1 Background

1. cohomological and geometric dimension
2. group splittings
3. Danny Wise VH-complexes
4. Guirardel Core

2 Problem Statement

Statement for Dimension 2:

Best case scenario: Given $N \geq 2$ tree actions (i.e. splittings) that are pairwise not in the same deformation space obtain a generalized-VH complex, called the core (e.g. VHD-complex for the case of three pairwise inequivalent actions) by deleting light quadrants from the tree product.

The core is generalized-VH, simply connected, and cocompact. The quotient modulo G is a compact, generalized-VH, graph-of-spaces decomposition of G along its (two-sided) hyperplanes. (i.e. in particular edge maps are injective on fundamental groups) The hyperplanes are also generalized-VH complexes. Using a theorem of Bieri we get that cd G is strictly greater than that of it's hyperplanes. Continuing inductively, we get that G has cd $\geq N$.

3 Progress

1. Freeness of product action:

   We're going to walk through the setup in the two dimensional case. Let G be a group with two locally finite type FP actions T_1 and T_2 lying in different deformation spaces. Consider the diagonal action on $T_1 \times T_2$, we will show that this action is free. We will show that for a given $g \in G$ that if g fixed a point in one tree then it doesn't fix a point in the other. By symmetry suppose g fixes a point in T_1. Consider G_v where v lies in T_1, as a subgroup of G it also acts on T_2. Suppose that G_v acts without a global fixed point on T_2 then

2. Hyperplane separability: Note that hyperplanes in the tree product are dual to edges in its one-skeleton and we have a diagonal action without edge inversions.

3. Using Bieri: If a group G has geometric dimension n and acts on a locally finite tree with FP_∞ (i.e. finite dimensional $K(G, 1)$) stabilizers then the stabilizers have geometric dimension $\leq n - 1$.

4. Using Howsen

5. Using "Quadrant-Convex" lemma of Guirardel

6. Using injectivity lemma for maps of NPC complexes
7. other: relevant facts on locally finite trees

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