Homework 3 has been posted

The date of Midterm ~~has~~ is: 9/23

In Class. During regular time.

~~Next~~ Coming Recitation: Solutions for HW2.

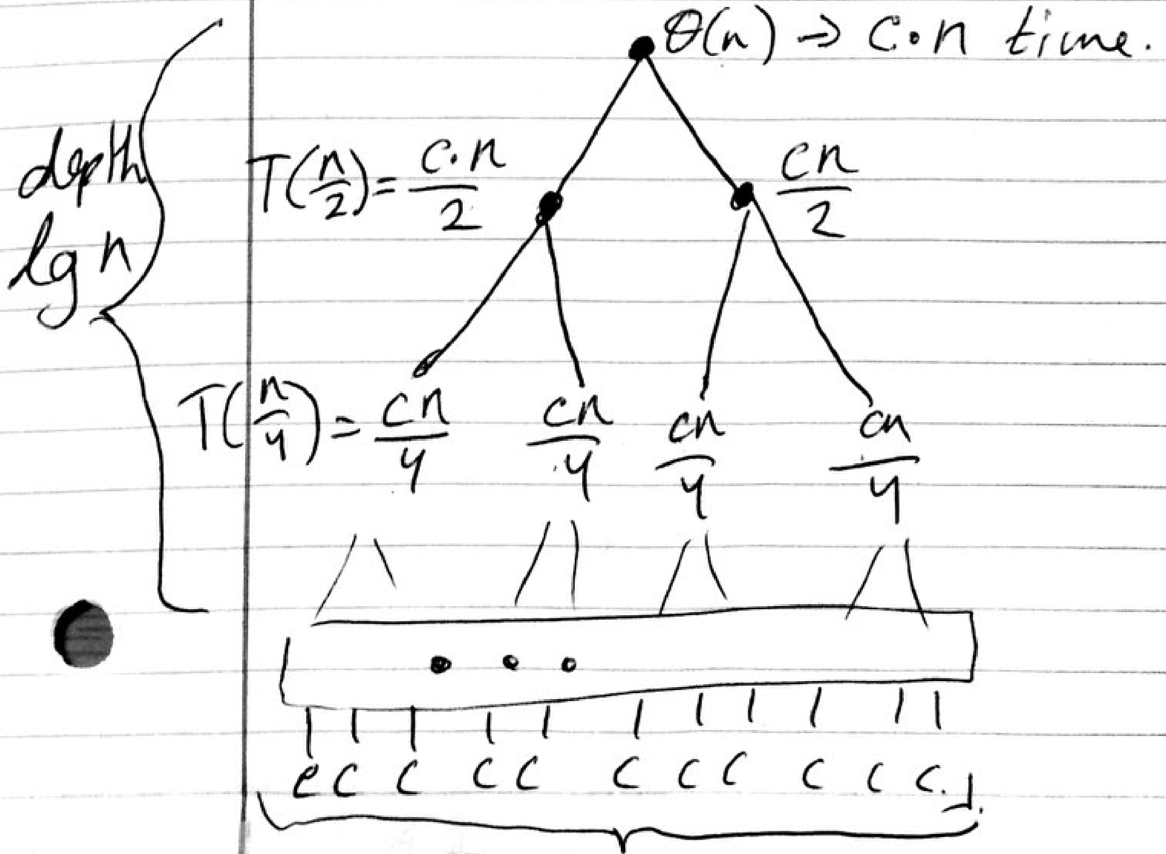
Solving Recurrence relations

$T(n) = 2T(\frac{n}{2}) + \Theta(n) = 2T(\frac{n}{2}) + cn$ for some c

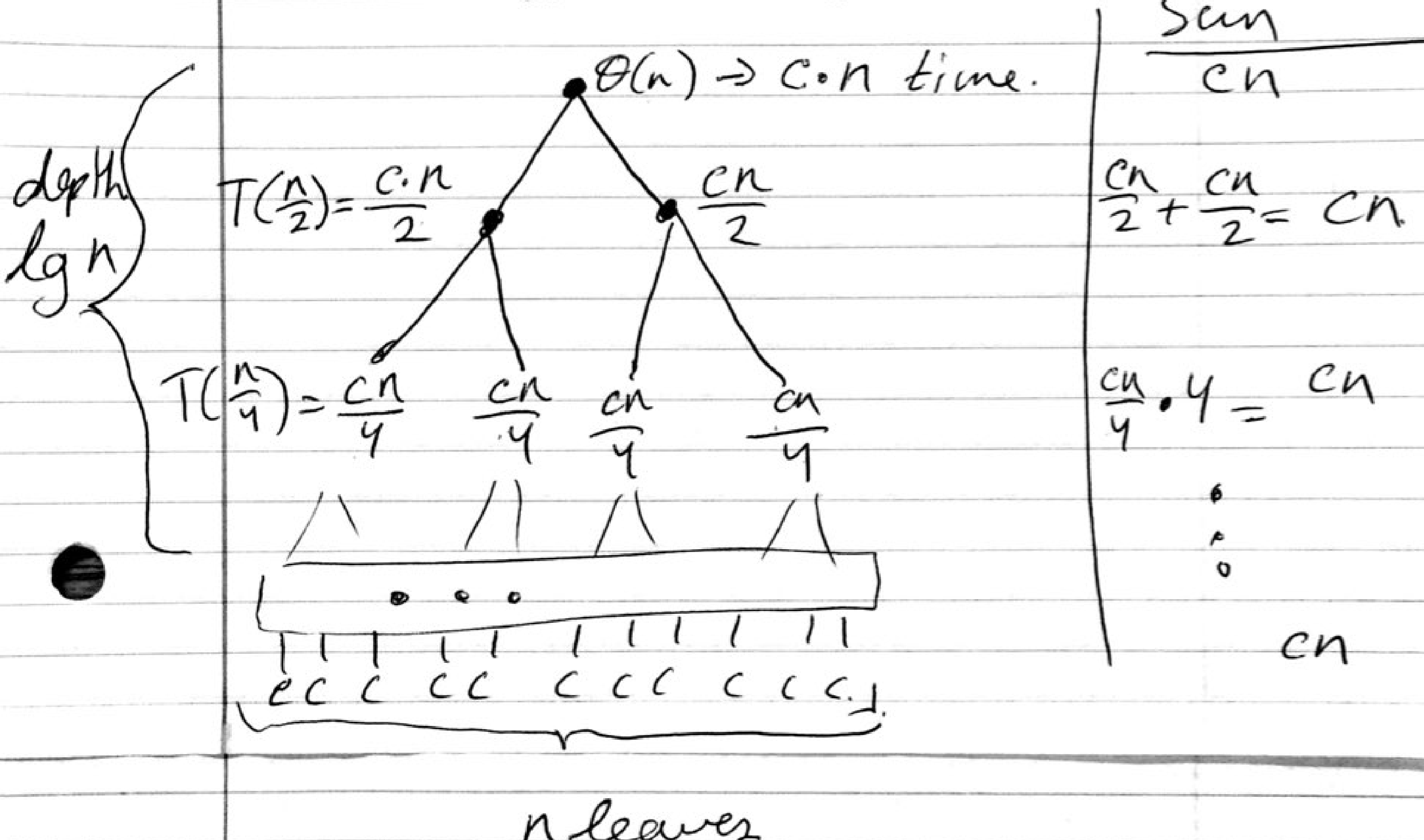
run time for inputs of size n

How to solve this

1st Step: Guess a solution using a recursion tree.



1st Step: Guess a solution using a recursion tree.



Each level of tree takes cn time.
there are $\lg n$ levels.
Total time = $\boxed{c \cdot n \cdot \lg n}$ ← This is our guess

Lets verify that $T(n) = \Theta(n \lg n)$.

Need to show two things.

$\underline{T(n) = O(n \lg n)}$ and $\underline{T(n) = \Omega(n \lg n)}$.

$T(n) = \underline{O(n \lg n)}$ ← lets prove this!

$T(n) \le d \cdot n \lg n$ for some constant d

We use induction on inputsize with IH

equal → $\bigcirc\!\!\!=$

$T(n) = 2\underline{T(\frac{n}{2})} + cn$

IH implies $T(\frac{n}{2}) \le d \cdot \frac{n}{2} \lg \frac{n}{2}$

$T(n) \le 2\left[\frac{dn}{2} \lg \frac{n}{2}\right] + cn.$

$\le dn \lg n$

need to show $2\left[\frac{dn}{2} \lg \frac{n}{2}\right] + cn = \underline{dn \lg \frac{n}{2}} + cn$

$\lg n - \lg 2$
$= \lg n - 1$

$= \underbrace{dn \lg n - dn + cn.}_{\text{residual should be } \le 0}$

$-dn + cn \le 0$

$cn \le dn$

$c \le d$

d cannot depend on n
but can depend on c.

Lets show $T(n) = \Omega(n \lg n)$.

