Introduction to Coding Theory Assignment 8

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1 Q1

If $A_q(n, d)$ denotes the maximum size of a q-ary code with length n and distance d, then $A_q(n, d) \geq \frac{q^n}{\sum_{j=0}^{d-1} \begin{pmatrix} n \\ j \end{pmatrix} (q-1)^j}$

For an example, let us take $n = 4$ and $d = 2$ for a binary code
$A_2(n, d) = 8$
$q^n = 16$
$\sum_{j=0}^{d-1} \begin{pmatrix} n \\ j \end{pmatrix} (q-1)^j = 1 + 4.12311$
Taking the floor value of the above, we get $\frac{16}{5} \leq A_2(n, d)$

2 Q2

For a binary linear code, the Griesmer bound is:

$$n \geq \sum_{i=0}^{k-1} \lceil \frac{d}{2^i} \rceil$$

3 Q3

For this, we will be checking each code against the following bounds:

Singleton Bound: If C is an [n,k,d] binary code, then $d \leq n - k + 1$

The Plotkin Bound: if $n < 2d$, $M \leq 2\lfloor \frac{d}{2d-n} \rfloor$

The Gilbert-Varshamov Bound

The Griesmer Bound

The Hamming Bound: $M \left(1 + \sum_{j=1}^{\lfloor \frac{d-1}{2} \rfloor} \left(\begin{array}{c} \text{n} \\ \text{j} \end{array} \right) \right) \leq 2^n$

(a) From the Hamming bound, we see that $M(1+n) = 261 > 256(2^n)$
Therefore, no such binary code exists.

(b) From the Plotkin bound, we get $M = 8$; $2\lfloor \frac{d}{2d-n} \rfloor = 4$
$M > 4$ therefore no such code exists.

(c) Again with the Plotkin bound, we get $2\lfloor \frac{d}{2d-n} \rfloor = 4, < M$
Thus, no such code exists.

(d) This code satisfies the Gilber-Varshamov bound, the Griesmer bound, the Singleton bound, and the Hamming bound.
Therefore, a linear code with these contraints may or may not exist, and if it does, it will be a perfect code.

4 Q4

This code satisfies the Gilber-Varshamov bound, the Griesmer bound, the Singleton bound, and the Hamming bound.
Therefore, a linear code with these contraints may or may not exist.

5 Q5

This code satisfies the Gilber-Varshamov bound, the Griesmer bound, the Singleton bound, and the Hamming bound.
Therefore, a linear code with these contraints may or may not exist.

6 Q6

This code satisfies the Gilber-Varshamov bound, the Griesmer bound, the Singleton bound, and the Hamming bound.
Therefore, a linear code with these contraints may or may not exist.

7 Q7

This code satisfies the Gilber-Varshamov bound, the Griesmer bound, the Singleton bound, and the Hamming bound.
Therefore, a linear code with these contraints may or may not exist.