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Example: Using non-square matrices to do a projection

Practice Quiz • 30 min • 7 total points

Congratulations! You passed!

Grade received 100% To pass 40% or higher

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1. This quiz is a bit more tricky than the others. We've lowered the pass grade accordingly. Do read through the feedback after submission to build your understanding.

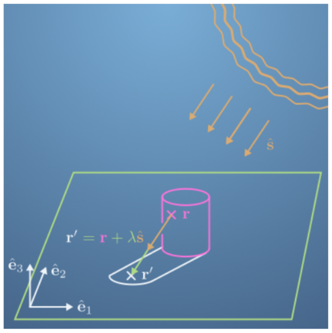
1 / 1 point

The quiz builds sequentially, so don't be afraid to submit and check your answers after tricky questions to make sure you're on the right track before moving on to later questions.

* * *

Shadows are an example of a transformation that reduces the number of dimensions. For example, 3D objects in the world cast shadows on surfaces that are 2D.

We can consider an example for looking at shadows using linear algebra.



The sun is sufficiently far away that effectively all of its rays come in parallel to each other. We can describe their direction with the unit vector $\hat{\mathbf{s}}$.

We can describe the 3D coordinates of points on objects in our space with the vector $\mathbf{r}$. Objects will cast a shadow on the ground at the point $\mathbf{r}'$ along the path that light would have taken if it hadn't been blocked at $\mathbf{r}$, that is, $\mathbf{r}' = \mathbf{r} + \lambda\hat{\mathbf{s}}$.

The ground is at $\mathbf{r}'_3 = 0$; by using $\mathbf{r}'.\hat{\mathbf{e}}_3 = 0$, we can derive the expression, $\mathbf{r}.\hat{\mathbf{e}}_3 + \lambda s_3 = 0$, (where $s_3 = \hat{\mathbf{s}}.\hat{\mathbf{e}}_3$).

Rearrange this expression for λ and substitute it back into the expression for $\mathbf{r}'$ in order to get $\mathbf{r}'$ in terms of $\mathbf{r}$.

- (●) $\mathbf{r}' = \mathbf{r} - \hat{\mathbf{s}}(\mathbf{r}.\hat{\mathbf{e}}_3)/s_3$
- ( ) $\mathbf{r}' = \mathbf{r} - \hat{\mathbf{s}}$
- ( ) $\mathbf{r}' = \mathbf{r} + \hat{\mathbf{s}}$
- ( ) $\mathbf{r}' = \mathbf{r} + \hat{\mathbf{s}}(\mathbf{r}.\hat{\mathbf{e}}_3)/s_3$

Correct
Well done!

2. From your answer above, you should see that $\mathbf{r}'$ can be written as a linear transformation of $\mathbf{r}$. This means we should be able to write $\mathbf{r}' = A\mathbf{r}$ for some matrix A.

2 / 2 points

To help us find an expression for A, we can re-write the expression above with Einstein summation convention.

Which of the answers below correspond to the answer to Question 1? (Select all that apply)

- [ ] None of the other options.
- [x] $r'_i = (I_{ij} - s_i[\hat{\mathbf{e}}_3]_j/s_3)r_j$

Correct
In this form, it's easier to see this as a matrix multiplication. The term in brackets has free indices i and j. Compare this to $[A\mathbf{r}]_i = A_{ij}r_j$.

- [x] $r'_i = r_i - s_i r_3 / s_3$

Correct
This answer is correct and concise, but more difficult to see it as a matrix multiplication on r.

- [x] $r'_i = (I_{ij} - s_i I_{3j} / s_3) r_j$

Correct
Another way to write the unit vectors is in terms of the identity matrix $[\hat{\mathbf{e}}_a]_j = I_{aj}$. Think about why this is true.

- [x] $r'_i = r_i - s_i [\hat{\mathbf{e}}_3]_j r_j / s_3$

Correct
This form probably flows most naturally from the previous question.

3. Based on your answer to the previous question, or otherwise, you should now be able to give an expression for A in its component form by evaluating the components A_{ij} for each row i and column j.

1 / 1 point

Since A will take a 3D vector, $\mathbf{r}$, and transform it into a 2D vector, $\mathbf{r}'$, we only need to write the first two rows of A. That is, A will be a 2×3 matrix. Remember, the columns of a matrix are the vectors in the new space that the unit vectors of the old space transform to - and in our new space, our vectors will be 2D.

What is the value of A?

- (●) $\begin{bmatrix} 1 & 0 & -s_1/s_3 \\ 0 & 1 & -s_2/s_3 \end{bmatrix}$
- ( ) $\begin{bmatrix} -s_1/s_3 & 0 & 0 \\ 0 & -s_2/s_3 & 0 \end{bmatrix}$
- ( ) $\begin{bmatrix} r_1 & 0 & -s_1/s_3 \\ 0 & r_2 & -s_2/s_3 \end{bmatrix}$
- ( ) $\begin{bmatrix} s_1/s_3 & 0 & -s_1/s_3 \\ 0 & s_2/s_3 & -s_2/s_3 \end{bmatrix}$

Correct
Well done!

4. A is a 2x3 matrix, but if you were to evaluate its *third* row, what would it's components be?

1 / 1 point

```
# Fill in the components of the 'third' row of A,
A3 = [0, 0, 0]
```

Run
Reset

```
[0, 0, 0]
```

Correct
Correct! the matrix has all zeros in its final row, as r' never has any value in the third direction.

5.

1 / 1 point

Assume the Sun's rays come in at a direction $\hat{\mathbf{s}} = \begin{bmatrix} 4/13 \\ -3/13 \\ -12/13 \end{bmatrix}$.

Construct the matrix, A, and apply it to a point, $\mathbf{r} = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$, on an object in our space to find the coordinates of that point's shadow.

Give the coordinates of $\mathbf{r}'$.

```
# Give the coordinates for r'
rp = [7, 1.25]
```

Run
Reset

```
[7, 1.25]
```

Correct
Well done!

6. Another use of non-square matrices is applying a matrix to a list of vectors.

1 / 1 point

Given our transformation $\mathbf{r}' = A\mathbf{r}$, this can be generalized to a matrix equation, $R' = AR$, where R' and R are matrices where each column are corresponding r' and r vectors, i.e.,

$$\begin{bmatrix} r_1' & s_1' & t_1' & u_1' & \dots \\ r_2' & s_2' & t_2' & u_2' & \dots \end{bmatrix} = A \begin{bmatrix} r_1 & s_1 & t_1 & u_1 & \dots \\ r_2 & s_2 & t_2 & u_2 & \dots \\ r_3 & s_3 & t_3 & u_3 & \dots \end{bmatrix}.$$

In Einstein notation, $r_i' = A_{ij} r_j$ becomes $R_{ia}' = A_{ij} R_{ja}$.

For the same $\hat{\mathbf{s}}$ as in the previous question, apply A to the matrix

$$R = \begin{bmatrix} 5 & -1 & -3 & 7 \\ 4 & -4 & 1 & -2 \\ 9 & 3 & 0 & 12 \end{bmatrix}.$$

Observe that it's the same result as treating the columns as separate vectors and calculating them individually.

```
Rp = [[8,  0,  -3,  11],
      [1.75,  -4.75,  1,  -5]]
```

Run

Reset

No Output

Correct

Well done!