

Image Filter

Three views of filtering:

- **Image filters in the spatial domain**
 - Filter is a **mathematical operation of a grid of numbers** S
 - smoothing, sharpening, measuring texture
- **Image filters in the frequency domain**
 - Filtering is a **way to modify the frequencies of images**
 - Denoising, sampling, image compression
- **Templates and Image Pyramids**
 - Filtering is a **way to match a template to the image**
 - Detection, coarse-to-fine registration

Templates & Image Pyramids

- Image downsampling
 - Anti-aliasing
- Gaussian image pyramid
- Laplacian image pyramid

Templates & Image Pyramids

- **Image downsampling**

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Image downsampling

- Why does a lower resolution image still make sense to us? What do we lose?

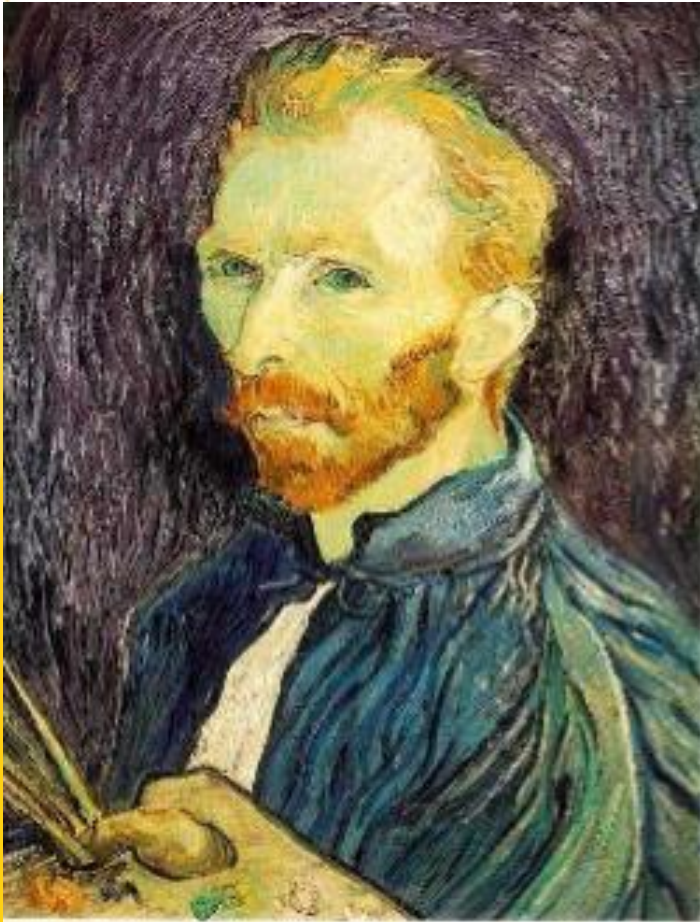


A close-up portrait of Vincent van Gogh, showing his face and upper torso. He has a thick, reddish-brown beard and mustache, and his eyes are a pale, greenish-blue. His hair is a mix of yellow and brown. He is wearing a dark blue jacket over a white shirt. The background is a dark, textured, brownish-purple. The painting is done in Van Gogh's characteristic style, with visible, expressive brushstrokes.

This image is too big to fit on the screen.
How would you reduce it to half its size?

Naïve image downsampling

Throw away half the rows and columns



$1/2$

delete even rows
delete even columns



$1/4$

delete even rows
delete even columns



$1/8$

What is the problem with this approach?

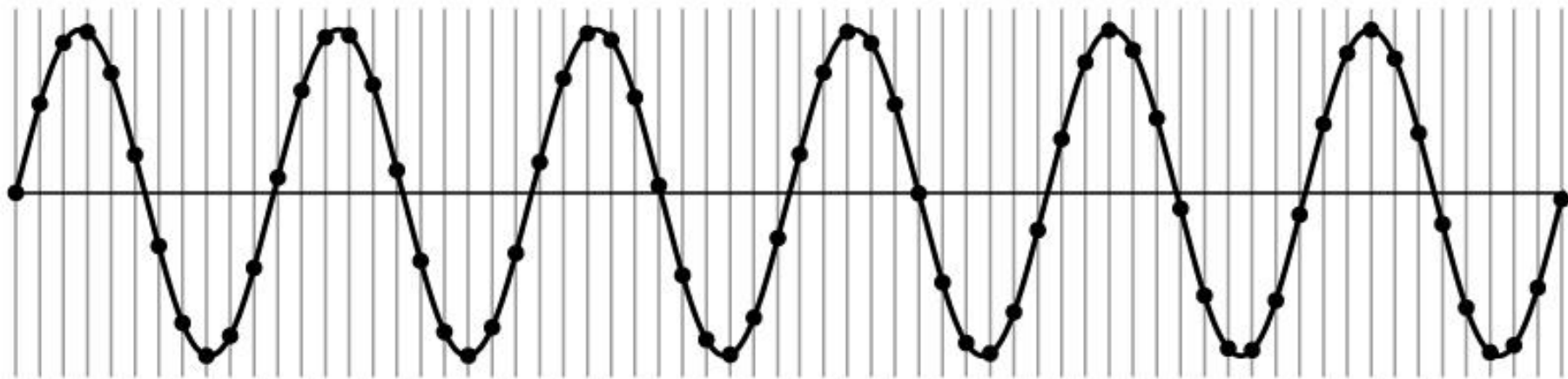
Naïve image downsampling

 $1/2$  $1/4$ (2x zoom) $1/8$ (4x zoom)

What is the $1/8$ image so pixelated (and do you know what this effect is called)?

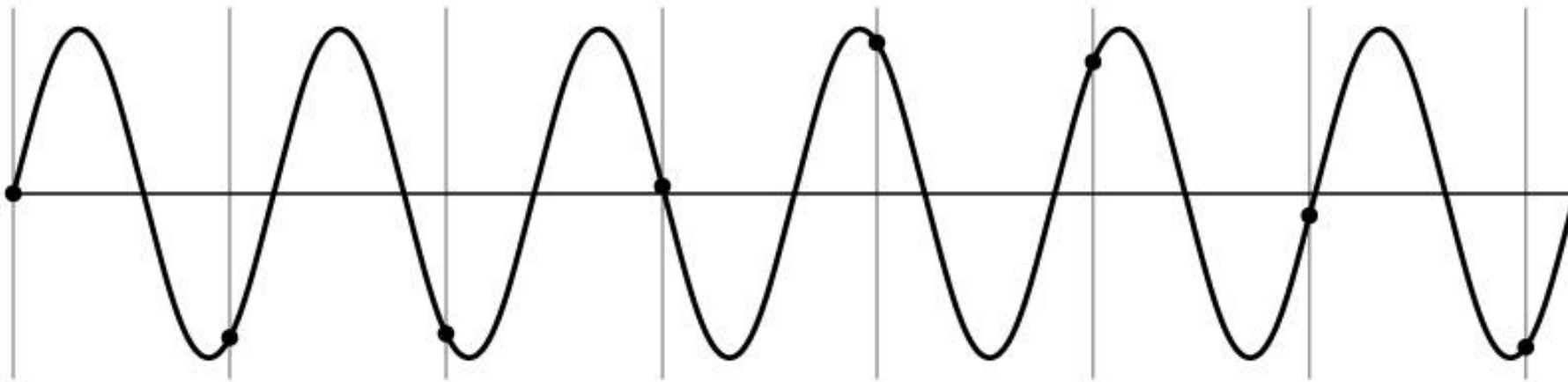
Sampling

Very simple example: a sine wave



Undersampling

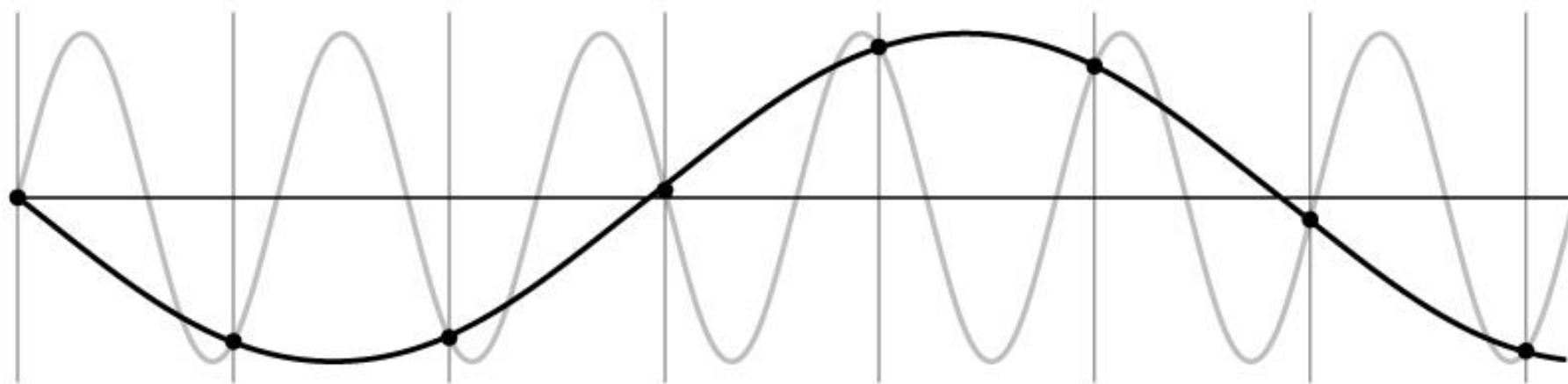
Very simple example: a sine wave



Unsurprising effect: information is lost.

