ĐẠI HỌC ĐÀ NẴNG
TRƯỜNG ĐẠI HỌC CÔNG NGHỆ THÔNG TIN VÀ TRUYỀN THÔNG VIỆT - HÀN
Vietnam - Korea University of Information and Communication Technology
http://vku.udn.vn

Image Filter

Lecturer: LeNga

Image filter

- **Image filtering**
  - Compute function of local neighborhood at each position
  - Replace each pixel with a weighted average of its neighborhood
  - The weights are called the **filter kernel**

- Really important!
  - **Enhance images**
    - Denoise, resize, increase contrast, etc.
  - **Extract information from images**
    - Texture, edges, distinctive points, etc.
  - **Detect patterns**
    - Template matching

$$g[\cdot,\cdot]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Example: box filter

Image filter

Three views of filtering:

- **Image filters in the spatial domain**
  - Filter is a **mathematical operation of a grid of numbers**S
  - moothing, sharpening, measuring texture
- **Image filters in the frequency domain**
  - Filtering is a **way to modify the frequencies of images**
  - Denoising, sampling, image compression
- **Templates and Image Pyramids**
  - Filtering is a **way to match a template to the image**
  - Detection, coarse-to-fine registration

Image filters in the spatial domain

- Linear filter
- Convolution filter
- Gaussian filter
- Derivative filter
- Laplace filter
- Sobel filter

Image filters in the spatial domain

- **Linear filter**
  - Convolution filter
  - Gaussian filter
  - Derivative filter
  - Laplace filter
  - Sobel filter

Cross-correlation filtering

- Let's write this down as an equation. Assume the averaging window is (2k+1)x(2k+1):

$$G[i,j] = \frac{1}{(2k+1)^2} \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i+u, j+v]$$

- We can generalize this idea by allowing different weights for different neighboring pixels:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

Cross-correlation filtering (Linear filter)

- We can generalize this idea by allowing different weights for different neighboring pixels:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u,j+v]$$

- This is called a **cross-correlation** operation and written:

$$G = H \otimes F$$

  - H is called the "**filter**," "**kernel**" or "**mask**"
  - The above allows negative filter indices. When you implement need to use:
    H[u+k,v+k] instead of H[u,v]

Linear filter

- **Neighborhood Operation**: replace each pixel by a *linear* combination of its neighbors (and possibly itself).
- The combination is determined by the filter's *kernel*.
- The same kernel is *shifted* to all pixel locations so that all pixels use the same linear combination of their neighbors.

Example: the box filter- the 2D rect filter also known as the square mean filter

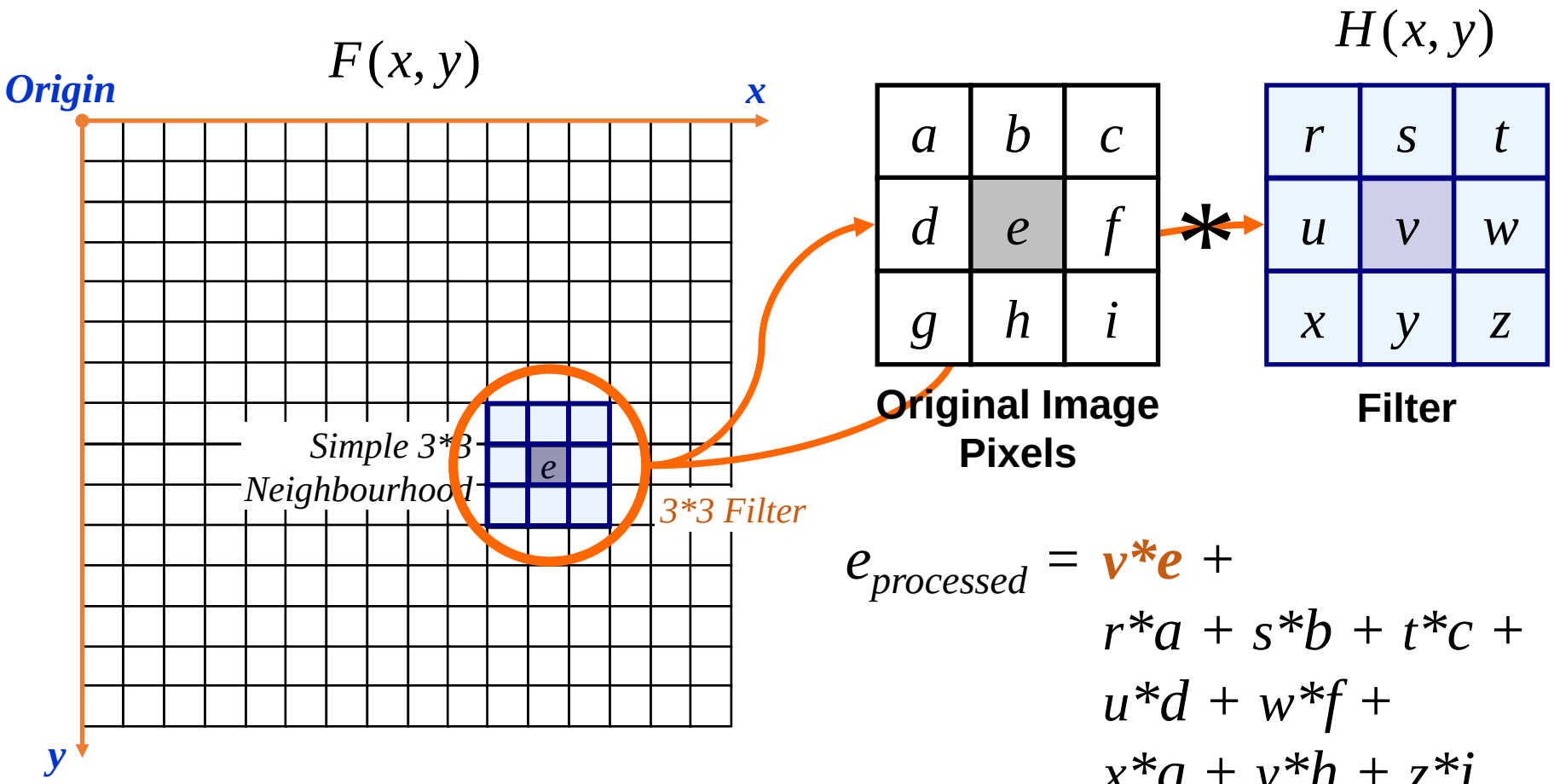
- Replaces each pixel with an average of its neighborhood
- Achieve smoothing (blurring) effect (remove sharp features)

$$H(x,y) = \frac{1}{9}.$$

1	1	1
1	1	1
1	1	1

kernel

…Linear filter



$$e_{processed} = v*e + r*a + s*b + t*c + u*d + w*f + x*g + y*h + z*i$$

Let's run the box filter

$H(x, y)$

kernel

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$F(x, y)$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

output

note that we assume that the kernel coordinates are centered

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

output filter image (signal)

Let's run the box filter

$H(x, y)$

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

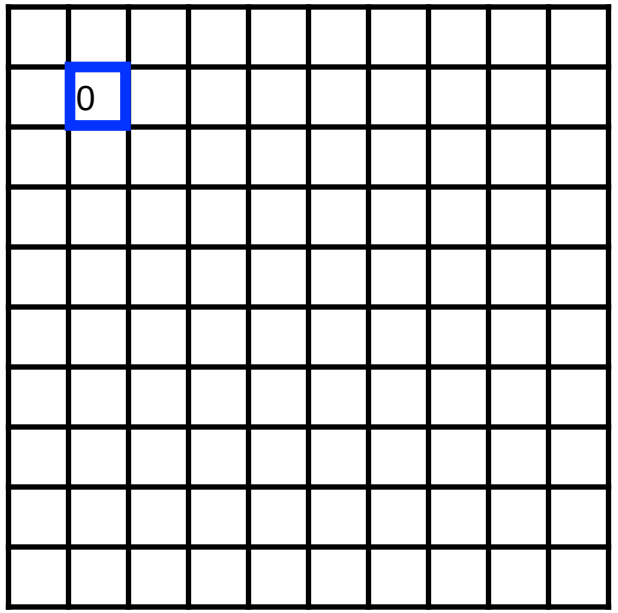
$F(x, y)$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

output



$$G[i, j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u, v] F[i + u, j + v]$$

output filter image (signal)

Let's run the box filter

$H(x, y)$

kernel

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

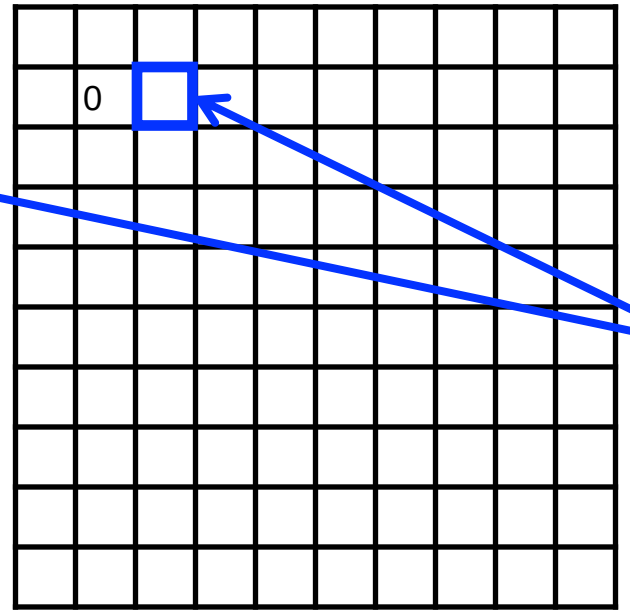
$F(x, y)$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

output



shift-invariant: as the pixel shifts, so does the kernel

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

output filter image (signal)

Let's run the box filter

$H(x, y)$

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$F(x, y)$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

output

	0	10							

$$G[i, j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u, v] F[i + u, j + v]$$

output filter image (signal)

Let's run the box filter

$H(x, y)$

kernel

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$F(x, y)$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

output

	0	10	20						

$$G[i, j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u, v]F[i + u, j + v]$$

output filter image (signal)

Let's run the box filter

$H(x, y)$

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$F(x, y)$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

output

	0	10	20	30					

$$G[i, j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u, v] F[i + u, j + v]$$

output filter image (signal)

Let's run the box filter

$H(x, y)$

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$F(x, y)$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

output

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30							

$$G[i, j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u, v] F[i + u, j + v]$$

output filter image (signal)

Let's run the box filter

$H(x, y)$

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$F(x, y)$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

output

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	0	10	20	30	30	30	20	10	
	10	10	10	10	0	0	0	0	
	10								

$$G[i, j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u, v] F[i + u, j + v]$$

output filter image (signal)

Let's run the box filter

$H(x, y)$

kernel

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$F(x, y)$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

output

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	0	10	20	30	30	30	20	10	
	10	10	10	10	0	0	0	0	
	10	10	10	10	0	0	0	0	

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

output filter image (signal)

… and the result is

$H(x, y)$

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$F(x, y)$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

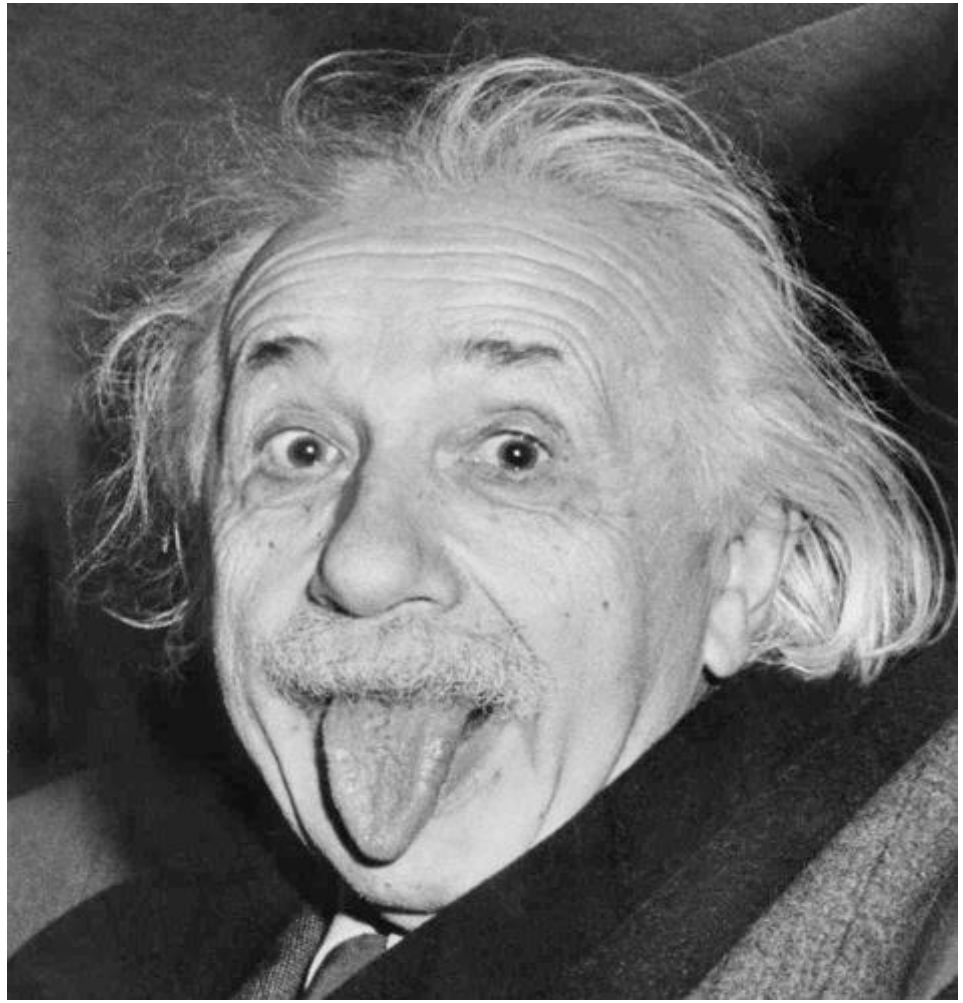
output

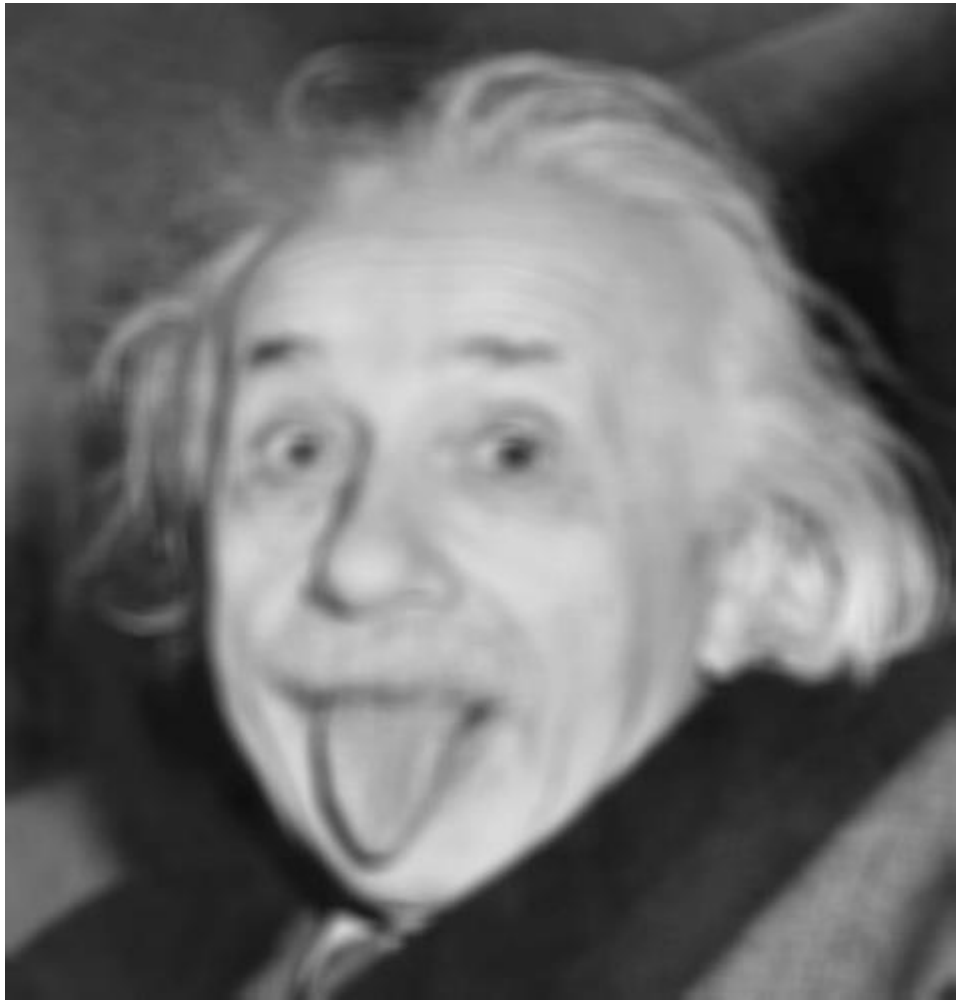
	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	0	10	20	30	30	30	20	10	
	10	10	10	10	0	0	0	0	
	10	10	10	10	0	0	0	0	

