New Models to Characterize and Predict Rare Events in Complex Physical Systems

The objective of this proposal is to assemble new mathematical tools and models to characterize and predict rare events in complex physical processes of interest to the United States Navy. The physical processes of interest include: (1) surface water waves, as generated by wind and tidal forces in earth's oceans, and as they affect the theatre of the US Navy's operations; (2) thermal convection, as the driving force behind both atmospheric dynamics and the ocean's large-scale circulation structure; (3) the earth's magnetosphere, as generated by interior liquid-core dynamics, and as it underlies navigational systems. The rare events in these systems respectively include: (1) anomalous or rogue water waves; (2) large-scale circulation reversals in the atmosphere/ocean; (3) magnetic field drifts and reversals.

I will briefly describe the goals for each of these three processes in order. First, anomalous or rogue water waves are those that are significantly higher than the surrounding wave field. They can appear spontaneously without warning. The power possessed by these waves can disable or destroy sea-going vessels or naval structures, potentially impacting naval operations in a severe way [1]. The PI has experience modeling anomalous water waves using a truncated-dynamical-systems approach, in particular performing deterministic and statistical analysis of the truncated Korteweg-De Vries (TKdV) PDE system [2, 3]. This work has provided a highly successful description of anomalous wave statistics that are induced by abrupt changes in water depth, for instance by subsurface shelves or ridges present in earth's oceans. Though many questions have been answered, many more arise. The next generation of theoretical work will see vastly improved numerical methods to sample wave events from the TKdV Gibbs ensemble. These methods, informed by the theoretical advances outlined in [4], will enable ultra-efficient sampling of the Gibbs ensemble to produce a database of wave events without the need to directly simulate dynamics. Such a database will enable rapid hypothesis testing as to the nuanced mechanisms for how bottom-topography triggers anomalous waves and potentially the prediction of such events and/or inverse-problem approaches to infer underlying structure from wave-field observations.

The second main objective of this proposal involves natural convection, a process that underlies atmospheric and oceanic circulation patterns. Despite the presence of small-scale turbulence, thermal convection often gives rise to large-scale circulation (LSC) patterns [5]. One prominent example is the Atlantic meridional overturning circulation (AMOC), which encompasses the Gulf Stream. Under some circumstances, LSC patterns are known to spontaneously reverse direction. The PI has recent experience modeling LSC reversals in the idealized context of thermal convection in an annular domain [6, 7]. Here, the PI constructed a first-principled model that accurately predicts a sequence of bifurcations with increasing Rayleigh number. The sequence includes: (1) the onset of convective motion giving rise to steady circulation; (2) the instability of steady circulation giving rise to chaotic LSC reversals; and (3) a return to order at high Rayleigh number, in which reversals recur periodically despite small-scale turbulence. Ultimately, the model reveals that the LSC-reversal phenomenon boils down to the motion of a damped, driven pendulum, with a precisely described driving term that acts to raise the fluid center-of-mass. Despite decades of research on thermal connection, this physically insightful, and precisely defined, link to a mechanical pendulum had been overlooked. The next grand goal will be to leverage this clean understanding of 2D convection to characterize large-scale convection in complex 3D environments and to identify the seeds of LSC reversals [8]. Such work could potentially relate to the observed weakening of the Gulf Stream and the associated evolution of the AMOC pattern.

The third, and most ambitious, objective of this proposal is to leverage the newly developed convective models, in conjunction with magnetohydrodynamics, to examine magnetosphere dynamics with an eye towards field reversals. It is generally accepted that the earth's magnetosphere arises from convective dynamics of conducting iron within the earth's liquid core [9]. Geological evidence suggests that magnetic field reversals occur on an intermittent basis, on a timescale of hundreds of kiloyears. Such a reversal, as well as the weakening of the magnetic field leading up to it, would have clear implications for navigational systems. The study of such dynamo processes is already a well-developed field, with much work on analyzing flow fields that can generate and sustain a strong magnetic field, as well as direct-numerical simulations of the fully-coupled dynamics [9-11]. The PI plans to examine such processes using the approach of truncated dynamical systems and analysis of statistical ensembles. Much like how annular convection represents a clean paradigm problem that is conducive to the development of systematic, first-principled, and physically insightful models, the PI aims to examine dynamo-action within an idealized framework that exhibits the essential physical processes but with minimal extraneous factors.

Problems of this complexity typically do not yield to a single approach. Instead, the PI will bring to bear the full range of analytical and computational tools, encompassing dynamical, statistical, asymptotic, and numerical analysis.

References

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