Undersampling

Very simple example: a sine wave

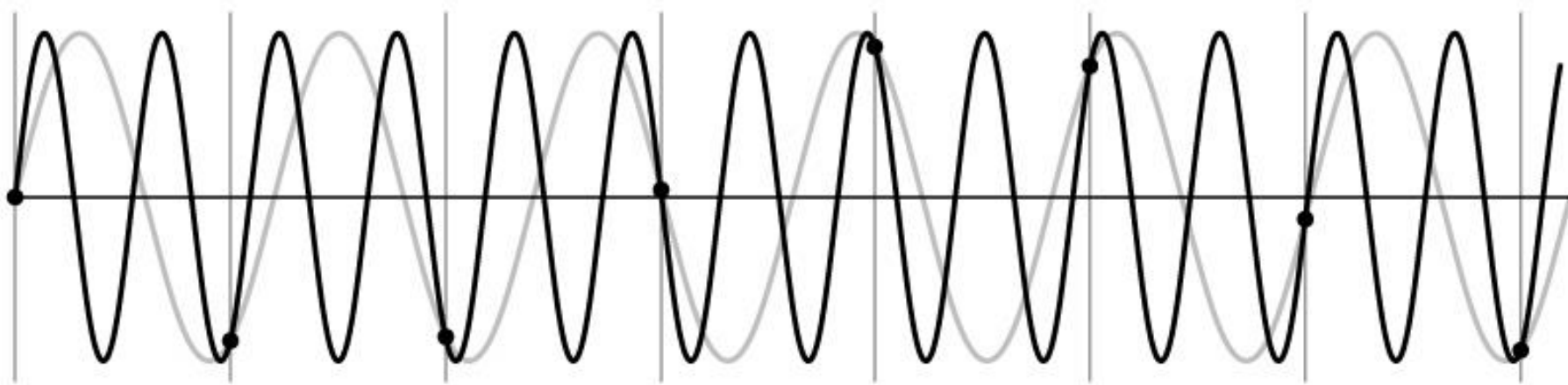


Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

Undersampling

Very simple example: a sine wave



Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

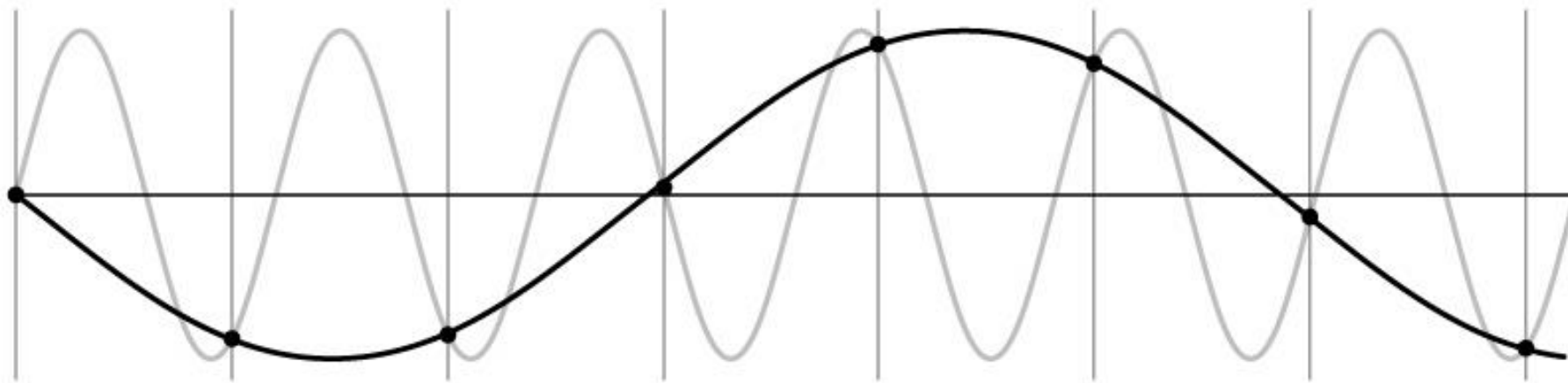
Note: we could always confuse the signal with one of *higher* frequency.

Templates & Image Pyramids

- Image downsampling
- **Anti-aliasing**
- Gaussian image pyramid
- Laplacian image pyramid

Aliasing

Fancy term for: *Undersampling can disguise a signal as one of a lower frequency*

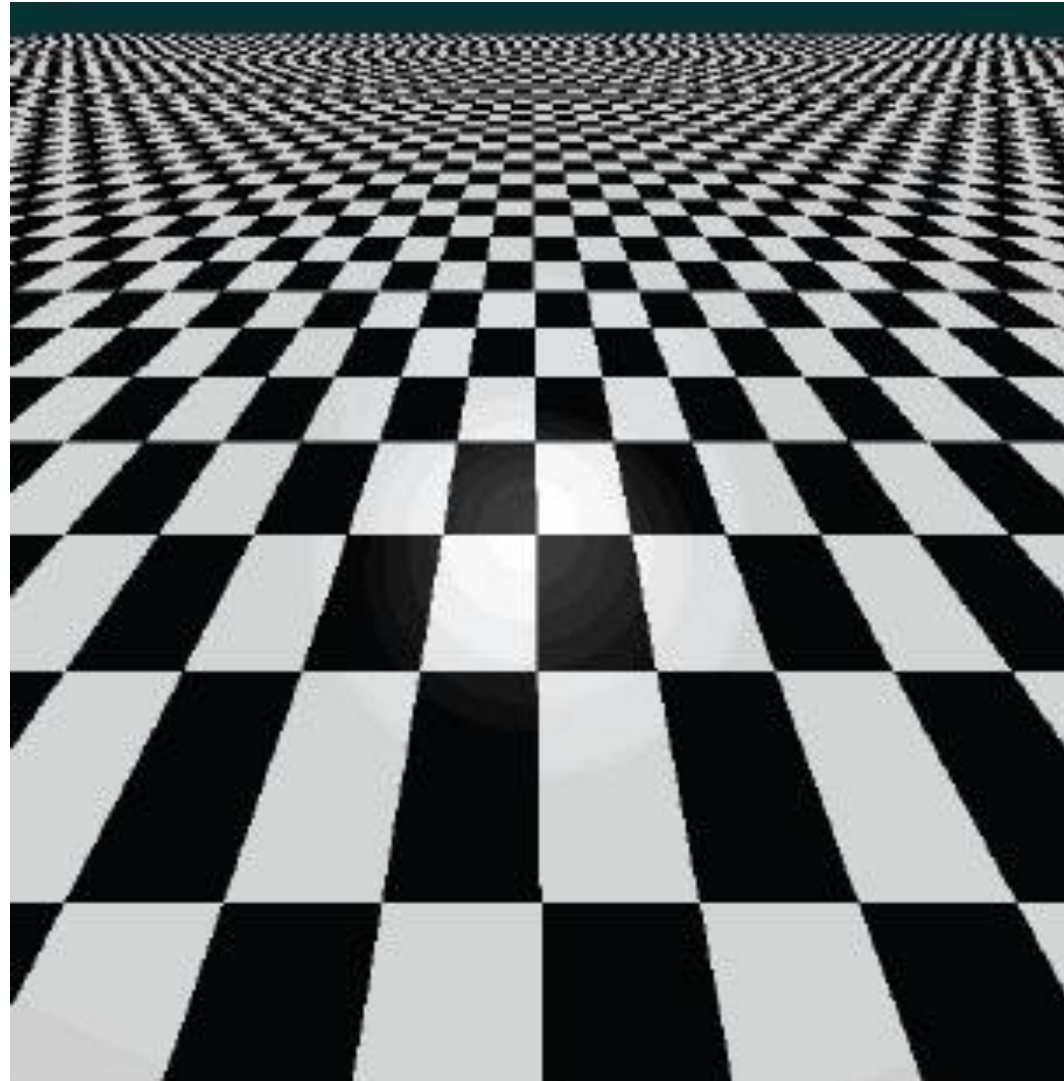


Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

Note: we could always confuse the signal with one of *higher* frequency.