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

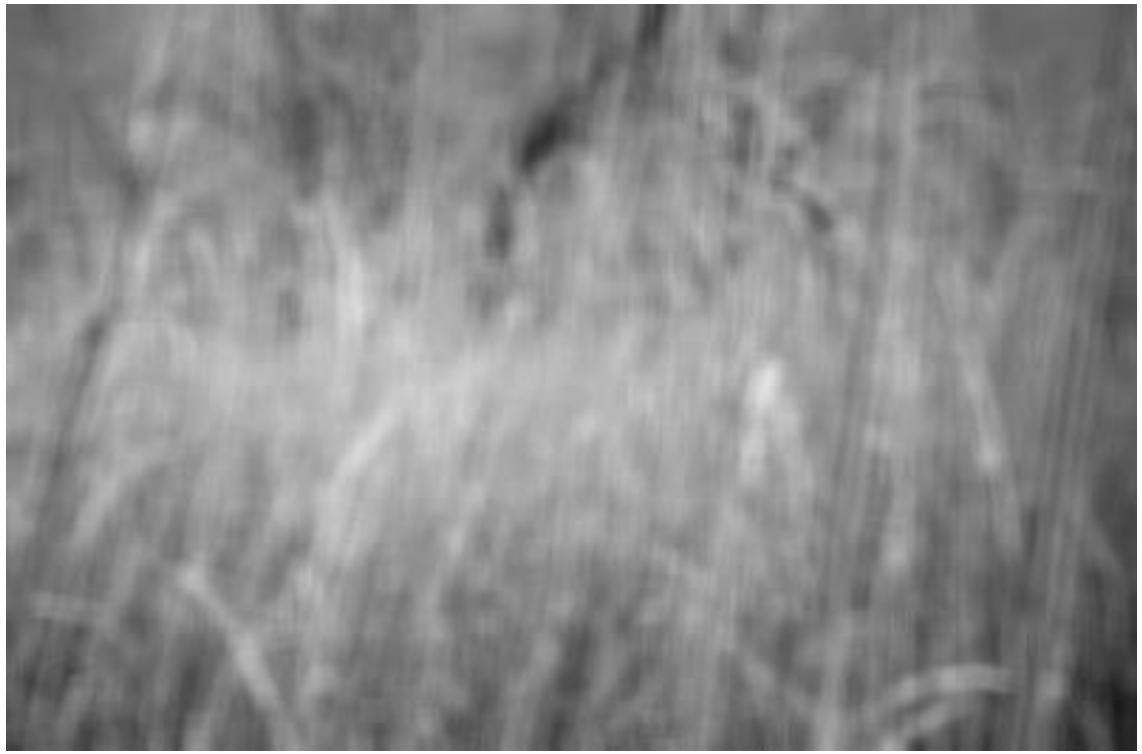
output — filter — image (signal)

Smoothing with box filter





Smoothing with box filter



Practice with linear filters

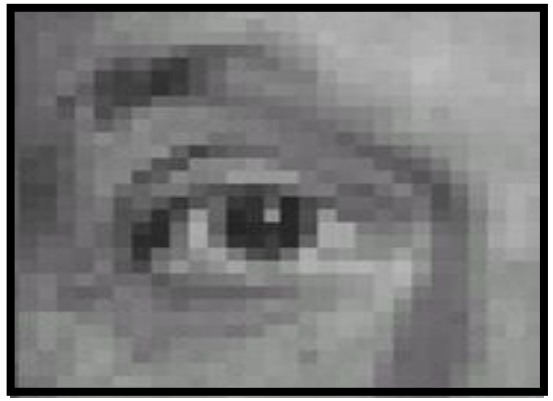
Original

0	0	0
0	1	0
0	0	0

Filtered (no change)

Original

0	0	0
0	0	1
0	0	0

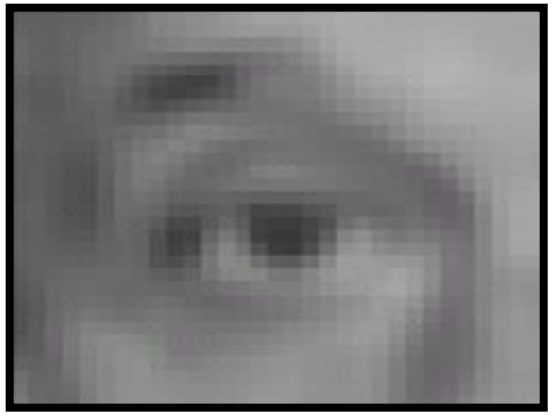


Shifted left by 1 pixel

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$



Blur (with a box filter)

Original

$$\begin{bmatrix}0 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

Sharpening filter

- accentuates differences with local average
- stress intensity peaks

Key properties of linear filters

- **Linearity:**

$$\text{filter}(f_1 + f_2) = \text{filter}(f_1) + \text{filter}(f_2)$$

- **Shift invariance:** same behavior regardless of pixel location

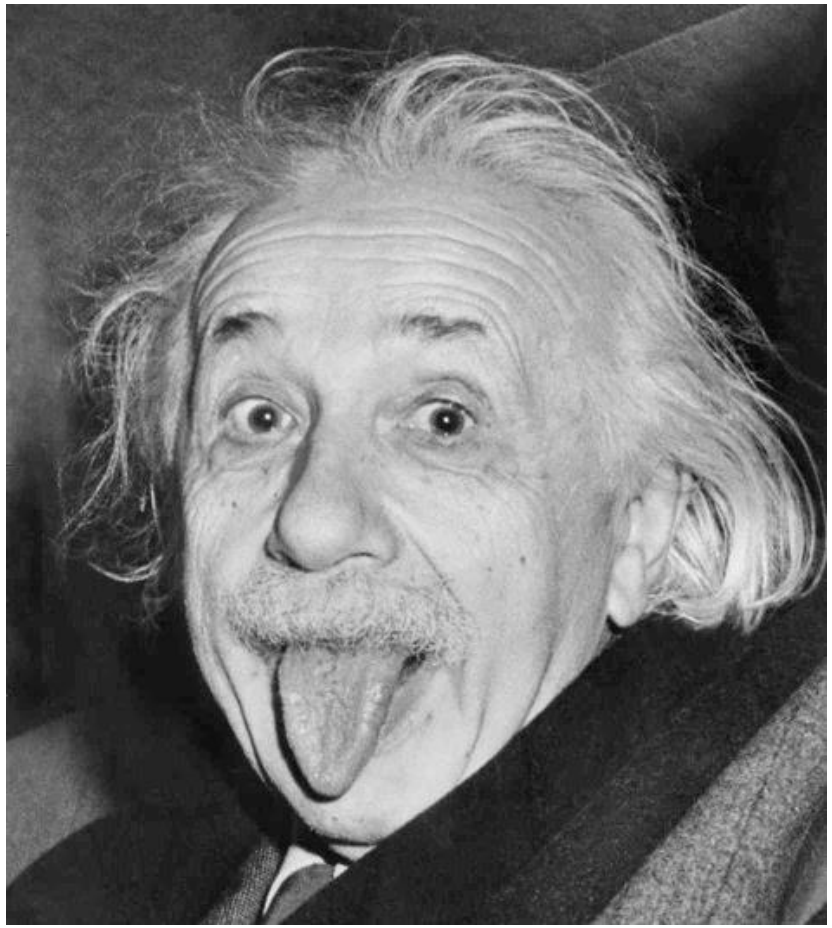
$$\text{filter}(\text{shift}(f)) = \text{shift}(\text{filter}(f))$$

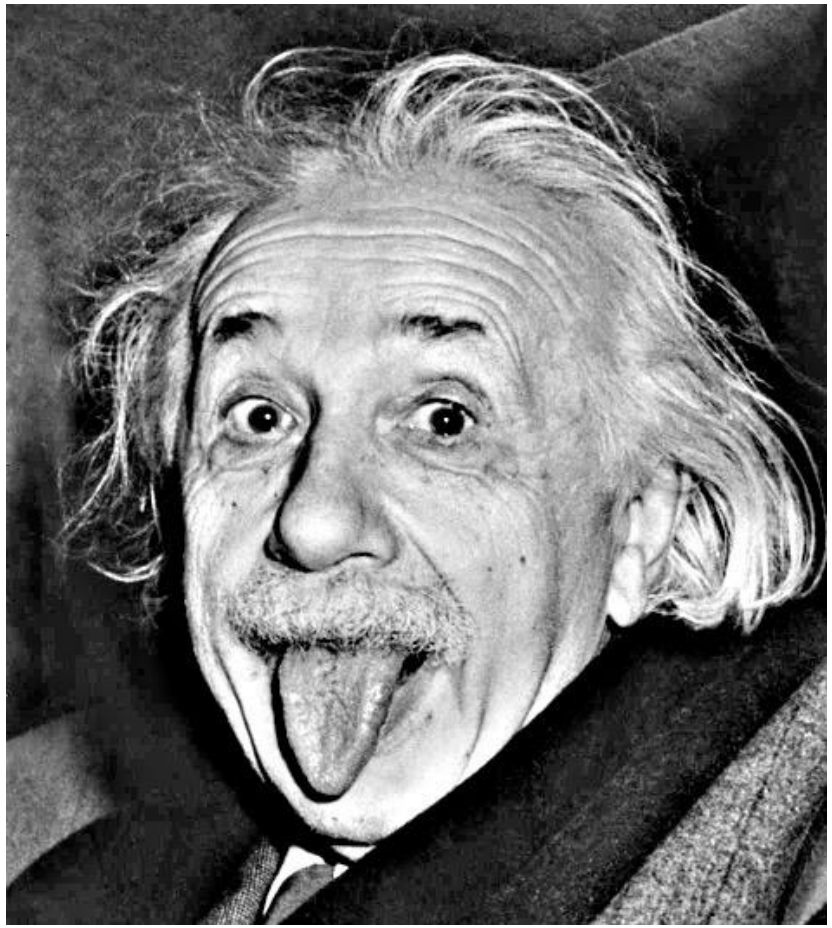
- Any linear, shift-invariant operator can be represented as a convolution

…More properties

- Commutative: $a * b = b * a$
  - Conceptually no difference between filter and signal
- Associative: $a * (b * c) = (a * b) * c$
  - Often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
  - This is equivalent to applying one filter: $a * (b_1 * b_2 * b_3)$
- Distributes over addition: $a * (b + c) = (a * b) + (a * c)$
- Scalars factor out: $ka * b = a * kb = k (a * b)$
- Identity: unit impulse $e = [0, 0, 1, 0, 0]$, $a * e = a$

…Sharpening





…Sharpening

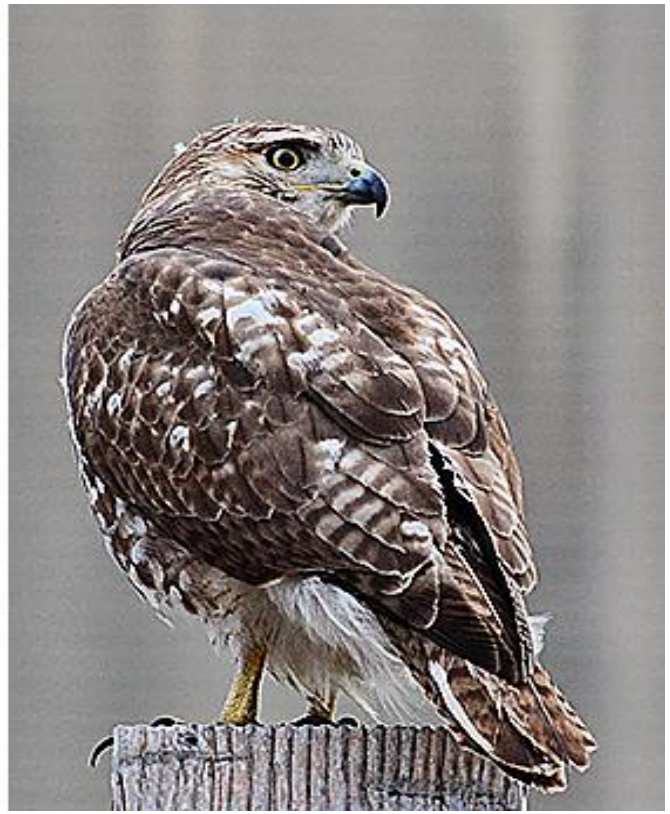
…Sharpening

…Sharpening

but….. Do not overdo it with sharpening…!!!

original

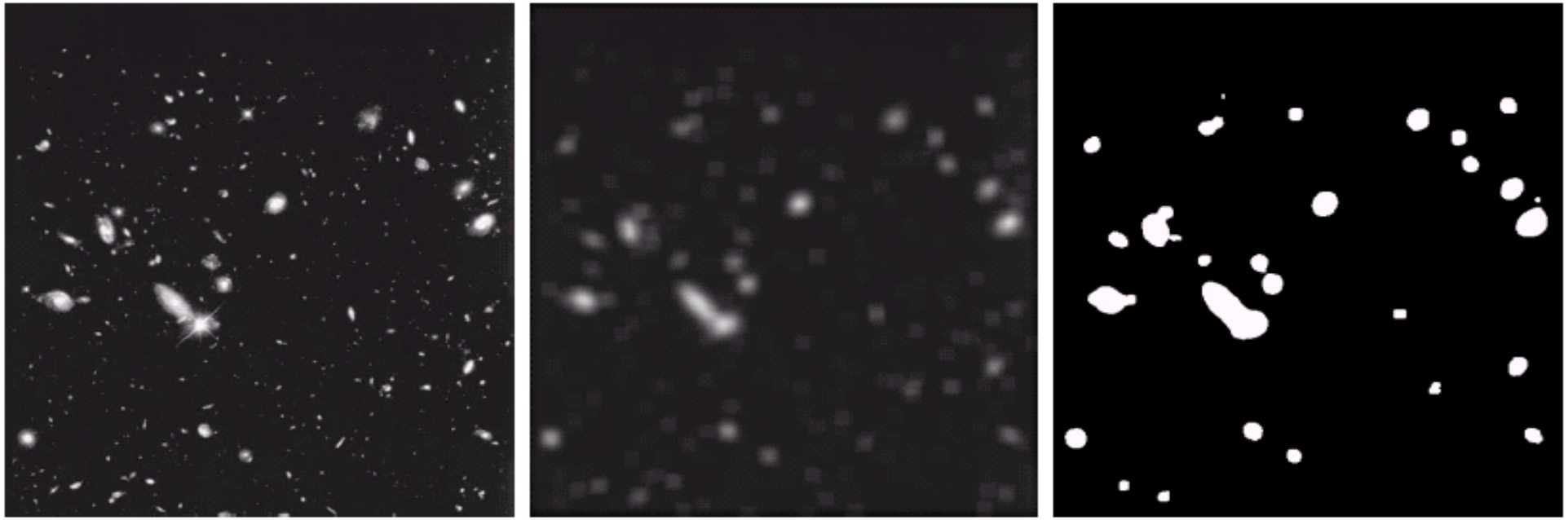
sharpened



oversharpened

What is wrong in this image?

…Linear filter



Original Image | Smoothed Image | Thresholded Image

Image filters in the spatial domain

- Linear filter
- **Convolution filter**
- Gaussian filter
- Derivative filter
- Laplace filter
- Sobel filter

Convolution (tích chập)

A **convolution** operation is a cross-correlation where the filter is flipped both horizontally and vertically before being applied to the image:

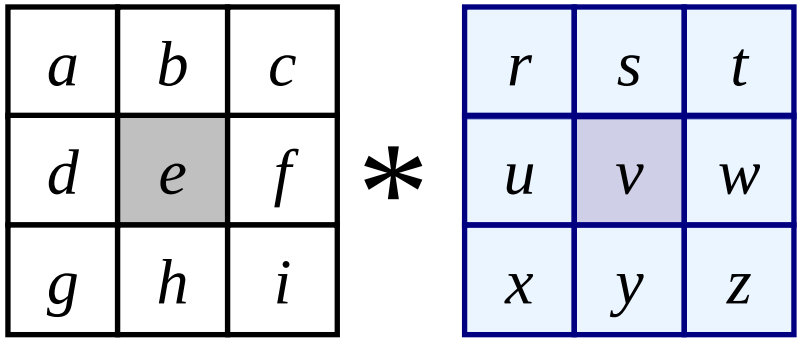
$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

It is written:

$$G = H * F$$

Convolution (tích chập)

A **convolution** operation is a cross-correlation where the filter is flipped both horizontally and vertically before being applied to the image:



Original Image Pixels | **Filter**

$$e_{processed} = \mathbf{v*e} + z*a + y*b + x*c + w*d + u*f + t*g + s*h + r*i$$

Filtering vs. Convolution

- 2D linear filter

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

- 2D convolution filter

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

- Most of the time won't matter, because our kernels will be symmetric.
  - Will be important when we discuss frequency-domain filtering.

