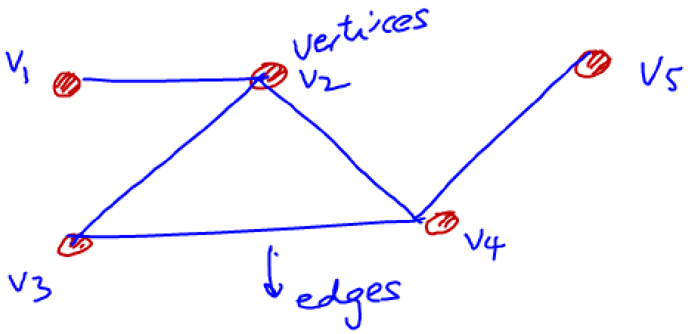
Graph theory :



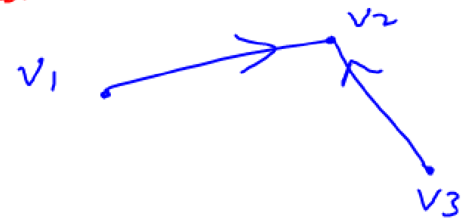
network

$$V = \{v_1, v_2, v_3, v_4, v_5\}$$

$$E \subseteq V \times V$$

pairs are ordered

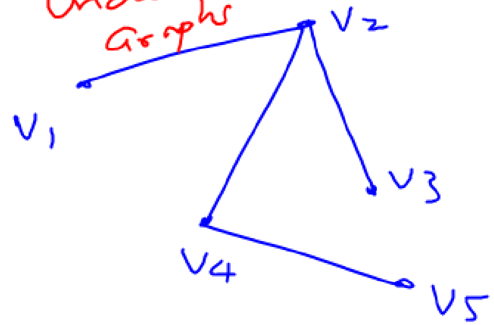
Directed Graphs

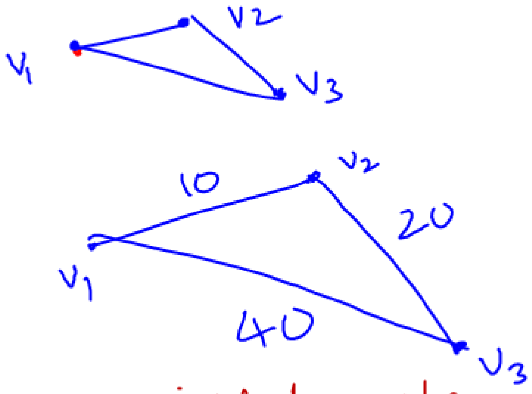


$v_1 \to v_2$, $v_3 \to v_2$

pair are unordered

Undirected Graphs

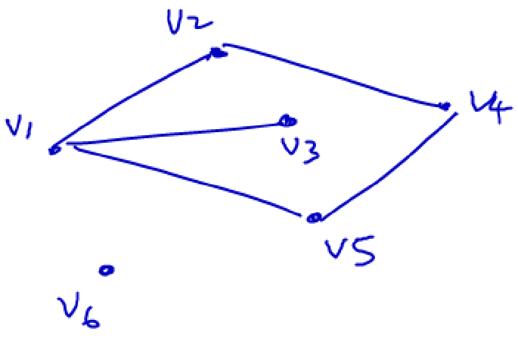




weighted graphs

Graph $G = (V, E)$
where $E \subseteq V \times V$

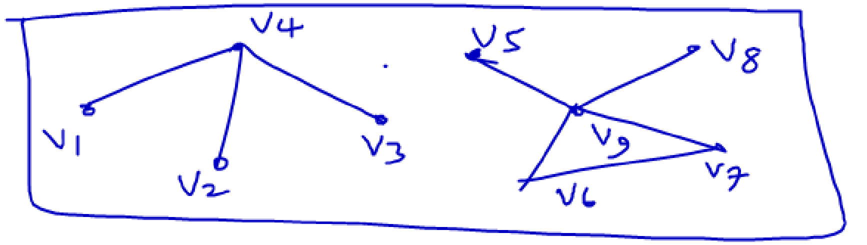
$\deg(v_i)$ = no. of edges from v_i = $d(v_i)$



$\deg(v_1) = 3$

$d(v_3) = 1$ Pendant

$d(v_6) = 0$
isolated vertex



Connected graph : $\forall\, v_i, v_j$ $\exists$ a path between v_i & v_j.