Aliasing in textures



Aliasing in photographs



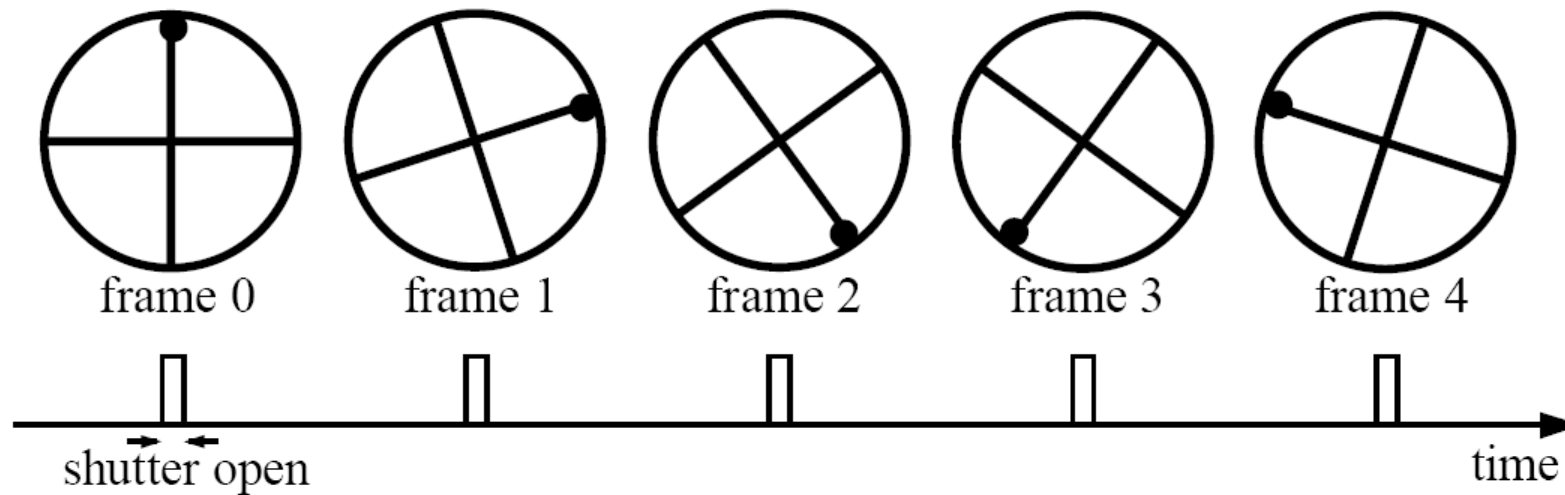
This is also known as “moire”

Temporal aliasing

Imagine a spoked wheel moving to the right (rotating clockwise).

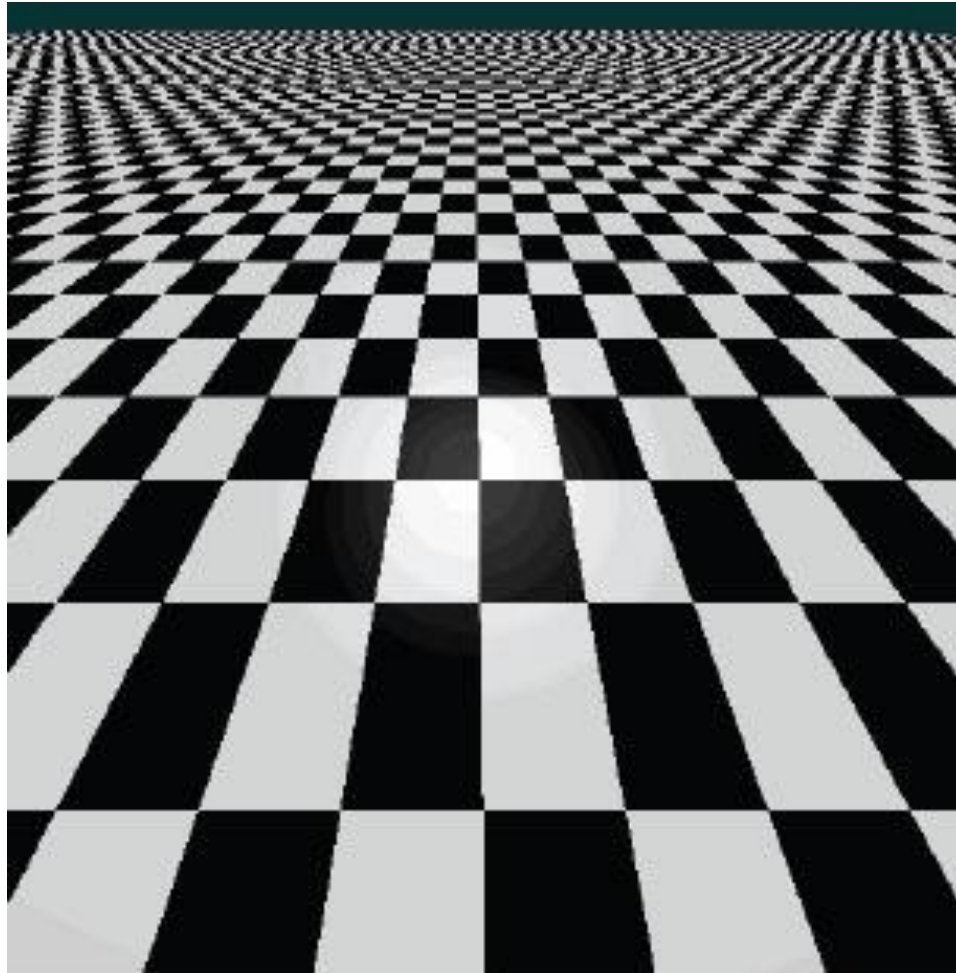
Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):

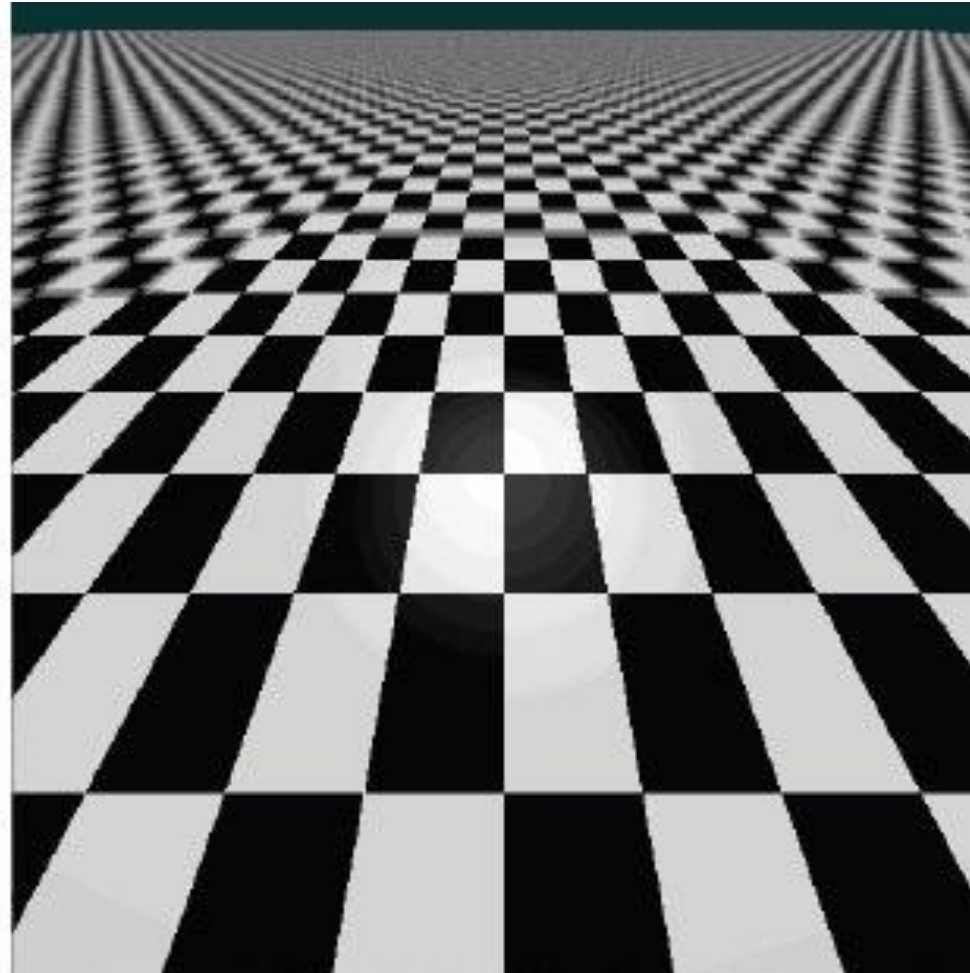


Without dot, wheel appears to be rotating slowly backwards!
(counterclockwise)

Anti-aliasing in textures



aliasing artifacts



anti-aliasing by oversampling

Question 1: How much smoothing do I need to do to avoid aliasing?