Equivalently, need to give a constant $d > 0$

s.t $\forall n$, $T(n) \geq dn\lg n$

$T(n) = 2T(\frac{n}{2}) + cn$

by IH. $\geq 2\left[d\frac{n}{2} \lg \frac{n}{2}\right] + cn$

$= dn\lg\frac{n}{2} + cn = dn\lg n \underbrace{-dn + cn}_{\text{residual should be } \geq 0}$

need to show $\geq dn \lg n$

$-dn + cn \geq 0$

$cn \geq dn$

$c \geq d$. ✓

Lets try to prove $T(n) = \Omega(n^3)$ and lets see what goes wrong.

IH: $T(n) \geq dn^3$

$T(n) = 2T(\frac{n}{2}) + cn$

$= 2\left(d\left(\frac{n}{2}\right)^3\right) + cn = \frac{2dn^3}{8} + cn = \frac{dn^3}{4} + cn$

need to show $\geq dn^3$

$\frac{dn^3}{4} + cn = dn^3 \underbrace{- \frac{3n^3 d}{4} + cn}_{\text{residual should be } \geq 0}$

$\boxed{\frac{dn^3}{4} = dn^3 - \frac{3dn^3}{4}}$

$$-\frac{3}{4}n^3d + cn \geq 0$$

$$cn \geq \frac{3}{4}n^3d$$

$$c \geq \frac{3n^2}{4}d.$$

$$d \leq \frac{4c}{3n^2} \quad (\forall n)$$

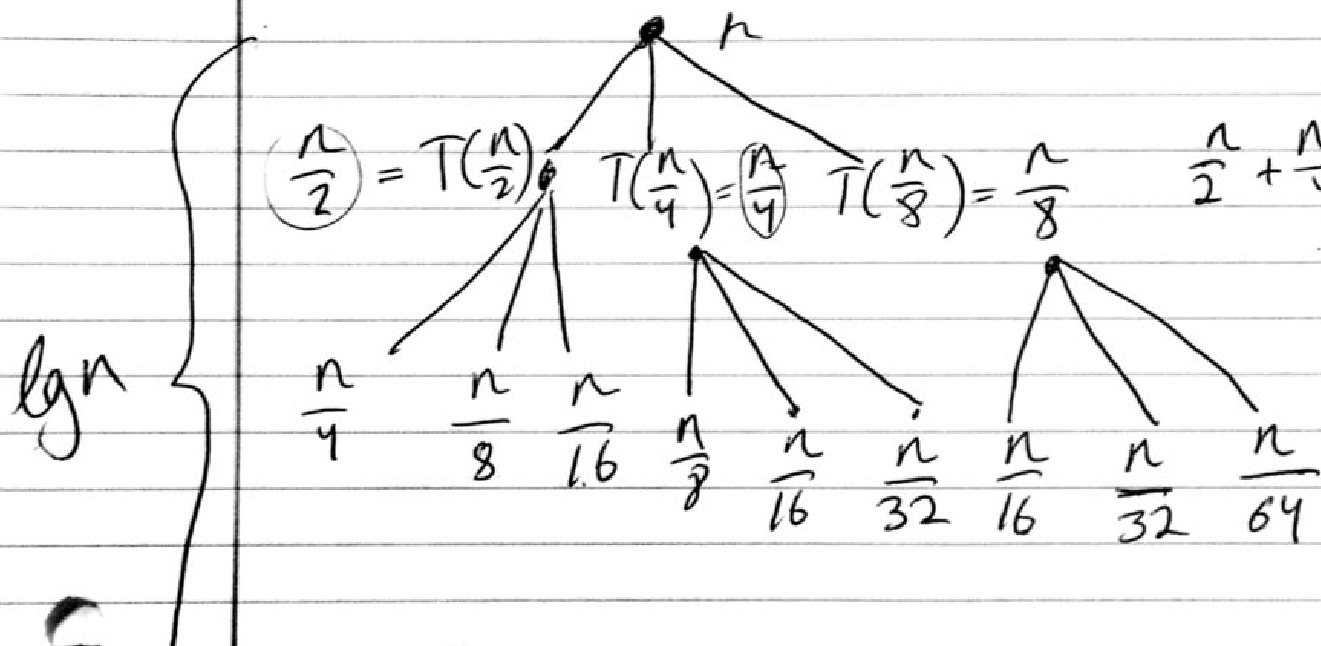
Remember - d cannot depend on n

Such a value of d cannot exist, therefore, we have <u>not</u> proven.

$$T(n) = \Omega(n^3)$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + T\left(\frac{n}{8}\right) + n$$

Recursion Tree for a guess



$$\frac{n}{2} + \frac{n}{4} + \frac{n}{8} = \frac{7}{8}n$$

$$= \frac{49}{64}n = \left(\frac{7}{8}\right)^2 n.$$