$$(1+x+\cdots+x^5)(1-x)^{-2} \longrightarrow \text{Coeff of } x^7$$

$$\left(\frac{1-x^6}{1-x}\right)(1-x)^{-2}$$

$$= (1-x^6)(1-x)^{-3}$$

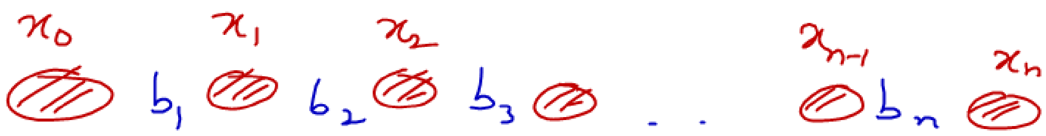
$$\text{Coeff of } x^7 \text{ in } \underbrace{(1-x^6)}(1-x)^{-3} = 1 \cdot \text{Coeff of } x^7 \text{ in } (1-x)^{-3}$$

$$-1 \cdot \text{Coeff of } x^{1} \text{ in } (1-x)^{-3}$$

$$= \ldots$$

HW 3 Q4 n boys k seats

Let the n boys sit without any restriction $= n!$



gaps $= (k-n)$

$$x_0 + x_1 + x_2 + \cdots + x_{n-1} + x_n = (k-n)$$

where $x_0, x_n \geq 0$

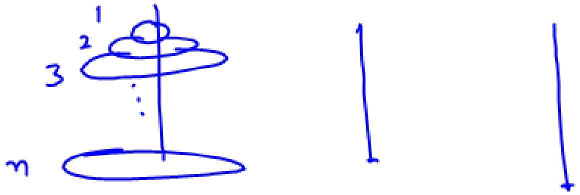
$x_1, x_2 \ldots x_{n-1} \geq 1$

let $x_1 = y_1 + 1$ etc.

$$x_0 + y_1 + y_2 + \cdots + y_{n-1} + x_n = (k-n) - (n-1)$$
$$= (k - 2n + 1)$$

. . .

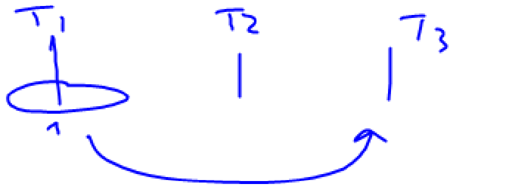
Recurrence relations:



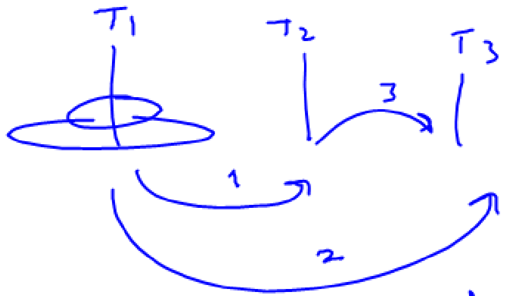
How many steps for shifting n disks from T_1 to T_3 (using T_2).

$H(n)$ = no of ways for moving n disks

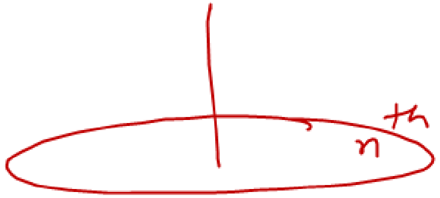
Ex:



no of steps = 1



no of steps = 3.



This configuration can be achieved in $H(n-1)$ steps

$$\underline{H(n) = 2 \cdot H(n-1) + 1} \quad , \quad H(1) = 1$$

$$= 2\left(2\, H(n-2) + 1\right) + 1$$

$$= 2^2\, H(n-2) + 2 + 1$$

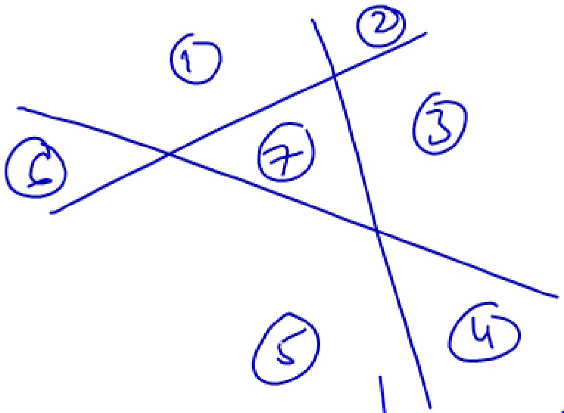
$$= 2^2\left(2H(n-3) + 1\right) + 2 + 1$$