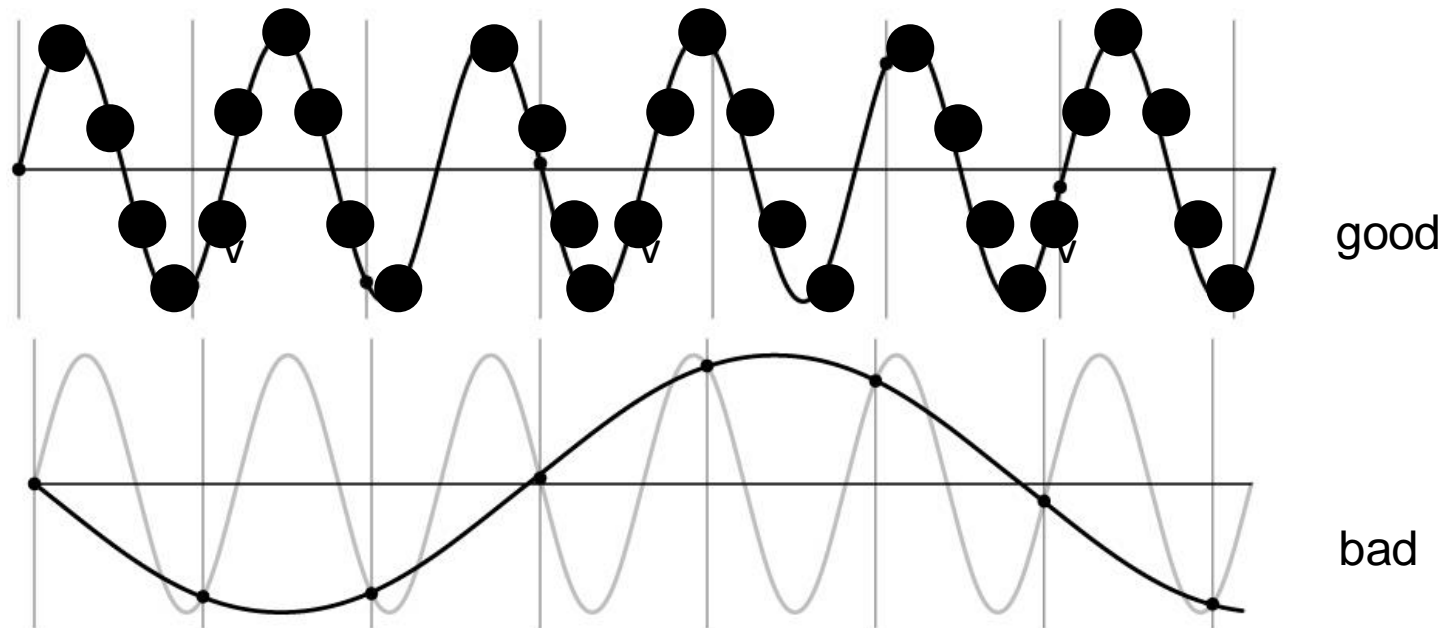
Question 2: How many samples do I need to take to avoid aliasing?

⇒ Answer to both: Enough to reach the Nyquist limit.

We'll see what this means soon.

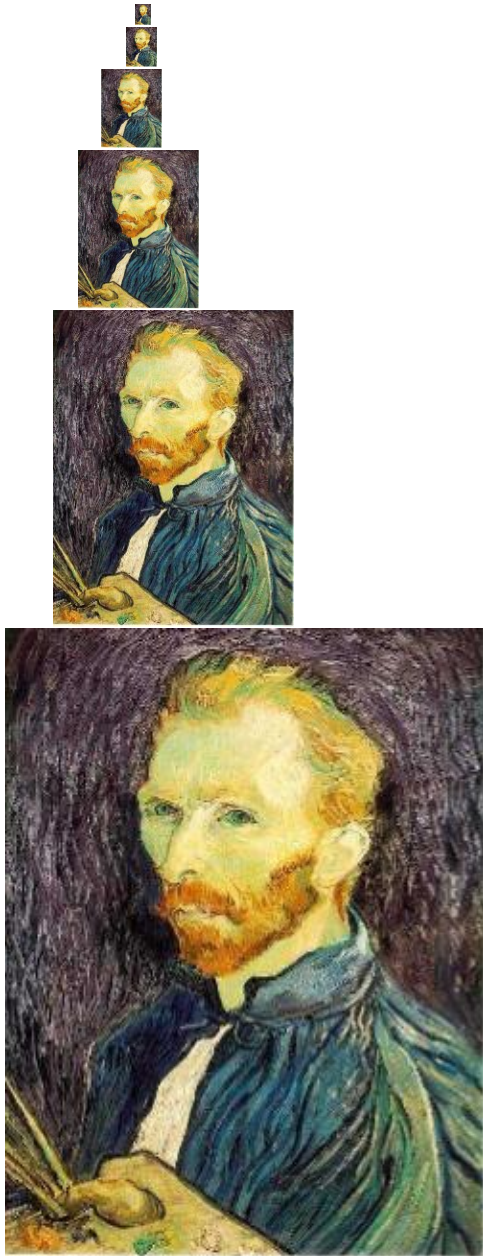
Nyquist-Shannon Sampling Theorem

- When sampling a signal at discrete intervals, the sampling frequency must be $\geq 2 \times f_{\max}$
- f_{\max} = max frequency of the input signal
- This will allow to reconstruct the original perfectly from the sampled version



Templates & Image Pyramids

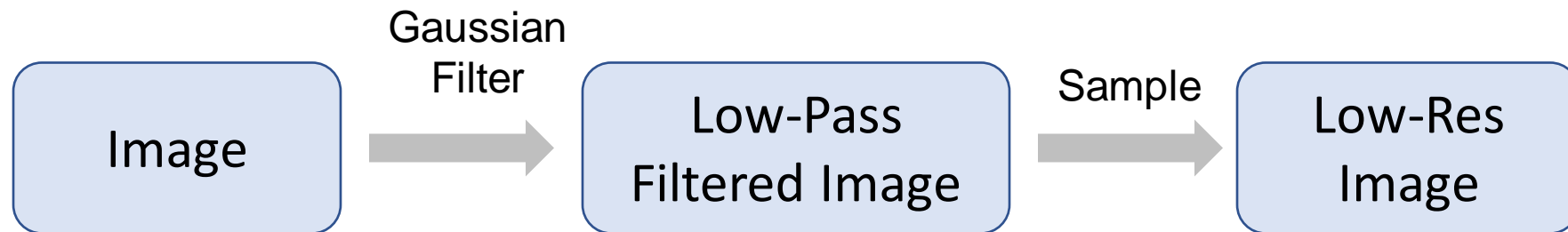
- Image downsampling
- Anti-aliasing
- **Gaussian image pyramid**
- Laplacian image pyramid



Gaussian image pyramid

The name of this sequence of subsampled images

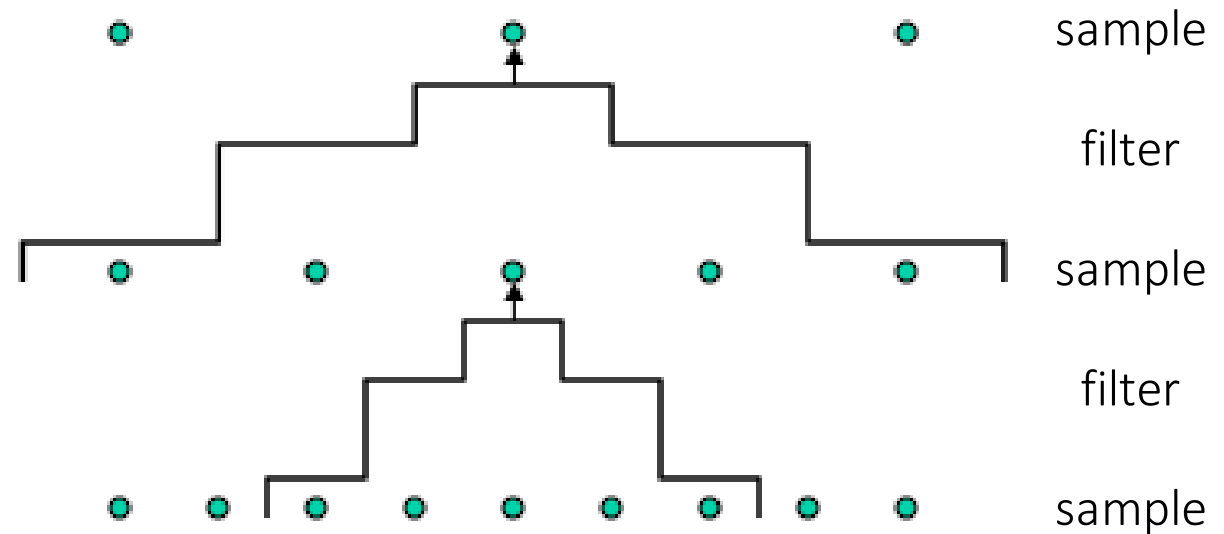
...Gaussian image pyramid



Algorithm

```

repeat:
  filter
  subsample
until min resolution reached
  
```

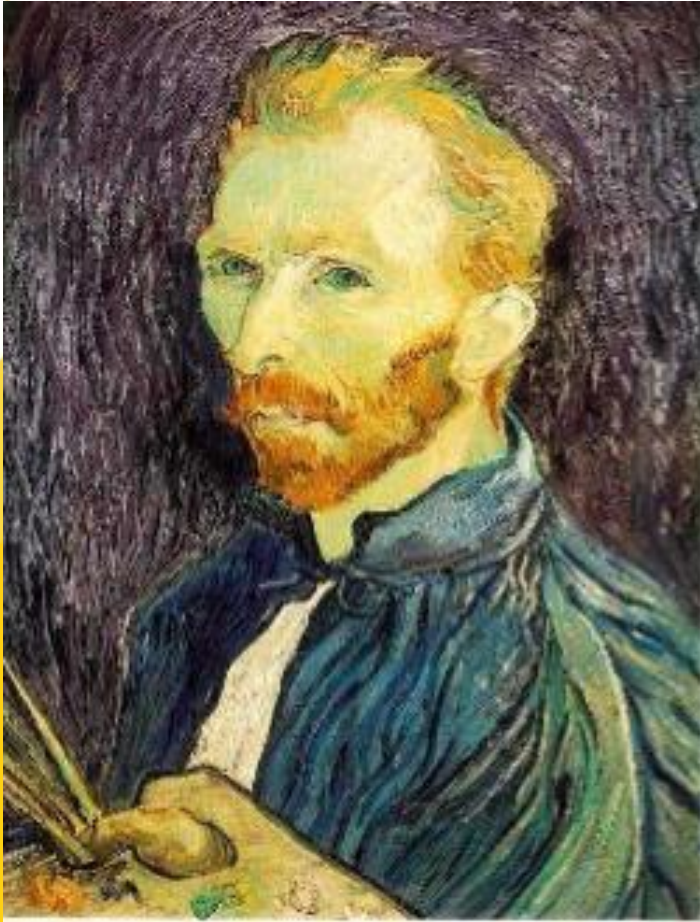


Question: How much bigger than the original image is the whole pyramid?