Filtering vs. Convolution

- Cross-correlation/convolution is useful for, e.g.,
  - Blurring
  - Sharpening
  - Edge Detection
  - Interpolation

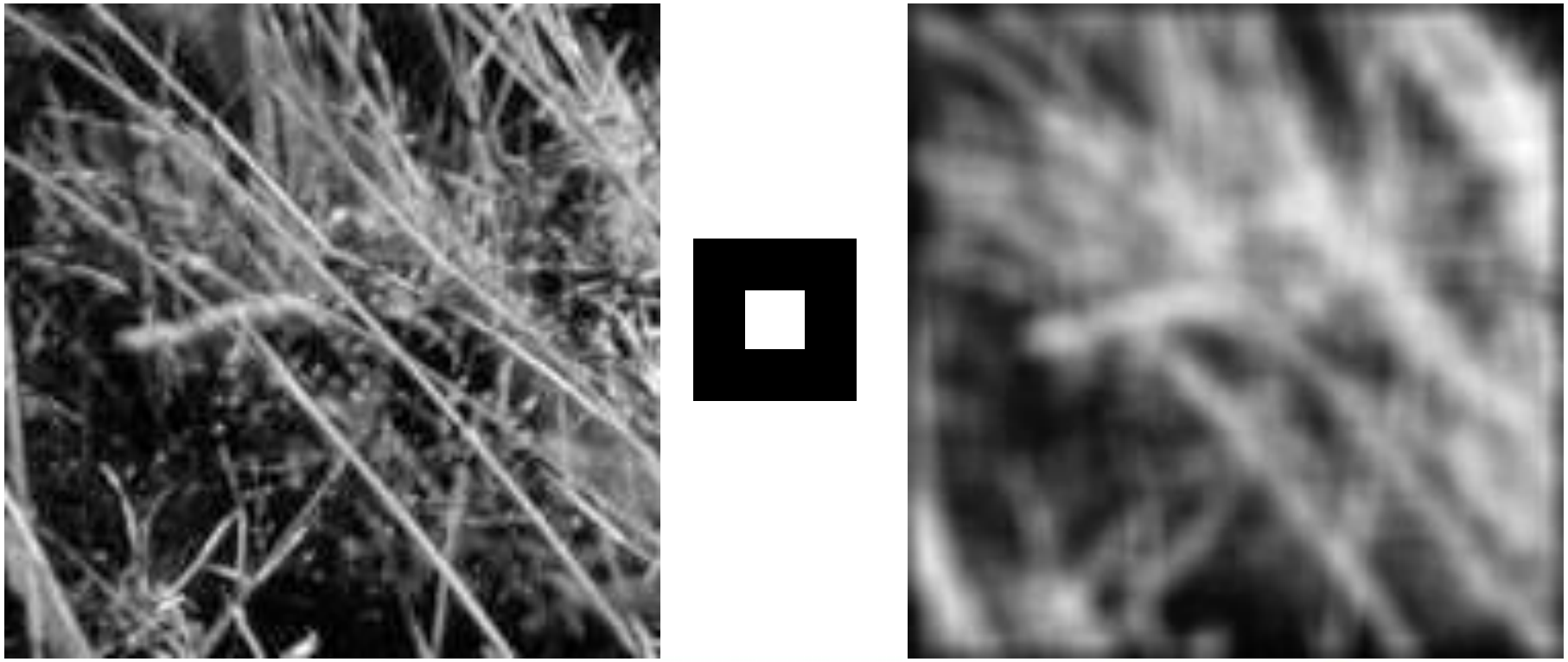
- Convolution has a number of nice properties
  - Commutative, associative
  - Convolution corresponds to product in the Fourier domain

Image filters in the spatial domain

- Linear filter
- Convolution filter
- **Gaussian filter**
- Derivative filter
- Laplace filter
- Sobel filter

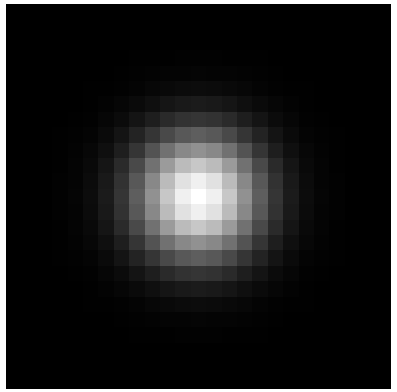
Gaussian filter

- Most obvious difference is that a single point of light viewed in a defocused lens looks like a fuzzy blob;
- …but the averaging process would give a little square



…Gaussian filter

- Better idea: to eliminate edge effects, weight contribution of neighborhood pixels according to their closeness to the center, like so:



"fuzzy blob"

⇨Important filter: **Gaussian filter**

A Gaussian kernel gives less weight to pixels further from the center of the window

kernel

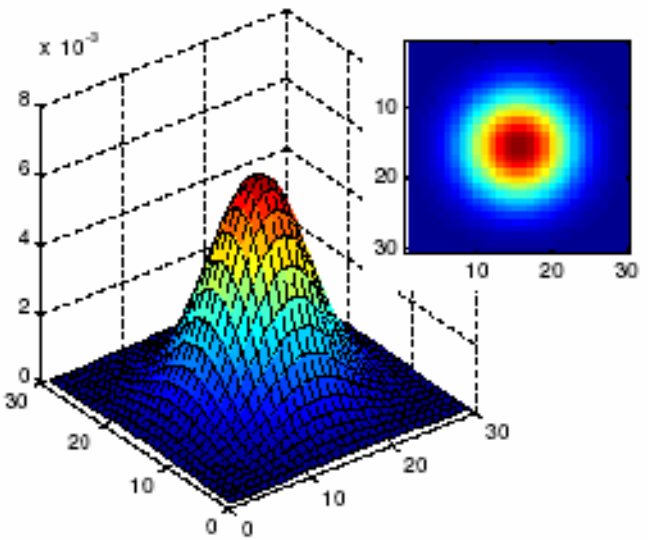
$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

…Gaussian filter

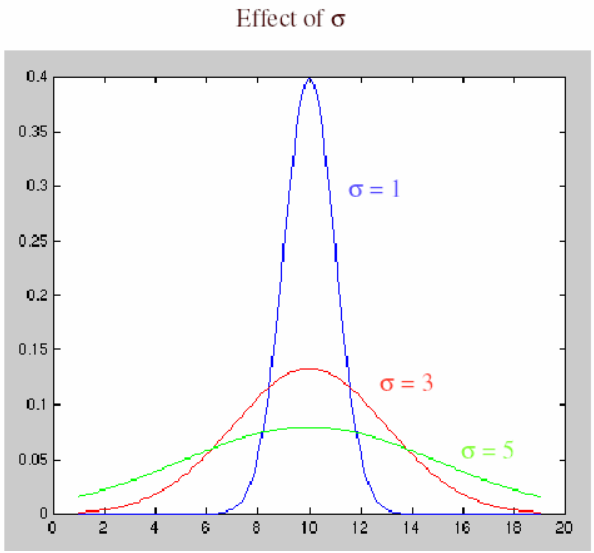
- Gaussian Kernel

$$h(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$

- Weight of neighboring pixels fall off with distance from center pixel



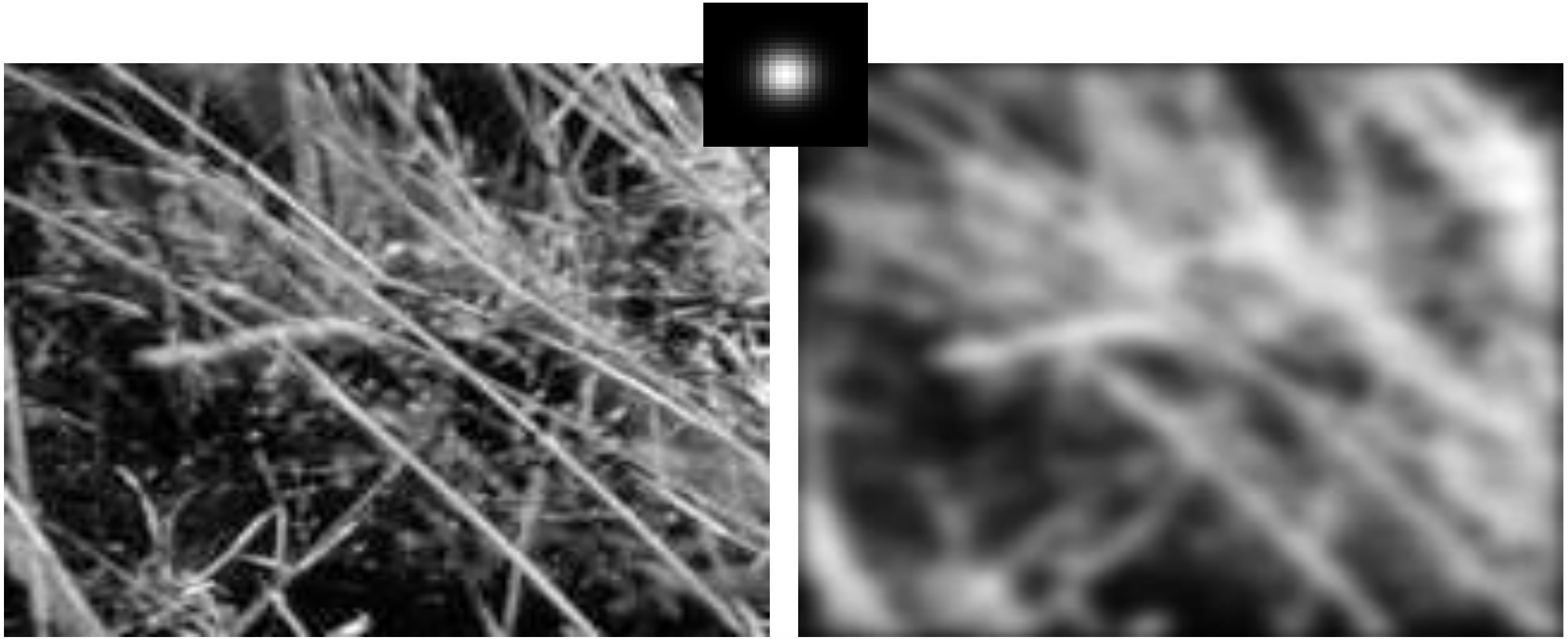
σ = 5 with 30x30 kernel



0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

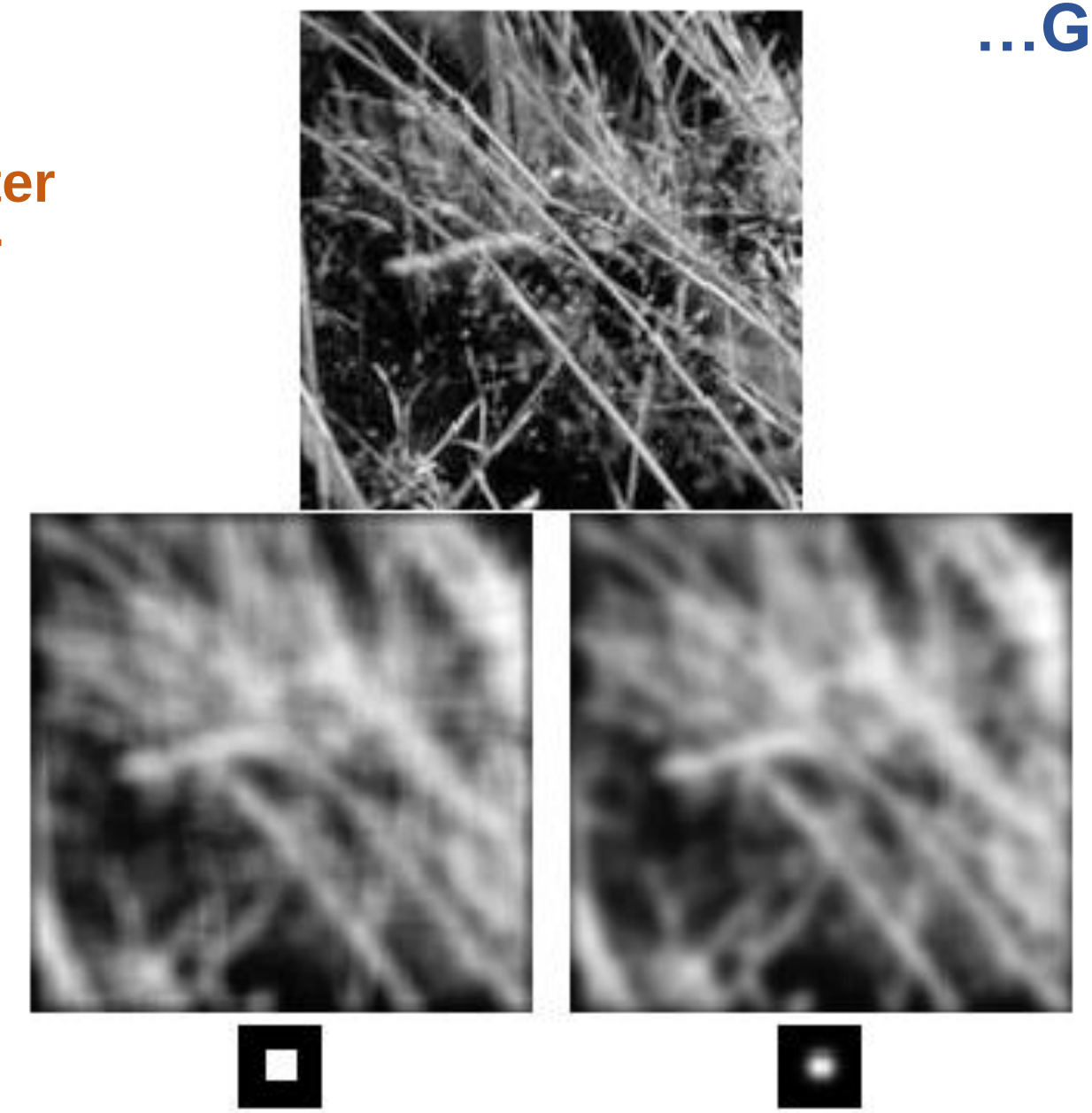
5 x 5, σ = 1

Smoothing with Gaussian filter



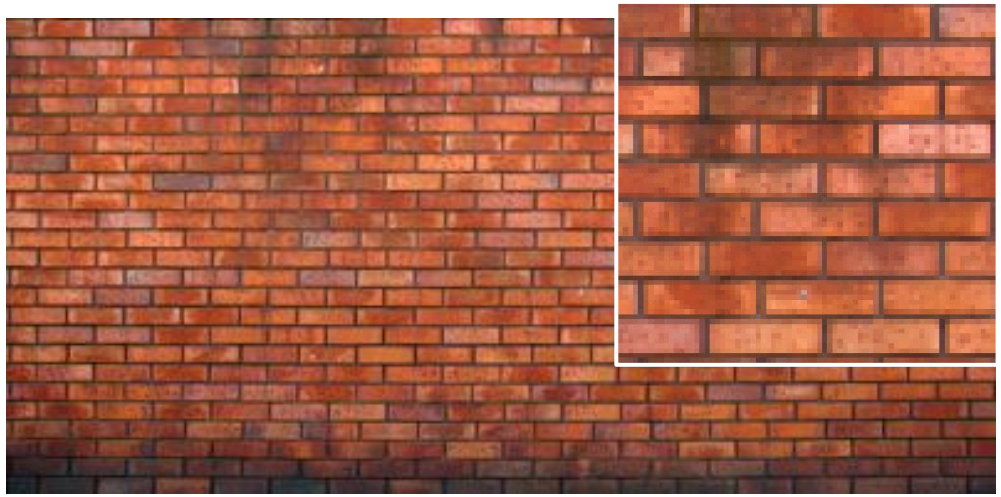
…Gaussian filter

Gaussian filter
vs. Box filter

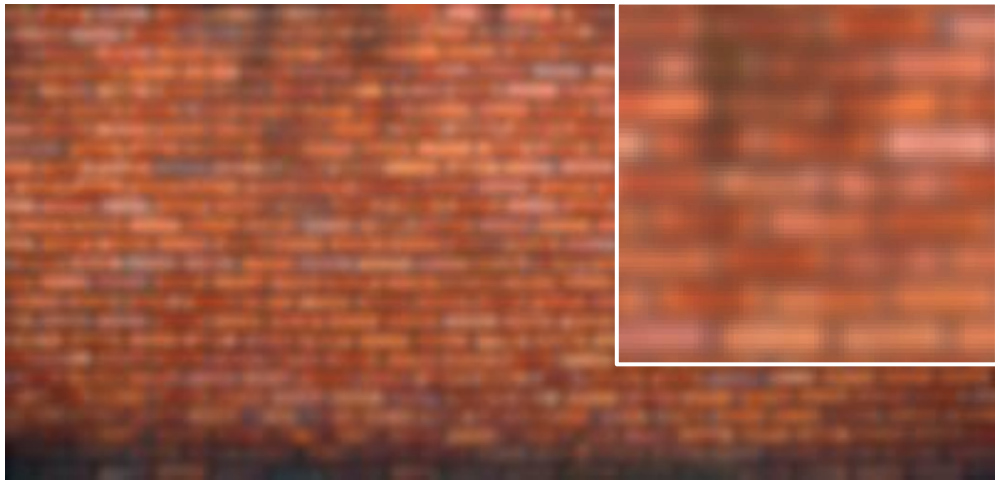


…Gaussian filter

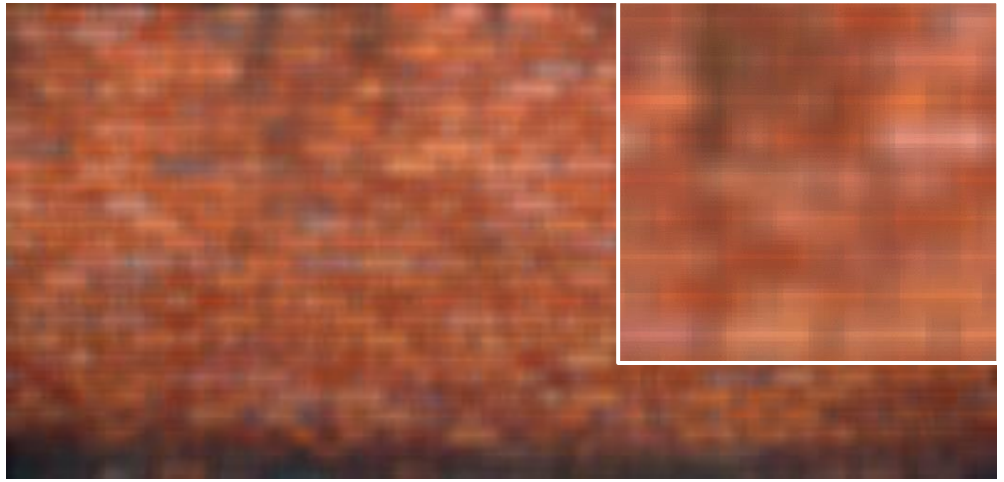
Gaussian vs Box filtering



original



7x7 Gaussian



7x7 box

Gaussian filter remove “high-frequency” components from the image (low-pass filter)

⇨ Images become more smooth

…Gaussian filter

- **Gaussian filter** removes “high-frequency” components from the image (low-pass filter)
  - Images become more smooth

