(1) Rule of sum

(2) Rule of product

(1) Drawing with repititions

(2) with/without order

	order	repititions	
Case I	✓	✓	n^m
Case II	✓	×	$n(n-1)(n-2) \cdots (n-m+1)$
Case III	×	×	
Case IV	×	✓	

first second

$\underline{n} \times \underline{n} \times \cdots \underline{\cdots}$

$= P(n, m)$

n objects $\longrightarrow$ m to be drawn

n objects $\longrightarrow$ m to be drawn

We are interested in "without order"

Let us consider .. with order : (Example $m = 3$)

1,2,3	3, 5, 7	, ...	
1, 3,2			
2, 3, 1			
2, 1, 3			
3, 1, 2			
3, 2, 1			

$(3\times2\times1) = 3!$

total entries in the ~~table~~ $= P(n,3)$

The no of groups $= \dfrac{P(n,3)}{3!}$

Generalize: Choosing m objects out of n in this fashion $= \dfrac{P(n,m)}{m!}$

$$P(n, m) = \underline{n(n-1)(n-2)\ldots(n-m+1)}$$

$$= n(n-1)(n-2)\ldots(n-m+1)\,\frac{(n-m)(n-m-1)\ldots 2\cdot 1}{(n-m)(n-m-1)\ldots 2\cdot 1}$$

$$= \frac{n!}{(n-m)!}$$

$$\Rightarrow$$

$$\frac{P(n, m)}{m!} = \frac{n!}{(n-m)!\, m!} = C(n, m)$$

Some things are repeating, others are not.

Given:

1, 1, 1, 2, 3, 4, 5

How many 7 digit no's can be formed? (You can only use what is available)

$1_a, 1_b, 1_c, 2, 3, 4, 5.$ $\longrightarrow$ 7! numbers

$1_a\ 2\ 3\ 1_c\ 5\ 1_b\ 4$, ... , ...

$1_b\ 2\ 3\ 1_a\ 5\ 1_c\ 4$

. . . .

. . .

3!

$$\text{Ans} = \frac{7!}{3!}$$

1, 1, 1, 2, 2, 3, 4

How many diff no's of 7 digits

$$= \frac{7!}{3!\ 2!}$$

m things of one type, n things of another type

t things of a third type & r different

$$\text{Total arrangements} = \frac{(m+n+t+r)!}{m!\ n!\ t!}$$

Case IV

Order	Repititions
X	✓

3 icecreams , 7 to be consumed

Box 1   Box 2   Box 3

1111 0   0 111

00 1111111

$$= \frac{(7+2)!}{7!\ 2!}$$

$$= {}^{9}C_{2} = C(9,2)$$

n types of objects $\longrightarrow$ m to be drawn

Zeros $= (n-1)$

ones $= m$

$$\text{ans} = \frac{(n-1+m)!}{(n-1)!\ m!} = {}^{(n+m-1)}C_m$$

$$= {}^{n+m-1}C_{n-1}.$$

3 types of icecreams, 7 to be chosen

+ Each type of icecream should be selected.

3 types, 4 to be chosen $\downarrow$ without restriction (+ 3 to be added later $\downarrow$ one of each type.)

$= {}^{6}C_{2}$

Ex:

3 types of icecreams, 7 to be chosen

But the shop has only 5 of the first type

Find the no. of non-negative integral solutions to the equation:

$$x + y + z = 7$$

where

$$0 \le x \le 5$$
$$0 \le y \le 7$$
$$0 \le z \le 7$$

$x + y + z = 7$

No. of integral solutions

$0 \le x \le 5$

$0 \le y$

$0 \le z$

first consider this

$p + q = 5$

$0 \le p \le 3$

$0 \le q$

$$(\alpha^0+\alpha^1+\alpha^2+\alpha^3)(\alpha^0+\alpha^1+\alpha^2+\alpha^3+\alpha^4+\alpha^5+\dots)$$

$$= 1 + 2\alpha + (\dots)\alpha^2 + (\dots)\alpha^3 + (\cdot)\alpha^4 + (\dots)\alpha^5 + \dots$$

$(3,2),\ (2,3),\ (1,4),\ (0,5)$

Coefficient of x^5 = how many ways exist to obtain x^5 where we take x^p from first bracket, x^q from second bracket

Ex: Team scores 15 runs in an over (no extra balls). How many ways?

Alternate

$$x_1 + x_2 + \cdots + x_6 = 15$$

Where $x_i \in \{0, 1, 2, 3, 4, 5, 6\}$

Coeff of x^{15} in $(1 + x + x^2 + \cdots + x^6)^6$

Ex:

Scores allowed are 2, 4, 6 only on each ball

to score 18. How many ways?

$$(x^2 + x^4 + x^6)^6 \longrightarrow \text{Coef of } x^{18}.$$

2 types of things $\longrightarrow$ 7 to be taken

$$\therefore \quad x + y = 7 \qquad 0 \le x, y \le 7$$

$$(1 + x + x^2 + \dots + x^7)^2 \longrightarrow \text{Coeff of } x^7$$