Answer: Just $\frac{4}{3}$ times the size of the original image!

...Gaussian image pyramid

Apply a smoothing filter first, then throw away half the rows and columns



1/2

Gaussian filter
delete even rows
delete even columns



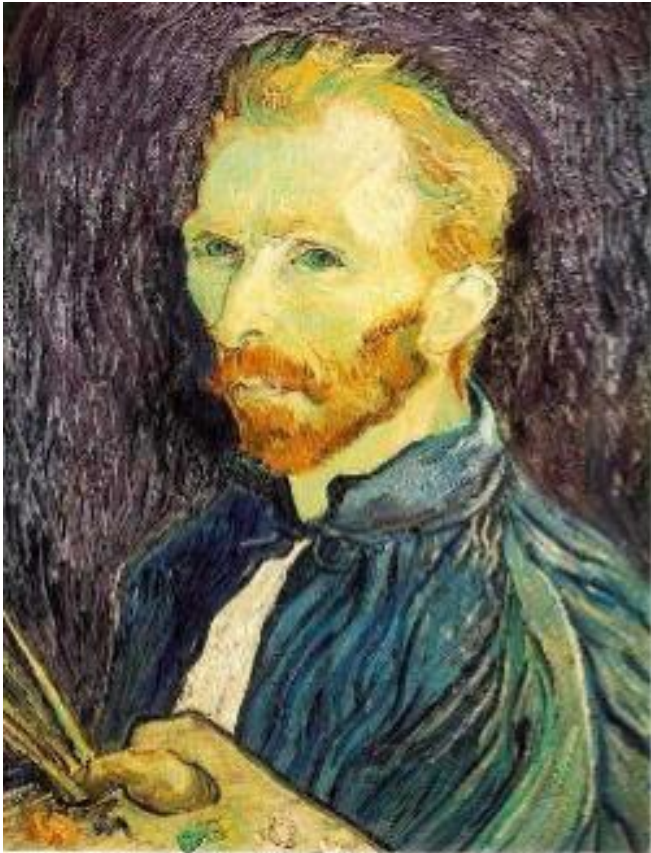
1/4

Gaussian filter
delete even rows
delete even columns

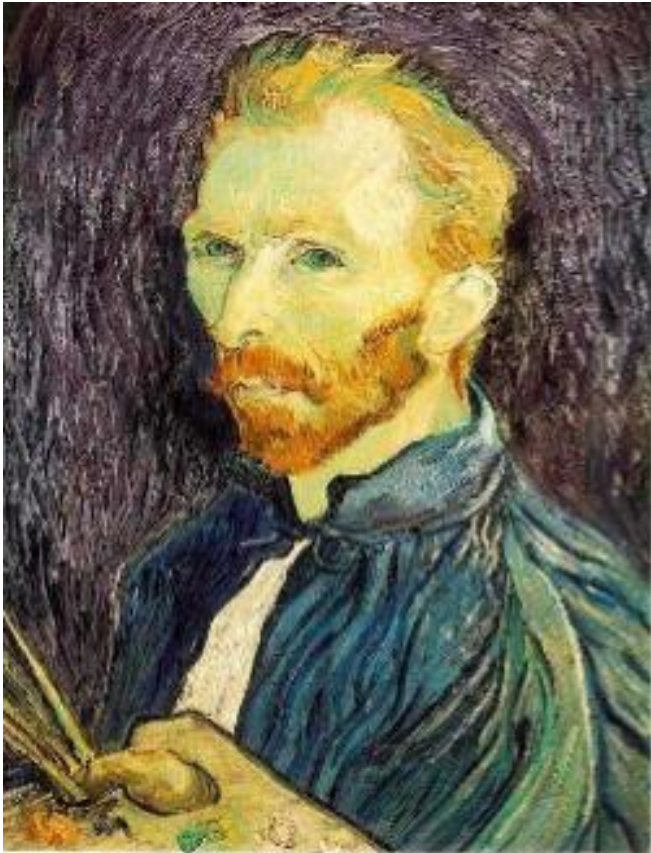


1/8

Better image downsampling

 $1/2$  $1/4$ (2x zoom) $1/8$ (4x zoom)

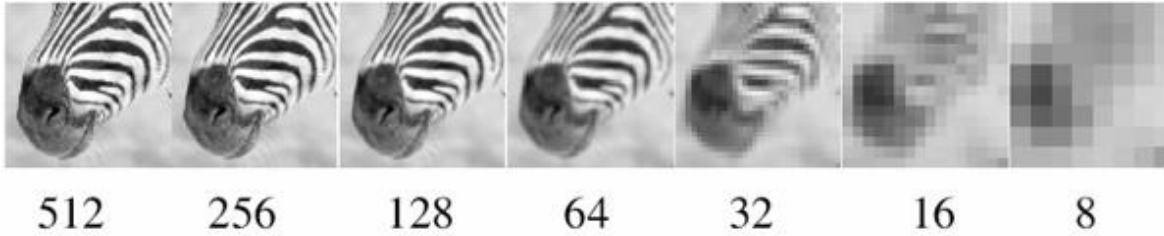
Naïve image downsampling

 $1/2$  $1/4$ (2x zoom) $1/8$ (4x zoom)

Better image downsampling with Gaussian pre-filtering

Some properties of the Gaussian pyramid

...Gaussian image pyramid



What happens to the details of the image?

- They get smoothed out as we move to higher levels.

What is preserved at the higher levels?

- Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?

- That's not possible.



Templates & Image Pyramids

- Image downsampling
 - Anti-aliasing
- Gaussian image pyramid
- **Laplacian image pyramid**

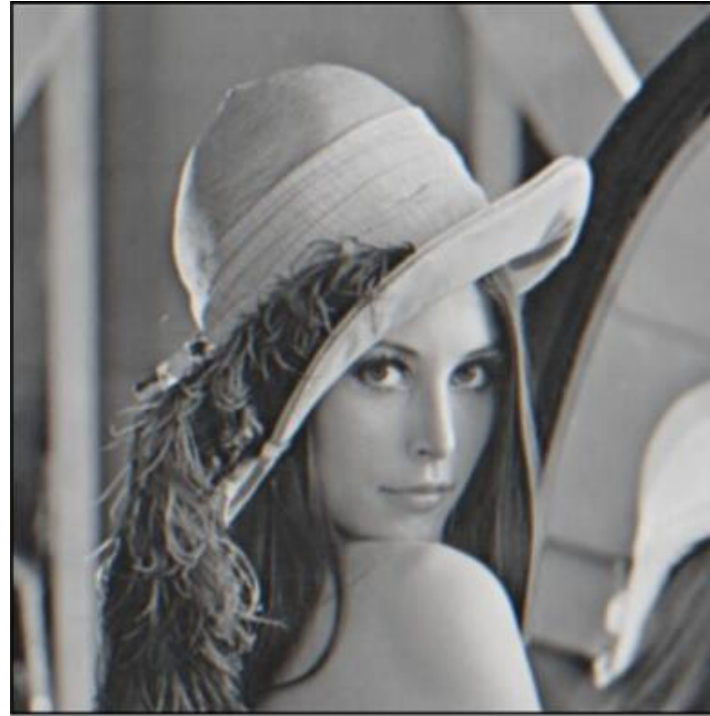
Blurring is lossy

Laplacian image pyramid



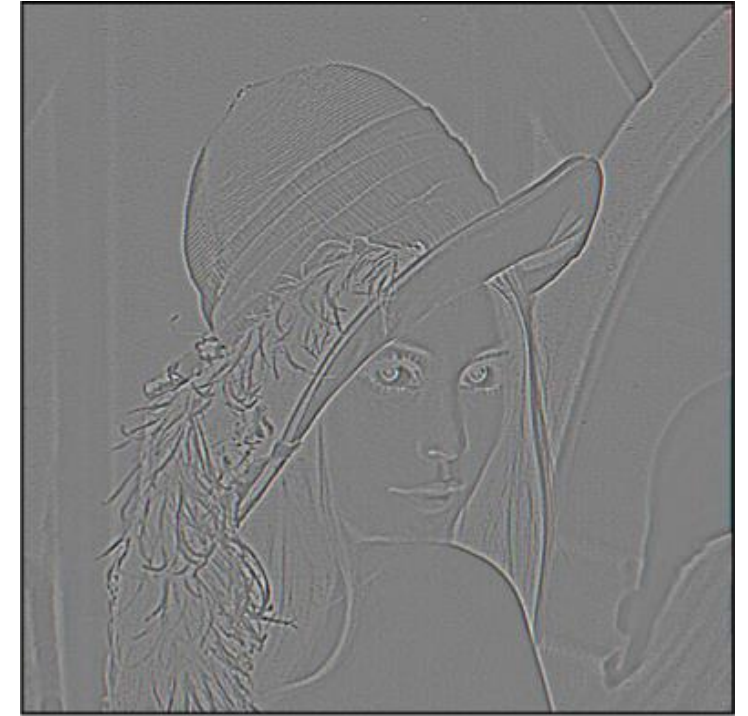
level 0

-



level 1 (before downsampling)