Path :

Subgraph:

$G = (V, E)$

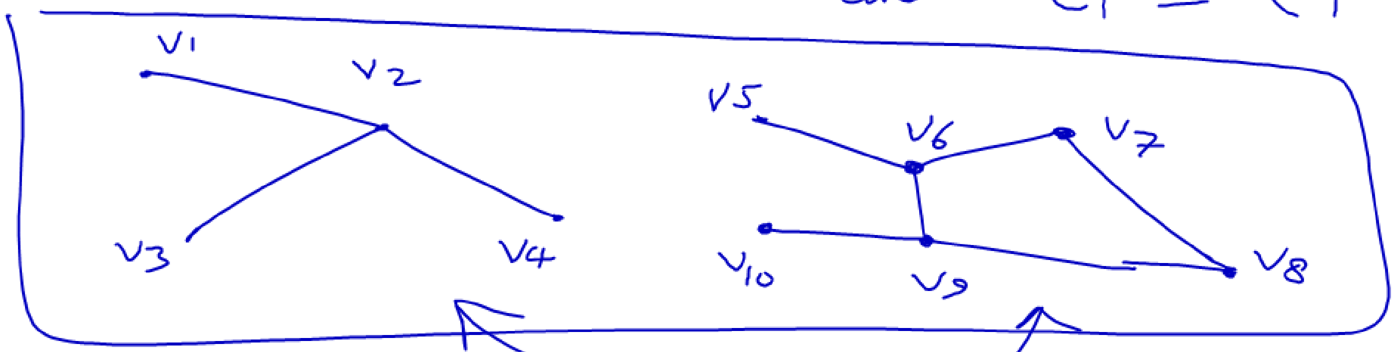
H is a subgraph of G

$H = (V_1, E_1)$

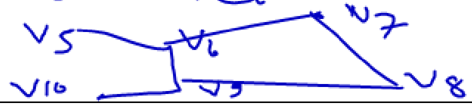
where $V_1 \subseteq V$

and $E_1 \subseteq (V_1 \times V_1) \cap E$

G:



Connected components : maximal subgraphs which are connected



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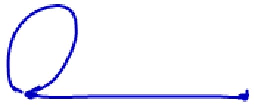


normally, we will not consider.

×

multiedges : ——— " ———

– no of edges $|E|$: e

– no of vertices $|V|$: n

$G = (V, E)$ is a graph (without loops & multiple edges)

then no. of connected components in G

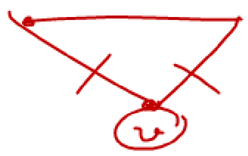
$$\boxed{\#\text{comp} \geq |V| - |E|}$$

Basis: • $|V| = 1$, $|E| = 0$ $\qquad \#\text{comp} = 1 \geq 1 - 0$

Hypothesis: any graph with k vertices.

Consider from: $(k+1)$ vertices

remove one vertex v

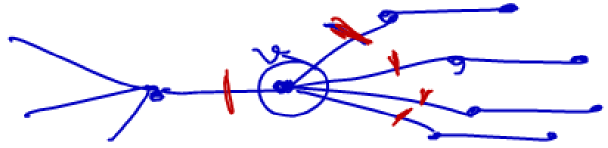


$d(v) = 0$

no. of connected components drops by 1,

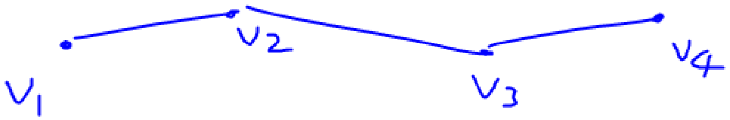
$d(v) > 0$

no of connected comp.

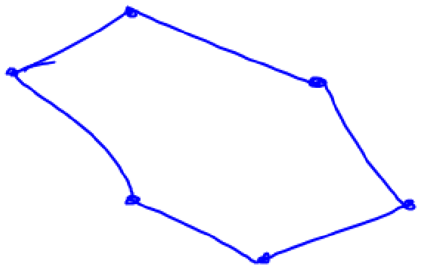


Corr. In a connected graph with n vertices & e edges,

$$\boxed{e \geq n-1}$$



Path : $(n-1)$ edges (P_n)

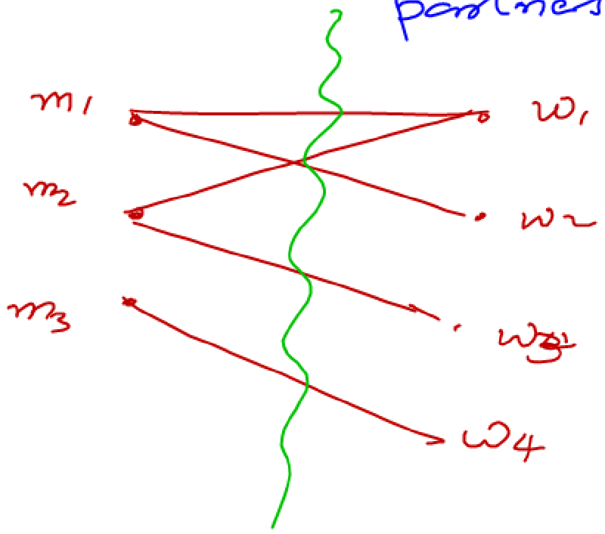


cycle : n edges (C_n)

