Introduction to Deep Learning

Review of Linear Regression

Example

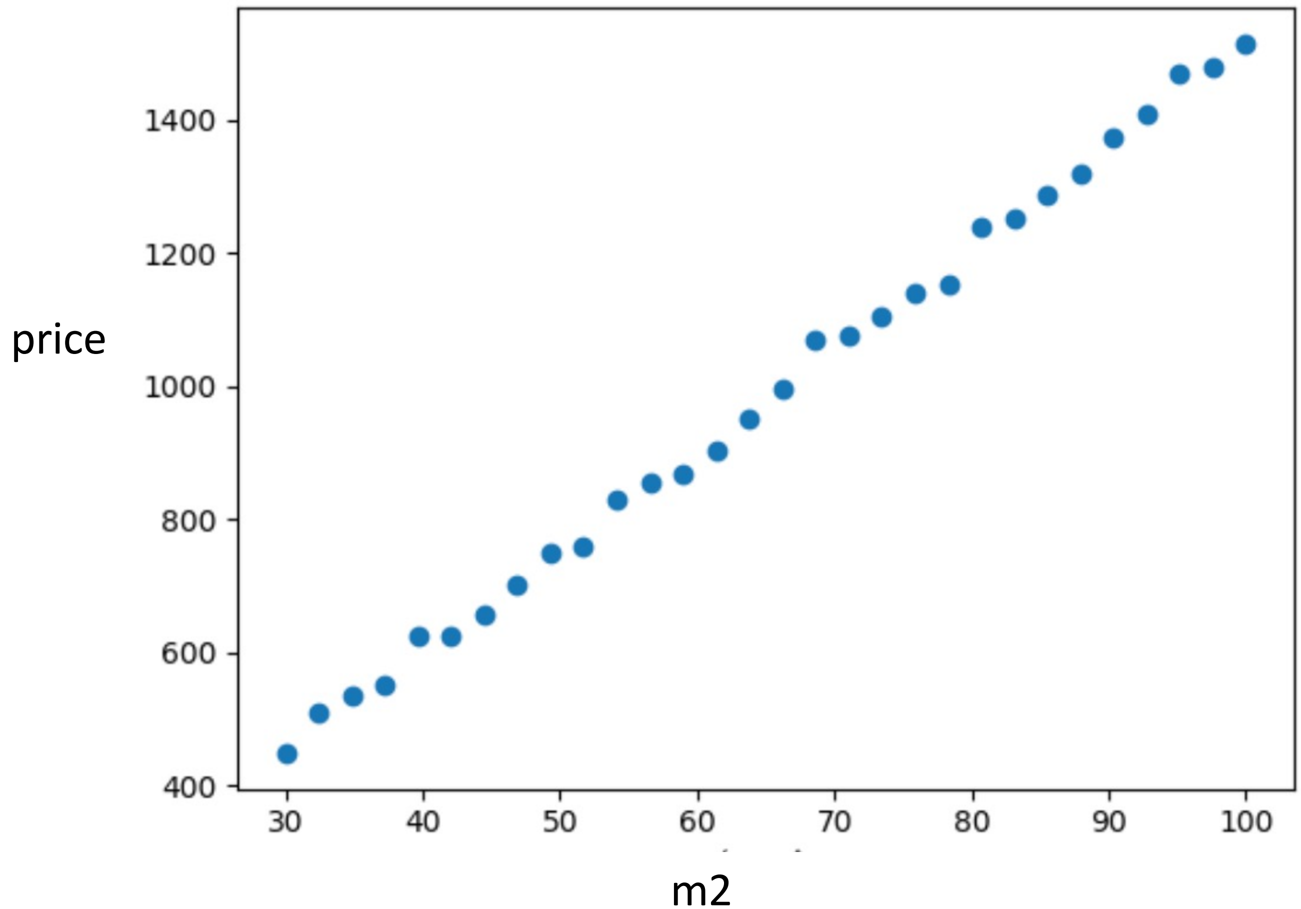
- Problem: House selling price prediction
- Input: Having data about areas and selling prices of 30 houses as in table 1 below:

Area (m2)	Selling price (Vnese million)
30	448.524
32.4138	509.248
34.8276	535.104
37.2414	551.432
39.6552	623.418
…	…

Table 1: Dataset for house selling price prediction

Visualization of table 1 data

Relationship between house selling price and house area



Example (next)

