w.o.p.

Round robin match between n players.

(i plays j exactly once, for all $i, j : i \neq j$)

cycle:

$P_i \rightarrow P_j \equiv P_i$ wins against P_j (no draws)

$$\begin{tikzcd} P_i \arrow[r] & P_j \arrow[r] & P_{j'} \arrow[r] & P_{j''} \arrow[lll, bend left] \end{tikzcd}$$

To prove: If there is a cycle of length m (where $m \geq 3$) then there must be a cycle of length 3 among these m players.

$S = \{$ cycle lengths which exist in the tournament $\}$

we know that (i) S contains +ve integers

(ii) $m \in S \Rightarrow S$ is non-empty.

$\Rightarrow \exists\ k$ s.t. k is the least cycle.

if $k = 3$ ✓

<u>let $k > 3$</u>

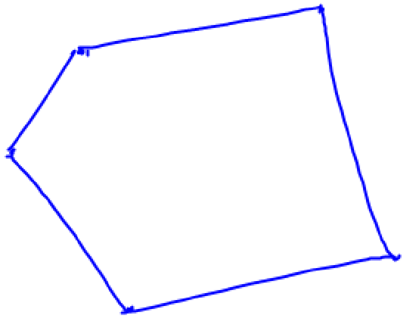
$P_1 \to P_2 \to P_3 \cdots \to P_k$

$P_1 \to P_2 \to P_3$ or $P_1 \to P_2 \to P_3$

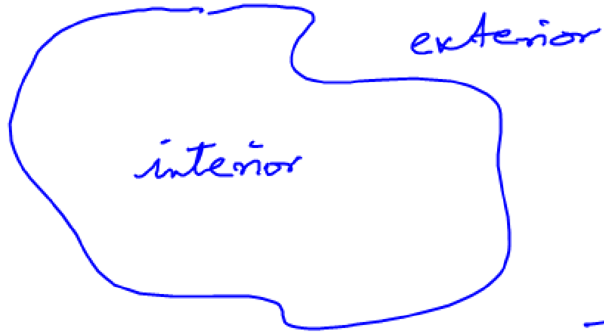
This is the 3-cycle

There is a cycle of length $(k-1)$.

Polygon:



Closed shape



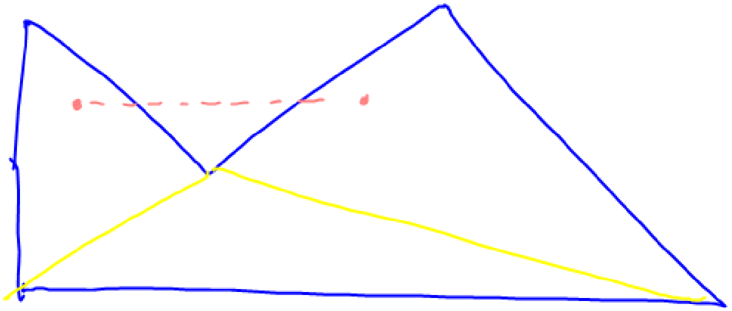
(Jordan Curve Theorem)

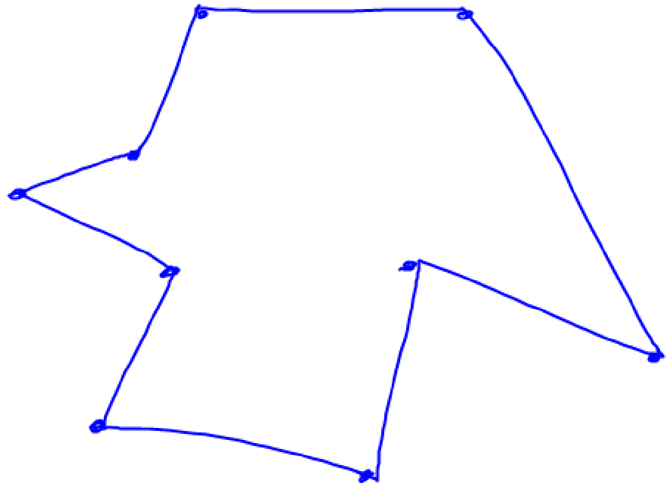
$\forall\, a, b$ a · - - - · b   both in the interior of a polygon,

all points on this line segment lie in the interior of the polygon $\longrightarrow$ Convex Polygon