- **Convolution** with self is another Gaussian
  - So can smooth with small-width kernel, repeat, and get same result as larger-width kernel would have
  - Convolving two times with Gaussian kernel of width σ is same as convolving once with kernel of width $\sigma\sqrt{2}$

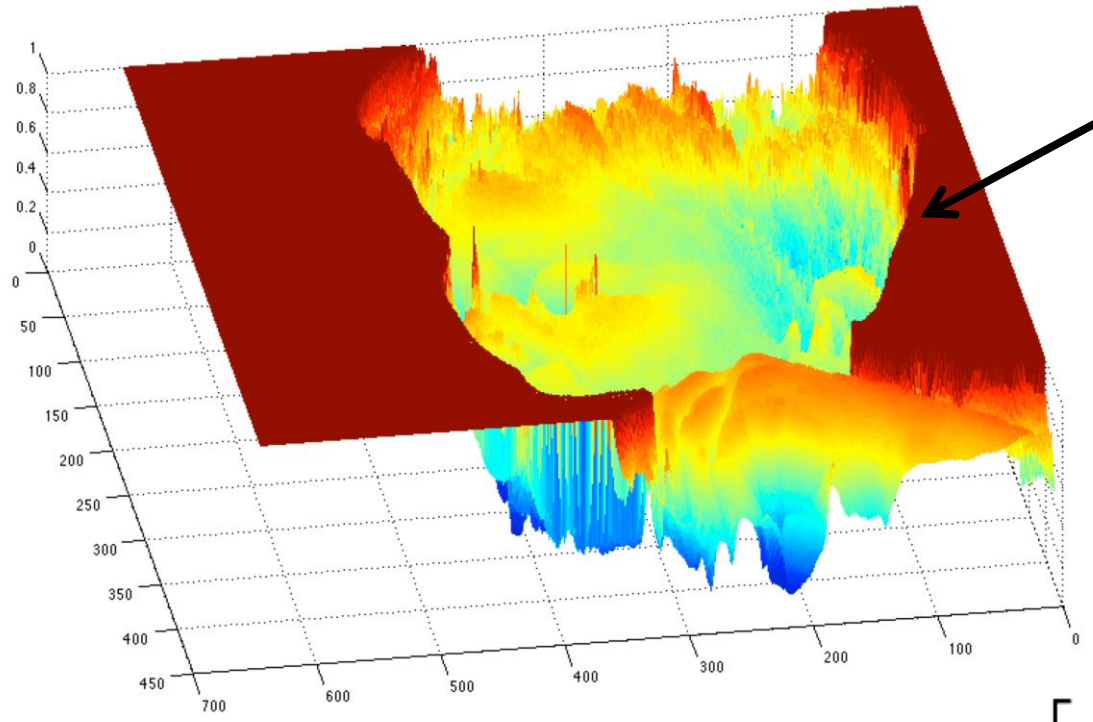
Image filters in the spatial domain

- Linear filter
- Convolution filter
- Gaussian filter
- **Derivative filter**
- Laplace filter
- Sobel filter

What are image edges?

grayscale image

$f(\boldsymbol{x})$



Very sharp discontinuities in intensity.

domain $\boldsymbol{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

Edge detection

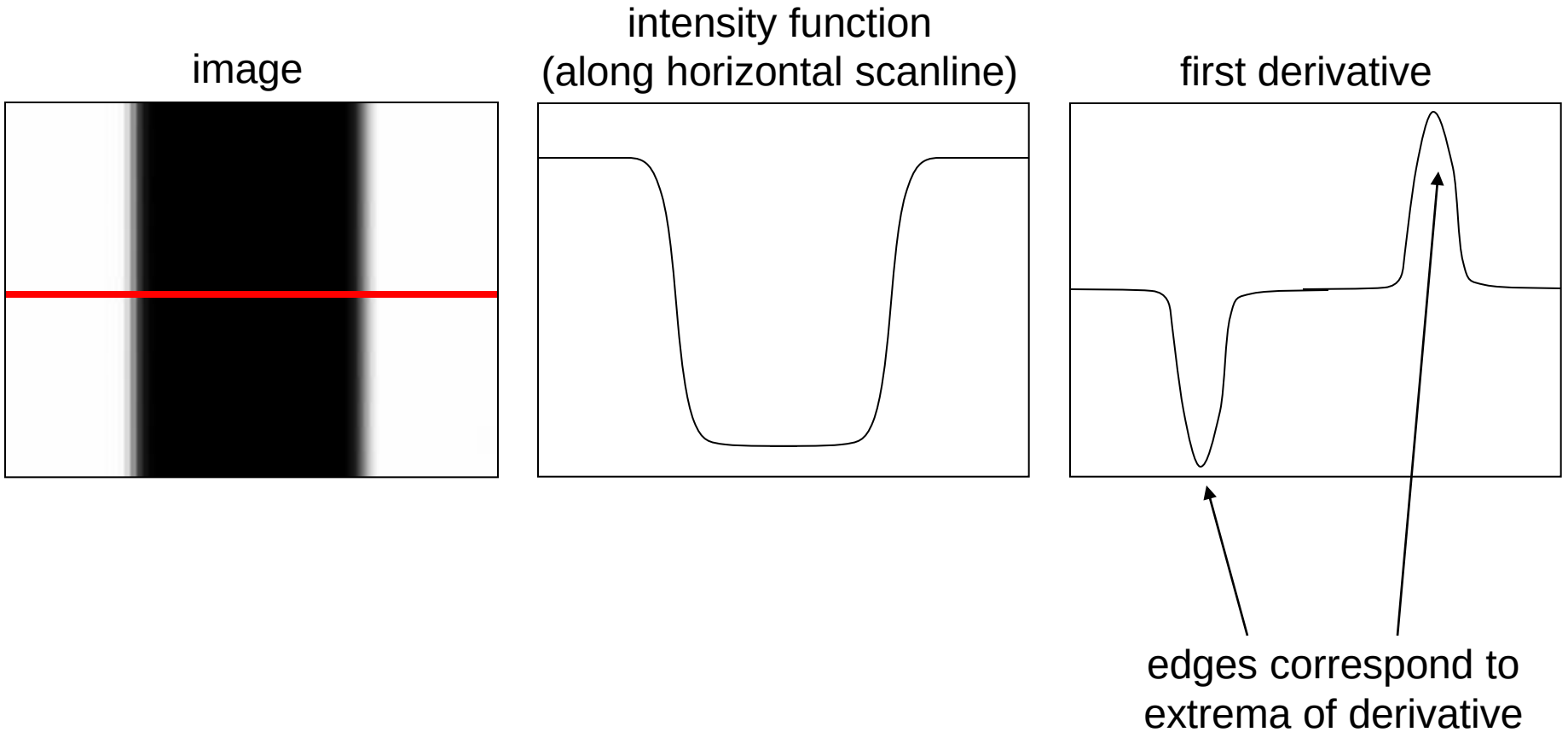
Goal: Identify sudden changes (discontinuities) in an image

- Detecting edges in an image (i.e., discontinuities in a function) ?
  ⇨ take derivatives: derivatives are large at discontinuities.

- Differentiate a discrete image (or any other discrete signal) ?
  ⇨ use finite differences.

Characterizing edges

- An edge is a place of rapid change in the image intensity function



Finite differences

- Definition of the first-order derivative using forward difference

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Alternative: use central difference

$$f'(x) = \lim_{h \to 0} \frac{f(x+0.5h) - f(x-0.5h)}{h}$$

- For discrete signals: Remove limit and set h = 2

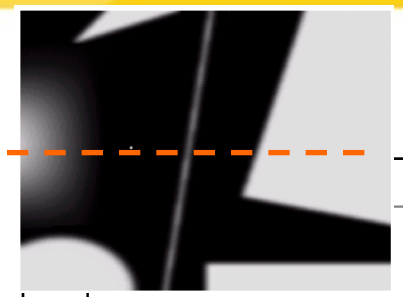
$$f'(x) = \frac{f(x+1) - f(x-1)}{2}$$

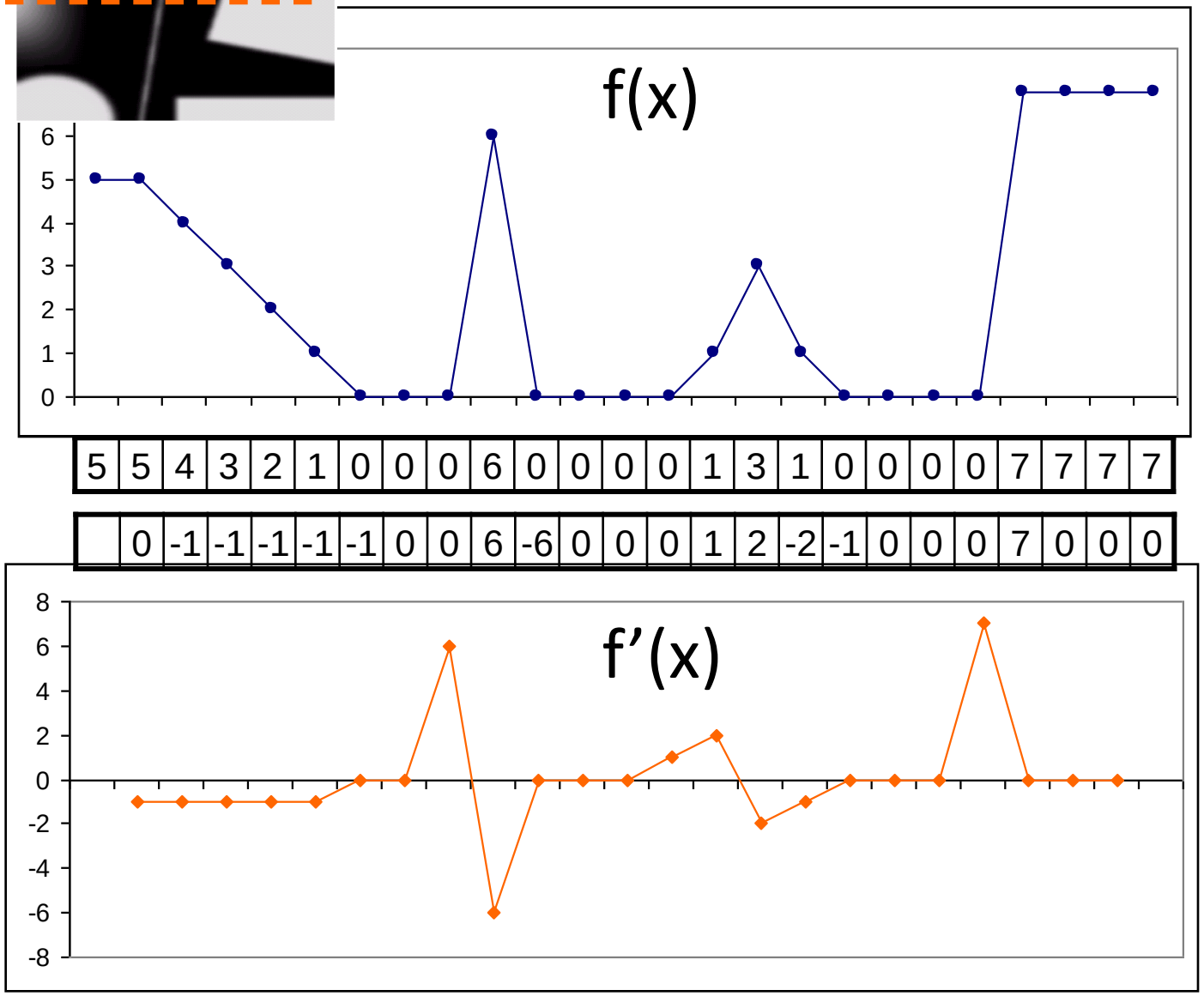
Finite differences

- first-order finite difference

$$f'(x) = \lim_{h \to 0} \frac{f(x + 0.5h) - f(x - 0.5h)}{h}$$

⇨ sự khác nhau của các giá trị liền kề → tốc độ thay đổi của hàm



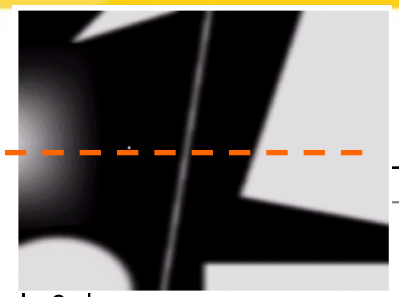


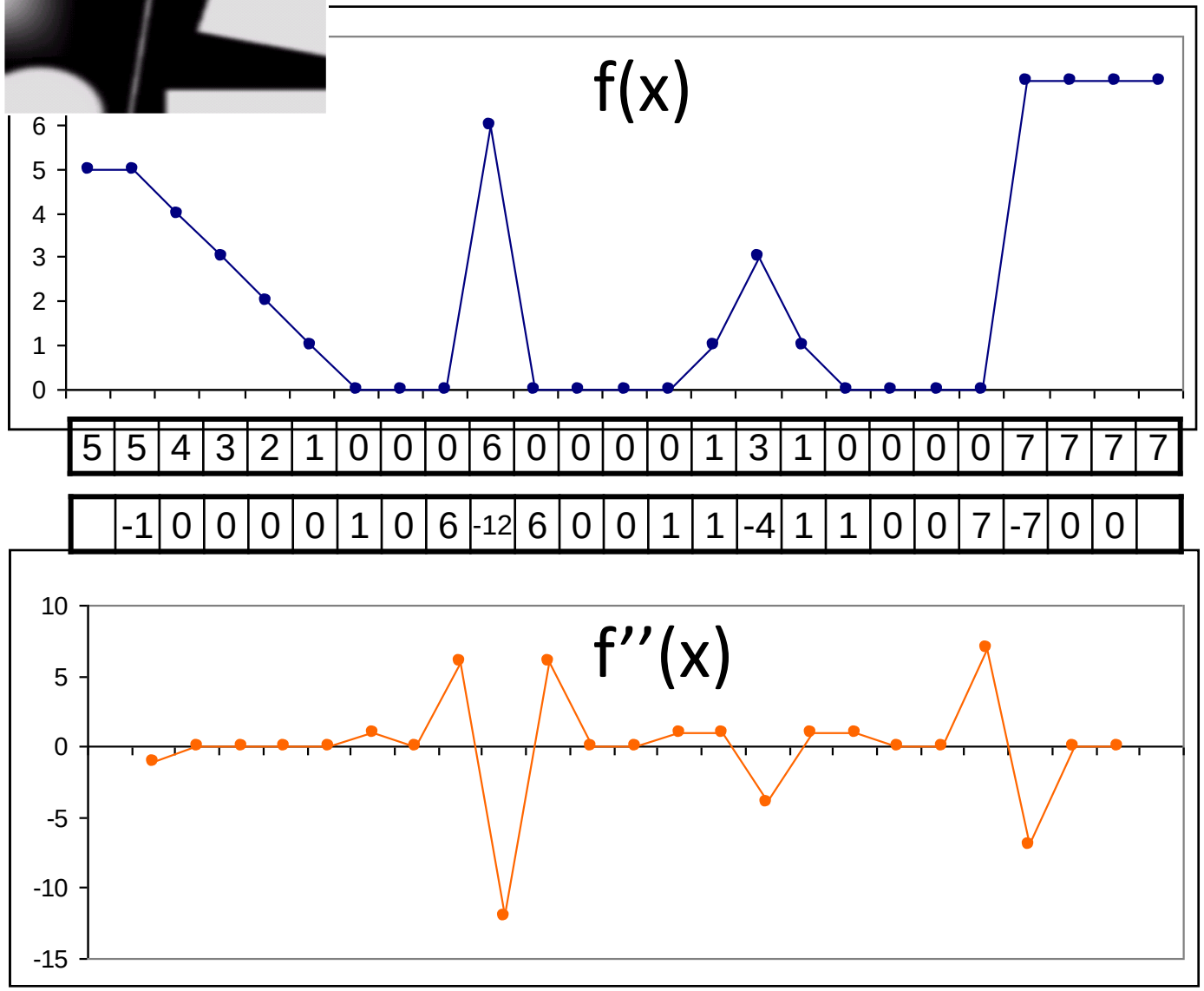
Finite differences

- second-order finite difference

$$f''(x) = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

⇨ sự khác nhau của giá trị trước, sau và giá trị hiện tại





Finite differences

first-order finite difference

$$f'(x) = \lim_{h \to 0} \frac{f(x + 0.5h) - f(x - 0.5h)}{h}$$

→ 1D derivative filter

1	0	-1

second-order finite difference

$$f''(x) = \lim_{h \to 0} \frac{f(x + h) - 2f(x) + f(x - h)}{h^2}$$

→ Laplace filter

1	-2	1

Several derivative filters

Sobel

1	0	-1
2	0	-2
1	0	-1

1	2	1
0	0	0
-1	-2	-1

Scharr

3	0	-3
10	0	-10
3	0	-3

3	10	3
0	0	0
-3	-10	-3

Prewitt

1	0	-1
1	0	-1
1	0	-1

1	1	1
0	0	0
-1	-1	-1

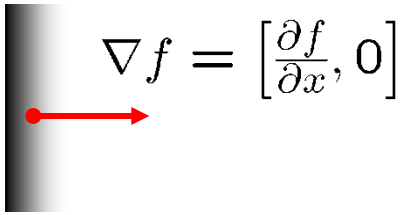
Roberts

0	1
-1	0

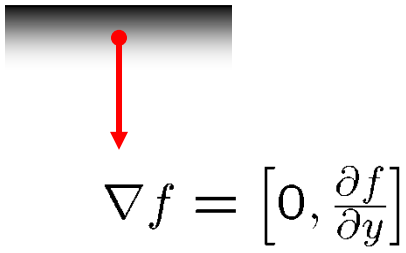
1	0
0	-1

Image gradient

- The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$



  - The gradient points in the direction of most rapid increase in intensity



$$\boldsymbol{S}_x = \begin{array}{|c|c|c|}\hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array} \qquad \boldsymbol{S}_y = \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