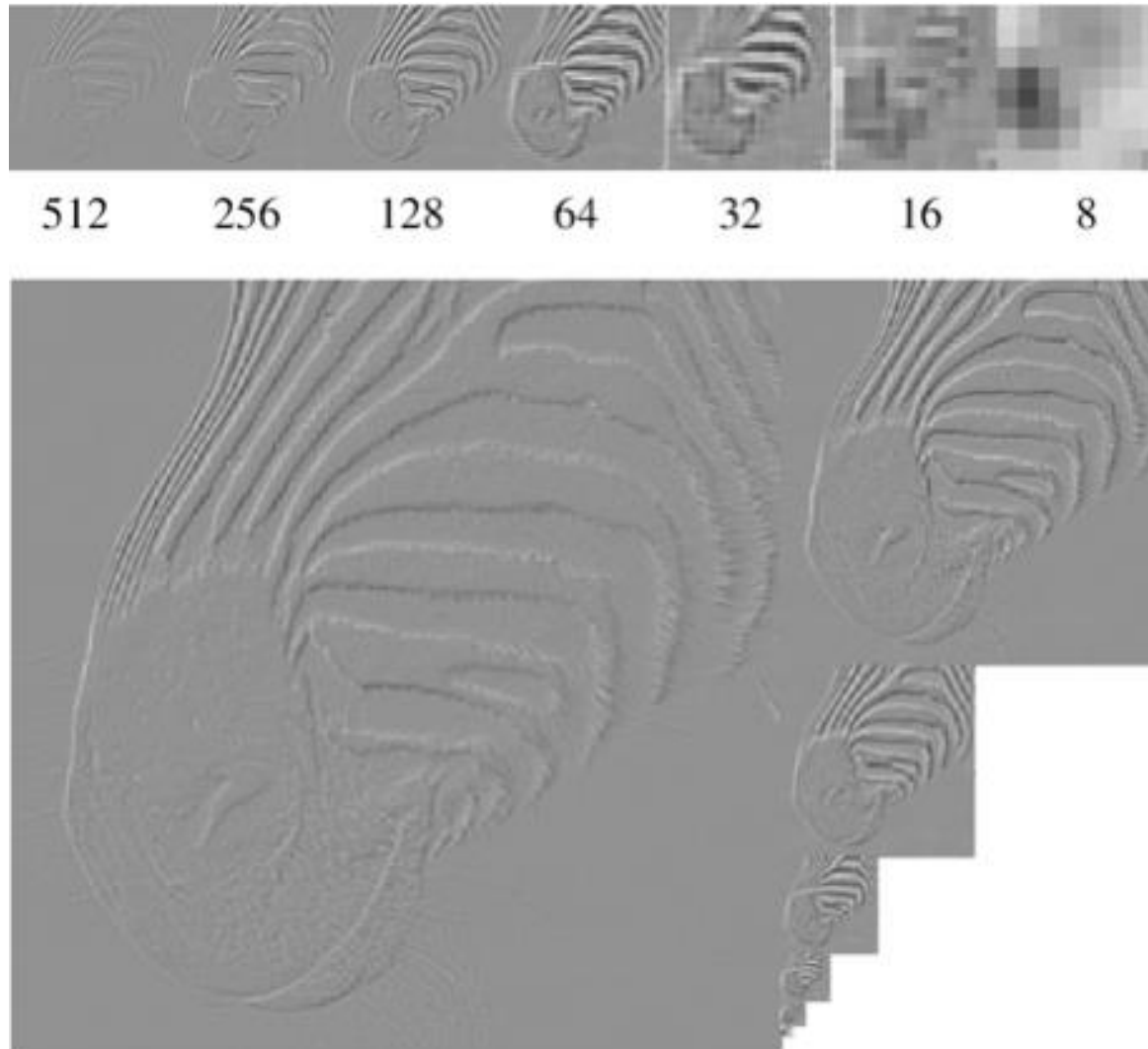
=



residual

Can we make a pyramid that is lossless?

...Laplacian image pyramid



At each level, retain the residuals instead of the blurred images themselves.

Can we reconstruct the original image using the pyramid?

- Yes we can!

What do we need to store to be able to reconstruct the original image?

...Laplacian image pyramid



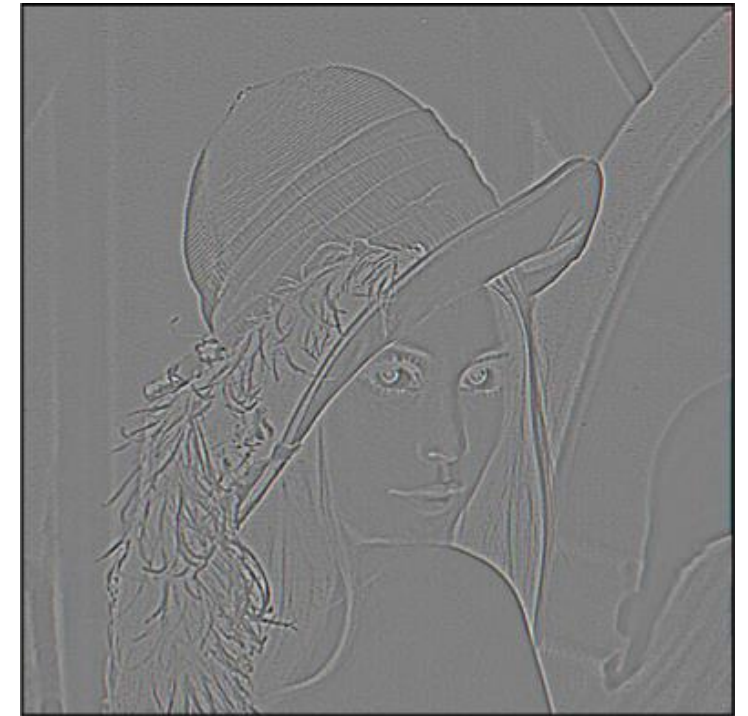
level 0

=



level 1 (upsampled)