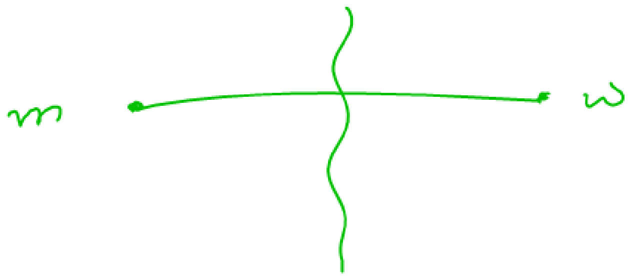
Univ. of Chicago :

Study. on average men have 74% more partners than women.

$\Downarrow$



$$(\text{av deg of } m \text{ vertex}) \geq 1.74 \, (\text{av degree of a } w \text{ vertex})$$



$$\sum \deg(m) = \sum \deg(w)$$

$$\sum d(m) = \sum d(\omega)$$

$$\Downarrow$$

$$\frac{\sum d(m)}{|m|} = \frac{\sum d(\omega)}{|\omega|} \cdot \frac{|\omega|}{|m|}$$

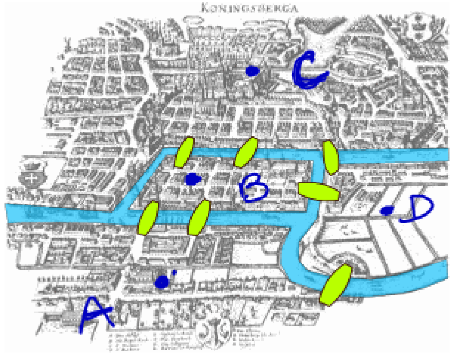
$$(\text{av deg of } m) = \frac{|\omega|}{|m|} \cdot (\text{av deg of } \omega)$$

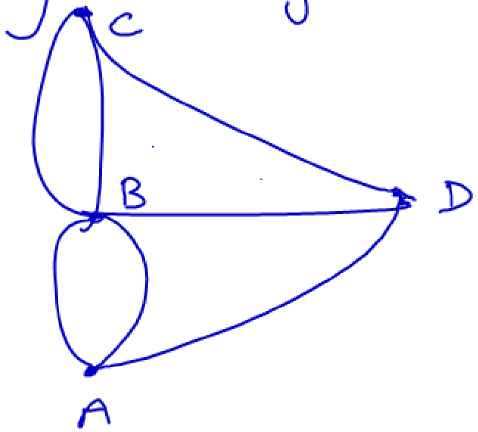
$$= 1.035 \; (\; \ldots \;)$$

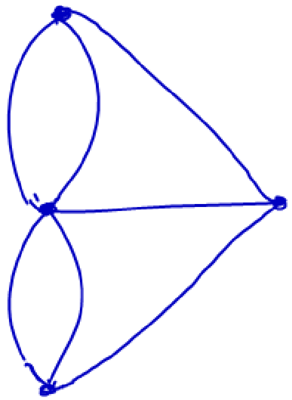
In a graph G with n vertices & e edges

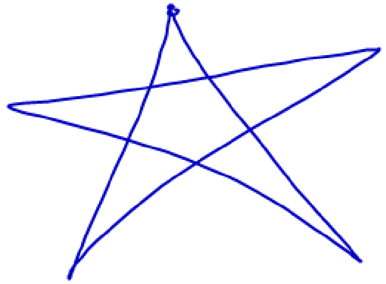
$$\boxed{\sum d(v_i) = 2e}$$

Euler: Konigsberg bridges

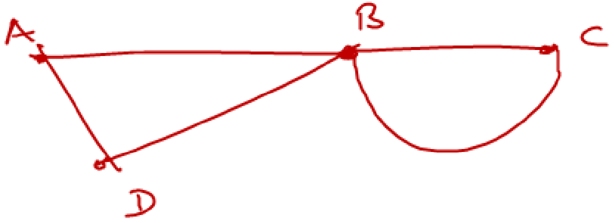








draw a figure without lifting hand, without covering any line twice



Graph is connected

Necessary Condition: Every vertex should have even degree

Sufficient condition.

[Euler]

Even degree of every vertex is sufficient for such a cycle to exist

Proof in next class