Algorithms 2

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1 Proofs

1.1 Question 2

Let us start with $n = 1$
$3^2 = 9$ and $2^{3^n} + 1 = 9$ in which 3^{n+1} divides $2^{3^n} + 1$

Let us assume that this is true for $n = k$, that is that a 3^{k+1} divides $2^{3^{k+1}} + 1$

For $n = k + 1$, we get $3^{k+1+1} = 3^{k+1} * 3^1$
We also get $2^{3^{k+1}} + 1 = 2^{3^k * 3} + 1$
Which can be written as $2^{3^{k+1}} + 1^3$
$= (2^{3^k} + 1)(2^{3^k + 2} + 1 - 2^{3^k})$
$= (2^{3^k} + 1)(3(2^{3^k}) + 1)$

1.2 Question 3

Let us start from $n = 1$
$2^1 * 2^1 = 4$
If we remove one tile, we are left with 3 tiles in an L-shape. This can easily be filled by 1 tromino.

Let us assume this is true for $n = k$, that is for a $2^k * 2^k$ board, the board can be perfectly covered by trominoes after removing one square from it.

For $n = k + 1$, a 2^{k+1} can be split into four boards of 2^k
We know that we can cover a board of 2^k by removing one square. To cover these boards, we would remove one piece each, for a total of 4 pieces.
Of these 4, we can cut it down to 1 piece, by covering the other 3 with a tromino.

Thus, we can perfectly cover a board of $2^{k+1} * 2^{k+1}$
Thus proven.

2 Grey Code

2.1 Pseudocode

create an empty string ans

for n iterations, do
create blank string temp
for n iterations, do
append '1' to string
append temp to ans

for n iterations, do
create blank string temp
for n iterations, do
append '0' to string
append temp to ans
return ans

2.2 Program

```
ans=''
for i in range(n):
        term=''
        for j in range(n):
                temp=termp+'1'
        ans=ans+temp

for i in range(n):
        term=''
        for j in range(n):
                temp=termp+'0'
        ans=ans+temp
return ans
```