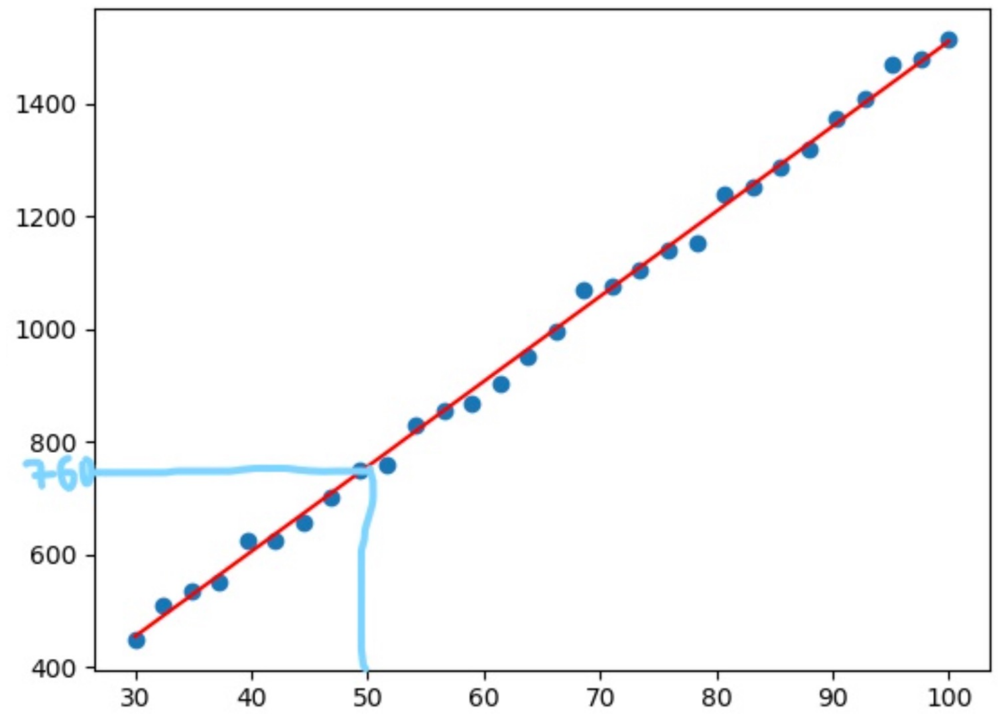
- Requirement: estimate the selling price of a 50 square meter
- Output: estimated price?

Solution: Idea

Solution: Draw a line closest to the data points and calculate the house price at 50

Estimated price of 50-m2 house

price



m2

Solution: Programming

- Step 1- Training: Find the line closest to the data points (called ***model***)

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- Step 2 - Prediction: Predict how much a 50-m2 house will cost based on the trained model

Formulating model

- Model formula: $y = w_1 \ * x + w_0$

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  → Linear Model

Formulating model

- Model formula: $y = w_1 \ * x + w_0$

  → Linear Model
- Problem becomes: find w_1, w_0
- Represent input data points as:

  {(x_i,y_i), i = 1...30}

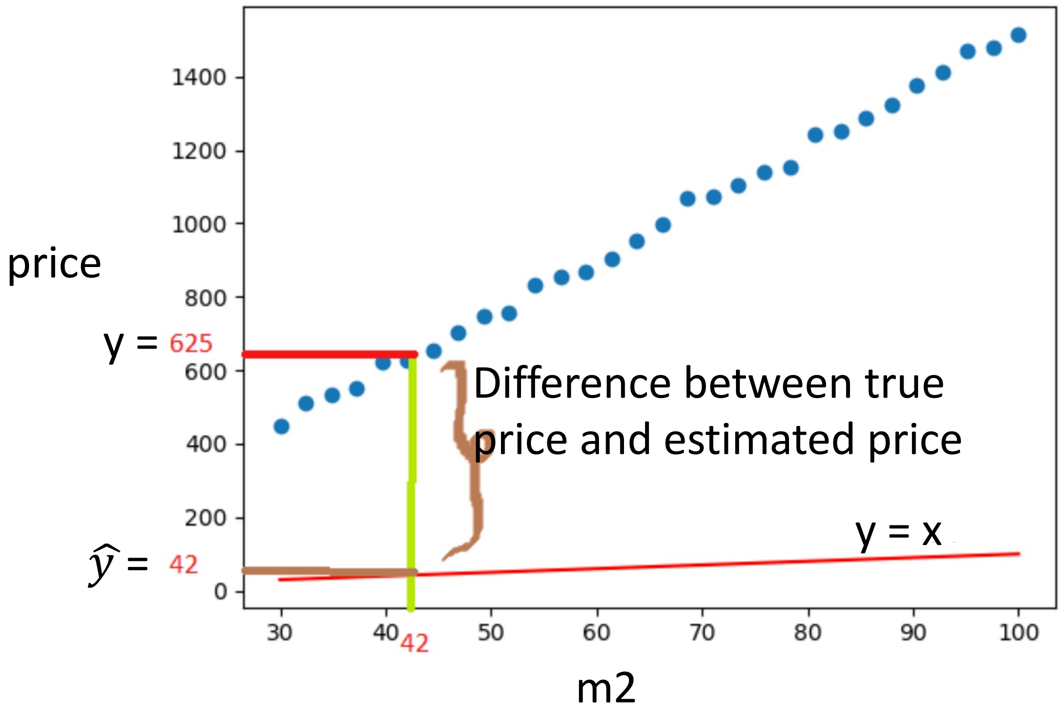
  In which: y_i = $w_1 \ * x_i + w_0$
- Represent estimated data point as:

  $\widehat{y}_i$ = $w_1 \ * xi + w_0$

Model training

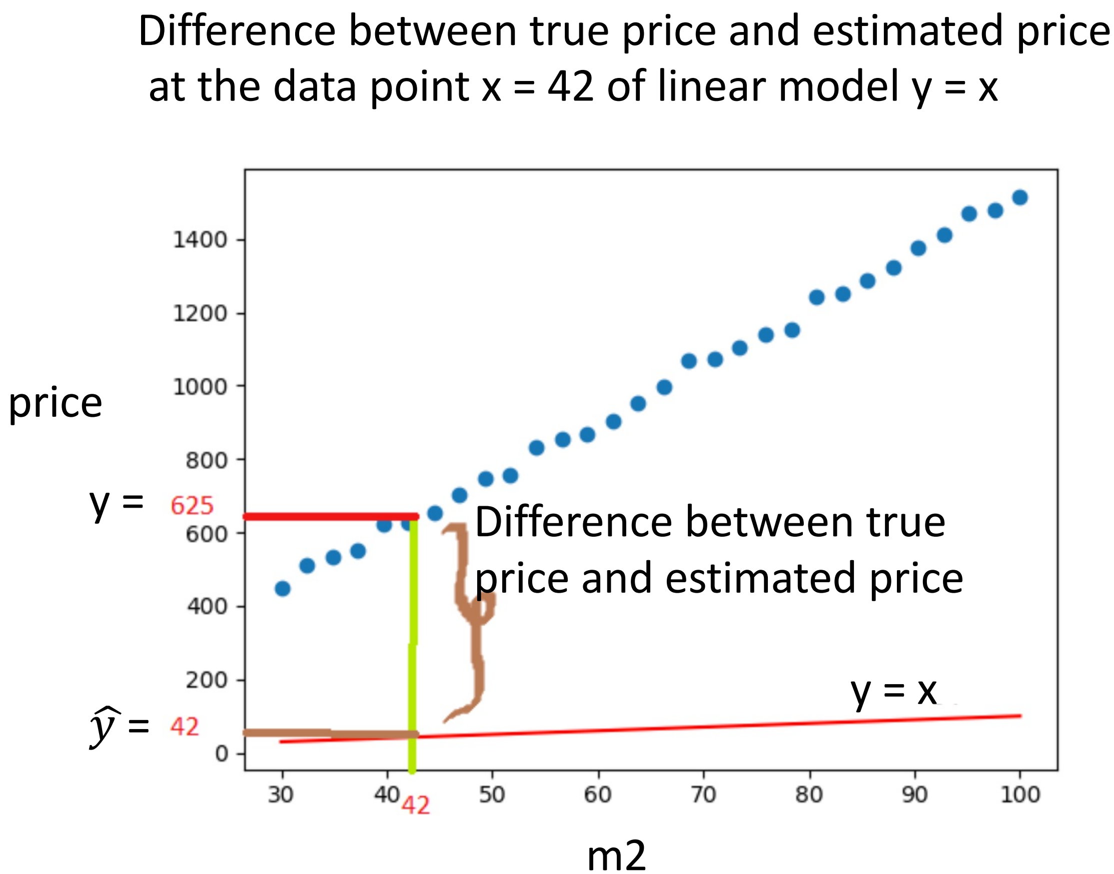
- Random initial data point: $w_1 = 1, w_0 = 0$ → Model becomes: y = x
- Model fine-tuning

