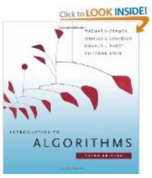
Data Structures and Algorithms *CMPSC* 465



LECTURE 4, 8/31

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Some of these slides are based on slides of S. Raskhodnikova, A. Smith, K. Wayne, E. Demaine and C. Leiserson

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- Transpose Symmetry:
  - $f(n) = O(g(n))$ *if and only if* $g(n) = \Omega(f(n))$
  - $f(n) = o(g(n))$ *if and only if* $g(n) = \omega(f(n))$
- If $f(n) = o(g(n))$ *then* $f(n) = O(g(n))$
- If $f(n) = \omega(g(n))$ *then* $f(n) = \Omega(g(n))$
- Transitivity of Θ: (same holds for O, Ω, o, ω)
  - If $f(n) = \Theta(g(n))$ *and* $g(n) = \Theta(h(n))$ *then* $f(n) = \Theta(h(n)))$
- Symmetry: $f(n) = \Theta(g(n))$ *if and only if* $g(n) = \Theta(f(n))$
- Additivity for Θ (same holds for O, Ω, o, ω):
  - $f(n) = \Theta(g(n))$ *and* $g(n) = \Theta(h(n))$, *then* $f(n) + g(n) = \Theta(h(n))$
- Multiplication: If $f(n) = \Theta(h_1(n))$ *and* $g(n) = \Theta(h_2(n))$ *then* $f(n) \cdot g(n) = \Theta(h_1(n) \cdot h_2(n))$
  - Same holds for O, Ω, o, ω

Summary

Notation	… means …	Think…	E.g.
$f(n)=O(n)$	$\exists c > 0, n_0 > 0, \forall n \geq n_0$: $0 \leq f(n) \leq cg(n)$	Upper bound "≤"	$100n^2 = O(n^3)$
$f(n)=\Omega(g(n))$	$\exists c > 0, n_0 > 0, \forall n \geq n_0$: $0 \leq cg(n) \leq f(n)$	Lower bound "≥"	$n^{100} = \Omega(2^n)$
$f(n)=\Theta(g(n))$	$f(n) \in \Omega(g(n))$ **and** $f(n) \in O(g(n))$	Tight bound "="	$\log(n!) = \Theta(n \log n)$
$f(n)=o(g(n))$	$\forall c > 0, \exists n_0 > 0, \forall n \geq n_0$: $0 \leq f(n) < cg(n)$	"<"	$n^2 = o(2^n)$
$f(n)=\omega(g(n))$	$\forall c > 0, \exists n_0 > 0, \forall n \geq n_0$: $0 \leq cg(n) < f(n)$	">"	$n^2 = \omega(\log n)$

Useful identities for logarithms

- For all x>0 and a,b >1:

1. $x = a^{\log_a(x)}$
2. $\log_a(b) = \dfrac{\log_2(b)}{\log_2(a)}$
3. $a^{\log_b(x)} = x^{\log_b(a)}$

Examples:

$$2^{\log_2(n)} = n$$

$$2^{\log_3(n)} = n^{\log_3(2)} \approx n^{0.63}$$

$$9^{\log_3(n)} = n^2$$

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Exercise: Sort by asymptotic order

1. $\sqrt{n}$
2. $n \log n$
3. $n^{1/\log n}$
4. n^2
5. $2^{\log n}$
6. $2^{\log_3 n}$
7. $2^{n/100}$
8. $\log(n!)$
9. $n\sqrt{n}$

Post questions to Piazza, instructors will answer