Counting:

$$p + q = 7$$

integral solutions

$$0 \le p$$
$$0 \le q$$

$$(x^0 + x^1 + x^2 + \cdots + x^7)\cdot(x^0 + x^1 + x^2 + \cdots + x^7)$$

Coeff of x^7 in the product.

$$= 1\cdot x^0 + 2x^1 + 3x^2 + 4x^3 + \cdots$$

$\{(1,0),(0,1)\}$ $\{(0,2),(1,1),(2,0)\}$ $\{(0,3),(1,2),(2,1),(3,0)\}$

Score in an over

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 18$$

$$x_i \in \{0, 1, 2, 4, 6\}$$

Batsmen can score 0, 1, 2, 4 or 6 runs on each ball. In how many ways can they score 18 runs in an over (assuming that no extra runs/balls).

$$\equiv \quad (x^0 + x^1 + x^2 + x^4 + x^6)^6 \longrightarrow \text{Coeff of } x^{18}.$$

$$x_1 + x_2 + \cdots + x_n = k$$

$$x_i \in \{0, 1\}$$

$$(x^0 + x^1)^n \longrightarrow \text{Coeff of } x^k$$

$$\equiv \quad (1+x)^n \longrightarrow \text{Coeff of } x^k$$

$${}^nC_k$$

3 types of icecreams, 7 to be taken

$$x_1 + x_2 + x_3 = 7$$

$$0 \le x_i \le 7$$

$$(x^0 + x^1 + x^2 + \cdot + x^7)^3 \longrightarrow \text{Coeff of } x^7$$

$$\equiv \text{Coeff of } x^7 \text{ in } (x^0 + x^1 + x^2 + \dots + x^7 + x^8 + x^9 + \dots)^3$$

$$\equiv \quad '' \quad (1-x)^{-3} = {}^{3+7-1}C_7$$

(from pg 6)

G.P

$$a + ax + ax^2 + \cdots + ax^{r-1} = a\left(\frac{x^r - 1}{x - 1}\right) = a\left(\frac{1 - x^r}{1 - x}\right)$$

$$a + ax + ax^2 \cdots\cdots = \frac{a}{1-x}$$

$$\boxed{\begin{array}{l}(1+0.01)^{365} = 1.37 \\ (1-0.01)^{365} =\end{array}}$$

$$S = a + ax + ax^2 \cdots + ax^{r-1}$$

$$xS = ax + ax^2 \cdots + ax^{r-1} + ax^r$$

$$(1-x)S = a - ax^r \Rightarrow S = \frac{a(1-x^r)}{1-x}$$

$*\quad (1-x)^{-3} = 1 + \underset{^3C_1}{3x} + \underset{^4C_2}{\frac{3.4}{1.2}} \cdot x^2 + \underset{^5C_3}{\frac{3.4.5}{1.2.3}} \cdot x^3 + \cdots\cdots$

$$\boxed{(1-x)^{-n} = \sum_{r=0}^{\infty} \left({}^{n+r-1}C_r \right) x^r}$$

$$\boxed{(1-x)^{-n} \equiv \sum_{r=0}^{\infty} \left({}^{n+r-1}C_{n-1} \right) x^r}$$

How many integral solutions to

$$x_1 + x_2 + \cdots + x_n = m$$

where

$$1 \le x_i$$

<u>Ans:</u>

$$\left(x^1 + x^2 + \cdots + x^{m-n+1}\right)^n \longrightarrow \text{Coeff of } x^m$$

$$\equiv x^n \cdot \left(1 + x + \cdots + x^{m-n}\right)^n \longrightarrow \text{Coeff of } x^m$$

$$\equiv \left(1 + x + \cdots + x^{m-n}\right)^n \longrightarrow \text{Coeff of } x^{m-n}$$

$$\equiv \left(1 + x + \cdots + \infty\right)^n \longrightarrow \text{''}$$

$${}^{m-1}C_{n-1}$$

$$(1-x)^{-n} \longrightarrow \text{Coeff of } x^{m-n} \equiv {}^{n+m-n-1}C_{n-1}$$