Difference between true price and estimated price
at the data point x = 42 of linear model y = x



Model training

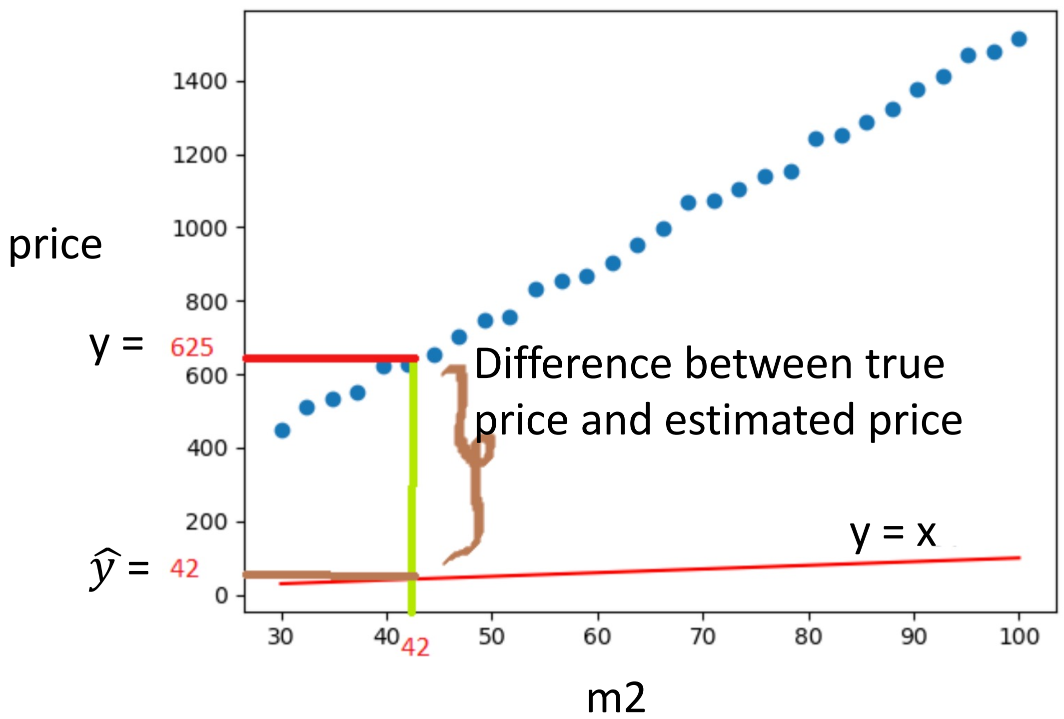
- Problem: estimated price is too far from true price. For example, at the point x = 42



Model training

- Need a metric to evaluate the linear model with parameter set: $(w_0,w_1) = (0,1)$

Difference between true price and estimated price at the data point x = 42 of linear model y = x



Loss Function

- For each data point (x_i, y_i), the difference between the actual price and the predicted price:

$$\frac{1}{2} * (\hat{y}_i - y_i)^2$$

- The difference across the entire data set as the sum of the differences of each data point:

$$J = \frac{1}{2} * \frac{1}{N} * \left(\sum_{i=1}^{N} (\hat{y}_i - y_i)^2\right)$$

Where N is number of data points

Loss Function

$$J = \frac{1}{2} * \frac{1}{N} * \left(\sum_{i=1}^{N} (\hat{y}_i - y_i)^2\right)$$

- J >= 0
- The smaller J is, the model is more close to the actual data points
- If J = 0 then the model passes through all data points

→ J is called the loss function

Loss Function

$$J = \frac{1}{2} * \frac{1}{N} * \left(\sum_{i=1}^{N} (\widehat{y}_i - y_i)^2\right)$$

- The problem transfers from: finding the linear model $y = w_1 * x + w_0$ cloest to the data points
- → to: finding the parameter (w_0,w_1) such that J obtains the minimum value

→ Use ***Gradient descent*** algorithm to find minimum value of J

Gradient Descent Algorithm

- Idea: use derivative to find the minimum value of a function f(x)
- Algorithm:

(1) Random initialization: $x = x_0$

(2) Assign: $x = x - learning\_rate * f'(x)$

(3) Re-compute f(x). Stop if f(x) is small enough, or repeat step (2) if not

Gradient Descent Algorithm

Note:

- learning_rate is non-negative constant
- step 2 will be repeated until a large enough number of times or f(x) is small enough

Gradient Descent: example

Problem: Find minimum value of $f(x) = x^2$ using gradient descent algorithm

Gradient Descent: example

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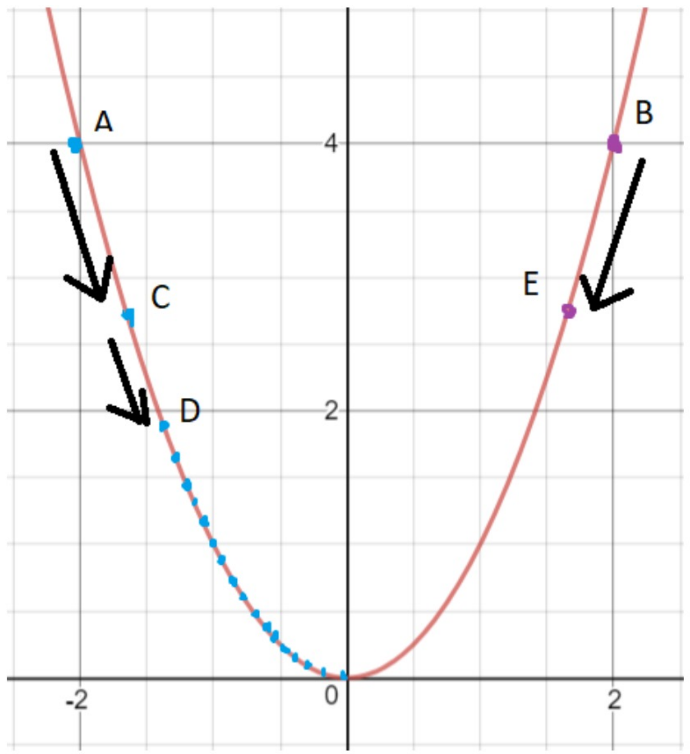
- Solution 1: use Linear Algebra
- Solution 2: use gradient descent algorithm

Gradient Descent: example

Step 1: Random initialization x= -2 (Point A)
Step 2: compute f'(x)
    then $x = x_A - \text{learning\_rate} * f'(x_A)$
Step 3: compute f(x) → still big → move to point C, and repeat Step 2



