Algorithms 6

Divij Singh

22/02/18

1

Consider $lim_{n\to\infty}\frac{n^b}{a^n}$
We can apply L'Hopital's rule, we get $lim_{n\to\infty}\frac{bn^{b-1}}{a^n ln|a|}$
Applying the rule $b-1$ times: $\frac{b!}{ln^b a}lim_{n\to\infty}\frac{1}{a^n}$
Applying the limit: $\frac{b!}{ln^b a} * \frac{1}{a^\infty} = \frac{b!}{ln^b a} * 0 = 0$

Thus, using the definition of o, we can conclude that since $lim_{n\to\infty}\frac{n^b}{a^n} = 0$, $n^b = o(a^n)$

2

To prove: $lim_{n\to\infty}\frac{lg^a n}{n^b} = 0$
Using L'Hopital's $a+1$ times, we get: $\frac{a!}{b^a}lim_{n\to\infty}\frac{1}{n^b}$
Applying limits, we get: $\frac{a!}{b^a} * 0 = 0$

Q.E.D

3

To prove: $lim_{n\to\infty}\frac{n!}{n^n} = 0$
Via Sterling's approximation of $n!$, we know that: $n! = \sqrt{2\pi} * n^{1/2} * (\frac{n}{c})^n * (1+\frac{c}{n})$
Applying that to our equetion, we get: $lim_{n\to\infty}\frac{n!}{n^n} = \sqrt{2\pi}(lim_{n\to\infty}\frac{n^{1/2}}{c^n})$
We know from Q1 that $lim_{n\to\infty}\frac{n^{1/2}}{c^n} = 0$, leaving us with $\sqrt{2\pi} * 0 = 0$
Q.E.D

4

To prove: $lim_{n\to\infty}\frac{n!}{2^n} = \infty$
Again we use Sterling's approximation, getting:
$lim_{n\to\infty}\frac{n!}{2^n} = \sqrt{2\pi}lim_{n\to\infty}\frac{n^n}{(2e)^n} * \sqrt{n} + \theta * lim_{n\to\infty}\frac{n^{n-0.5}}{(2e)^{n-0.5}} * 2e^{0.5}$

Applying limits, we get: $\sqrt{2\pi} * \infty + \infty = \infty$
Q.E.D

5

6

The order is as follows: $\frac{n}{logn}; 2^{\sqrt{lg(n)}}; 2^{lg(n)}; \frac{n}{lg(n)}; n^n$

7