- Convolve with the image to compute derivatives.

$$\frac{\partial \boldsymbol{f}}{\partial x} = \boldsymbol{S}_x \otimes \boldsymbol{f} \qquad \frac{\partial \boldsymbol{f}}{\partial y} = \boldsymbol{S}_y \otimes \boldsymbol{f}$$

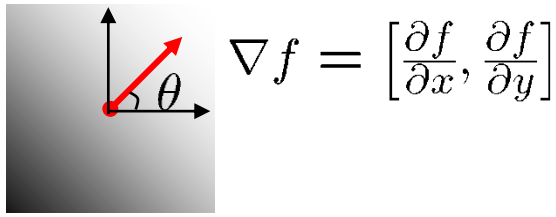
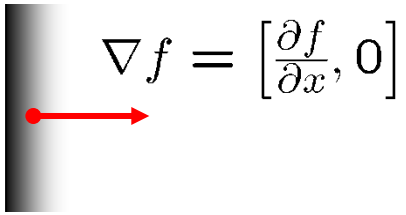


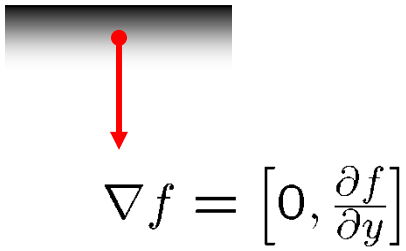
Image gradient

- The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$



- Form the image gradient, and compute its direction and amplitude.
  - Direction

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$



  - Amplitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

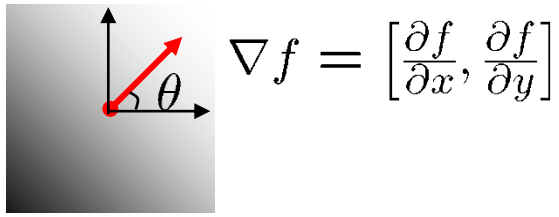
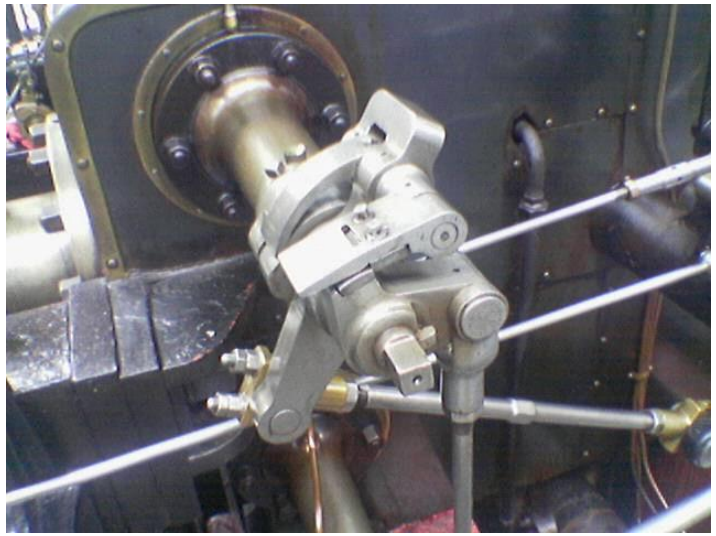
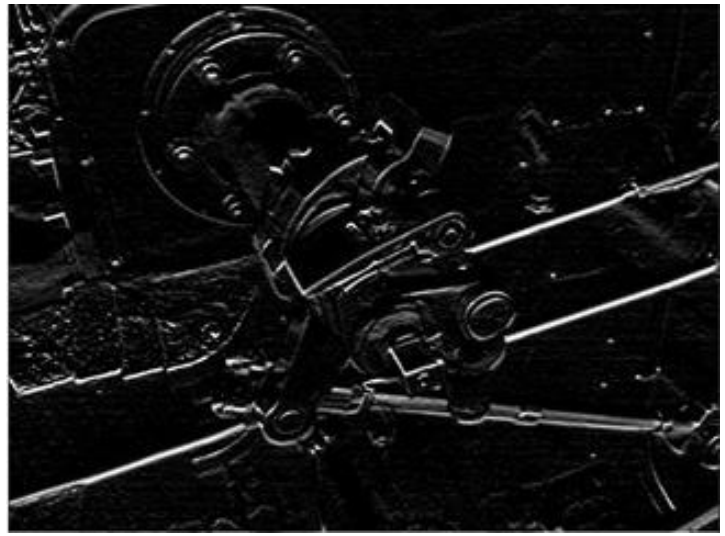


Image gradient

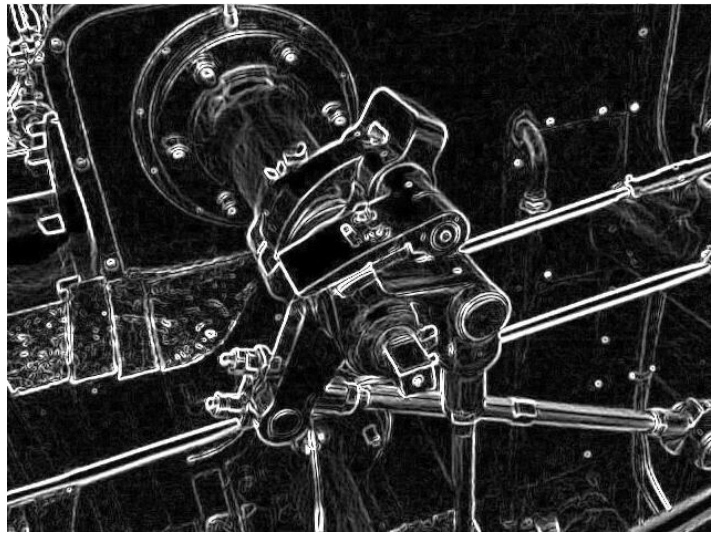
original



vertical derivative



gradient amplitude

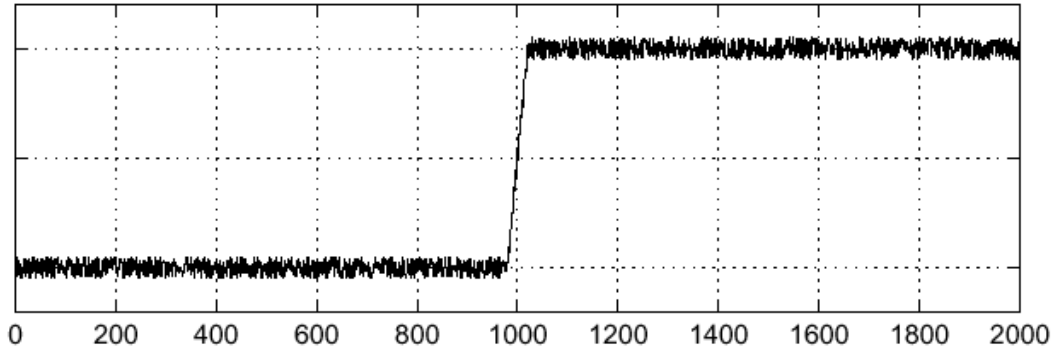


horizontal derivative

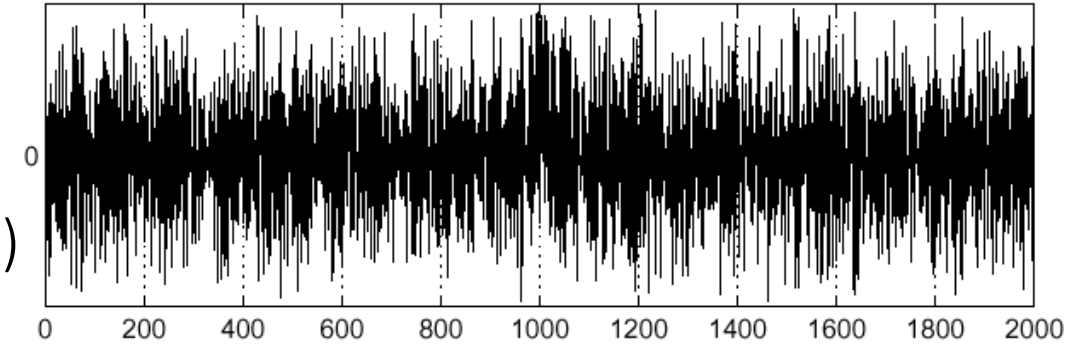
Effects of noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal

$f(x)$
(Intensity)



$\frac{d}{dx}f(x)$
(derivative)



Where is the edge?

Effects of noise: Differentiation is very sensitive to noise

- Finite difference filters respond strongly to noise
  - Image noise results in pixels that look very different from their neighbors
  - Generally, the larger the noise the stronger the response
- What is to be done?
  - Smoothing the image should help, by forcing pixels different to their neighbors (=noise pixels?) to look more like neighbors

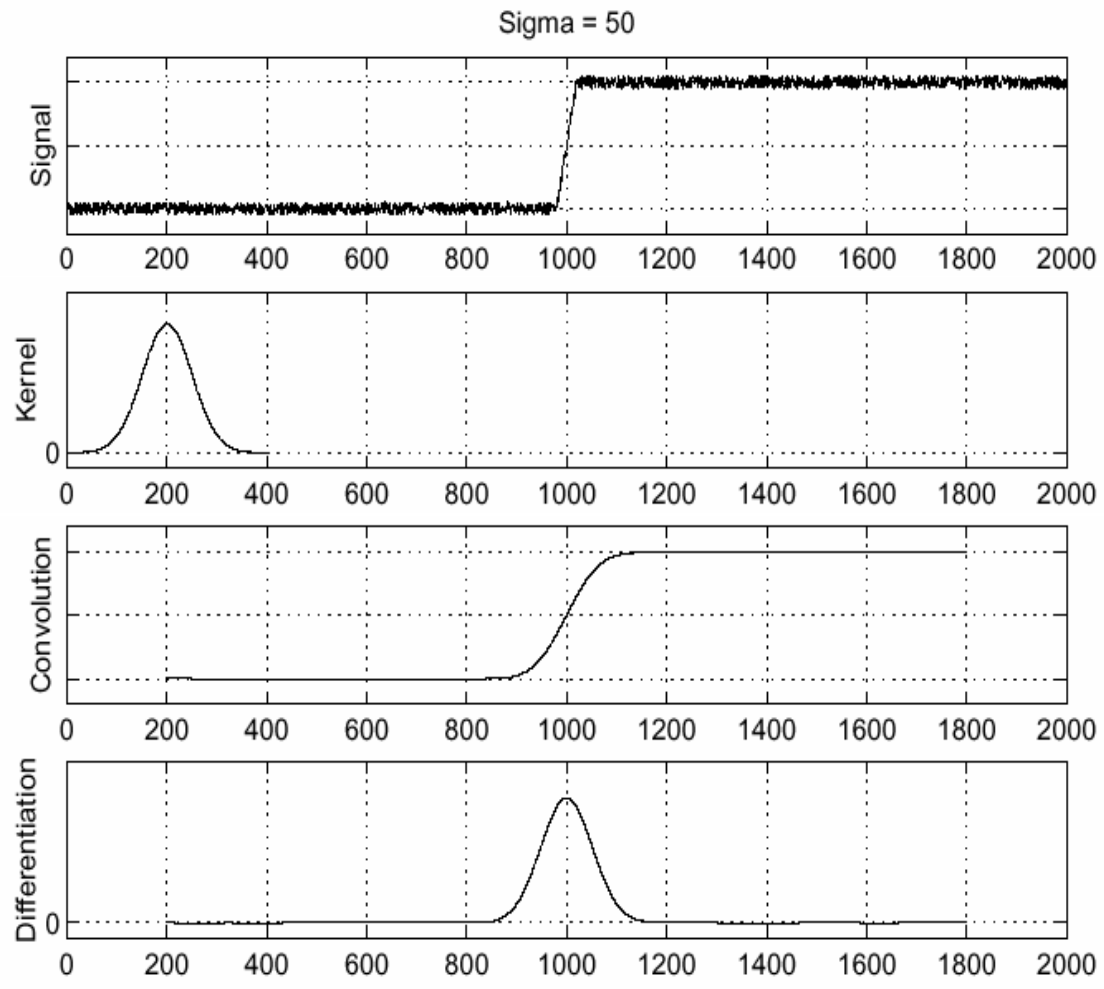
Solution: blur/smooth first

input

Gaussian g

blurred $f * g$

derivative of blurred $\frac{\partial}{\partial x}(f * g)$



How much should we blur?

⇨ To find edges, look for peaks in $\frac{\partial}{\partial x}(f * g)$

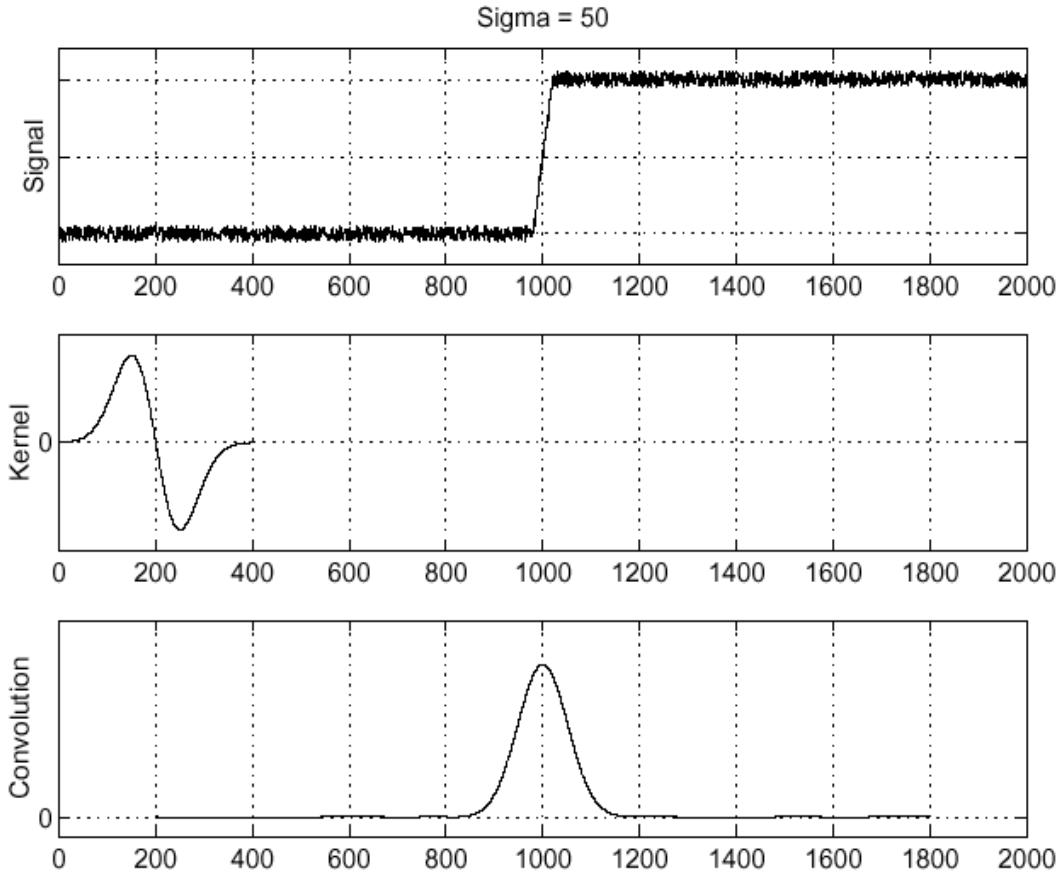
Derivative of Gaussian (DoG) filter

- Derivative theorem of convolution: $\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$

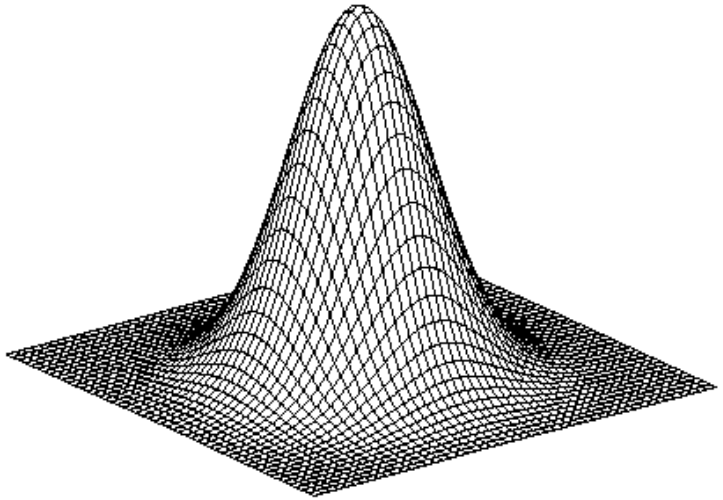
input f

derivative of Gaussian $\frac{\partial}{\partial x}h$

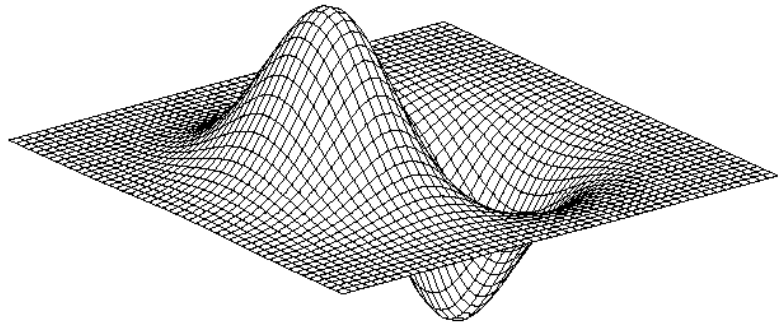
output (same as before) $(\frac{\partial}{\partial x}h) * f$



Derivative of Gaussian filter

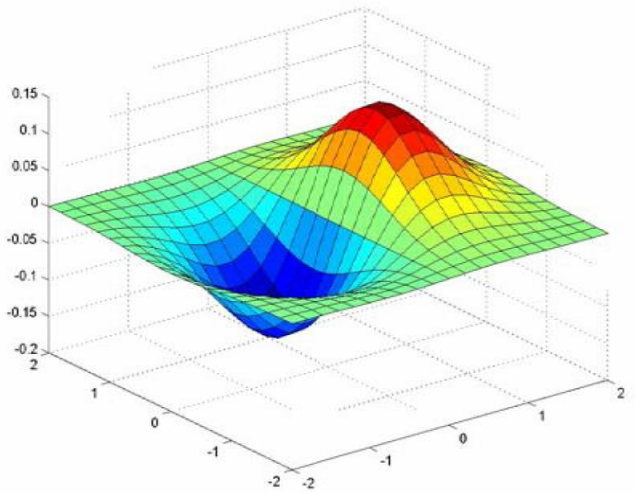


* [1 0 -1] =

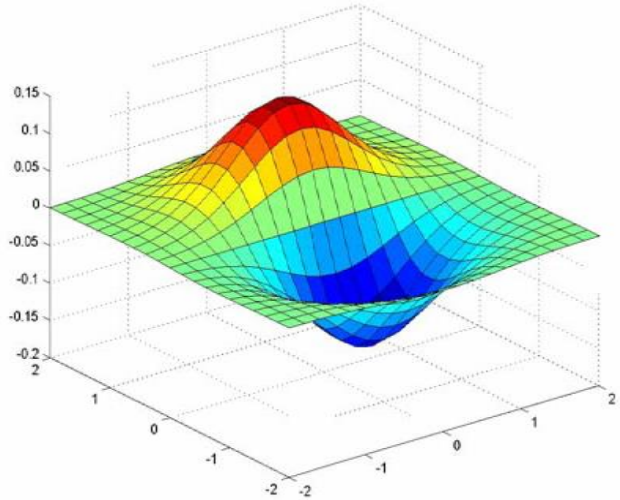


- Is this filter separable?