$$= 2^3 \cdot H(n-3) + 2^2 + 2 + 1$$

$$= \cdots$$

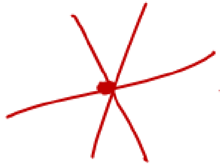
$$= 2^{n-1} \cdot H\left(n-(n-1)\right) + 2^{n-2} + 2^{n-3} + \cdots + 2 + 1$$

$$= 2^{n-1} + 2^{n-2} + \cdots + 2 + 1 = \left(2^n - 1\right).$$



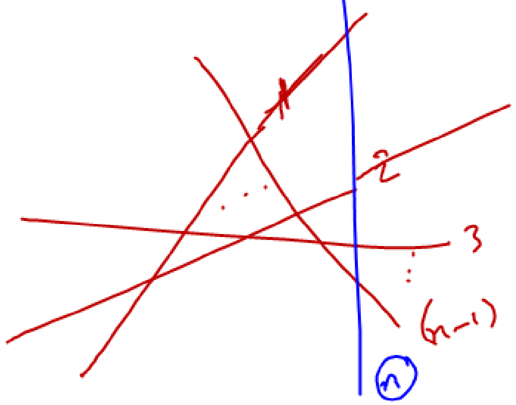
lines — no two parallel →

no three collinear →



this is excluded

no. of regions by n lines $= R(n)$



$$\boxed{R(n) = R(n-1) + n.}$$

$$R(1) = 2$$

Fibonacci's Rabbits.

- no rabbit dies
- Each pair takes 2 months to mature
- Each mature pair produces 1 pair every month.

$f(n)$ = no. of pairs after n months.

$f(0) = 1, \; f(1) = 1$

$f(n) = \; ?$

$= f(n-1) +$ those born at the end of n^{th} month

$= f(n-1) +$ no. of pairs at the end of $(n-2)$ months.

$= f(n-1) + f(n-2)$

$$f(n) = f(n-1) + f(n-2)$$

$$x^n = x^{n-1} + x^{n-2}$$

$$\Rightarrow \quad x^2 - x - 1 = 0 \quad \Rightarrow \quad x = \frac{+1 \pm \sqrt{5}}{2}$$

$$\text{roots} = \frac{1+\sqrt{5}}{2}, \quad \frac{1-\sqrt{5}}{2}$$

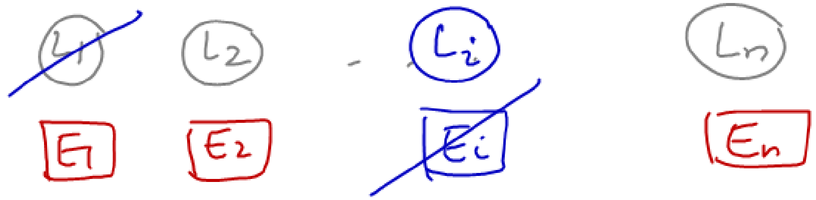
gen solutions $\Rightarrow$ $f(n) = c_1 \cdot \left(\frac{1+\sqrt{5}}{2}\right)^n + c_2 \cdot \left(\frac{1-\sqrt{5}}{2}\right)^n$

find c_1, c_2 by initial conditions

distinct roots

if repeated root then

$$f(n) = (c_1 + c_2 n) \cdot (\text{root})^n$$



$D(n)$ = no. of ways in which <u>every</u> letter goes to a wrong envelope

$$D(n) = (n-1) * \left(\begin{array}{c} \text{no of ways in which remaining} \\ (n-1) \text{ letters all go wrong} \end{array} \right)$$

