Calculuus - The Reckoning

Krishna ;)

Contents

Chapter 1

1.1 The Tests

Definition 1.1.1: The Geometric Series Test

Suppose you have a series

$$\sum_{n=1}^{\infty} a(r)^{n-1}$$

or any other series with a common ratio r and an initial a, involving a multiplication of that variate value r over the indexes of said infinite series $\sum a_n$. The series converges if:

$$r < 1$$

and diverges when:

$$r > 1$$

If the infinite series $\sum a_n$ converges the sum can be found with:

$$S = \frac{a}{1-r}$$

And the interval of converges can also be found if r is a variate quantity with the inequality $|r| < 1$. Using this method, the *endpoints* must be tested to see if the bounds are inclusive of exclusive.

Definition 1.1.2: Telescoping Series Test

Suppose you have a series $\sum a_n$ of the form

$$a_n = b_n - b_{n+1}$$

where

$$\sum a_n = \sum_{n=1}^{\infty} b_n - b_{n+1} = (b_1 - \cancel{b_2}) + (\cancel{b_2} - \cancel{b_3}) + (\cancel{b_3} - b_4) + \ldots$$

$$\implies S = b_1 \therefore S_k = b_1 - b_{k+1}$$

Definition 1.1.3: n^{th} term test

Suppose you have a series $\sum a_n$

$$L = \lim_{n\to\infty} a_n$$

$\sum a_n$ diverges if:

$$L \neq 0$$

Definition 1.1.4: The Integral Test

Suppose you have a series $\sum_{n=j}^{\infty} a_n$, where $a_n = f(x)$, and $f(x)$ is continuous, positive and decreasing on the interval $[j, \infty)$.

$$\sum_{n=j}^{\infty} a_n \text{ converges if}$$

$$L = \int_j^{\infty} f(x) = \lim_{b\to\infty} \int_j^b f(x)$$

where L is some positive, finite value meeting the condition: $0 < L < \infty$

The sum diverges if L diverges (DNE counts)