P&S Assignment 3

Divij Singh

04/10/17

1 Q1

1.1

We know that $f(\alpha, \beta) \to R$
We take $S = a_1, a_2, ...a_n$
Since we know that $X \in S$, we also know that $\alpha \leq X \leq \beta$
$E[f(X)]$ can also be written as $\Sigma_{i=\alpha}^{\beta} S_i f(X_i)$, where $i \in (\alpha, \beta)$
Consequently, $f(E[X])$ can also be written as $f(\Sigma_{i=\alpha}^{\beta} S_i X_i)$

Thus, by applying the fact from Definition 1, we know that
$f(\Sigma_{i=\alpha}^{\beta} S_i X_i) \leq \Sigma_{i=\alpha}^{\beta} S_i f(X_i)$

I.e, $f(E[X]) \leq E[f(X)]$

1.2

Yeah... no, I don't know this.

1.3

$E[X] \geq e^{E[\log X]}$
$E[X]$ can also be written as $\Sigma_{z \in S} a P_x(z)$
$e^{E[\log X]}$ can also be written as $e^{\Sigma_{z \in S} log(z) P_x(z)}$

$\Sigma_{z \in S} z P_x(z) = (z_1 P_x(z_1) + z_2 P_x(z_2) ... a_k P_x(z_k)) = \frac{z_1 + z_2 + z_3 ... z_k}{k}$ (because $P_x = \frac{1}{k}$)

Similarly, $e^{\Sigma_{z \in S} log(z) P_x(z)} = e^{log(z_1) P_x(z_1) ... log(z_k) P_x(z_k)} = e^{\frac{\log(z_1 * z_2 ... * z_k)}{k}}$ (due to properties of logarithms)
Which becomes $\frac{(z_1 * z_2 ... * z_k)}{k}$ (because $e^{\log x} = x$)
Which becomes $(\prod_{i=1}^{k} z_i)\frac{1}{k}$

Thus, $\frac{\Sigma_{i=1}^{k} z_i}{k} \geq (\prod_{i=1}^{k} z_i)\frac{1}{k}$