Homework 4 – due Oct 9th, 4:40pm

Reminders:

- For the Vocareum assignment, you must work alone. For the minimart question, you may work either alone or in teams of two. If working in pairs, you should hand-in a single assignment, with both of your names on it.
- The solution to Question 2 must be typed (Word is recommended). No handwritten work will be accepted or graded (though you are allowed to draw an illustration if you feel it helps).
- Remember to use the header page provided on Angel.
- Late homework will not be accepted, but you are allowed to drop one lowest homework grade at the end of the semester.
- You are not allowed to discuss any aspects of the homework solutions problem with anyone. The only exception is for the minimart question, if you are working in a team of two, in which case you may naturally discuss it with your partner. Finding answers to problems on the Web or from other outside sources (these include anyone not enrolled in the class) is strictly forbidden.

Problems:

1. **Dynamic programming assignment on Vocareum**
2. **(Minimarts)** You are managing the construction of minimarts along a stretch of road. The possible sites for the minimarts are given by real numbers $x_1, ..., x_n$, each of which specifies the position along the road measured in miles from its beginning. Assume that the road is a straight line. If you place a minimart at location x_i, your company will make a profit of $r_i > 0$ dollars.

   Regulations require that every pair of minimarts be at least 5 miles apart. You'd like to place minimarts at a subset of the sites so as to maximize total profit, subject to this restriction. The input is given as a list of n pairs $(x_1, r_1), ..., (x_n, r_n)$ where the x_i's are sorted in increasing order.

   (a) You first consider some very simple approaches to the problem, which you realize will not work well. For each of the following approaches, give an example of an input on which the approach does not find the optimal solution:

      i. *Next available location:* put a minimart at $i = 1$. From then on, put a minimart at the smallest index i which is more than five miles from your most recently placed minimart.

ii. *Most profitable first*: Put a minimart at the most profitable location. From then on, place a minimart at the most profitable location not ruled out by your current minimarts.

(b) Give a dynamic programming algorithm for this problem. Analyze the time complexity of your algorithm. To make it easier to present your answer clearly, follow the steps below:

i. Clearly define the subproblems that you will solve recursively.

ii. Give a recursive formula for the solution to a given subproblem in terms of smaller subproblems. Explain why the formula is correct.

iii. Give a bottom-up pseudocode for an algorithm that calculates the profit of the optimal solution. Analyze the time complexity of your algorithm.

iv. Give pseudocode for an algorithm that uses the information computed in the previous part to output an optimal solution. Analyze the time complexity of your algorithm.