Big Oh notation

$n^2 \in O(n^3)$

$n^2 \in O(n^2)$

$\in$

$n^3 \in \Omega(n^2)$

$\Omega(g(n))$

Big Omega.

"$\geq$"

such that.

$\Omega(g(n)) = \{f(n) : \exists c > 0, n_0 > 0, \text{ s.t. }$
$0 \leq c \cdot g(n) \leq f(n) \ \forall n \geq n_0\}.$

Transpose symmetry property.

$f(n) \in O(g(n))$

iff ← if and only if.

$g(n) \in \Omega(f(n))$

$\left[\begin{array}{c} a \leq b \\ \text{iff} \\ b \geq a \end{array}\right]$

$\frac{n^2}{2} - 3n \in \Omega(n^2).$

iff.

$n^2 = O(\frac{n^2}{2} - 3n)$

Every polynomial function can be written as.

$$a_d n^d + a_{d-1} n^{d-1} + \dots + a_1 n + a_0 = \Omega(n^d)$$

$$\in O(n^d)$$

$3n^2 + \frac{n}{2} + 5$. $d = 2$. $\in \Theta(n^d)$.

$a_2 = 3$

$a_1 = 1/2$

$a_0 = 5$

$\Theta(g(n)) = O(g(n)) \cap \Omega(g(n))$

set intersection

$n^3 + 2n^2 + 5 = n^3 + \Theta(n^2)$

placeholder for some anonymous function that we don't care to name, but in belongs to $\Theta(n^2)$

$n^3 = \Theta(n^3)$

Log base does not matter asympt.

i.e.

$\forall a>0, \forall b>0.\ \log_a n = \Theta(\log_b n).$

log identity. $\log_a n = \dfrac{\log_b n}{\log_b a}$

$$\log_a n = \frac{\log_b n}{\log_b a} = \Theta\left(\frac{\log_b n}{\log_b a}\right) = \Theta(\log_b n)$$

"reflexivity"

$f(n) \in \Theta(f(n))$

$f(n) \in O(f(n))$

$f(n) \in \Omega(f(n)).$

$n = \Theta(2n)$

Little oh $\quad o$ ("<") $\quad O$ ("≤").

$o(g(n)) = \{f(n) : \forall c > 0, \exists n_0 > 0, \forall n \geq n_0$

$0 \leq f(n) < c \cdot g(n) \}$.

alt definition

$f(n) \in o(g(n))$ iff $\lim_{n \to \infty} \frac{f(n)}{g(n)} \to 0$

$f(n) = \lg n \leftarrow$ "lg" means $\log_2$

$g(n) = n^d$, for some $d > 0$.

$\lg n = o(n^d)$ ✓

L'Hopital

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{\lg n}{\boxed{n^d}} = \lim \frac{\lg e}{n \cdot d \cdot n^{d-1}}$$

$$= \lim \left(\frac{\lg e}{d}\right) \cdot \frac{1}{n^d} \to 0$$

$$\frac{d(\lg(n))}{dn} = \frac{d}{dn}(\ln n \cdot \lg e) \quad \Bigg| \quad \frac{d(\ln n)}{dn} = \frac{1}{n}$$

$$= \lg e \cdot \frac{1}{n}.$$

$$\frac{d(n^d)}{dn} = d n^{d-1}$$

In general, any log grows slower than any polynomial.

$f(n) = n$

$g(n) = n \lg n.$

$n = o(n \lg n).$

Need to show. $\forall c > 0 \; \exists n_0 > 0 \; \forall n \geq n_0$

$$0 \leq n < c \cdot n \lg n.$$

always holds for $n \geq 0$

$$\frac{n}{n} < \frac{c n \lg n}{n}$$

$$1 < c \lg n.$$

$$\frac{1}{c} < \lg n.$$

$$2^{1/c} < 2^{\lg n} = n.$$

$$n > 2^{1/c}$$

$$n_0 = 2^{1/c} + 0.01$$

or

$$2^{1/c} + 100$$

$f(n) = n^2$

$g(n) = 2^n$. $\qquad n^2 = o(2^n)$

Need to show. $\forall c > 0, \exists n_0 > 0 \; \forall n \geq n_0$

$$0 \leq n^2 < c \cdot 2^n.$$

$$n^2 < c2^n$$

$$\frac{n^2}{2^n} < c$$

$$\lg\left(\frac{n^2}{2^n}\right) < \lg c.$$

$$\lg n^2 - \lg 2^n < \lg c$$

$$2\lg n - n \cancel{\lg 2}^{1} < \lg c.$$

$$2\lg n - n < \lg c.$$

$$\boxed{2\lg n - n < 2\lg n}$$

pick a value for n to satisfy.

$$2\lg n < \lg c$$

$$\cancel{n < \sqrt{c}}$$

$2 \lg n < \lg c$

$\cancel{n < \sqrt{c}}$

$\boxed{2^{2 \lg n}} < 2^{\lg c}$

$2^{\lg n \cdot 2} = (2^{\lg n})^2 = n^2$

$n^2 < c$

$n < \sqrt{c}$

In general, $n^d = o(a^n)$ $\forall d > 0$ $\forall$ $a > 1$

$\sqrt{n} = o(1.1^n)$

"polynomials grow slower than exponential functions"

$2^n = o(3^n)$ $1.1^n = o(1.2^n)$

In general, $\forall a > 0$, $\forall b > a$, s.t. $a < b$.

$a^n = o(b^n)$

$n^3 = \omega(n^2)$

$n^3 \neq \omega(n^3)$

Transpose symmetry:

$f(n) \in o(g(n))$ $a < b$

iff   iff

$g(n) \in \omega(f(n))$ $b > a$

To prove $n^3 \in \omega(n^2)$, it is the same as

$n^2 \in o(n^3)$

$n! = n \cdot (n-1) \cdot \cdots \cdot 2 \cdot 1.$

$n! = 2^{\Theta(n \lg n)}$

$\lg(n!) = \Theta(n \lg n).$