$$= (n-1) \cdot \left(\begin{array}{c} \text{no of ways in which } (n-2) \text{ letters go wrong} \\ \& \\ (L_i \text{ goes to } E_1) \text{ or } (L_1 \text{ does not go to } E_1) \end{array} \right)$$

$$= (n-1) \cdot \left(D(n-2) + \cdots\cdots \right)$$

$\cancel{L_1}$ $\quad L_2 \quad \cdots \quad L_i \quad \cdots \quad L_n$

$\boxed{E_1}$ $\quad \boxed{E_2} \quad \cdots \quad \cancel{\boxed{E_i}} \quad \cdots \quad E_n$

missing term in the previous formula for $D(n)$.

& L_i does not go to E_1.

$\boxed{E_1}$ is behaving as if it is the correct envelope for L_i

$$= D(n-1)$$

$$\Rightarrow \boxed{D(n) = (n-1)\big(D(n-2) + D(n-1)\big)}$$

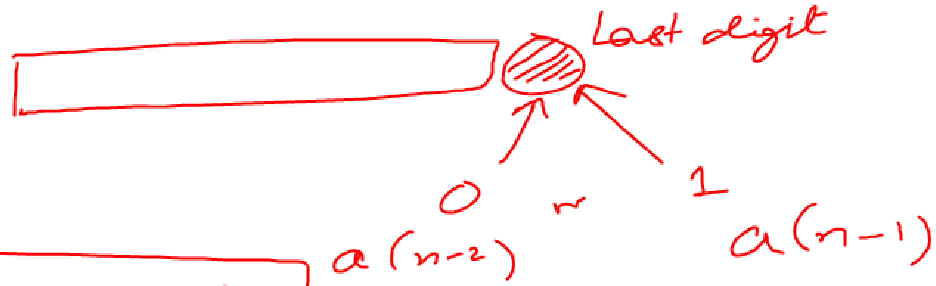
$D(1) = 0$

$D(2) = 1$

Bit strings of length n.

How many bit strings of length n not having two consecutive zeroes.

Let $a(n)$ = no of such bit strings.



$$\boxed{a(n) = a(n-1) + a(n-2)}$$

$a(1) = 2$, $a(2) = 3$.

$x_0 \cdot x_1 \cdot x_2 \cdot \cdot \cdot \cdot x_n$

nos in this order

In How many diff ways can we multiply?

$x_0 \; x_1 \; x_2 \; x_3 \longrightarrow ((x_0 \cdot x_1) \cdot (x_2 \cdot x_3))$ $n=3$.

$x_0 \cdot ((x_1 \cdot x_2) \cdot x_3)$

$n=1$.. $x_0 \cdot x_1 \rightarrow$ only 1 way.

$n=2$: $x_0 (x_1 \cdot x_2)$, $(x_0 \cdot x_1) \cdot x_2 \rightarrow$ 2 ways

$$(x_0 \cdot \; x_1 \cdots \; x_i) \cdot (x_{i+1} \quad x_n)$$

top most multiplication

$C(n) =$ ans for $(n+1)$ variables $x_0, x_1, \ldots x_n$

$$\boxed{C(n) = \sum_{i=0}^{n-1} C(i) \cdot C(n-i)}$$

Catalan numbers.

For every integer n there exists an integral multiple which can be written by only 0^s & 1^s (in decimal)

Pf:

$1, 11, 111, 1111, \dots \underbrace{111\dots1}_{n\text{-times}}.$

if we divide by n, what remainders can we get? $\longrightarrow 0, 1, 2, \dots (n-1)$

Case 1: There is a number with remainder 0 ----> Done.

Case 2: All the remainder are between 1 to (n-1) only.
In this case, two numbers have the same remainder r. (Pigeonhole principle)
Let these numbers be i and j. Then i = n(k1) +r and j = n(k2) +r

Clearly, (i-j) is a number which is a multiple of n.
Further, it contains only 1's and 0's in its representation.
---> Done.