Q: Let $S = \{x_1, x_2, x_3, \dots x_n\}$

To prove: no. of subsets of $S = 2^n$.

Basis: $n = 1$. $S = \{x_1\}$

Subsets of $S = \phi, \{x_1\}$

$|\mathcal{P}(S)| = 2 = 2^1$. ✓

Powerset of S.

Hypothesis: Let the given statement be true for $n = k$

if $S^* = \{y_1, y_2, \ldots y_k\}$

then $|\mathcal{P}(S^*)| = 2^k$

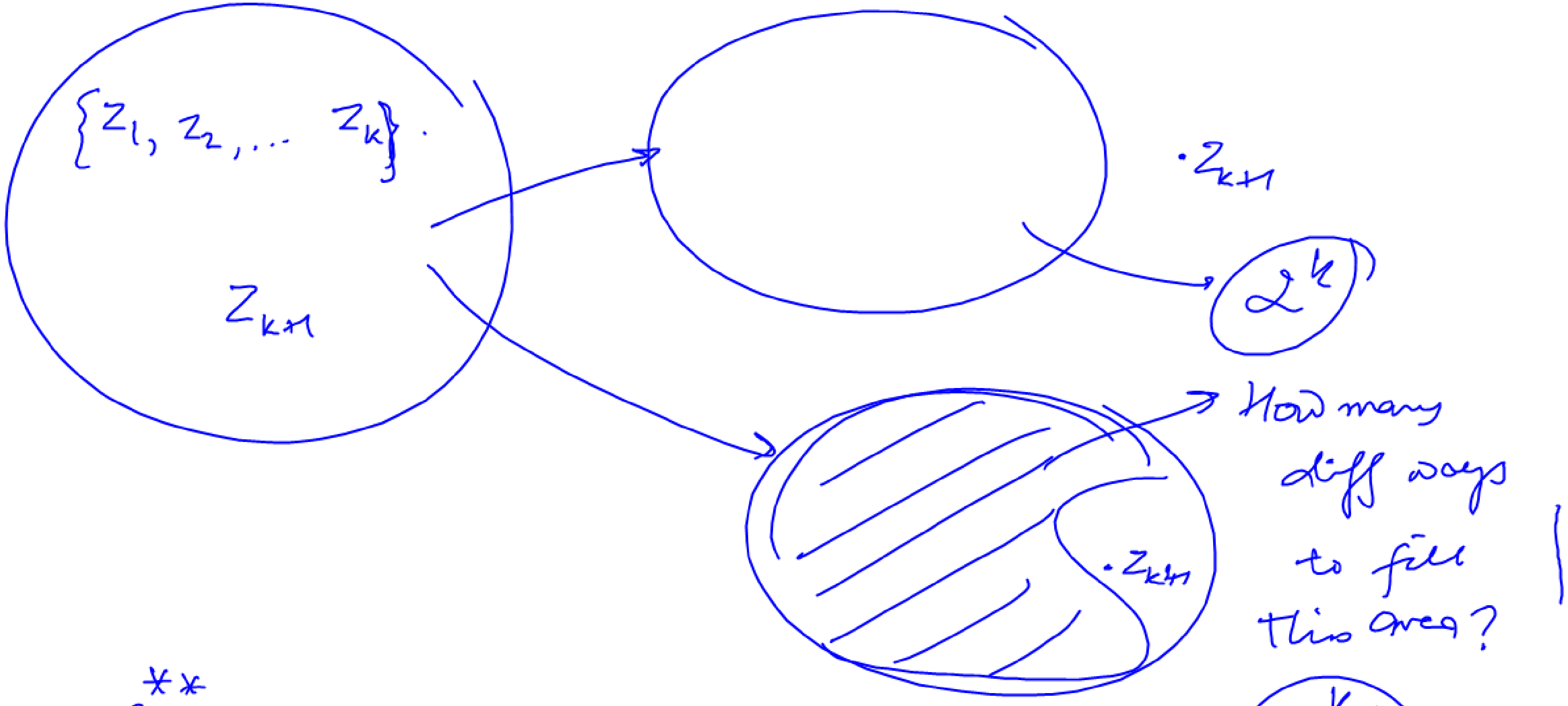
Induction:

$S^{**} = \{\underbrace{z_1, z_2, \ldots\ldots z_k}, \underbrace{z_{k+1}}\}$

any subset $Q \subseteq S^{**}$

$z_{k+1} \in Q \rightarrow$

$z_{k+1} \notin Q \rightarrow$ no. of Q's $= 2^k$



$S^{**} = \ldots$ subset of S^{**} $= 2^K$

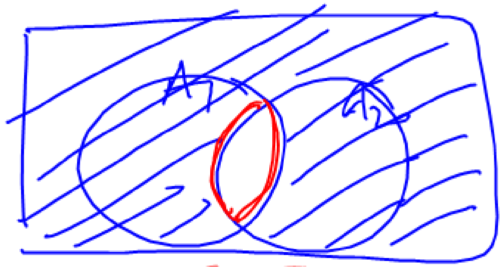
total subsets $= 2^k + 2^k = 2^{k+1}$

Q.

$$\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$$

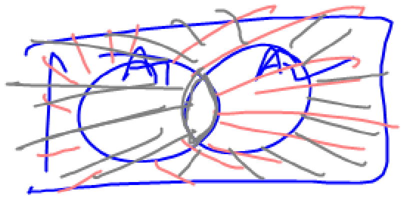
Basis: $n=2$

LHS: $\overline{A_1 \cap A_2}$



Intuition

RHS = $\overline{A_1} \cup \overline{A_2}$



Formal: To show $S_1 = S_2$

if $x \in S_1 \rightarrow x \in S_2$ $(S_1 \subseteq S_2)$ $\Rightarrow S_1 = S_2$

and if $x \in S_2 \rightarrow x \in S_1$ $(S_2 \subseteq S_1)$

Let $P(k)$ be true

$$\overline{\bigcap_{i=1}^{k} A_i} = \bigcup_{i=1}^{k} \overline{A_i}$$

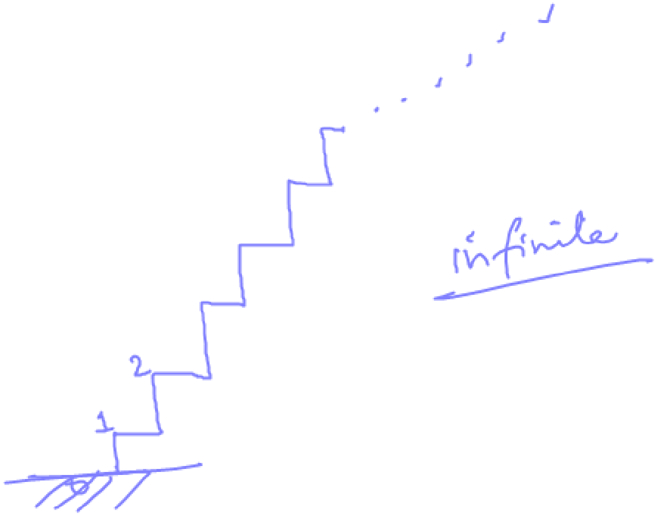
Proof step (Induction step).

Aim: $\overline{\bigcap_{i=1}^{k+1} A_i} = \bigcup_{i=1}^{k+1} \overline{A_i}$

$$\text{LHS} = \overline{\bigcap_{i=1}^{k+1} A_i} = \overline{\bigcap_{i=1}^{k} A_i \cap A_{k+1}} = \overline{B \cap A_{k+1}} \quad \text{(use of basis step)}$$

$$\bigcap_{i=1}^{k} A_i = B$$

$$= \left(\overline{B} \cup \overline{A_{k+1}} \right)$$



(i) One can step to 1 or 2

(ii) Whenever one can go to k, she can also go to $(k+2)$.

Strong principle of M. I.

(i) $1 \in S$

(ii) whenever $1, 2, 3, \ldots k \in S$

then $(k+1) \in S$

$\Rightarrow \quad S = \mathbb{N}$.

Q: Any natural no > 1 can be written as a product of prime numbers.

Basis: $n = 2$ which is prime.

