

Arguing that the S_{ij} have the right properties.

There's an old lemma about $N_R(W)$ being CCP if W CCP
we may not need this any more

T_1, T_2, T_3

Transverse Lemma
 X_{12}, X_{13}, X_{23} compact VH cube complexes
We get embeddings X_{ij} into T_{ij} **Wise**

By construction but
some writing is needed

S_{ij} is an invariant, cocompact, subcomplex, CCP
sitting in the product of three trees.

I want to think about to make S_{12} again.

Think of X_{12} as sitting in $T_1 \times T_2$

$X_{12} \sim \text{----} \text{Guirardel map } g \text{ ----} \rightarrow T_3$

$X_{12} \sim + = \text{coning off } X_{12} \text{ a 2-complex}$

Make a map from $X_{12} \sim + \text{-----} f_{12} \text{-----} \rightarrow T_1 \times T_2 \times T_3$

$\text{Im } f_{12}$ is contained in $(\text{cocompact}) \times T_3$

On $X_{12} \sim$ inside $X_{12} \sim +$ f_{12} is inclusion

Should be "Graph(g)" [note 1]

$\text{Im } g$ is in $X_{12} \sim \times T_3$, and $X_{12} \sim$ is cocompact

$\text{Im } f_{12}$ is in R -nbhd of $\text{Im } g$, some R

so $\text{Im } f_{12}$ is in $N_R(X_{12} \sim) \times T_3$, which is
cocompact $\times T_3$

By lemma 3.2 in original GGT file
Guir \Rightarrow fibers conn
lemma \Rightarrow planes are homeo fibers

Need "compactness in factors" to get that filling
in the x , then y , then z directions remains cocompact. ✓

Consider S_{23} ... fill in x direction

Since S_{23} in $T_1 \times (\text{cocompact})$ so $S_{23}x$ is cocompact by our lemma
and CCP by another lemma. ~~We want $S_{23}x$ to be in an R -nbhd of S_{23} (true because
we added a finite bunch)~~ ✓

At this point, due to local finiteness, $S_{23}x$ is cocompact and within and R -nbhd of S_{23}

Need $S_{23}x$ contained in $(\text{cocompact}) \times T_2$

Know S_{13} is in $(\text{cocompact}) \times T_2$ and S_{23} is in a nbhd of S_{13} ,
and therefore $S_{23}x$ is in a nbhd of S_{13} . ✓

Try upgrading to lemma: Should be true for any G -invariant
cocompact subcomplexes

Hence, $S_{23}x$ in $(\text{cocompact}) \times T_2$

$S_{23}xy$ is CCP, cocompact (by line above and our lemma), and it's a nbhd of $S_{23}x$
reapply the lemmas

$S_{23}xyz$ is cocompact, and 1-dim fiber convex, subcomplex of $T_1 \times T_2 \times T_3$

by switching lemma aka "Part 1"

Set the core $C = S_{23}xyz$.

In the end, it's true and we might need it:
 C is finite hausdorff distance from S_{12} ($S_{12}x$, $S_{12}xy$, S_{23} , $S_{23}x$, etc.)