Introduction to Coding Theory Assignment 19

Divij Singh

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1 Q1

(a)
$z_1 = \alpha^3, z_2 = \alpha^{11} = \alpha^3 + \alpha^2 + \alpha$
$\beta = \frac{z_2}{z_1^3} + 1$
$= \alpha^2 + 1$
$= [1\ 0\ 1\ 0].\begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \\ \alpha^3 \end{bmatrix}$

$$\begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \\ \alpha^3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha^7 \\ \alpha^{13} \\ \alpha^{13} \\ \alpha^{11} \end{bmatrix}$$

Let $\delta = 0$. Then with $y_0 = \delta, y_1 = y_0 + b_1, y_2 = y_1 + b_2, y_3 = y_2 + b_3$
$Y = \alpha^{11}$, $x = z_1 Y = \alpha^3 + 1 = i$

Thus $i = \alpha^3 + 1, j = 1$

(b)
$z_1 = \alpha^3 + \alpha^2, z_2 = \alpha^3 + \alpha^2 + 1$

$$\beta = \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \\ \alpha^3 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \\ \alpha^3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha^7 \\ \alpha^{13} \\ \alpha^{13} \\ \alpha^{11} \end{bmatrix}$$

$Y = y_0\alpha^7 + y_1\alpha^{14} + y_2\alpha^{13} + y_3\alpha^{11} = \alpha^{13}$
Then $x = \alpha + 1$
Thus, $i = \alpha + 1, j = \alpha^3 + \alpha^2 + \alpha + 1$

(c)
$z_1 = \alpha, z_2 = \alpha^{13}$

$$\beta = \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \\ \alpha^3 \end{bmatrix}$$

$Y = \alpha^{13}$
$x = \alpha^3 + 1$

Thus, $i = \alpha^3 + 1, j = \alpha^3 + \alpha + 1$

(d)
$z_1 = \alpha^{14}, z_2 = 0$

$$\beta = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \\ \alpha^3 \end{bmatrix}$$

$Y = \alpha^{10}, x = \alpha^3 + \alpha$

Thus, $i = \alpha^3 + \alpha, j = \alpha^3$