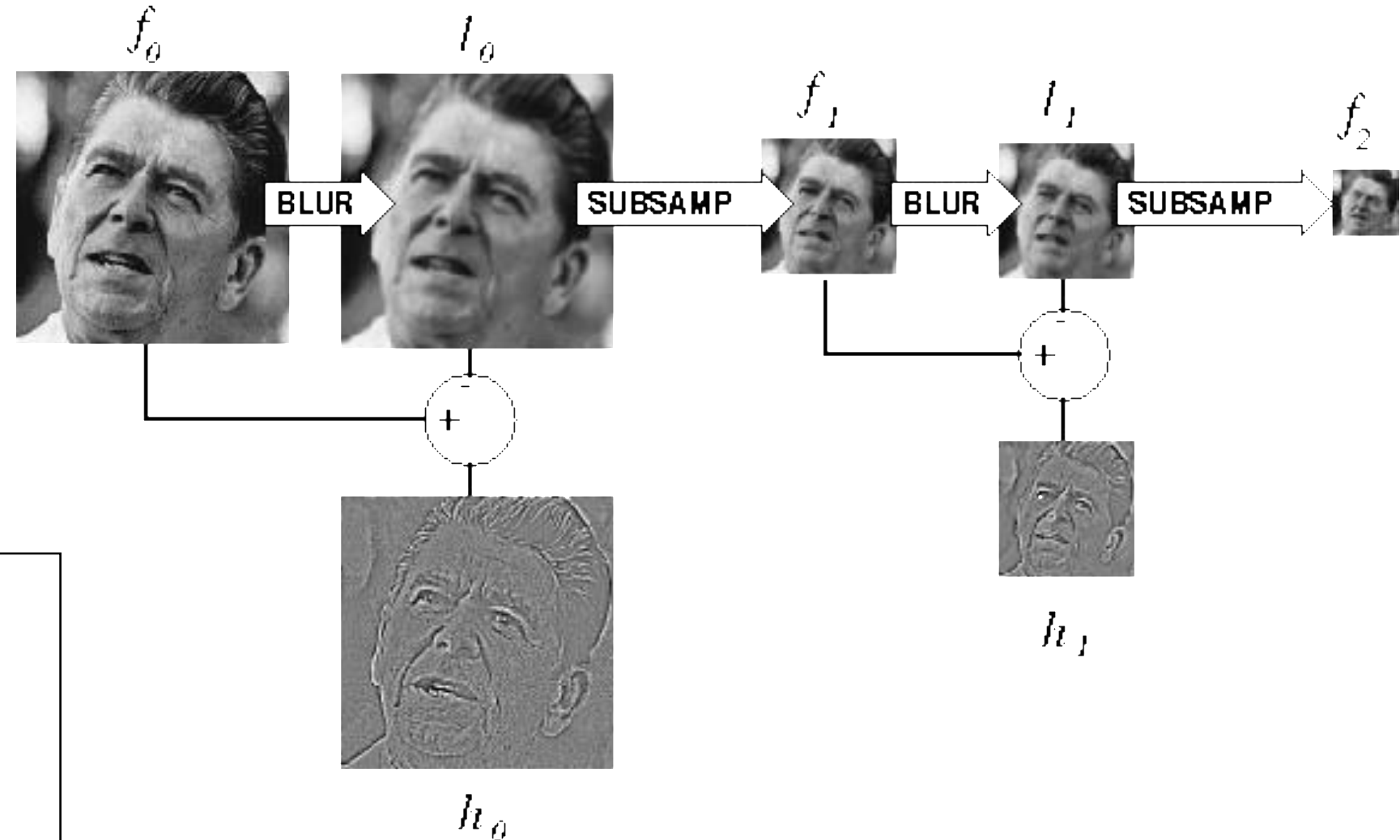
+



residual

Does this mean we need to store both residuals and the blurred copies of the original?

...Laplacian image pyramid



Algorithm

repeat:

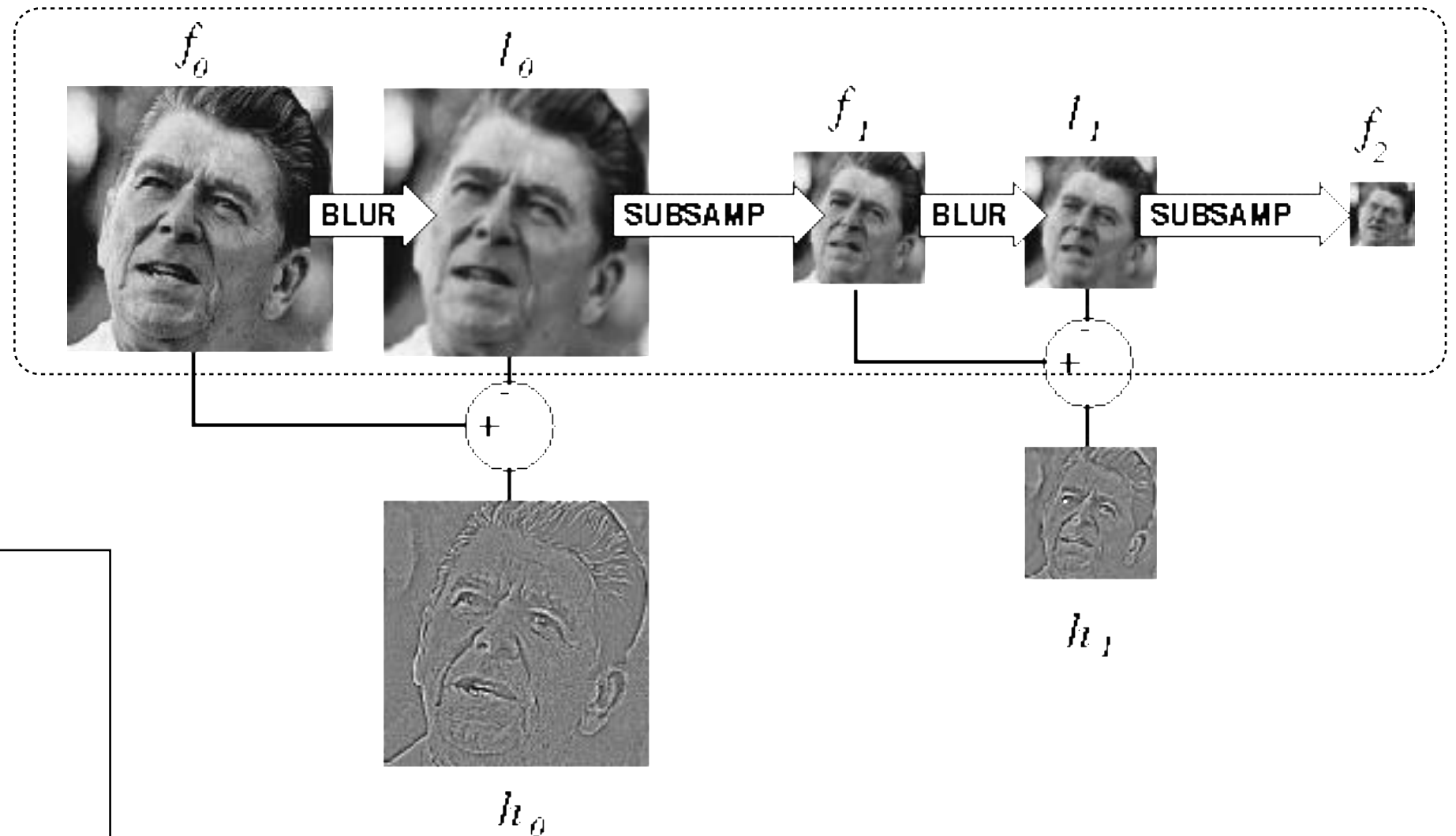
filter

compute residual

subsample

until min resolution reached

It's a Gaussian pyramid.



Algorithm

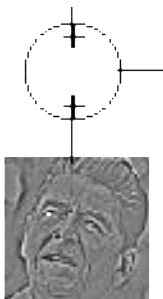
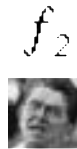
```
repeat:
    filter
    compute residual
    subsample
until min resolution reached
```

What do we need to construct the original image?



What do we need to construct the original image?

(2) smallest
image



h_1

(1) residuals

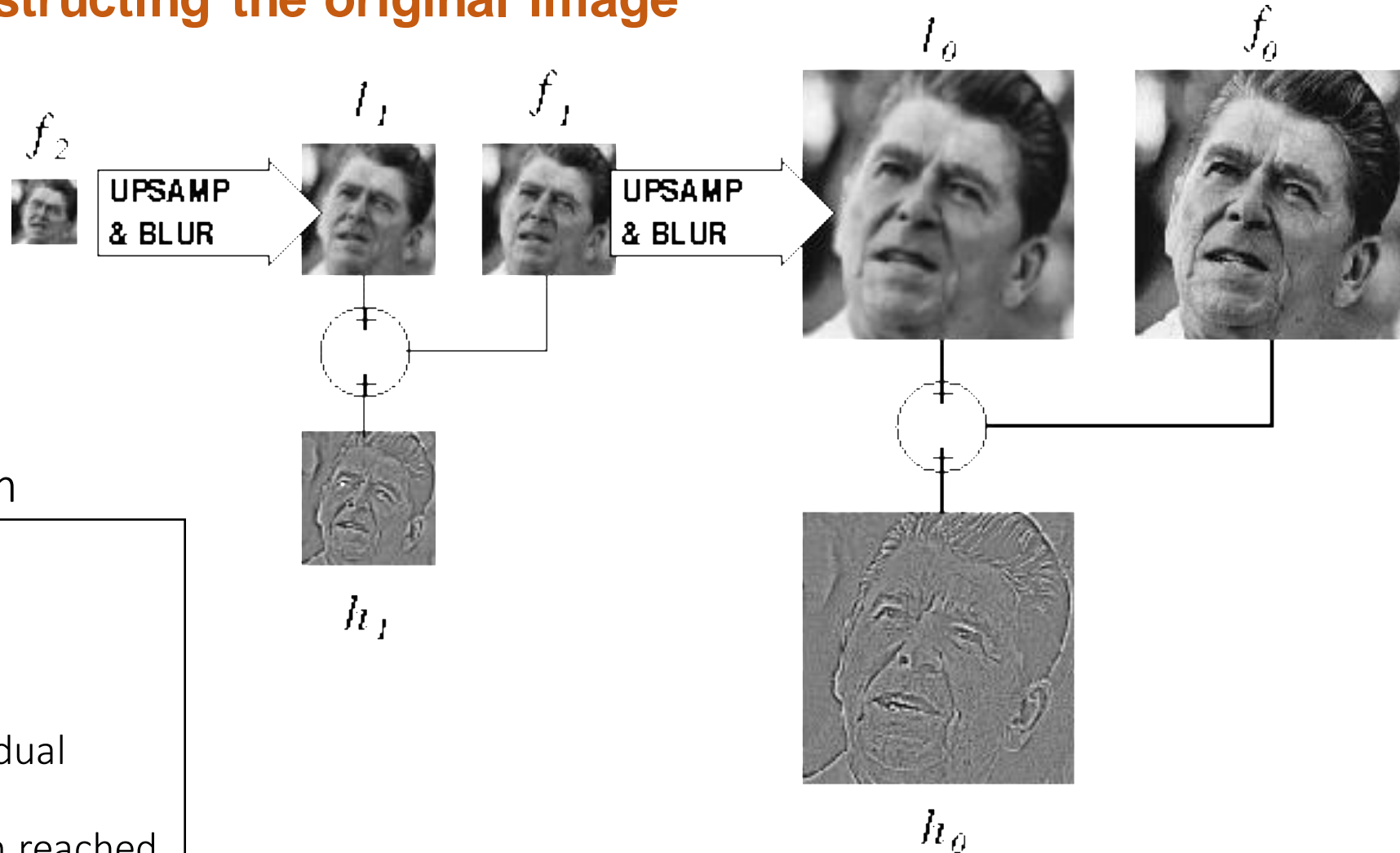


h_0



f_0

Reconstructing the original image



Algorithm

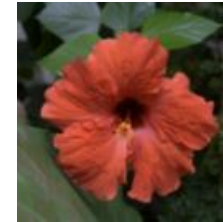
repeat:

