1 Aigrain et al. 2012 — FF' Notes

1.1 Location and Flux of a Spot

Let there be two reference frames of a star, S and S'. S is the original position of the star whereas S' is after the star has rotated/changed to the position as we see it now. Thus, we are in the S' frame and all measurements influenced by a spot on the surface of the star depend on that frame's coordinates. **Note: the coordinates used are longitude and latitude (ϕ and δ), not polar coordinates (θ and ϕ).**

Falling from spherical coordinates, the location of a spot on the surface of a star (with radius R_*, rotation period P_{rot}, and inclination i) is:

$$S = \frac{1}{R_*}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\delta\,\cos\phi \\ \cos\delta\,\sin\phi \\ \sin\delta \end{bmatrix}, \tag{1}$$

where δ is the latitude of spot, and $\phi(t) = 2\pi t/P_{\mathrm{rot}} + \phi_0$ with ϕ_0 as the zero phase of the spot at time $t = 0$.

If we rotate the spot about the y-axis by and angle of $\theta = -(\pi/2 - i)$ (i.e., multiply this position vector by the standard rotation $R_y(\theta)$ matrix, see subsection 1.4), then the spot's new position vector becomes:

$$\begin{aligned} S' = \frac{1}{R_*}\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} &= R_y(\theta)\begin{bmatrix} \cos\delta\,\cos\phi(t) \\ \cos\delta\,\sin\phi(t) \\ \sin\delta \end{bmatrix}, \\ &= \begin{bmatrix} \cos\delta\,\cos\phi(t)\,\sin i + \sin\delta\,\cos i \\ \cos\delta\,\sin\phi(t) \\ -\cos\delta\,\cos\phi(t)\,\cos i + \sin\delta\,\sin i \end{bmatrix}. \end{aligned} \tag{2}$$

Here, they define the vector along the viewer line-of-sight as the x' axis. Thus, the angle defined between the spot normal and the line-of-sight, $\beta(t)$ (which is the angle that defines how we view the spot), is just the x' component of the rotated position vector divided by R_*:

$$\cos\beta(t) = \frac{x'}{R_*} = \cos\delta\,\cos\phi(t)\,\sin i + \sin\delta\,\cos i. \tag{3}$$

The (relative drop in) flux from the star's surface due to a spot follows the same time dependence and is thus defined as:

$$F(t) = f\ \cos\beta(t) = f\,(\cos\delta\ \cos\phi(t)\ \sin i + \sin\delta\ \cos i)\,, \tag{4}$$

where $f = 2(1-c)(1-\cos\alpha)$ is the relative flux drop at the spot center and has contributions from the angular size of the spot, α, and the spot contrast ratio, c.

1.2 RV Signature of a Spot

1.2.1 Rotational Motion

As the star rotates, spots suppress flux on the portion of the stellar surface they reside on. Thus, the RV signature from the spot due to rotation is the **projected area of the spot**, $F(t)$, multiplied by that rotational motion. This rotational motion at the position of the spot is defined as:

$$V_{\mathrm{rot}} = \begin{bmatrix} -V_{\mathrm{eq}}\cos\delta\ \sin\phi(t) \\ V_{\mathrm{eq}}\cos\delta\ \cos\phi(t) \\ 0 \end{bmatrix}, \tag{5}$$

where $V_{\mathrm{eq}} = 2\pi R_*/P_{\mathrm{rot}}$ is the amplitude of the equatorial velocity, and the rest comes from taking the derivative of the position vector (S) with respect to t.

Doing the **exact** same rotation as in Equation 2, we obtain the rotational velocity vector of the spot in the S' frame:

$$V'_{\mathrm{rot}} = \begin{bmatrix} -V_{\mathrm{eq}}\cos\delta\ \sin\phi(t)\ \sin i \\ V_{\mathrm{eq}}\cos\delta\ \cos\phi \\ V_{\mathrm{eq}}\cos\delta\ \sin\phi\ \cos i \end{bmatrix}. \tag{6}$$

Following the same argument as the position, the time dependence for rotational motion of the spot depends only on the x' direction, giving us:

$$\Delta\mathrm{RV}_{\mathrm{rot}}(t) = -F(t)\ V_{\mathrm{eq}}\cos\delta\ \sin\phi(t)\ \sin i. \tag{7}$$

1.2.2 Suppression of Convective Blueshift

Accounting for the suppression of the convective blueshift around the magnetized area of the starspot gives us a positive RV contribution dependent on the strength of the magnetic field (set by the ratio of magnetized area to the stellar surface, κ, and the velocity difference between the magnetized region and the surrounding photosphere, δV_c) and the line of sight:

$$\Delta RV_c(t) = +F(t)\ \kappa\ \delta V_c \cos\beta(t). \tag{8}$$

1.3 FF'

Since $F(t) = f\,(\cos\delta\,\cos\phi(t)\,\sin i + \sin\delta\,\cos i)$, then we can take its time derivative to get:

$$\begin{aligned} F'(t) &= -f\,\sin\phi(t)\ \phi'(t)\,\cos\delta\,\sin i, \\ &= -f\,\sin\phi(t)\,\cos\delta\,\sin i\ \left(\frac{2\pi}{P_{\rm rot}}\right), \\ &= -f\,\sin\phi(t)\,\cos\delta\,\sin i\ \left(\frac{V_{\rm eq}}{R_*}\right). \end{aligned} \tag{9}$$

where we can substitute $\phi' = 2\pi/P_{\rm rot} = V_{\rm eq}/R_*$.

Great, but since the RV signature was essentially the derivative of the flux contribution multiplied by the rotational motion, we see that the second part of F' looks suspiciously like the second part of $\Delta RV_{\rm rot}$. This leads us to conclude:

$$\Delta \mathrm{RV}_{\rm rot}(t) = -F(t)\ F'(t)\ \frac{R_*}{f}. \tag{10}$$

Similarly, for the convective blueshift term, we note the contribution from $\cos\beta(t)$, which is present in the definition of $F(t)$, so we find:

$$\Delta \mathrm{RV}_c(t) = F^2(t)\ \frac{\kappa\ \delta V_c}{f}. \tag{11}$$

TLDR; this simple model works because the spot flux signature is reduced to the spot area, $F(t)$, and position and the RV signature is treated as the spot area, $F(t)$, multiplied by the velocity vector (which is the derivative of the position vector, $F'(t)$).

1.4 Extra Angle Definitions

1.4.1 $R_y(\theta)$

Rotation about the y-axis by an angle of $\theta = -(\pi/2 - i)$:

$$\begin{aligned} R_y(\theta = -(\pi/2 - i)) &= \begin{bmatrix} \cos\left(-(\pi/2 - i)\right) & 0 & \sin\left(-(\pi/2 - i)\right) \\ 0 & 1 & 0 \\ -\sin\left(-(\pi/2 - i)\right) & 0 & \cos\left(-(\pi/2 - i)\right) \end{bmatrix} \\ &= \begin{bmatrix} \sin i & 0 & +\cos i \\ 0 & 1 & 0 \\ -\cos i & 0 & \sin i \end{bmatrix} \end{aligned} \tag{12}$$