Introduction to Coding Theory Assignment 7

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1 Q1

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0| & 0 & 0 & 1 & 1 & 1| & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1| & 1 & 1 & 0 & 0 & 0| & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0| & 1 & 1 & 0 & 0 & 1| & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1| & 0 & 1 & 0 & 1 & 0| & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Syndrome	Coset Leader
0000	00000—00000—00000
0001	00010—00000—00000
0010	00100—00000—00000
0011	00000—01000—00000
0100	01000—00000—00000
0101	00000—00001—00000
0110	00000—10000—00000
0111	00000—00000—01000
1000	10000—00000—00000
1001	00000—00000—00001
1010	00000—00010—00000
1011	00000—00000—00010
1100	00001—00000—00000
1101	00000—00100—00000
1110	00000—00000—10000
1111	00000—00000—00100

(a) Let the received codeword be r
$r = c + n$, where c is the original codeword, and n is noise.
We first multiply r^{tr} with the parity check matrix H, to receive the four digit syndrome.
$Hr^{tr} = 0110$
We then add the corresponding coset leader to r.
We receive $01010 - 11010 - 01000$ as the decoded codeword.

(b) We apply the same method as above.
$Hr^{tr} = 1010$
Decoded codeword: $11100 - 01100 - 00111$

(c) We apply the same method as above.
$Hr^{tr} =$
Decoded codeword: $11001 - 11011 - 11000$

2 Q2

The same method as is used for binary Hamming codes can be used for ternary Hamming codes.
That is, for an $r \geq 2$, $n = 2^r - 1$, $k = 2^r - r - 1$, and $d = 3$
The same method is used for decoding, where we construct a parity check matrix H. We then get the syndrome of the received vector r from Hr^{tr}
We then find the corresponding coset leader, and add it to the received vector to find the decoded codeword.

3 Q3

A single error correcting code is a perfect code if there is no coset leader with a weight greater than 1.
For Hammng code, sin $n = 2^r - 1$, there are $n + 1 = 2^r$ vectors for each codeword. The number of codewords is given by 2^k, which is 2^{2^r-r-1} codewords.
Multiply the number of codewords by the number of vectors, and you get $2^{2^r-r-1+r} = 2^{2^r-1} = 2^n$ vectors.
Thus, every vector of length n is contained in a codeword, and the code is perfect.