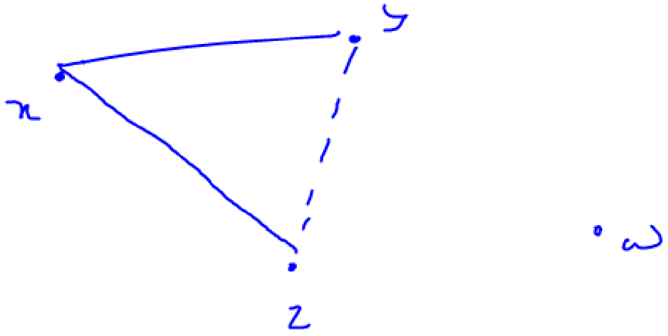
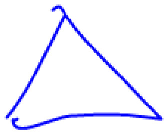
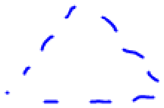
Frank Ramsey

x , y either friends ——

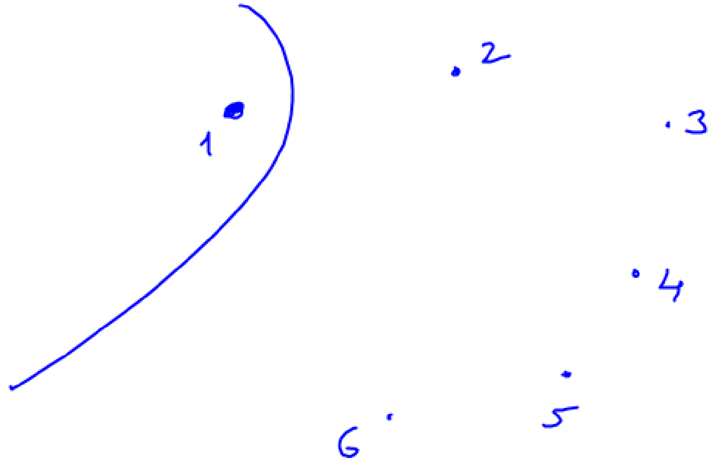
or enemies - - - -



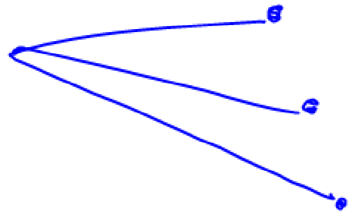
How many points do I need to get a $\triangle$ of similar sides

(i.e.  or )

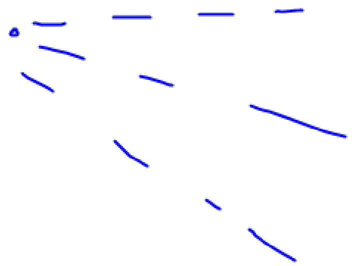
Claim: 6 people are enough



P.P $\Rightarrow$



or



$$R(3,3) = 6.$$

$$\boxed{R(m,n) = \ ?}$$

<u>Ramsay numbers</u>

Recurrence relations

$$f(n) = 3f(n-1) - 4f(n-2)$$

$f(1) = 0$

$f(2) = 2$

$x^n = 3x^{n-1} - 4x^{n-2}$

Ch Eq: $\boxed{x^2 = 3x - 4}$

$x^2 - 3x + 4 = 0$ $\quad (x-3)(x-1) = 0$

$f(n) = c_1 \cdot 3^n + c_2 \cdot 1^n$

$$f(n) = 4f(n-1) - 4f(n-2)$$

$$f(0) = 1$$

$$f(1) = 3$$

$$x^n = 4x^{n-1} - 4x^{n-2}$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)^2 = 0$$

roots $= 2, 2$

Soln:

$$f(n) = (c_1 + c_2 n) \cdot 2^n$$

$$f(n) = f(n-1) + f(n-2) + f(n-3)$$

$$x^3 - x^2 - x - 1 = 0$$

roots $= r_1, r_2, r_3$

Soln: $f(n) = c_1 \cdot r_1^n + c_2 r_2^n + c_3 r_3^n$

Homogeneous rec. relations:

$$f(n) = 2 f(n-1) + 1$$

Wed 22nd March. In class — Mid Term exam

Generalized Pigeonhole Principle:

If there are n pigeons & k holes

then atleast one box has $\left\lceil \frac{n}{k} \right\rceil$ pigeons.

Ex: 100 students, 12 months

$\Rightarrow$ at least $\left\lceil \frac{100}{12} \right\rceil = 9$ students born in the same month.

random seq. of distinct integers : (n^2+1) integers

8, 11, 9, 1, 4, 6, 12, 10, 5, 7

there will be a subsequence of $(n+1)$ integers which is either increasing or decreasing.

Ex: 1, 4, 6, 10 or 11, 9, ~~8~~, 6, 5

or - - . .

$a_1, \; a_2, \; a_3, \; \ldots \; a_{n^2+1}$

$\downarrow \quad \downarrow \qquad\qquad\qquad \downarrow$

$$\underbrace{\begin{pmatrix} i_1, \\ d_1 \end{pmatrix} \begin{pmatrix} i_2, \\ d_2 \end{pmatrix}, \ldots \ldots \begin{pmatrix} i_{n^2+1} \\ d_{n^2+1} \end{pmatrix}}_{} \quad (n^2+1) \text{ pairs}$$

Where i_k = longest increasing subseq. starting from k^{th} no.

d_k = .. decreasing ....

Assume, no $(n+1)$ length subseq. of the reqd. type exists

Then $1 \leq i_k \leq n$, $1 \leq d_k \leq n$

(i_k, n_k) pairs possible $= n^2$.

$$(i_k, d_k) = (i_j, d_j)$$

there must be distinct k & j s.t.

x_k $\quad$ x_j

k $\quad$ j

1. if $x_k > x_j$ then $x_k, \underbrace{\ldots\ldots}_{\text{whatever was used in } d_j}$

2. if $x_k < x_j$ then

$x_k \underbrace{\qquad\qquad}_{i_j}$

Prove that among any $(n+1)$ positive integers not exceeding $2n$ there must be an integer that divides one of the other integers.

$n=2$

3 integers $\in [1,4]$

$(1, 2, 4)$

$(2, 3, 4)$

$(1, 2, 3)$

any no. $n = 2^i \times k$

$k \downarrow$ odd.