Algorithms 8

Divij Singh

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Let P be a connected weighted graph.
At every iteration, we must find an edge that connects a vertex in the subgraph to a vertex outside it. Since P is connected, there is always a path to every vertex.
Let Y_1 be a minnimum spanning tree of graph P. If $Y_1 = Y$, then Y is a minnimum spanning tree, else let E be the first edge added during construction of tree Y that is not in Y_1, and V be the set of vertices connected by the edge before E.
Then, one end of E is in V, and one is not. Since Y_1 spans the tree of graph P, there is a path in Y_1 joining both points. As you follow the path, there will be an edge F that joins a vertex in V to one outside V.
Since F was not added at the step when edge E was added to Y, we conclude that $w(F) \leq w(E)$.

Let tree Y_2 be the graph we get from removing F from Y_1, and adding edge E to Y_1.
Since we can show that tree Y_2 is connected, has the same number of edges as Y_1, and the weights of its edges is not greater than Y_1, it is also a minimum spanning tree of graph P, and contains edge E and all other edges added while creating set V.

By repeating these steps, we will eventually get a minimum spanning tree of graph P that is identical to tree Y, showing that Y is a minimum spanning tree. This allows for the first subset of the sub-region to be expanded into subset X, which we assume as the minimum.