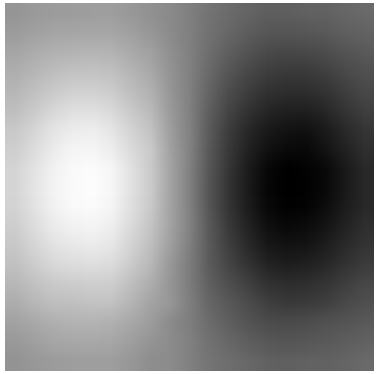
Derivative of Gaussian filter



x-direction



y-direction



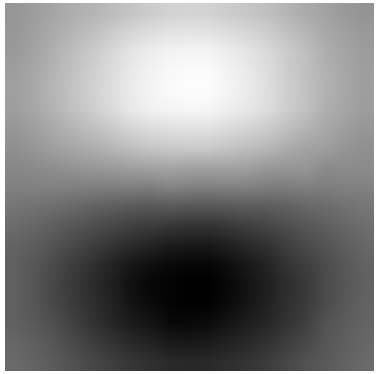


Image filters in the spatial domain

- Linear filter
- Convolution filter
- Gaussian filter
- Derivative filter
- **Laplace filter**
- Sobel filter

Laplace filter

- Laplacian được viết lại dạng

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

- Trong đó đạo hàm thành phần bậc 1 theo phương x, y:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

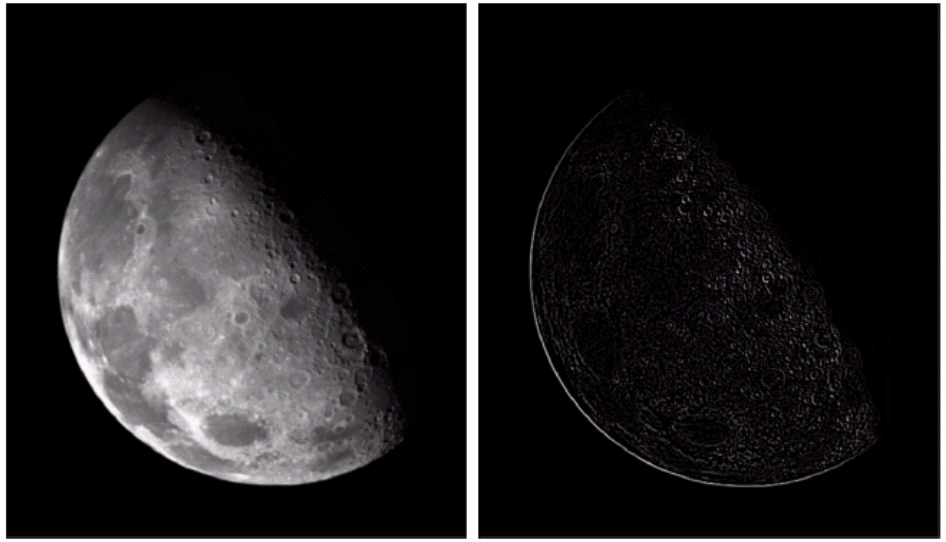
$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

…Laplace filter

- ⇨ $$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] - 4f(x,y)$$

- ⇨

0	1	0
1	-4	1
0	1	0



Original Image

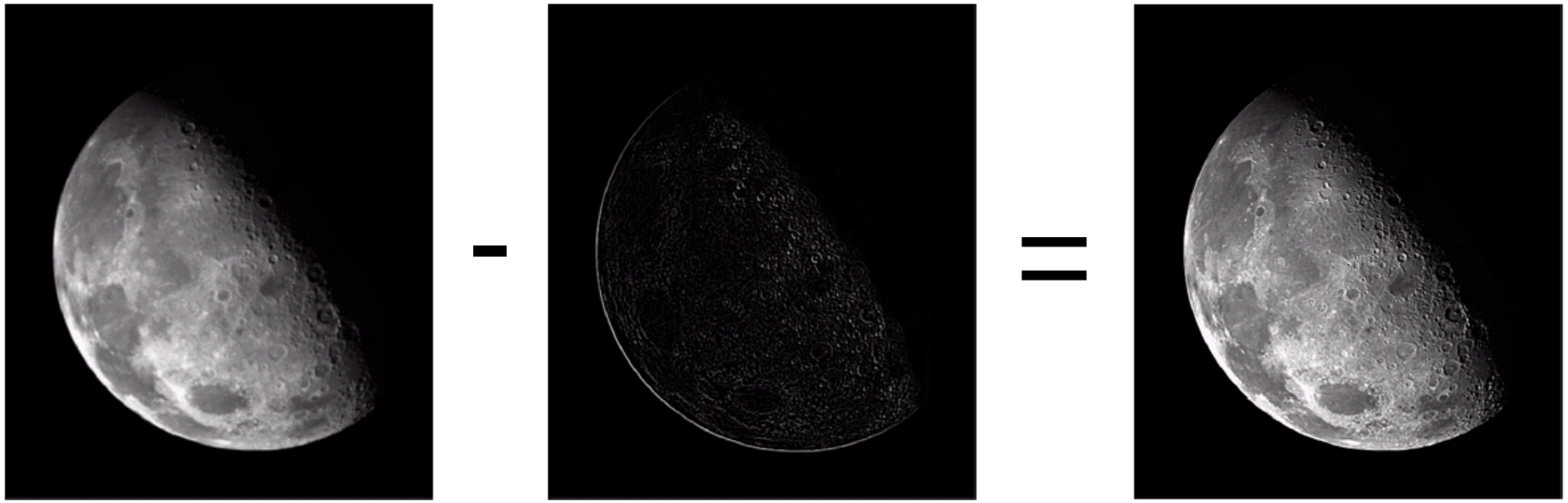
Laplacian Filtered Image

…Laplace filter

- Kết quả của lọc Laplacian chưa phải là một ảnh cải thiện

⇨Trừ ảnh ban đầu cho ảnh Laplacian để được ảnh cải thiện sắc nét

$$g(x,y) = f(x,y) - \nabla^2 f$$



Original Image | Laplacian Filtered Image | Sharpened Image

…Laplace filter

…Laplace filter

- Đơn giản hóa việc cải thiện ảnh

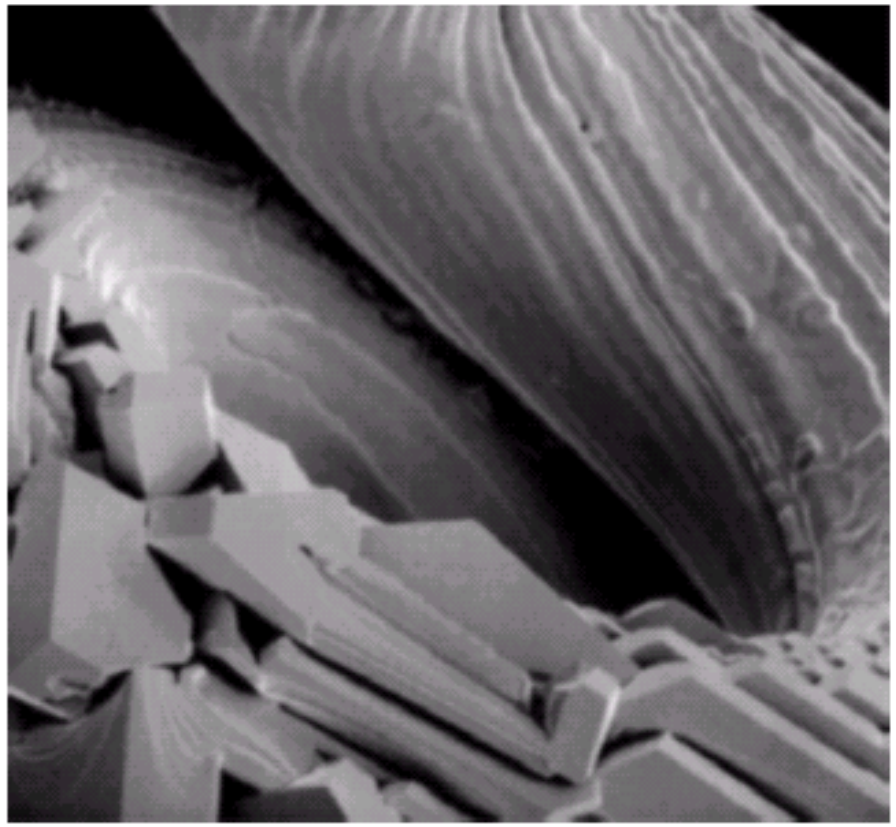
$$g(x,y) = f(x,y) - \nabla^2 f$$

$$= f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)]$$

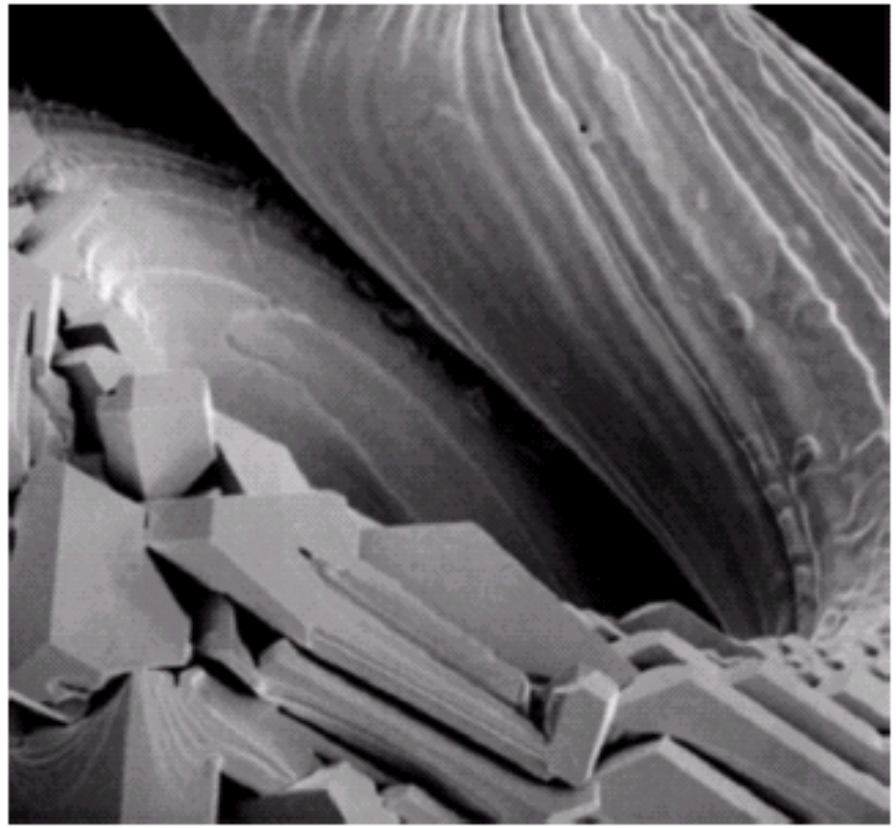
$$= 5f(x,y) - f(x+1,y) - f(x-1,y) - f(x,y+1) - f(x,y-1)$$

0	-1	0
-1	5	-1
0	-1	0

…Laplace filter



0	-1	0
-1	5	-1
0	-1	0



…Laplace filter

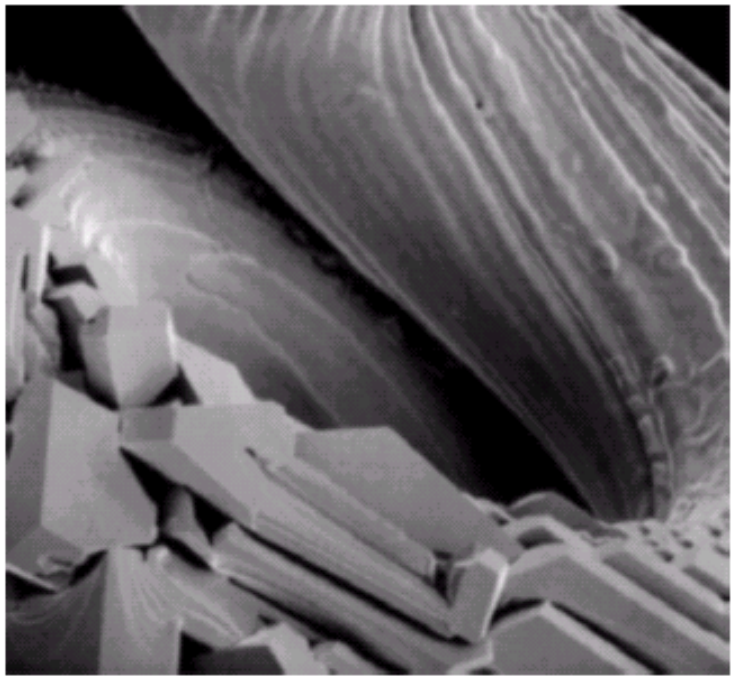
- Biến thể của Laplacian

0	1	0
1	-4	1
0	1	0

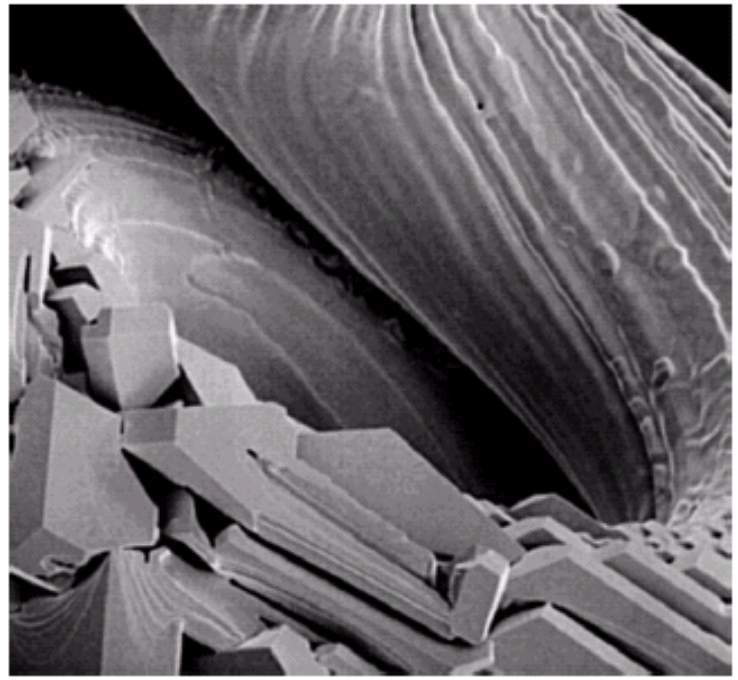
Simple Laplacian

1	1	1
1	-8	1
1	1	1

Variant of Laplacian



-1	-1	-1
-1	9	-1
-1	-1	-1



…Laplace filter

- **Differentiation is very sensitive to noise**

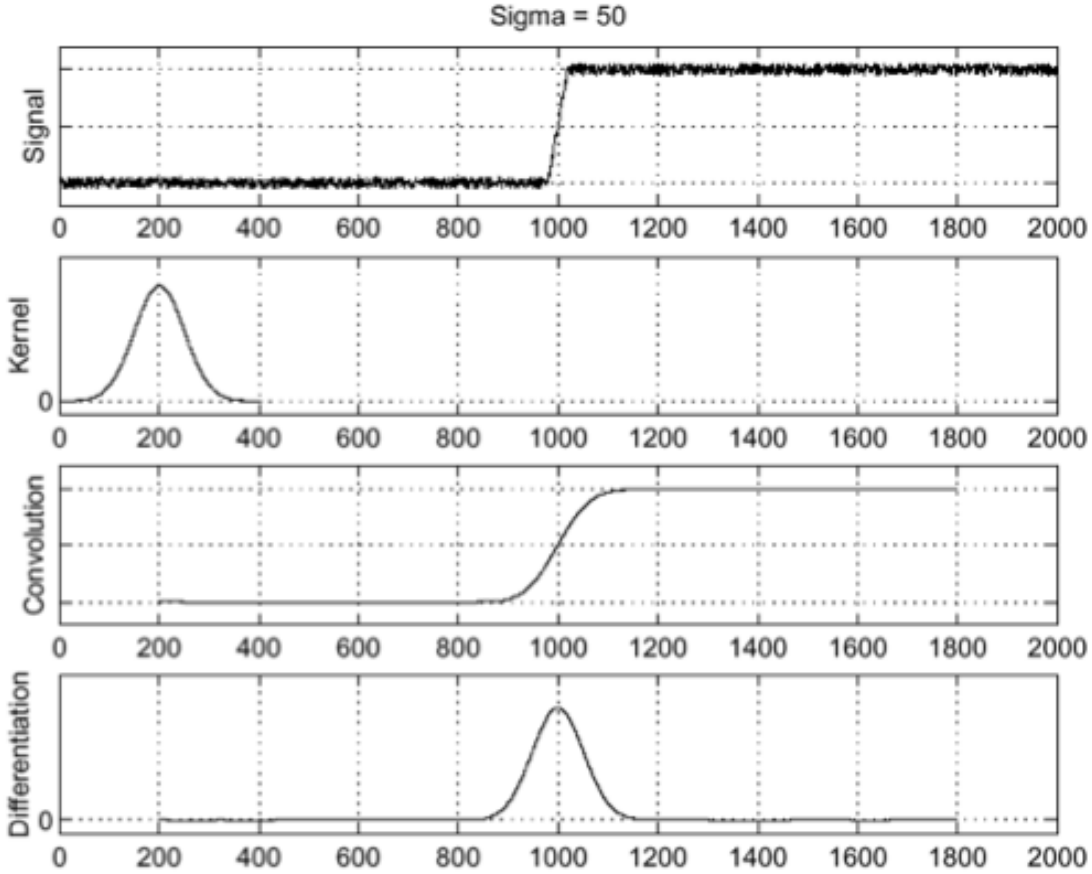
When using derivative filters, it is critical to blur first!

input

Gaussian

blurred

derivative of blurred



How much should we blur?

