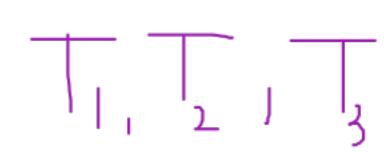
## Arguing that the Sij have the right properties.

There's an old lemma about N\_R(W) being CCP if W CCP we may not need this any more



X12, X13, X23 compact VH cube complexes

We get embeddings Xij into Tij Wise

S\_ij is an invariant, cocompact, subcomplex, CCP sitting in the product of three trees.

I want to think about to make S\_12 again.

Think of X12 as sitting in T1 x T2

X12~ ---- guirardel map g ----> T3

 $X12 \sim + = coning off X1i2 a 2-complex$ 

Make a map from X12~+ ----- f12 ----> T1 x T2 x T3n f12 is in R-nbhd of Im g, some R

Im f12 is contained in (cocompact) x T3

On X12~ inside X12~+ f12 is inclusion

By construction but some writing is needed

Should be "Graph(g)" [note 1]

Im g is in X12~ x T3, and X12~ is cocompact

Tan f12 is in D mind of im a some D

so lm f12 is in N\_R(X12~) x T3, which is cocompact x T3

Need "compactness in factors" to get that filling in the x, then y, then z directions remains cocompact.

Consider S23 ... fill in x direction

Since S23 in T1 x (cocompact) so S23x is cocompact by our lemma and CCP by another lemma. We want S23x to be in an R-nbd of S23 (true because

we added a finite bunch.

At this point, due to local finiteness, S23x is cocompact and within and R-nbhd of S23

Need S23x contained in (cocompact) x T2

Know S13 is in (cocompact) x T2 and S23 is in a nbhd of S13, and therefore S23x is in a nbd of S13.

Hence, S23x in (cocompact) x T2

cocompact subcomplexes

reapply the lemmas

S23xy is CCP, cocompact (by line above and our lemma), and it's a nbd of S23x

S23xyz is cocompact, and 1-dim fiber convex, subcomplex of T1xT2xT3

by switching lemma aka "Part 1"

Try upgrading to lemma: Should be true for any G-invariant

Set the core C = S23xyz.

In the end, it's true and we might need it: C is finite hausdorff distance from S12 (S12x, S12xy, S23, S23x, etc.)

By lemma 3.2 in original GGT file Guir => fibers conn lemma => planes are homeo fibers