Variational Auto-Encoders

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"You use a glass mirror to see your face; you use works of art to see your soul" – George Bernard Shaw

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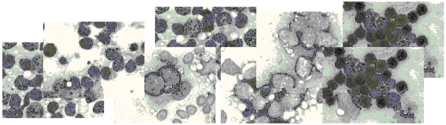

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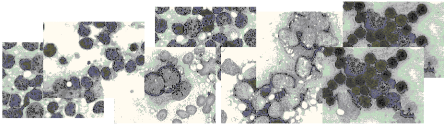

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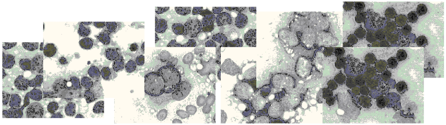

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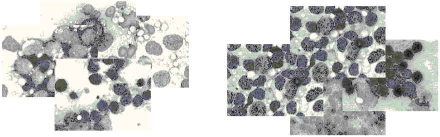

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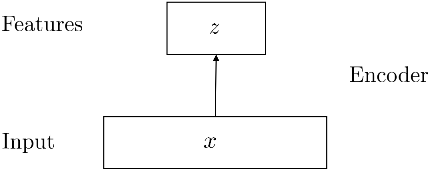

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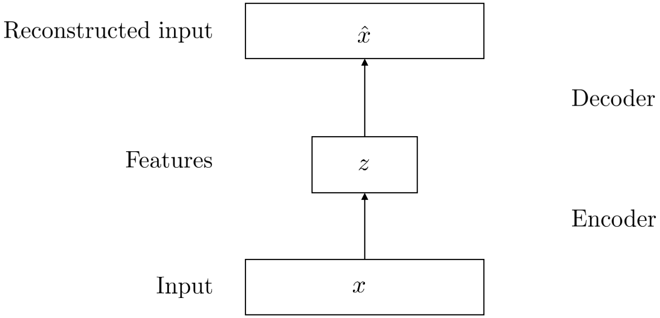

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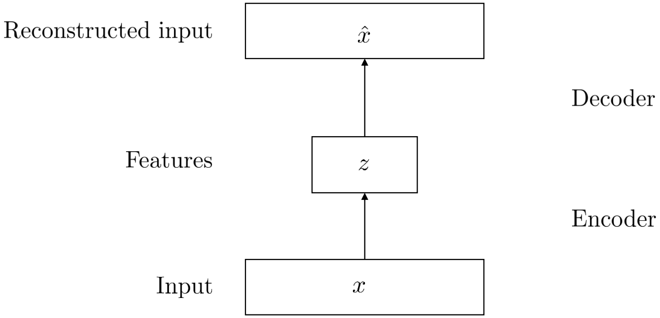

- Sometimes, when very little labeled data is available, one can use features obtained using this auto-encoder approach and a following layer (trained using the limited data that is available) for pattern classification

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  - This forces the space of latent variables to be continuous
- Decoding will require sampling from each latent variable distribution to generate a vector which can be used by the decoder

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- So, we approximate $p(z|x)$ with a tractable distribution, $q(z|x)$
- To be able to make inferences, $q(z|x)$ must be close to $p(z|x)$. So, we can choose $q(z|x)$ to minimize,
$$\min \mathrm{KL}(q(z|x) \parallel p(z|x)) \tag{3}$$

$$
\begin{aligned}
\mathrm{KL}(q(z|x) \parallel p(z|x)) &= -\sum q(z|x) \log \frac{p(z|x)}{q(z|x)} \\
&= -\sum q(z|x) \log \frac{\frac{p(x,z)}{p(x)}}{q(z|x)} \\
&= -\sum q(z|x) \left[\log \frac{p(x,z)}{q(z|x)} - \log p(x)\right] \\
&= -\sum q(z|x) \log \frac{p(x,z)}{q(z|x)} + \sum q(z|x) \log p(x) \\
&= -\sum q(z|x) \log \frac{p(x,z)}{q(z|x)} + \log p(x) \qquad (4)
\end{aligned}
$$

So,

$$
\log p(x) = \mathrm{KL}(q(z|x) \parallel p(z|x)) + \underbrace{\sum q(z|x) \log \frac{p(x,z)}{q(z|x)}}_{\text{Variational Lower Bound (VLB)}} \qquad (5)
$$

$p(x)$ is a constant and minimizing $\mathrm{KL}(\cdot)$ is the same as maximizing VLB. Since $\mathrm{KL}(\cdot)$ is non-negative, VLB cannot be larger than $\log p(x)$

$$
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\sum q(z|x) \log \frac{p(x,z)}{q(z|x)} &= \sum q(z|x) \log \frac{p(x|z)\, p(z)}{q(z|x)} \\
&= \sum q(z|x) \left[\log p(x|z) + \log \frac{p(z)}{q(z|x)}\right] \\
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&= E_{q(z|x)} \log p(x|z) - \mathrm{KL}(q(z|x) \parallel p(z)) \qquad (6)
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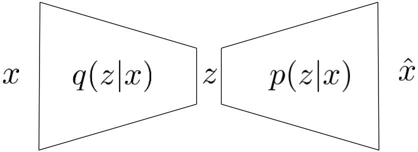

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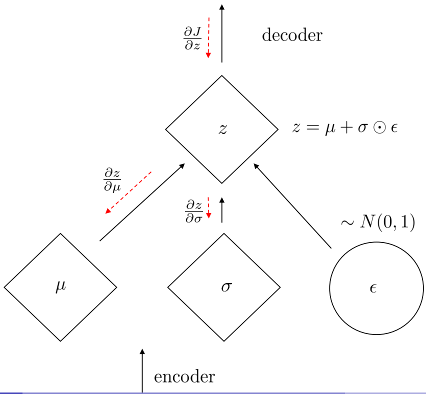

Demo

https://www.siarez.com/projects/variational-autoencoder

http://vdumoulin.github.io/morphing_faces/