…Laplace filter

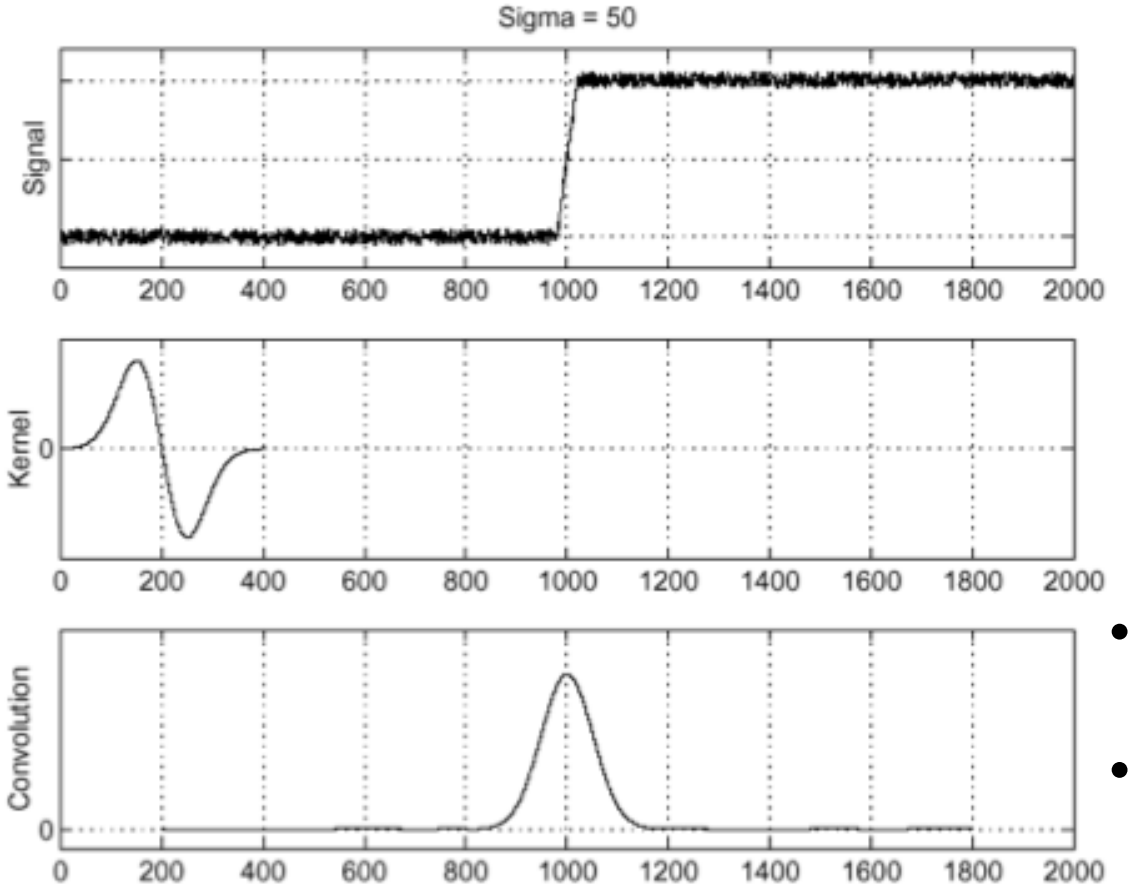
- **Derivative of Gaussian filter (DoG)**

Derivative theorem of convolution: $\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$

input

derivative of Gaussian

output (same as before)



- How many operations did we save?
- Any other advantages beyond efficiency?

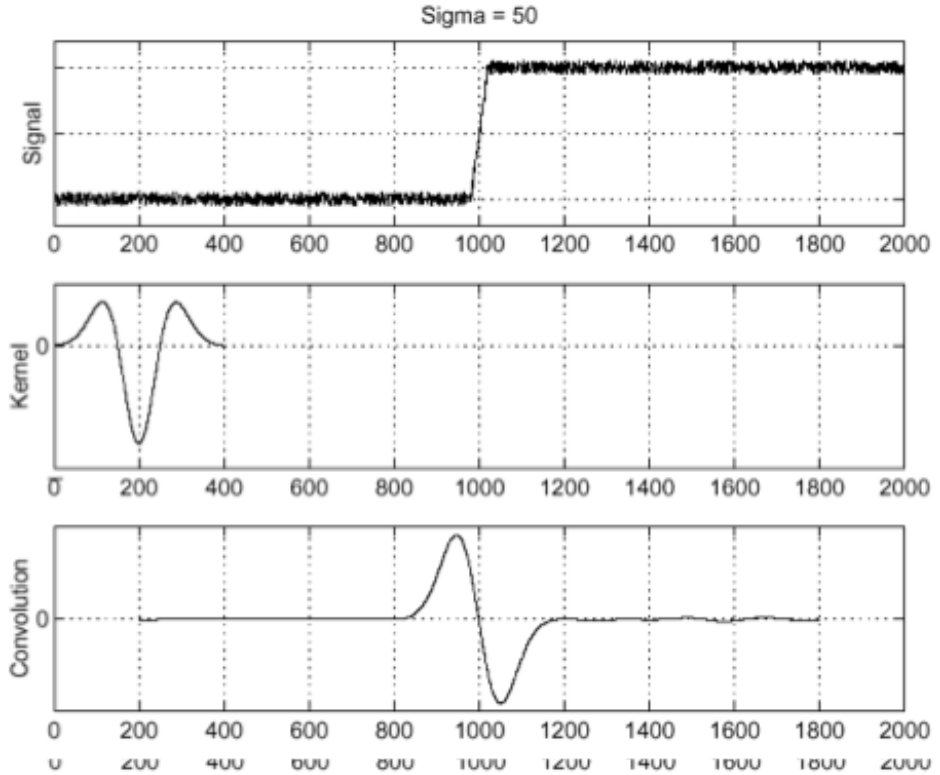
- **Laplacian of Gaussian filter (LoG)**

As with derivative, we can combine Laplace filtering with Gaussian filtering

input

Laplacian of Gaussian

output



"zero crossings" at edges

…Laplace filter

- **Laplace vs LoG filtering examples**

Laplacian of Gaussian filtering

Laplace filtering

…Laplace filter

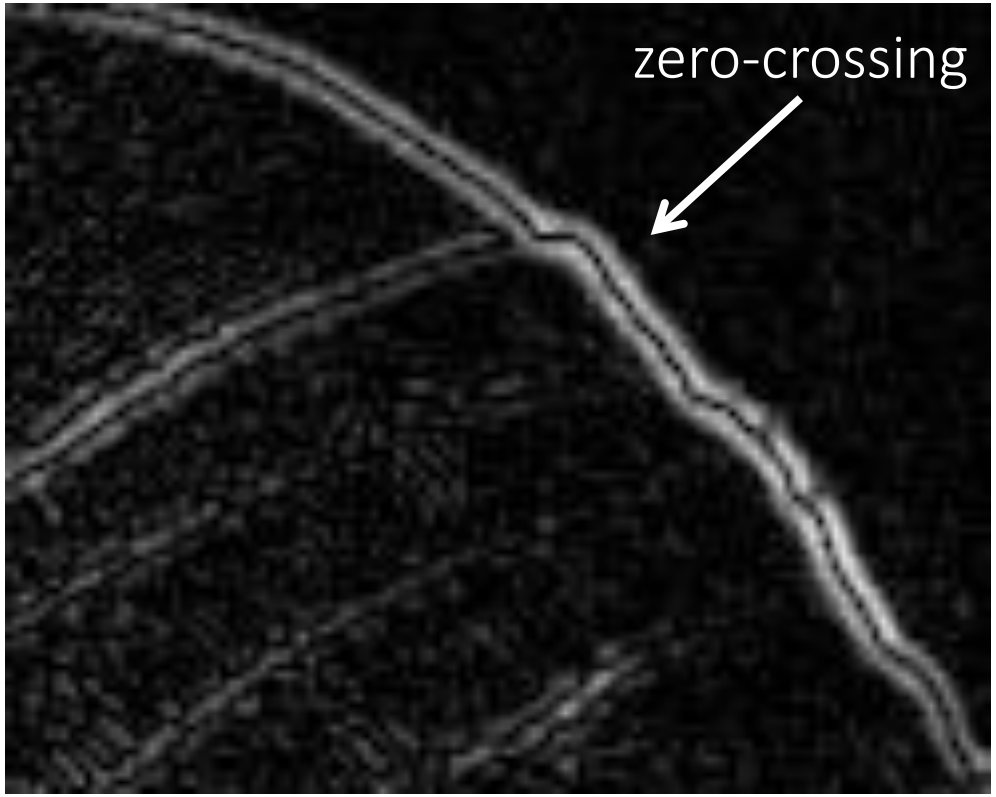
- **Laplacian of Gaussian vs Derivative of Gaussian**

Laplacian of Gaussian filtering

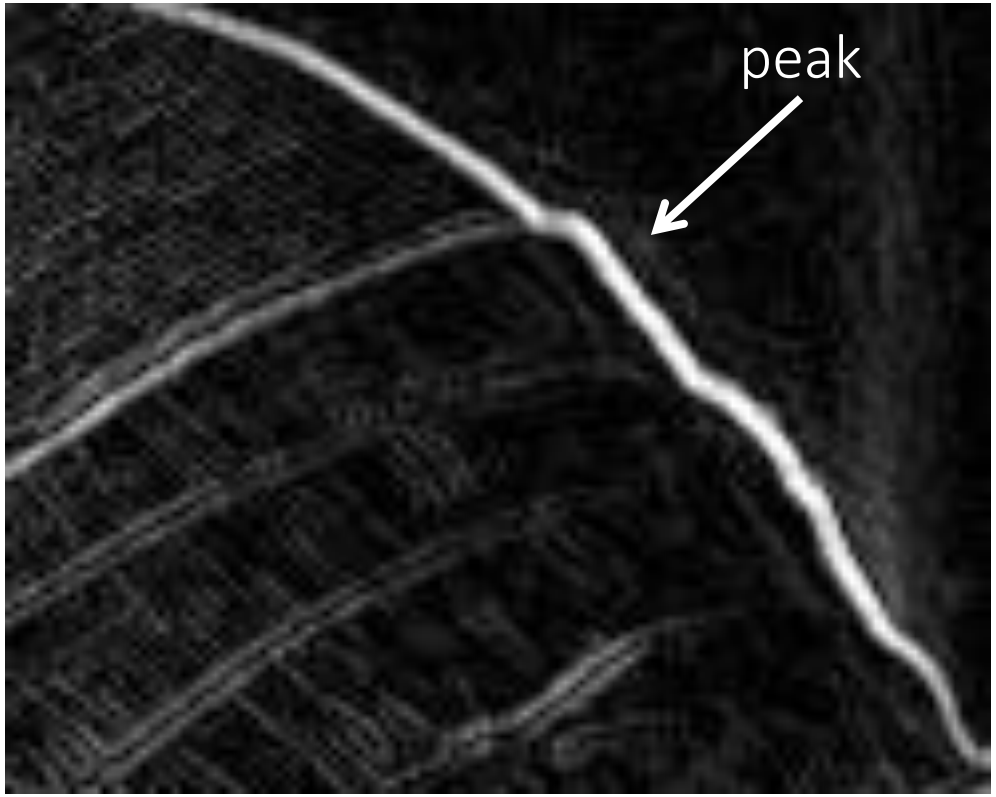
Derivative of Gaussian filtering

…Laplace filter

- **Laplacian of Gaussian vs Derivative of Gaussian**



Laplacian of Gaussian filtering

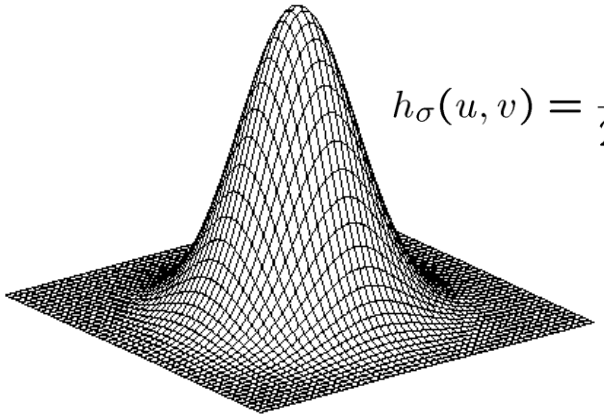


Derivative of Gaussian filtering

Zero crossings are more accurate at localizing edges (but not very convenient).

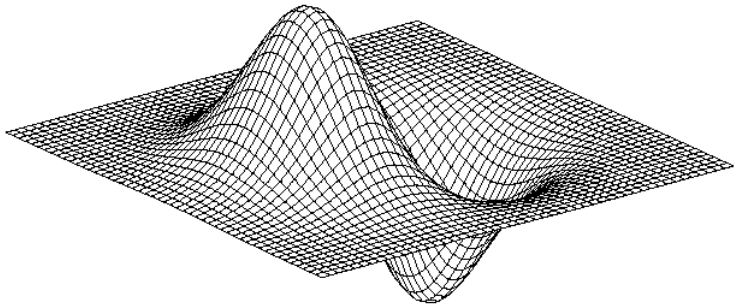
…Laplace filter

$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



Gaussian

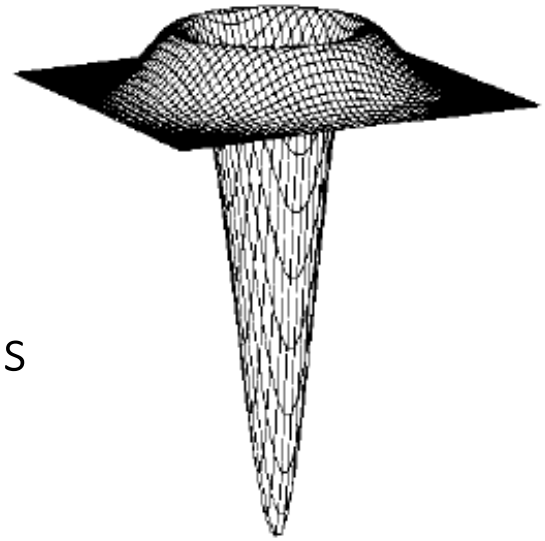
$$\frac{\partial}{\partial x} h_\sigma(u,v)$$



Derivative of Gaussian

how does this relate to this lecture's cover picture?

$$\nabla^2 h_\sigma(u,v)$$



Laplacian of Gaussian

Image filters in the spatial domain

- Linear filter
- Convolution filter
- Gaussian filter
- Derivative filter
- Laplace filter
- **Sobel filter**

Separable filters

- a 2D filter is separable if it can be written as the product of a “column” and a “row”.

example:
box filter

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

column row

- a 2D separable filter is equivalent to two 1D convolutions (with the “column” and “row” filters).
- If the image has M x M pixels and the filter kernel has size N x N:
  - What is the cost of convolution with a non-separable filter? → $M^2 \times N^2$
  - What is the cost of convolution with a separable filter? → $2 \times N \times M^2$

…Sobel filter

2D convolution
(center location only)

$$\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix} * \begin{bmatrix}2&3&3\\3&5&5\\4&4&6\end{bmatrix}$$

$$=2 + 6 + 3 = 11$$
$$= 6 + 20 + 10 = 36$$
$$= 4 + 8 + 6 = 18$$
$$65$$

The filter factors
into a product of 1D filters

$$\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix} = \begin{bmatrix}1\\2\\1\end{bmatrix} \times \begin{bmatrix}1&2&1\end{bmatrix}$$

