Nondimensionalization of Lipid Dynamics Model

December 18, 2020

Length scale: a half of lipid length $[L] = 1.25$ nm

water viscosity at 20°C= 293.15°K: $\mu = 1$ cP $= 10^{-12}\ \frac{\text{kg}}{\text{sec}\cdot\text{nm}}$

Line tension $\gamma = \frac{45}{11}$ mJ/nm= 1 kT/nm ≈ 4.1 pN nm

set force scale $[F] = \gamma$, we obtain

$$\text{time scale } [T] = \frac{\mu[L]^2}{[F]} = \frac{11 \cdot 10^{-12}\ \frac{\text{kg}}{\text{sec}\cdot\text{nm}} \cdot 1.25^2\ \text{nm}^2}{4.5 \cdot 10^{-2}\ \frac{\text{kg}\cdot\text{nm}}{\text{sec}^2}} \approx 3.82 \times 10^{-10}\ \text{sec}$$

Energy scale $[E] = [F][L] = 1 \cdot 1.25$ kT $= 5.13$ pN nm dimensionless shear rate $\chi = \dot{\gamma}[T]$

(ref. Finken, Eur. Phys. J. E. **25**, 2008)

For $N = 50$ vesicle simulations, the initial radius of the vesicle is $R_0 = 6.75[L] = 8.4375$ nm, the bending rigidity $\kappa = 8.51$ kT ≈ 35 pN nm (SIAM MMS paper)

$$\chi = \dot{\gamma} \cdot \frac{\mu R_0^2}{\kappa} = \dot{\gamma} \cdot \frac{10^{-12} \frac{\text{kg}}{\text{sec} \cdot \text{nm}} \times 8.4375^3 \text{ nm}^3}{35 \times 10^{-3} \frac{\text{kg} \cdot \text{nm}^2}{\text{sec}^2}} = \dot{\gamma} \cdot 17.16 \times 10^{-9} \text{ sec} = \dot{\gamma} \cdot 44.92[T].$$

(ref. Brandner et al.) In Figure 7, the shear rates are
(a) $\dot{\gamma} = 3.7 \times 10^7 \ s^{-1}$; (b) $\dot{\gamma} = 1.9 \times 10^9 \ s^{-1}$; (c) $\dot{\gamma} = 3.7 \times 10^9 \ s^{-1}$

By applying our timescale [T], we have all dimensionless shear rates:
(a) $\chi = 0.0141$; (b) $\chi = 0.7258$; (c) $\chi = 1.41$.

If we adopt the scaling law from Finken's paper with $R_0 = 10$ nm, then we have $\chi = \dot{\gamma} \cdot \frac{\mu R_0^2}{\kappa} \approx \dot{\gamma} \cdot 75[T]$.