Introduction to Coding Theory Assignment 5

Divij Singh

07/01/19

1 Q1

$00 \rightarrow 0000$
$01 \rightarrow 0121$
$02 \rightarrow 0212$
$10 \rightarrow 1022$
$11 \rightarrow 1110$
$12 \rightarrow 1201$
$20 \rightarrow 2011$
$21 \rightarrow 2102$
$22 \rightarrow 2220$

Message	00	01	02	10	11	12	20	21	22	Syndrome
codeword	0000	0121	0212	1022	1110	1201	2011	2102	2220	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
codeword + 0001	0001	0122	0210	1020	1111	1202	2012	2100	2221	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
codeword + 0002	0002	0120	0211	1021	1112	1200	2010	2101	2222	$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$
codeword + 0010	0010	0101	0222	1002	1120	1211	2021	2112	2200	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
codeword + 0011	0011	0102	0220	1000	1121	1212	2022	2110	2201	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
codeword + 0012	0012	0100	0221	1001	1122	1210	2020	2111	2202	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$
codeword + 0020	0020	0111	0202	1012	1100	1221	2001	2122	2210	$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$
codeword + 0021	0021	0112	0200	1010	1101	1222	2002	2120	2211	$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$
codeword + 0022	0022	0110	0201	1011	1102	1220	2000	2121	2212	$\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

2 Q2

000 → 000000
001 → 001110
010 → 010101
011 → 011011
100 → 100011
101 → 101101
110 → 110110
111 → 111000

Message	000	001	010	011	100	101	110	111	Syndr
codeword	000000	001110	010101	011011	100011	101101	110110	111000	$\begin{pmatrix}0\\0\\0\end{pmatrix}$
codeword + 000001	000001	001111	010100	011010	100010	101101	110111	111001	$\begin{pmatrix}0\\0\\1\end{pmatrix}$
codeword + 000010	000010	001100	010111	011001	100001	101110	110100	111010	$\begin{pmatrix}0\\1\\0\end{pmatrix}$
codeword + 000011	000011	001101	010110	011000	100000	101111	110101	111011	$\begin{pmatrix}0\\1\\1\end{pmatrix}$
codeword + 000100	000100	001010	010001	011111	100111	101000	110010	111100	$\begin{pmatrix}1\\0\\0\end{pmatrix}$
codeword + 000101	000101	001011	010000	011110	100110	101001	110011	111101	$\begin{pmatrix}1\\0\\1\end{pmatrix}$
codeword + 000110	000111	001001	010010	011100	100100	101011	110001	111111	$\begin{pmatrix}1\\0\\1\end{pmatrix}$
codeword + 000111	000101	001011	010000	011110	100110	101001	110011	111101	$\begin{pmatrix}1\\1\\1\end{pmatrix}$

3 Q3

Let us take a code C, such that $C = c_1, c_2, c_3, ...c_n$

We can define its cosets as C_1 and C_2, such that $C_1 = x_1, x_2, ...x_n$ and $C_2 = y_1, y_2, ...y_n$ and that $x_i = y_j$ for some $1 \leq i, j \leq n$

We can express x_i as $x_i = x_1 + z_i$ and $y_j = y_1 + z_j$, giving us $z_i + x_1 = z_j + y_1$

$z_i - z_j = y_1 = x_1$, which means $y_1 - x_1$ belong to C.

Let us have $y_1 - x_1 = z_k$ for some k.

This gives us $x_k = y_1 - x_1 + x_1 = y_k$

As the cosets are linear, C_1 and C_2 must be equal if there is a point of intersection between them.

If they are not equal, there is no point of intersection, this they are disjoint.

4 Q4

Let us take a binary linear code as C, where $C = c_1, c_2 ... c_n$
For $a \notin C$, let the code be X, where $X = C \cup (a + C)$

Let us take two variables, x and y, such that $x, y \in X$
Now, there are three cases where $x, y \in X$

1. $x, y \in C$: Then we know that $x + y \in C$ as C is a linear code.

2. $x \in C, y \in (a + C)$: Then we know that $y = a + z_i$ for some i (from the previous question)

So thus far we have $x + y = x + a + z_i$ which can be written as $x + y = z_k + a$ $(z_k = x + z_i, and c_k \in C$ as C is linear)
$c_k + a \in a + C$

3. $x, y \in (a + C)$: We can take $x = z_i + a$ and $y = z_j + a$
This gives us $x + y = z_i + z_j + 2a$, which means that $z_i + z_j \in C$, as the code is binary.

Thus, $C \cup (a + C)$ is a binary linear code if C is.

Now, what if C is not a binary code, but, say, a ternary code?

Let us return to the formula $x + y = z_i + z_j + 2a$
In this case, the a does not cancel out, leaving us with $z_i + z_j + 2a \notin C$