

- ① Rule of sum  
② Rule of product

- ① Drawing with repetitions  
② with/without order

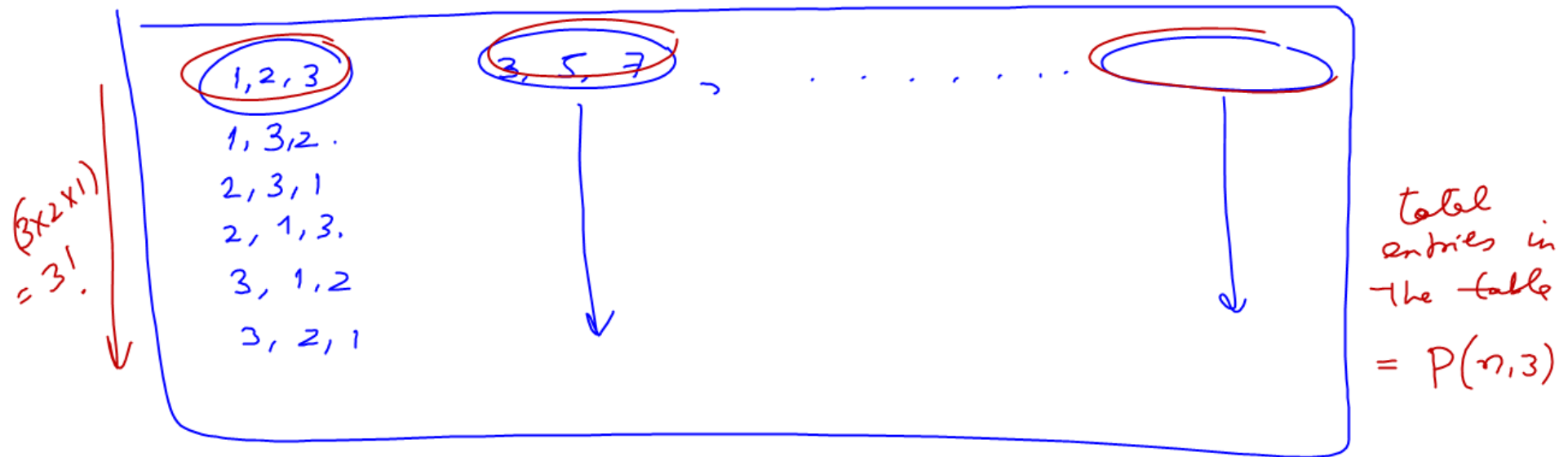
|          | order | repetitions |                             |
|----------|-------|-------------|-----------------------------|
| Case I   | ✓     | ✓           | $n^m$                       |
| Case II  | ✓     | ×           | $n(n-1)(n-2) \dots (n-m+1)$ |
| Case III | ×     | ×           |                             |
| Case IV  | ×     | ✓           |                             |

first second  
 $\overline{n} \times \overline{n} \times \dots$   
 $= P(n, m)$

$n$  objects  $\rightarrow$   $m$  to be drawn

$n$  objects  $\longrightarrow$   $m$  to be drawn  
 we are interested in "without order"

Let us consider, -- with order : (Example  $m=3$ )



The no of groups =  $\frac{P(n, 3)}{3!}$

Generalize: choosing  $m$  objects out of  $n$  in this fashion =  $\frac{P(n, m)}{m!}$

$$P(n, m) = \frac{n(n-1)(n-2) \dots (n-m+1)}{1}$$

$$= \frac{n(n-1)(n-2) \dots (n-m+1) \cdot \frac{(n-m)(n-m-1) \dots 2 \cdot 1}{(n-m)(n-m-1) \dots 2 \cdot 1}}{1}$$

$$= \frac{n!}{(n-m)!}$$

$\Rightarrow$

$$\frac{P(n, m)}{m!} = \frac{n!}{(n-m)! \cdot m!} = C(n, m)$$

Some things are repeating, others are not.

