Subject - Chapter 1

Chapter

Topic 1: Real Numbers and Number Systems

Real Numbers and Number Systems

Definition of Real Numbers: Real Numbers are all numbers that can be represented on the number line. They include both rational and irrational numbers.

Classification of Number Systems:

1. NATURAL NUMBERS (N)

Definition: Numbers used for counting objects. Set: N = {1, 2, 3, 4, 5, 6, ...}

Properties: • Smallest natural number is 1 • No largest natural number (infinite set) • Used for counting discrete objects • Closure property under addition and multiplication • Not closed under subtraction and division

Examples: • Counting books: 1, 2, 3, 4, 5 books • Number of students in class: 30, 45, 50 • Age of a person: 15, 20, 25 years

2. WHOLE NUMBERS (W)

Definition: Natural numbers including zero. Set: W = {0, 1, 2, 3, 4, 5, ...}

Properties: • Smallest whole number is 0 • Zero is the additive identity: 0 + a = a + 0 = a • Every natural number is a whole number • Closed under addition and multiplication • Not closed under subtraction and division

Importance of Zero: • Represents 'nothing' or 'absence' • Makes place value system possible • Additive identity element • Invented in India by Brahmagupta

3. INTEGERS (Z)

Definition: Whole numbers along with negative numbers. Set: Z = {..., -3, -2, -1, 0, 1, 2, 3, ...}

Types of Integers: • Positive integers: +1, +2, +3, ... (same as natural numbers) • Negative integers: -1, -2, -3, ... • Zero: Neither positive nor negative

Properties: • For every positive integer, there exists a corresponding negative integer • Additive inverse property: a + (-a) = 0 • Closed under addition, subtraction, and multiplication • Not closed under division • Multiplicative identity: 1 × a = a × 1 = a

Applications: • Temperature: -5°C, 0°C, 25°C • Floors in building: -2 (basement), 0 (ground), +5 (fifth floor) • Profit and loss: +^{1}500 (profit), -^{1}200 (loss)

4. RATIONAL NUMBERS (Q)

Definition: Numbers that can be expressed in the form p/q, where p and q are integers and q "` 0.

Set: Q = {p/q : p, q " Z, q "` 0}

Forms of Rational Numbers:

a) Fractions: • Proper fractions: 1/2, 3/4, 5/7 (numerator < denominator) • Improper fractions: 5/3, 7/4, 9/2 (numerator "e denominator) • Mixed numbers: 2½, 3¾, 4!T

b) Integers: • Any integer can be written as fraction: 5 = 5/1, -3 = -3/1

c) Terminating Decimals: • Decimals that end after finite digits • 0.5 = 1/2, 0.25 = 1/4, 0.125 = 1/8 • 0.75 = 3/4, 0.875 = 7/8

d) Non-terminating Repeating Decimals: • Decimals with repeating pattern • 0.333... = 1/3 (1/3 = 0.3) • 0.666... = 2/3 (2/3 = 0.6) • 0.142857142857... = 1/7 (1/7 = 0 (1/11 = 0.09)

Identifying Terminating/Non-terminating: A rational number p/q (in lowest terms) has: • Terminating decimal if q has only factors of 2 and 5 • Non-terminating repeating decimal otherwise

Examples: • 3/8: q = 8 = 2^3 !' Terminating (0.375) • 7/20: Terminating (0.35) • 1/6: q = 6 = 2 × 3 !' Non-terminating 12 = 2^2 × 3 !' Non-terminating (0.41666...)

5. IRRATIONAL NUMBERS

Definition: Numbers that cannot be expressed in p/q form, where p and q are integers and q "` 0.

Characteristics: • Non-terminating non-repeating decimal expansion • Cannot be written as fractions • Infinite decimal digits without repeating pattern

Examples:

a) Square roots of non-perfect squares: • "2 = 1.41421356... (never "5 = 2.23606797... • "7 = 2.64575131...

b) Mathematical constants: • À (pi) = 3.14159265358979... (ratio of (Euler's number) = 2.718281828459045... • Golden ratio Æ = 1.61803

c) Cube roots of non-perfect cubes: • "2 = 1.25992104989487... • "5

Proving "2 is Irrational: Using contradiction method: 1. Assume "2 is terms) 2. Then 2 = p^2/q^2, so $2q^2 = p^2$ 3. This means p^2 is even, so p is even 4. Let p = 2k, then $2q^2 = 4k^2$, so $q^2 = 2k^2$ 5. This means q^2 is even, so q is even 6. Both p and q are even, contradicting lowest terms assumption 7. Therefore, "2 is irrational

RELATIONSHIP BETWEEN NUMBER SETS:

Hierarchy: N ", W ", Z ", Q ", R

This means: • Every natural number is a whole number • Every whole number is an integer • Every integer is a rational number • Every rational number is a real number • Some real numbers are irrational

REAL NUMBERS = RATIONAL NUMBERS "* IRRATIONAL NUMBERS

Number Identification Table:

| Number | Natural | Whole | Integer | Rational | Irrational | Real |
|---------|---------|-------|---------|----------|-----------| ---- | | 7 | ' | '
-5 | ' | ' | ' | ' | ' | ' | | 3/4 | ' | ' | ' | ' | ' | ' | | 0.75 | ' | ' | ' | ' | '
| ' | ' | ' | ' | ' | ' |

IMPORTANT PROPERTIES:

1. Density Property: • Between any two rational numbers, there exist infinitely many rational numbers • Between any two real numbers, there exist infinitely many real numbers • Between any two irrational numbers, there can be rational numbers

2. Closure Properties: • Real numbers are closed under addition, subtraction, multiplication, and division (except by zero) • The sum or product of rational and irrational numbers can be either rational or irrational

3. Examples of Operations: • Rational + Rational = Rational (1/2 + 1/3 = 5/6) • Irrational + Irrational = Can be rational or irrational - "2 + "2 = 2"2 (irrational) - "2 + (-"2
Irrational = Irrational (except when rational = 0) - 3 × "2 = 3"2 (irra

APPLICATIONS: • Real numbers are used in measurements, calculations, and scientific computations • Rational numbers are used in fractions, percentages, and ratios • Irrational numbers appear in geometry (À in circles, "2 in squares) • Understanding num
algebra and higher mathematics