Congratulations! You passed!

Grade received 100% To pass 80% or higher

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1. As mentioned in the previous video, the Taylor series approximation can also be viewed as a power series, in which these approximations are used to build functions that are often simpler and easier to evaluate, particularly when using numerical methods. In the following questions, we are looking at developing our understanding of how the increasing order of a power series allows us to develop further information of a function.

1 / 1 point

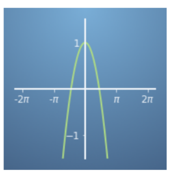
Below are three graphs highlighting the zeroth, second and fourth order approximations of a common trigonometric function. Observe how increasing the number of approximations in the power series begins to build a better approximation, and determine which function these approximations represent.

Zeroth order approximation:

$$f_0(x) = 1$$

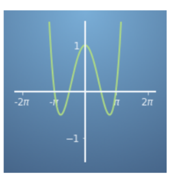
Second order approximation:

$$f_2(x) = 1 - \frac{x^2}{2}$$

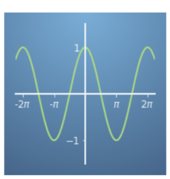


Fourth order approximation:

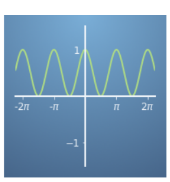
$$f_4(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$$



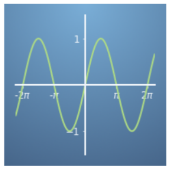
- (●) $f(x) = \cos(x)$



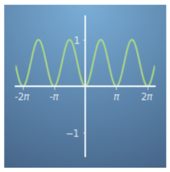
- ( ) $f(x) = \cos^2(x)$



○ $f(x) = \sin(x)$



○ $f(x) = \sin^2(x)$



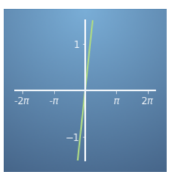
Correct
The function $f(x) = \cos(x)$ is symmetric about the line $x = 0$. Furthermore our approximation of this function is when $x = 0$, known as a Maclaurin series. At the point $x = 0$, $f(0) = 1$ which is shown in the zeroth order approximation.

2. Below are three graphs highlighting the first, third and fifth order approximations of a common trigonometric function. Observe how the power series begins to build the function, and determine which function these approximations represent.

1 / 1 point

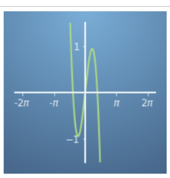
First Order:

$$f_1(x) = 2x$$



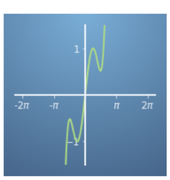
Third Order:

$$f_3(x) = 2x - \frac{4x^3}{3}$$

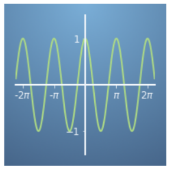


Fifth Order:

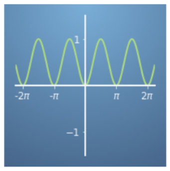
$$f_5(x) = 2x - \frac{4x^3}{3} + \frac{4x^5}{15}$$



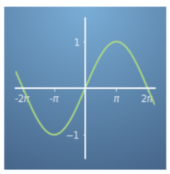
○ $f(x) = \cos(2x)$



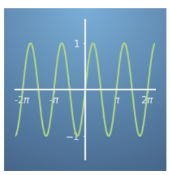
○ $f(x) = \sin^2(x)$



○ $f(x) = \sin(\frac{x}{2})$

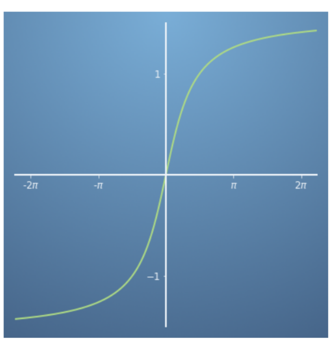


◉ $f(x) = \sin(2x)$

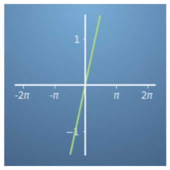


Correct
The function $f(x) = sin(2x)$ has rotational symmetry about the origin. Furthermore, we can see that the period is much shorter, also evident from the three approximations shown.

3. The graph below shows the function $f(x) = \tan^{-1}(x)$, select all the power series approximations that can be used to obtain an approximation for this function. **1 / 1 point**



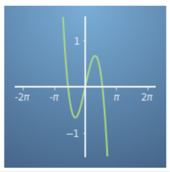
- [x] $f(x) = x \ldots$



Correct

We can see this approximation goes through the origin and also looks as if it fits the function well between $-0.5 < x < 0.5$. As this is a linear function, this is a first-order approximation.

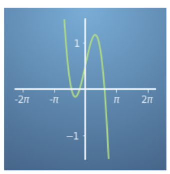
- [x] $f(x) = x - \frac{x^3}{3} \ldots$



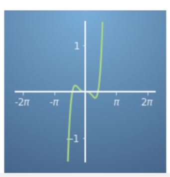
Correct

We can see this approximation goes through the origin and also looks as if it fits the function well between $-0.5 < x < 0.5$.

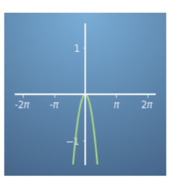
- [ ] $f(x) = \frac{1}{2} + x - \frac{x^3}{3} \ldots$



- [ ] $f(x) = -\frac{x^3}{3} + \frac{x^5}{5} \ldots$

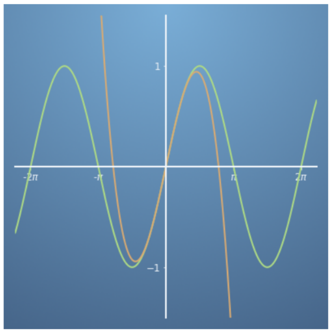


- [ ] $f(x) = -x^2 \ldots$



4. The sinusoidal function $f(x) = \sin(x)$ (green line) centered at $x = 0$ is shown in the graph below. The approximation for this function is shown through the series $f(x) = x - \frac{x^3}{6} \ldots$ (orange line). Determine what polynomial order is represented by the orange line.

1 / 1 point

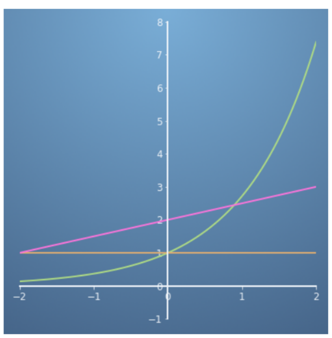


- ○ Zeroth Order
- ○ First Order
- ◉ Third Order
- ○ Fifth Order
- ○ None of the above

Correct
The highest power of x in the approximation is 3, therefore this approximation is a third order approximation.

5. The graph below shows the function $f(x) = e^x$ (green line), the exponential function so widely used in science and mathematics today. The orange line represents the zeroth order approximation for the exponential function, centred at $x = 0$. Determine if the pink line shown on the graph is, in fact, an approximation and if so, what order is this approximation.

1 / 1 point



- ○ First Order
- ○ Second Order
- ○ Third Order
- ◉ Not a correct approximation

Correct
The approximation shown is not tangent to this point and is, therefore, a poor approximation of the function.