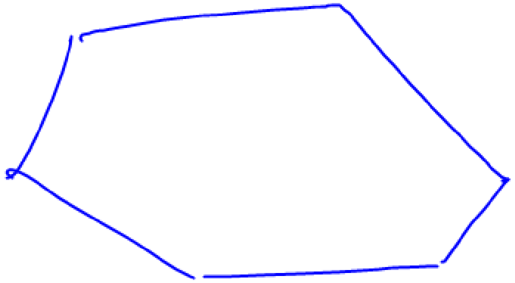
else

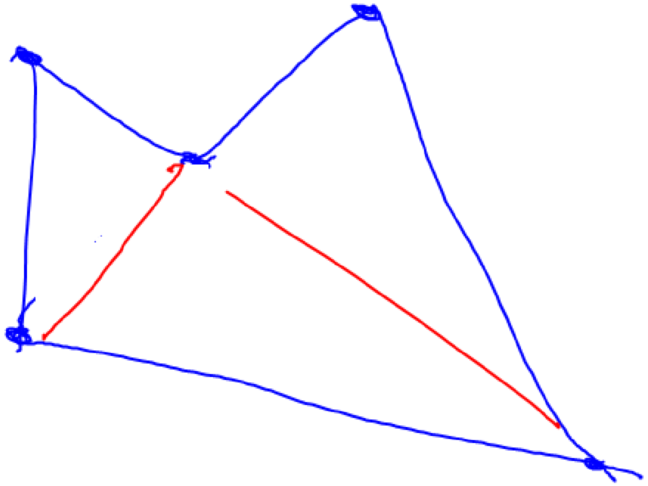
concave polygon or non-convex polygon





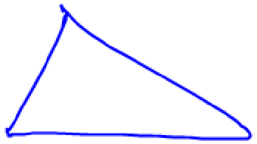
n sided polygon





To prove: an n-sided polygon can be triangulated into $(n-2)$ triangles. $(n \geq 3)$

Base:



$n = 3$. 1 $\triangle$

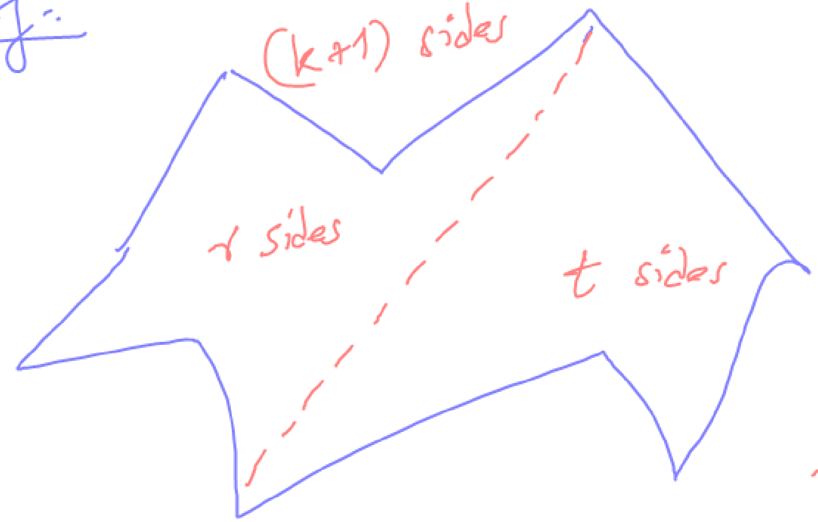
Hypothesis: triangulation into $(k-2)$ $\triangle$ is possible for a k sided polygon

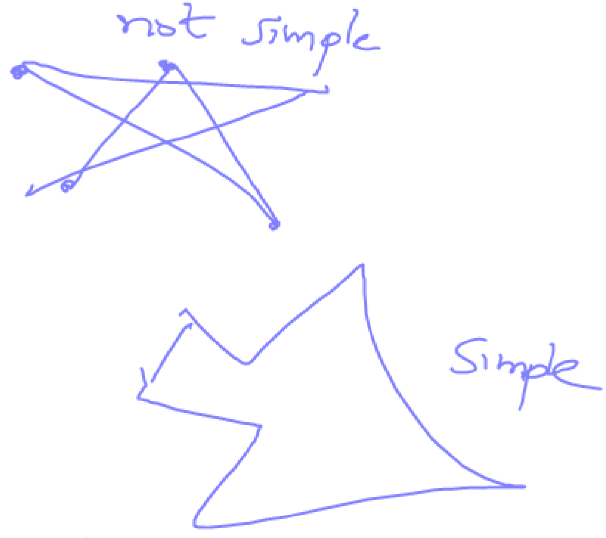
also true for . $4, 5, \ldots k$.

Generalized principle of M. I.

Lemma: Every simple polygon has an interior diagonal

Proof:





$r + t = (k+3)$