 upsample

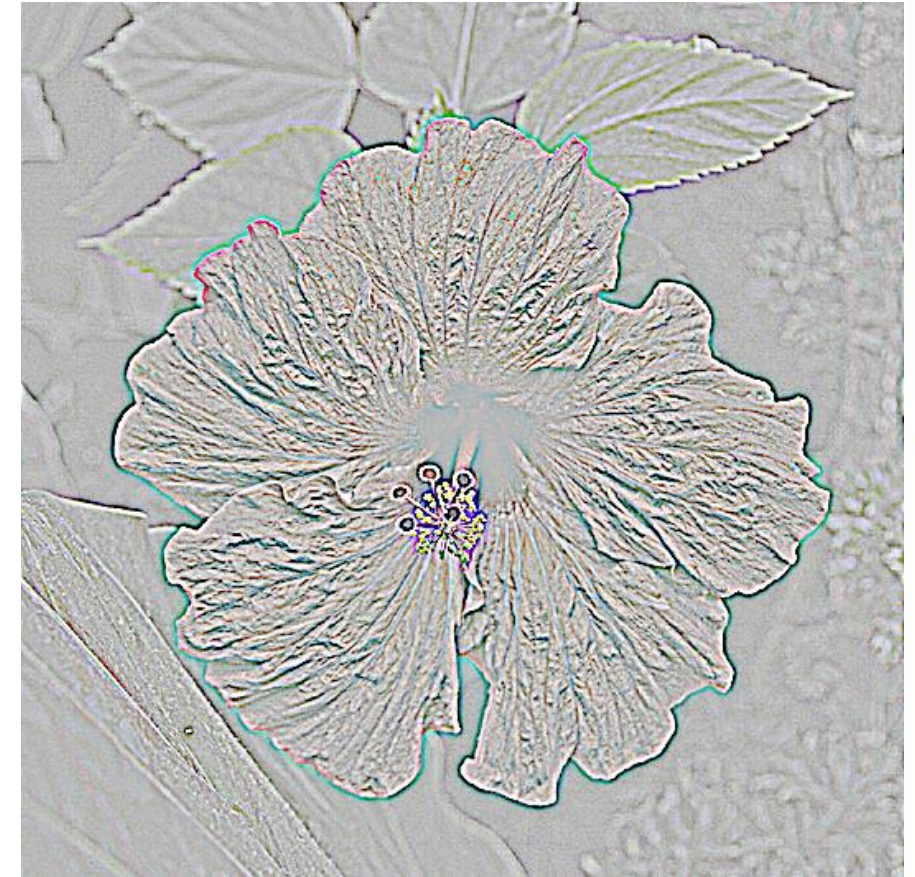
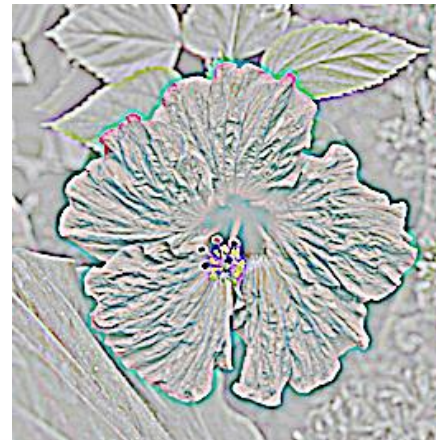
 sum with residual

until orig resolution reached

Gaussian vs Laplacian Pyramid



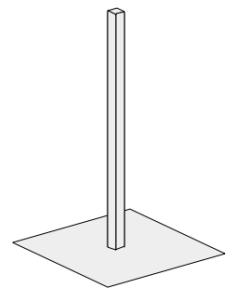
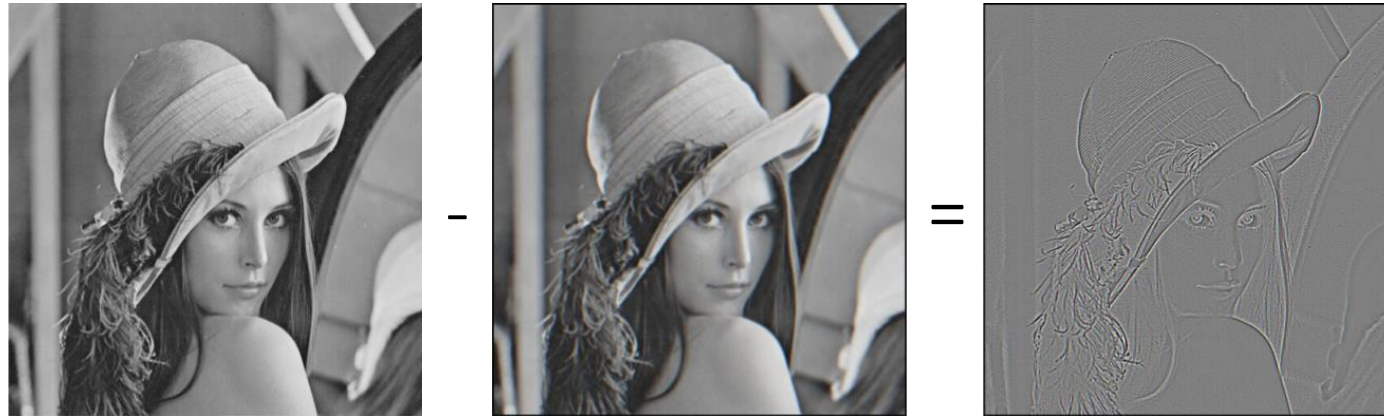
Shown in opposite order for space.



Which one takes more space to store?

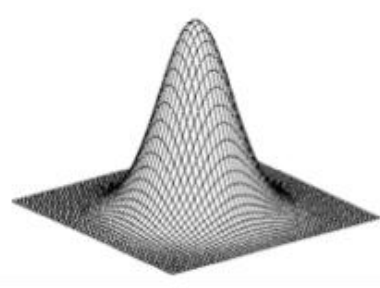


Why is it called a Laplacian pyramid?



unit

-



Gaussian

≈



Laplacian

Difference of Gaussians approximates the Laplacian

What are image pyramids used for?

image compression



multi-scale
texture mapping

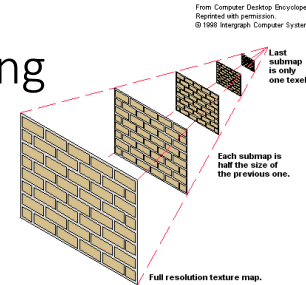
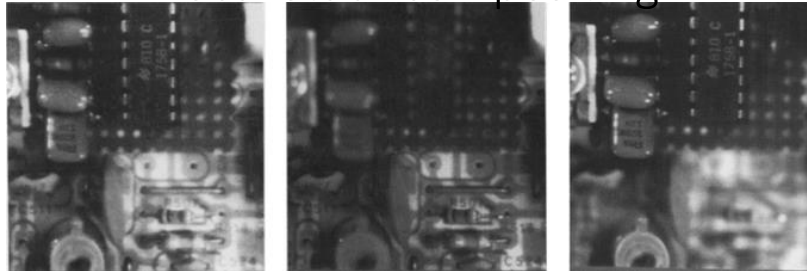


image blending



focal stack compositing



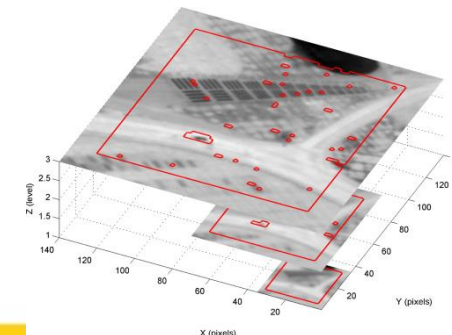
denoising



multi-scale detection



multi-scale registration







Thank You...!