Q 1.

(a)

(b)

Q 2.

Q 3.

(a)

(b)

(c)

Q 4.

(a)

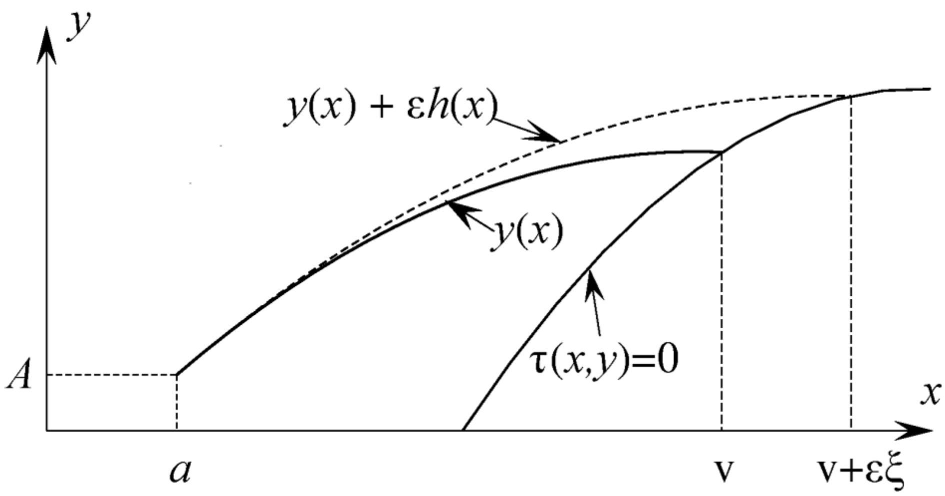

Figure 1: Diagram showing the stationary path (solid line) and a varied path (dashed line) for a problem in which the left-hand end is fixed, but the other end is free to move along the line defined by $\tau(x, y) = 0$.

Given the perturbed path

$$y_\epsilon(x) = y(x) + \epsilon h(x), \tag{4.1}$$

the Taylor series to the first-order of (4.1) at point $x = v$ (see figure 1) is given in (4.2).

The point $x = v$ is known as the point of expansion. HB p8.

$$y_\epsilon(x) = (y(v) + \epsilon h(v)) + (y'(v) + \epsilon h'(v))\,(x - v) + \mathcal{O}\left((x - v)^2\right). \tag{4.2}$$

Now, determining the value of (4.2) at $v_\epsilon = v + \epsilon\xi + \mathcal{O}\left(\epsilon^2\right)$ where v_ϵ is the perturbed value of v:

$$\begin{aligned} y(v_\epsilon) = {} & y(v) + \epsilon h(v) \\ & + \left(\cancel{v} + \epsilon\xi + \mathcal{O}\left(\epsilon^2\right) \cancel{- v}\right)(y'(v) + \epsilon h'(v)) \\ & + \mathcal{O}\left(\left(\cancel{v} + \epsilon\xi + \mathcal{O}\left(\epsilon^2\right) \cancel{- v}\right)^2\right), \end{aligned}$$

$$\begin{aligned} y(v_\epsilon) = {} & y(v) + \epsilon h(v) \\ & + \left(\epsilon\xi + \mathcal{O}\left(\epsilon^2\right)\right)(y'(v) + \epsilon h'(v)) \\ & + \mathcal{O}\left(\left(\epsilon\xi + \mathcal{O}\left(\epsilon^2\right)\right)^2\right), \end{aligned}$$

$$
\begin{aligned}
y(v_\epsilon) = y(v) &+ \epsilon h(v) \\
&+ \epsilon\xi\left(y'(v) + \epsilon h'(v)\right) \\
&+ \mathcal{O}\left(\epsilon^2\right)\left(y'(v) + \epsilon h'(v)\right) \\
&+ \mathcal{O}\left(\left(\epsilon\xi + \mathcal{O}\left(\epsilon^2\right)\right)^2\right),
\end{aligned}
$$

$$
\begin{aligned}
y(v_\epsilon) = &\; y(v) + \epsilon\left(h(v) + \xi y'(v)\right) \\
&+ \underbrace{\epsilon^2\xi h'(v) + \mathcal{O}\left(\epsilon^2\right)\left(y'(v) + \epsilon h'(v)\right) + \mathcal{O}\left(\left(\epsilon\xi + \mathcal{O}\left(\epsilon^2\right)\right)^2\right)}_{\text{These are all second-order terms in } \epsilon.}.
\end{aligned}
$$

Thus,

$$
y_\epsilon(v_\epsilon) = y(v) + \epsilon\left(h(v) + \xi y'(v)\right) + \mathcal{O}\left(\epsilon^2\right), \tag{4.3}
$$

as required.

(b)

(c)

Q 5.

(a)

(b)

(c)

(d)