Perform convolution
along rows

$$\begin{bmatrix}1&2&1\end{bmatrix} * \begin{bmatrix}2&3&3\\3&5&5\\4&4&6\end{bmatrix} = \begin{bmatrix}&11&\\&18&\\&18&\end{bmatrix}$$

Followed by convolution
along the remaining column

$$\begin{bmatrix}1\\2\\1\end{bmatrix} * \begin{bmatrix}&11&\\&18&\\&18&\end{bmatrix} = \begin{bmatrix}&&\\&65&\\&&\end{bmatrix}$$

…Sobel filter

1	0	-1
2	0	-2
1	0	-1

Sobel filter

$=$

1
2
1

Blurring

$*$

1	0	-1

1D derivative filter

- In a 2D image, Sobel filter responses along horizontal or vertical lines

⇨ Các bộ lọc Sobel thường được sử dụng để phát hiện nếp gấp

…Sobel filter

- In a 2D image, Sobel filter responses along horizontal or vertical lines

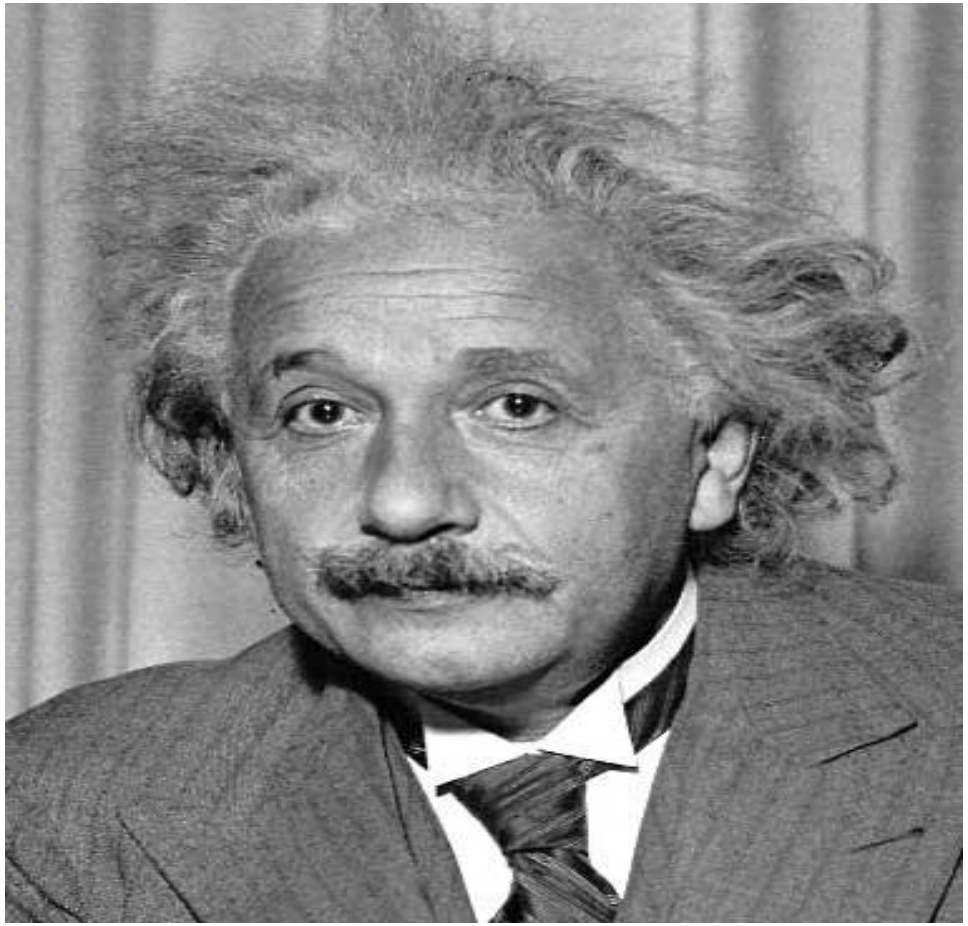
Horizontal Sober filter:

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

Vertical Sobel filter:

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

…Sobel filter

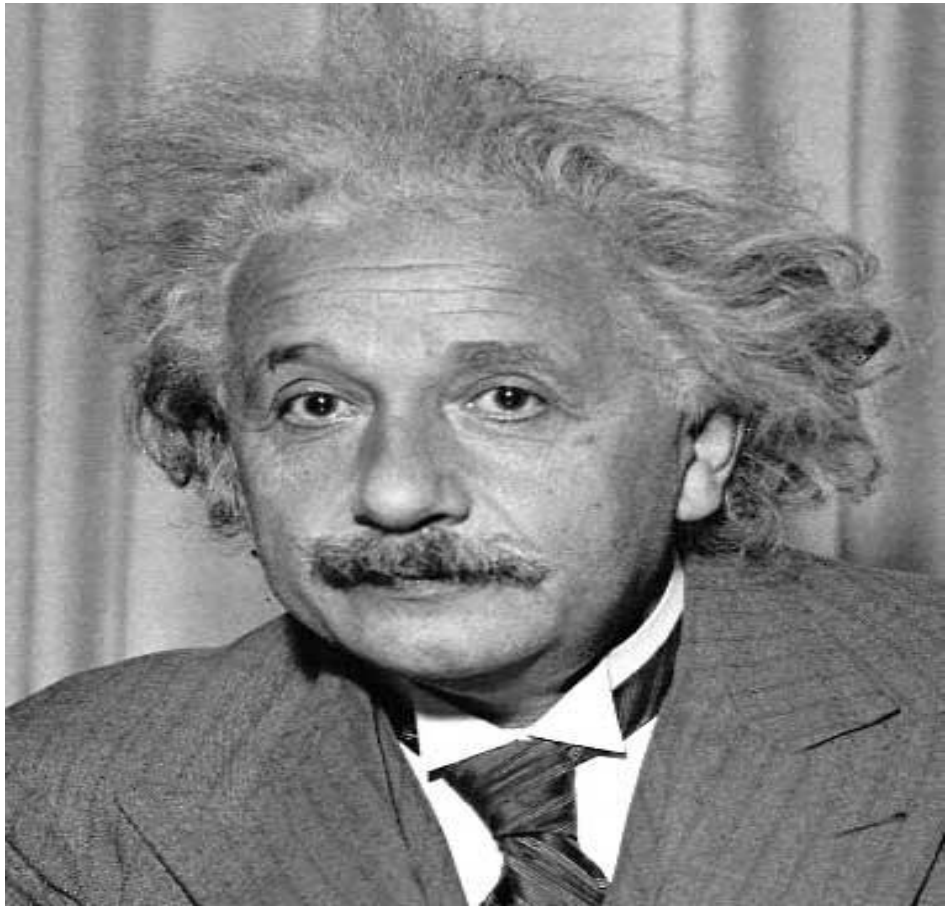


1	0	-1
2	0	-2
1	0	-1

Horizontal Sober filter

Vertical Edge
(absolute value)

…Sobel filter



1	2	1
0	0	0
-1	-2	-1

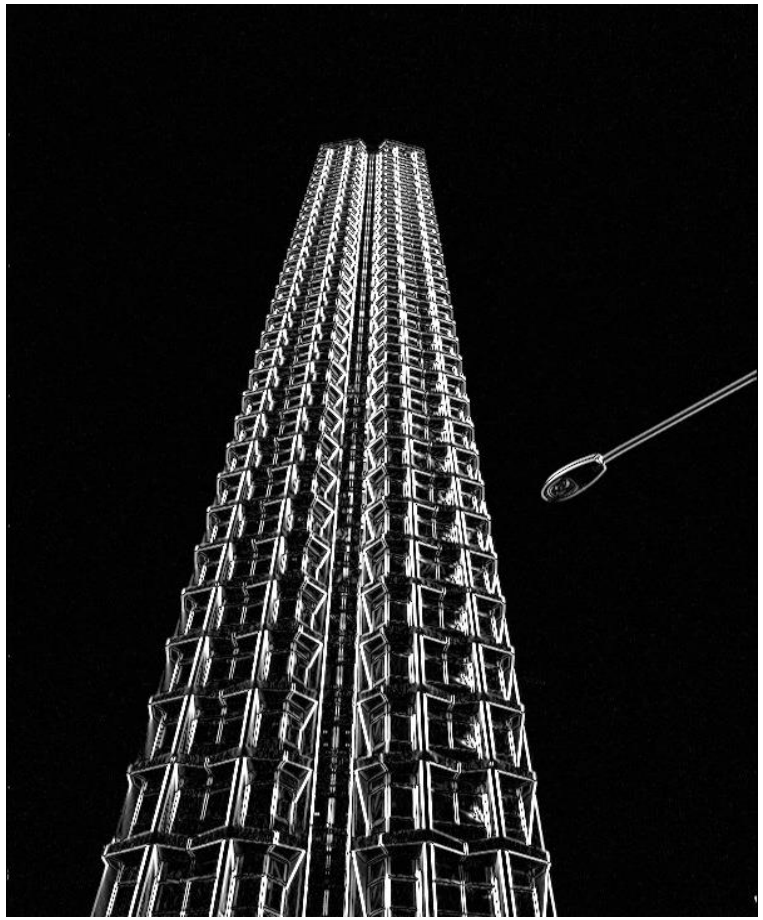
Vertical Sobel filter:

Horizontal Edge
(absolute value)

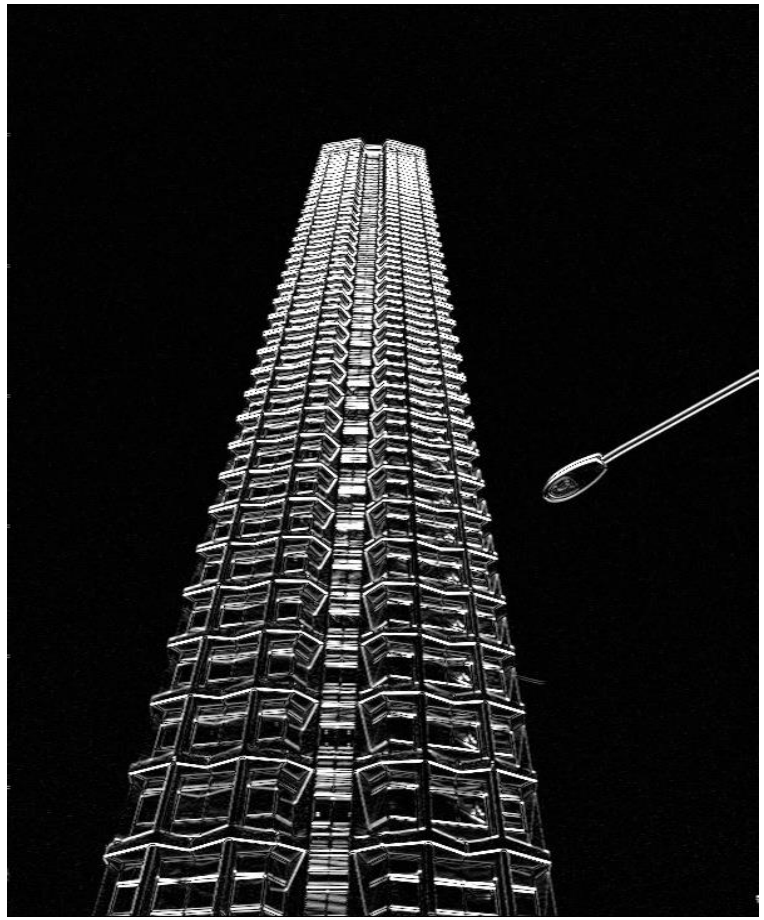
…Sobel filter

example

original



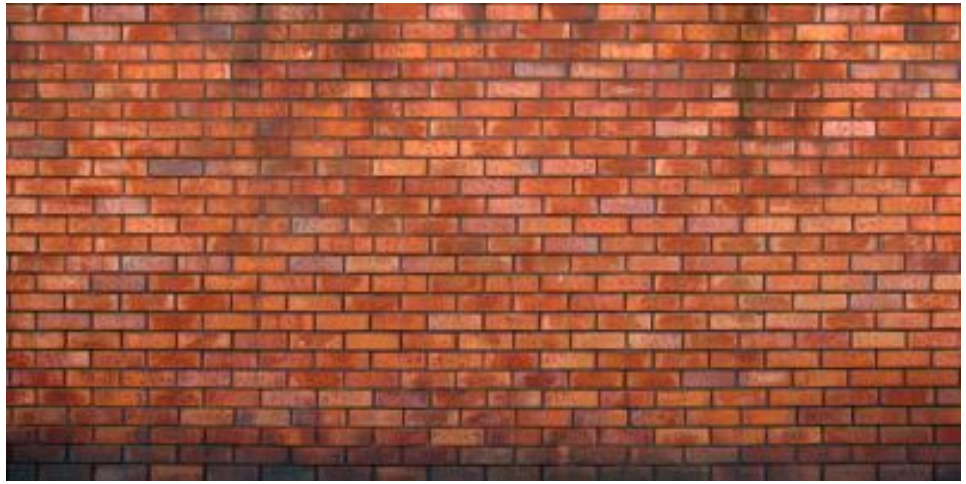
horizontal Sobel filter



vertical Sobel filter

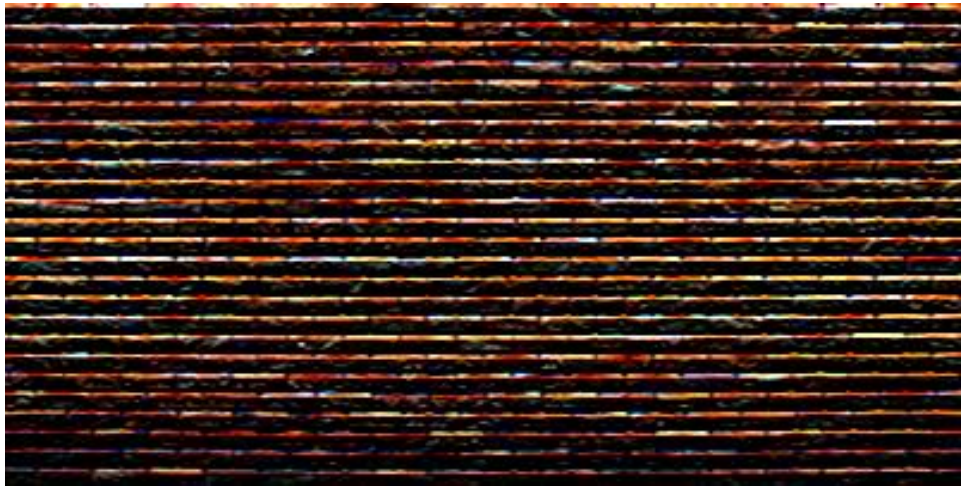
…Sobel filter

example



original

horizontal Sobel filter



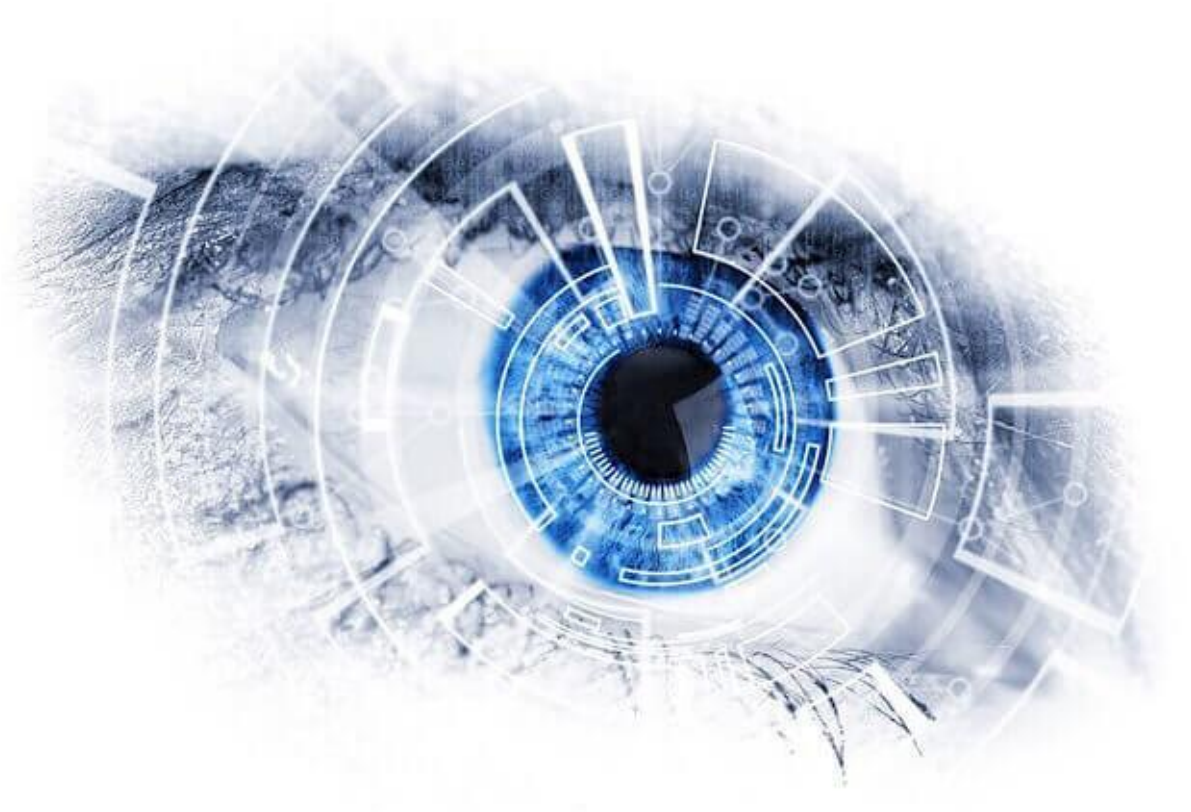
vertical Sobel filter

Basic reading:

Szeliski textbook, Sections 3.2 - 3.5

Image Filter

Computer Vision



Thank You…!