

1 Definitions

1.1 DEFINITION (Graph - Abstract). An *Abstract Graph* Γ is a set (V, E, ∂, i) where V and E are non-empty sets and $\partial : E \rightarrow V$ and $i : E \rightarrow E$ are functions satisfying $i^2(e) = e$ and $i(e) \neq e$ for all $e \in E$.

We set $o(e) := \partial(e)$ and $t(e) := (\partial \circ i)(e)$ for origin and terminal vertices and put $\bar{e} := i(e)$. We call the orbits of i the *undirected edges* of G .

1.2 DEFINITION (Collapse Move). If we have a group G acting on a tree T with an edge e such that $G_e = G_v$ where $v = o(e)$ and $o(e)$ and $t(e)$ are in different G -orbits then we can form T_e a new tree with $V(T_e) = V(T) \setminus Gt(e)$ and $E(T_e) = E(T) \setminus Ge$. Then for all edges f with $o(f) = t(ge)$ for some $g \in G$ we define $o(f) = gv$ in T_e .

(Alternatively, if q was the map that paired the initial and terminal vertices of ge and left the others alone then we could define the new tree by taking $E(T_e) := E(T) \setminus Ge$ and $V(T_e) := q(V)$ with attaching map $q \circ \partial$.

If \mathcal{G} is a graph of groups decomposition of G with $\varphi_e : G_e \rightarrow G_{\partial e}$ an isomorphism then define \mathcal{G}_e by removing e , \bar{e} and ∂e and for every edge f with $\partial(f) = \partial(e)$ replace φ_f with $\varphi_{\bar{e}} \circ \varphi_e^{-1} \circ \varphi_f$ and set $\partial f = \partial \bar{e}$. This corresponds to taking the edge e from the tree description above and folding $[e]$ in the graph.

1.3 DEFINITION (Fold). Given a tree with $\partial e = \partial f$ identify e with f as well as \bar{e} and \bar{f} and $\bar{\partial e}$ with $\bar{\partial f}$ and do so equivariantly. (In \mathcal{G} this corresponds to moves of type A or B with subtype I, II, or III.)

CLAIM. A reduced not locally finite tree remains not locally finite after folding.