$$x_1 + x_2 + \cdots + x_n = m$$

$$1 \leq x_i$$

Let

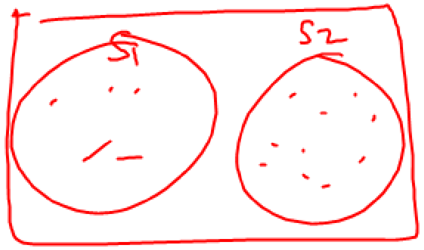
$$x_1 = 1 + y_1 , \quad x_2 = 1 + y_2 , \quad \ldots \quad x_n = 1 + y_n$$

$$0 \leq y_1, y_2, \ldots$$

$$y_1 + y_2 + \cdots + y_n = (m - n)$$

where $\quad y_i \geq 0$

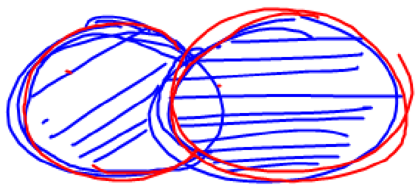
Axiom: $\left| S_1 \cup S_2 \right| = |S_1| + |S_2|$ if S_1 & S_2 are disjoint



What if S_1 & S_2 are not disjoint ?

Lemma:

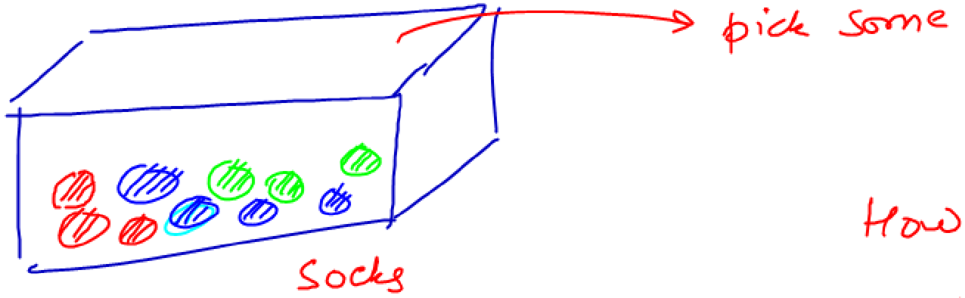
$$\left| S_1 \cup S_2 \right| = |S_1| + |S_2| - |S_1 \cap S_2|$$



$$|S_1 \cup S_2 \cdots \cup S_n| = \sum |S_i| - \sum_{\substack{\forall i,j \\ i \neq j}} |S_i \cap S_j| + \sum_{\substack{i,j,k \\ i \neq j \neq k}} |S_i \cap S_j \cap S_k| - \cdots\cdots$$

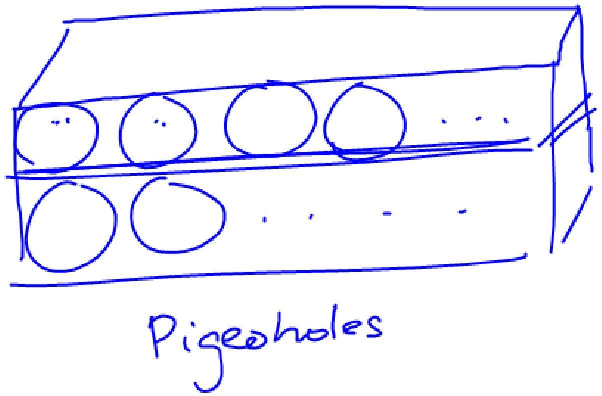
Principle of Inclusion & Exclusion

Proof: using induction. (Homework)



How many to pick, so that you have a good pair?

Ans = 4.



If there are $(n+1)$ pigeons & n pigeonholes, then at least one pigeonhole has more than 1 pigeon.

$\boxed{\text{Pigeonhole principle}}$

$$x+y+z=7$$

$$0 \le x \le 5$$

$$0 \le y$$

$$0 \le \le z$$

$$\Rightarrow (1+x+x^2+\cdots+x^5)(1+x+\cdots+x^7)^2$$

$\longrightarrow$ Coeff of x^7

$$(1+x\cdots x^5)\left(\overline{1-x}\right)^{2} \longrightarrow x^7$$

$$= \sum_{i=0}^{5} x^{7-i} \text{ in } \left(\overline{1-x}\right)^{2} = \cdots\cdots,$$