

Calculus - The Reckoning

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Chapter 1

1.1 The Tests

Definition 1.1.1: The Geometric Series Test

Suppose you have a series

$$\sum_{n=1}^{\infty} a(r)^{n-1}$$

or any other series with a common ratio r and an initial a , involving a multiplication of that variate value r over the indexes of said infinite series $\sum a_n$. The series converges if:

$$r < 1$$

and diverges when:

$$r > 1$$

If the infinite series $\sum a_n$ converges the sum can be found with:

$$S = \frac{a}{1-r}$$

And the interval of converges can also be found if r is a variate quantity with the inequality $|r| < 1$. Using this method, the *endpoints* must be tested to see if the bounds are inclusive or exclusive.

Definition 1.1.2: Telescoping Series Test

Suppose you have a series $\sum a_n$ of the form

$$a_n = b_n - b_{n+1}$$

where

$$\begin{aligned} \sum a_n &= \sum_{n=1}^{\infty} b_n - b_{n+1} = (b_1 - \cancel{b_2}) + (\cancel{b_2} - \cancel{b_3}) + (\cancel{b_3} - b_4) + \dots \\ \implies S &= b_1 \therefore S_k = b_1 - b_{k+1} \end{aligned}$$

Definition 1.1.3: n^{th} term test

Suppose you have a series $\sum a_n$

$$L = \lim_{n \rightarrow \infty} a_n$$

$\sum a_n$ diverges if:

$$L \neq 0$$

Note:-

This test only tests for divergence.

Definition 1.1.4: The Integral Test

Suppose you have a series $\sum_{n=j}^{\infty} a_n$, where $a_n = f(x)$, and $f(x)$ is continuous, positive and decreasing on the interval $[j, \infty)$.

$$\sum_{n=j}^{\infty} a_n \text{ converges if}$$

$$L = \int_j^{\infty} f(x) = \lim_{b \rightarrow \infty} \int_j^b f(x)$$

where L is some positive, finite value meeting the condition: $0 < L < \infty$

The sum diverges if L diverges (DNE counts)

Definition 1.1.5: p-series Test

Suppose you have a series of the form:

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

This series converges if $p > 1$ and diverges if $0 < p \leq 1$

Definition 1.1.6: The Alternating Series Test

Suppose you have a series:

$$\sum a_n = \sum_{n=1}^{\infty} (-1)^{n+k} b_n$$

$$||a_n|| = b_n$$

If:

$$\lim_{n \rightarrow \infty} b_n = 0$$

$$b_n > b_{n+1}$$

The series $\sum a_n$ converges.

Note:-

This test only tests for convergence.

Definition 1.1.7: The Ratio Test

Suppose you have a series $\sum a_n$

$$L = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$$

The series converges if:

$$L < 1$$

Diverges if:

$$L > 1$$

and fails when:

$$L = 1$$

Note:-

This test is especially useful when dealing with factorials and exponential, as the powers reduce. There is the off-chance that you find a product such as $2 * 5 * 7 * 9 * (2n - 1)$, where this technique tends to work well, just as it does in the factorial pattern.

1.2 The Tests of Character