Arguing that the Sij have the right properties.

T_1, T_2, T_3

Transverse Lemma

X12, X13, X23 compact VH cube complexes

We get embeddings Xij into Tij   Wise

S_ij is an invariant, cocompact, subcomplex, CCP sitting in the product of three trees.

By construction but some writing is needed

I want to think about to make S_12 again.

Think of X12 as sitting in T1 x T2

X12~ ---- guirardel map g ----> T3

X12~+ = coning off X12 a 2-complex

Make a map from X12~+ ------- f12 -------> T1 x T2 x T3

Im f12 is contained in (cocompact) x T3

On X12~ inside X12~+ f12 _is_ inclusion

Im g is in X12~ x T3, and X12~ is cocompact

Im f12 is in R-nbhd of Im g, some R

so Im f12 is in N_R(X12~) x T3, which is cocompact x T3

There's an old lemma about N_R(W) being CCP if W CCP
we may not need this any more

Need "compactness in factors" to get that filling in the x, then y, then z directions remains cocompact.

Consider S23 ... fill in x direction
Since S23 in T1 x (cocompact) so S23x is cocompact by our lemma
and CCP by another lemma. ~~We want S23x to be in an R-nbd of S23 (true because we added a finite bunch)~~

At this point, due to local finiteness, S23x is cocompact and within and R-nbhd of S23

Need S23x contained in (cocompact) x T2

Know S13 is in (cocompact) x T2 and S23 is in a nbhd of S13,
and therefore S23x is in a nbd of S13.

Try upgrading to lemma: Should be true for any G-invariant cocompact subcomplexes

Hence, S23x in (cocompact) x T2

reapply the lemmas

S23xy is CCP, cocompact (by line above and our lemma), and it's a nbd of S23x

S23xyz is cocompact, and 1-dim fiber convex, subcomplex of T1xT2xT3

by switching lemma aka "Part 1"

Set the core C = S23xyz.

In the end, it's true and we might need it:
C is finite hausdorff distance from S12 (S12x, S12xy, S23, S23x, etc.)

By lemma 3.2 in original GGT file
Guir => fibers conn
lemma => planes are homeo fibers