Hypothesis: let $1, 2, \ldots k$ satisfy this

Induction: $(k+1) = ?$

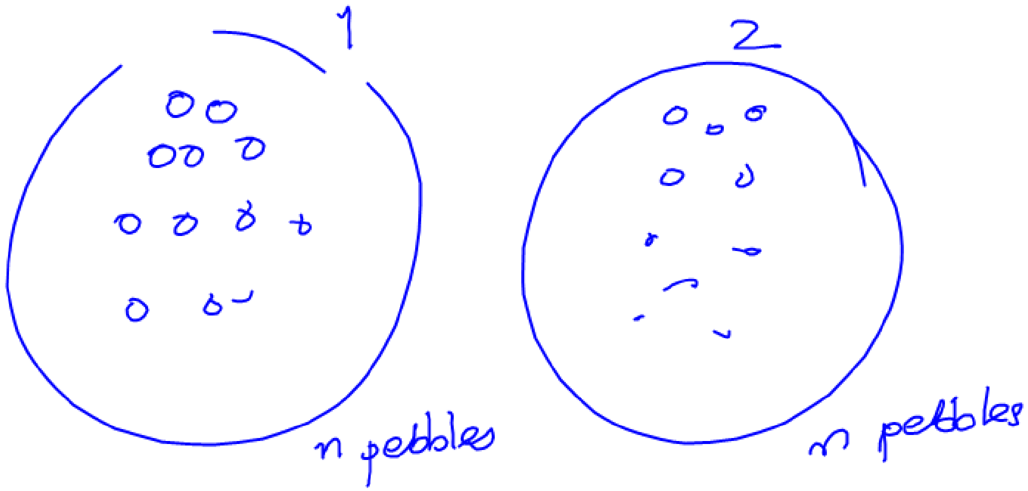
prime ✓

composite

$(k+1) = a \cdot b$

where $1 < a, b \leq k$

W.O.P $\longrightarrow$ P.M.I. $\longrightarrow$ Strong P.M.I.



Players A, B

Whoever removes the last pebble is the winner

Basis:

0 $\quad$ 0

A picks any one, B picks the other one & wins

Hypothesis

$\le k$ $\quad$ $\le k$

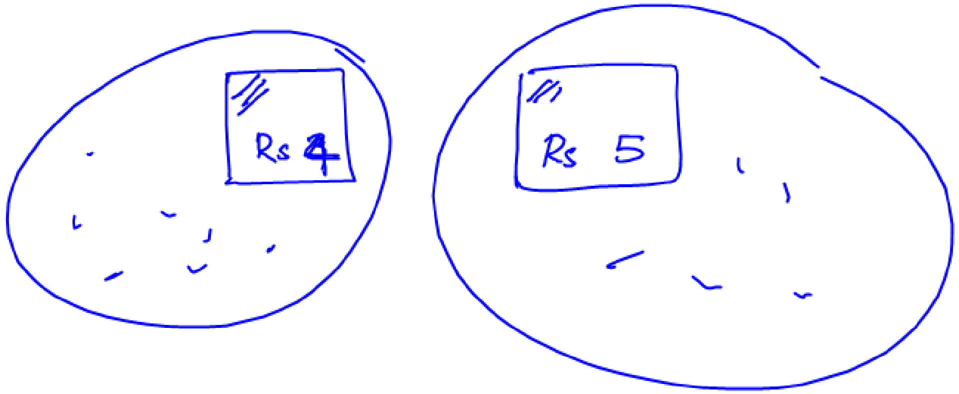
Inductive:

A : picks one pile & removes r from it

(we know $(k+1-r) \le k$)

$k+1-r$ $\quad$ $k+1$

Stamps



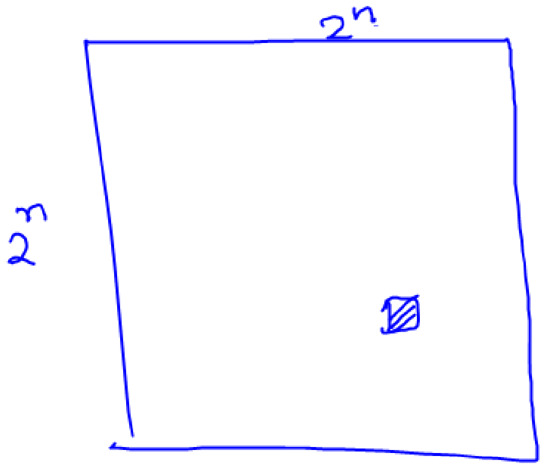
Prove that any amount ≥ 12 can be obtained using these stamps.

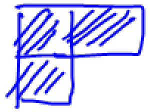
$k \longrightarrow k+1$

1. contains atleast 1 $\boxed{Rs\ 4}$ $\longrightarrow$ remove & replace with $\boxed{Rs\ 5}$

2. no $\boxed{Rs\ 4}$.

$k \geq 12 \Rightarrow$ there are atleast 3 $\boxed{Rs\ 5}$



$\longrightarrow$ remaining board can be covered by 

Base: 2×2 ✓

Assumption $2^k \times 2^k$ ✓

Induction: $2^{k+1} \times 2^{k+1}$