, \\
\text{thus,} \quad \frac{\mathrm{d}x}{\mathrm{d}t} &= -x - \frac{D}{1+D}, \qquad \text{where } -\infty < x \leq 0.
\end{aligned}$$

(b)

(c)

(d)