Midterm solutions and grading key on Angel.

Regrading policy

Longest Common Subsequence Problem.

Given ~~to~~ two sequences

$$X = \langle x_1, \dots, x_m \rangle$$

$$Y = \langle y_1, \dots, y_n \rangle.$$

Find a subsequence common to both whose length is greatest

subsequence: its a subset of the elements in the same order, not necessarily consecutive.

Ex: X = johny.

$\langle j, h, y \rangle$ is a subsequence of X.

$\langle j, o, h, n, y \rangle$ is a subsequence of X

Ex: X = johny.

$\langle j, h, o \rangle$ is <u>NOT</u> a subseq. of X

X = springtime

Y = pioneer

Ex common subseq. is $\langle p, i \rangle$

$\langle p, i, ne \rangle$ is the longest common subseq.
(LCS)

X = abcde   abcde

Y = aebcd.   aebcd

X contains $O(2^m)$ subsequences

Brute Force: Try every possible subseq. of X and check if its also a subseq of Y.
Take the longest one as your answer.

X_i = prefix of length i of X, i.e.

$\langle x_1, \dots, x_i \rangle$.

Y_i = prefix of length i of Y ...

spring

pion

Define $c(i,j)$ = length of LCS of X_i and Y_j

same as OPT before

two parameters instead of 1

The solution to the original problem is $c(m,n)$

$c(i,j)$ where $i<m$, and $j<n$ is a smaller subproblem.

The new info you have is x_m and y_n.

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$X = \boxed{a\ b\ c\ a}\ \boxed{d}$

$Y = \boxed{a\ c\ b\ a}\ \boxed{d}$

If $x_m = y_n$, then claim: Let $Z = \langle z_1, \dots, z_k \rangle$ be the LCS, then $z_k = x_m = y_n$ Why? we can always add x_n to the end of any solution and it can only help Claim: $\cancel{Z}\langle z_1, \dots, z_{k-1} \rangle$ is an LCS of X_{i-1}, Y_{j-1}

$X = $ a b c a ~~d~~

$Y = $ d c b a ~~e~~

Claim: if $z_k \neq x_m$, then Z is an LCS of X_{m-1} and Y

a b c a d

a c b a ~~d~~

Claim: if $z_k \neq y_n$, then Z is an LCS of X, and Y_{n-1}

$$c(i,j) = \begin{cases} 0 & \text{if } i=0 \text{ or } j=0 \\ 1 + c(i-1, j-1) & \text{if } i>0, j>0, \text{ and } x_i = y_j \\ \max\begin{pmatrix} c(i-1,j) \\ c(i,j-1) \end{pmatrix} & \text{if } i>0, j>0 \text{ and } x_i \neq y_j \end{cases}$$