Sample Midterm 1 Solutions
CMPSC 465, Fall 2015

Problem 1: t

Problem 2: Lets assume, as the induction hypothesis, that there exists a $c > 0$ such that $T(i) \leq ci \log i$ for all inputs of size $i < n$. We need to show that $T(n) \leq cn \log n$:

$$T(n) = 4T\left(\frac{n}{4}\right) + 3n \leq 4\left(\frac{cn}{4}\log\frac{n}{4}\right) + 3n = cn\log\frac{n}{4} + 3n = cn\log n - 2cn + 3n$$

This quantity is less than or equal to $cn \log n$ when $-2cn + 3n \leq 0$. This holds for any value of $c \geq \frac{3}{2}$, completing the induction step and proof.

Problem 3.I: $T(n) = \Theta(n^2)$. This follows from Case 3 of the Master Theorem.

Problem 3.II: $T(n) = \Theta(n^3 \log n)$. This follows from Case 2 of the Master Theorem.

Problem 3.III: $T(n) = \Theta(n \log n)$. This follows from Case 3 of the Master Theorem.

Problem 4: We need to find a value of c and n_0 such that that for all $n \geq n_0$, the following holds:

$$7n \leq cn \log n$$

We divide both sides by n to get

$$7 \leq c \log n$$

The correct answer is then any value of c and n_o for which the inequality holds. Just pick one. For example, one possible correct answer is $c = 7$ and $n_0 = 2$.

Problem 5

$f(n) = 3$	$g(n) = \log n$	a or b or c
$f(n) = \log^2 n$	$g(n) = \log n$	a or b or c
$f(n) = n^2$	$g(n) = \frac{n^2}{\log n}$	a or b or c
$f(n) = n^2 \log n$	$g(n) = \log(n!^2)$	a or b or c
$f(n) = 3^n$	$g(n) = 2^{\sqrt{n}}$	a or b or c

Problem 6: See solution to HW3, Problem 1a.