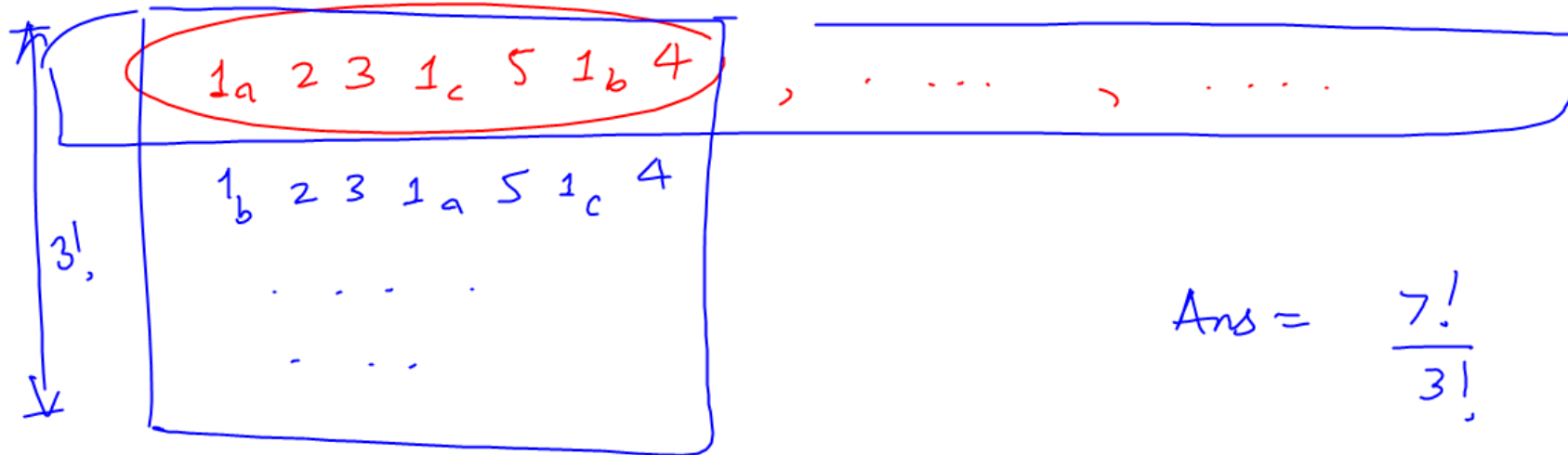
Given:

4, 1, 1, 2, 3, 4, 5

How many 7 digit nos can be formed ? (You can only use what is available)

$1_a, 1_b, 1_c, 2, 3, 4, 5.$

→ 7! numbers



$$\text{Ans} = \frac{7!}{3!}$$

1, 1, 1, 2, 2, 3, 4

How many diff no<sup>s</sup> of 7 digits

$$= \frac{7!}{3! 2! 1!}$$

[ m things of one type, n things of another type  
t things of a third type & r different

Total arrangements = 
$$\frac{(m+n+t+r)!}{m! n! t! r!}$$

Case IV

| Order | Repetitions |
|-------|-------------|
| X     | ✓           |

3 icecreams , 7 to be consumed

① ① ① ①  
Box 1

                      
Box 2

① ① ①  
Box 3

1 1 1 1 0

0 1 1 1

0 0 1 1 1 1 1 1

$$= \frac{(7+2)!}{7! 2!}$$

$$= {}^9C_2 = C(9,2)$$

$n$  types of object  $\rightarrow m$  to be drawn

$$\text{Zeros} = (n-1)$$

$$\text{ones} = m$$

$$\begin{aligned} \text{ans} &= \frac{(n-1+m)!}{(n-1)! m!} = \frac{(n+m-1)!}{(n-1)! m!} \\ &= \binom{n+m-1}{m} \end{aligned}$$

3 Types of icecreams , 7 to be chosen  
+ Each Type of icecream should be  
selected.

3 types , 4 to be chosen  $\left( + 3 \text{ to be added later} \right)$   
 $\downarrow$   
without restriction  
 $\downarrow$   
one of each type.  
 $= {}^6C_2$



Ex:

3 types of icecreams, 7 to be chosen

But the shop has only 5 of the first type

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Find the no. of non-negative integral solutions to the equation:

$$x + y + z = 7$$

where

$$0 \leq x \leq 5$$

$$0 \leq y \leq 7$$

$$0 \leq z \leq 7$$

$$x + y + z \equiv 7$$

$$0 \leq x \leq 5$$

$$0 \leq y$$

$$0 \leq z$$

No. of integral solutions

first  
consider this

$$p + q = 5$$

$$0 \leq p \leq 3$$

$$0 \leq q$$

$$\begin{aligned}
 & (\alpha^0 + \alpha^1 + \alpha^2 + \alpha^3) (\alpha^0 + \alpha^1 + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \dots) \\
 & = 1 + 2\alpha + (\dots)\alpha^2 + (\dots)\alpha^3 + (\dots)\alpha^4 + (\dots)\alpha^5
 \end{aligned}$$

$$(3, 2), (2, 3), (1, 4), (0, 5)$$

Coefficient of  $x^5$  = how many ways exist  
 to obtain  $x^5$  where  
 we take  $x^p$  from first bracket  
 $x^q$  from second bracket

Ex:

Team scores 15 runs in an over  
(no extra balls) . How many ways ?

Determine

---

$$x_1 + x_2 + \dots + x_6 = 15$$

Where  $x_i \in \{0, 1, 2, 3, 4, 5, 6\}$

---

Coeff of  $x^{15}$  in  $(1 + x + x^2 + \dots + x^6)^6$

Ex:

Scores allowed are 2, 4, 6 only on each ball  
to score 18. How many ways?

$$(x^2 + x^4 + x^6)^6 \longrightarrow \text{Coeff of } x^{18}.$$

---

2 types of things  $\rightarrow$  7 to be taken

$$: \quad x + y = 7 \quad 0 \leq x \leq 7$$

y

$$(1 + x + x^2 + \dots + x^7)^2 \longrightarrow \text{Coeff of } x^7.$$