1 Q1

The code $C = \{000000, 001110, 010101, 011011, 100011, 101101, 110110, 111000\}$
The are are 7 cosets, and the code itself, with coset leaders being: $\{000000, 000001, 000010, 100000, 000100, 01$

Let us take crossover probability as p.

Thus, $P_{error} = 1 - [(1-p)^6 + p(1-p)^5 + p(1-p)^5 + p(1-p)^5 + p(1-p)^5 + p(1-p)^5 + p(1-p)^5 + p^2(1-p)^4]$

Which gives us $1 - [(1-p)^6 + 6p(1-p)^5 + p^2(1-p)^4]$

2 Q2

For this code, $k = 2$, and $n = 8$, giving us an information rate of 1/4 .
Let us assume there exists a more optimal code, i.e a [7,2,5] code. We can check this via the Plotkin bound.

$M \leq 2\lfloor \frac{d}{2d-n} \rfloor$

$\lfloor \frac{d}{2d-n} \rfloor = 2\lfloor \frac{5}{10-7} \rfloor = 2 * 1$
$2 \not> M$ where $M = 4$

Thus the code is optimal, as no [7,2,5] code exists.

3 Q3

For this code, $k = 3$, and $n = 6$, giving us an information rate of 1/2.
Let us see if a [5,3,3] code exists.

$2\lfloor \frac{3}{6-5} \rfloor = 2 * 3 = 6$
$6 \not> M$, where $M = 8$

Thus the code is optimal, as no [5,3,3] code exists.