Consider Equation (1) from Cole's thesis

$$\Delta u + 2ik\frac{\partial u}{\partial z} + k^2\left(\frac{n^2}{n_0^2} - 1\right) = 0 \qquad (x,y)\in\mathbb{R}^2 \quad z>0$$

$$u(x,y,0) = u_0(x,y)$$

In the case that $k, n, + n_0$ are all constant, we can solve this PDE in Laplace space. Let

$$U(x,y,s) = \mathcal{L}[u] = \int_0^\infty u(x,y,z)e^{-sz}\,dz$$

Then,

$$\Delta U + 2ik\left(-sU + u_0(x,y)\right) + k^2\left(\frac{n^2}{n_0^2} - 1\right)U = 0$$

$$\Rightarrow \Delta U + \left(-2iks + k^2\left(\frac{n^2}{n_0^2} - 1\right)\right)U = -2iku_0(x,y)$$

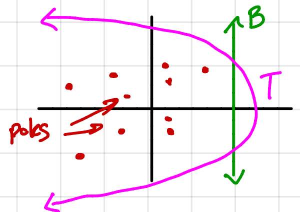
Instead of a Gaussian for u_0, imagine u_0 is a point source

$$\Rightarrow \Delta U + \left(-2iks + k^2\left(\frac{n^2}{n_0^2} - 1\right)\right)U = -2ik\delta(x-x_0, y-y_0)$$

This is an elliptic linear PDE with an exact solution that I'll call $U(x,y;s)$. It is a scaled version of the PDEs fundamental solution. To recover $u(x,y,z)$, we need to invert $\mathcal{L}$.

$$u(x,y,z) = \mathcal{L}^{-1}[U] = \frac{1}{2\pi i}\int_B U(x,y;s)e^{sz}\,ds$$

where B is a vertical line in $\mathbb{C}$ to the right of all poles of $U(x,y;s)$. In my papers with Jake + Alan Lindsay, we deform B to be a Talbot contour which gives a much nicer integrand to integrate numerically



We could also include Dirichlet or Neumann boundary conditions in (x,y), but I'm not sure there is a physical need for this.