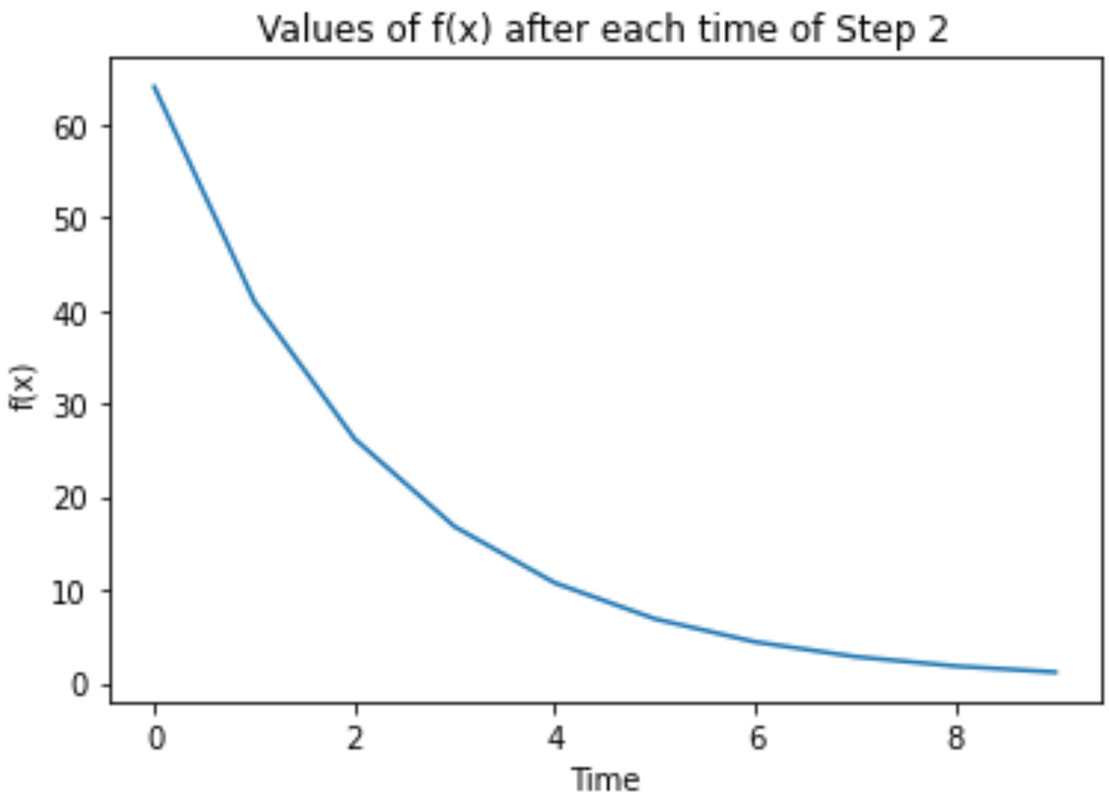
Gradient Descent: example

In detail: if we choose initial value: x = 10, learning_rate = 0.1, then the values of step 2 and step 3 will be as in the following table:

Time	**x**	**f(x)**
1	8.00	64.00
2	6.40	40.96
3	5.12	26.21
4	4.10	16.78
5	3.28	10.74
6	2.62	6.87
7	2.10	4.40
8	1.68	2.81
9	1.34	1.80
10	1.07	1.15

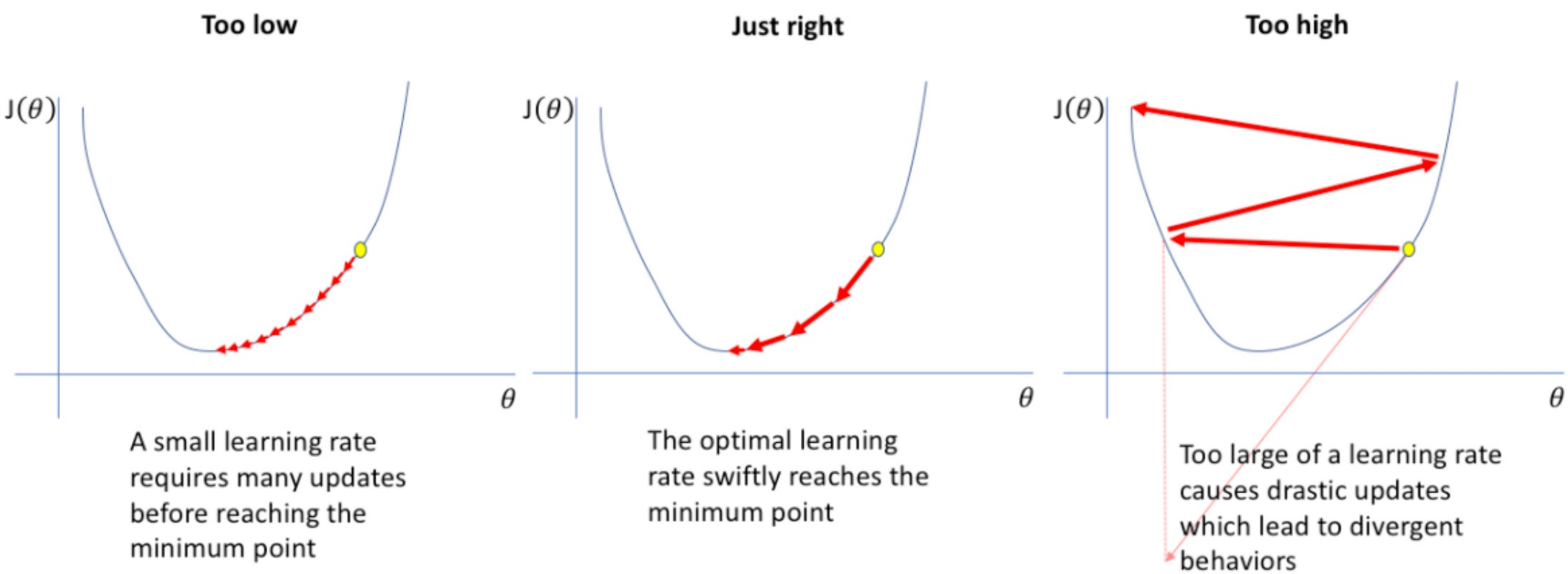
Table 2: values of f(x) after 10 times of step 2 calculation

Gradient Descent: example



Visualization of Table 2

Effect of Learning Rate Selection

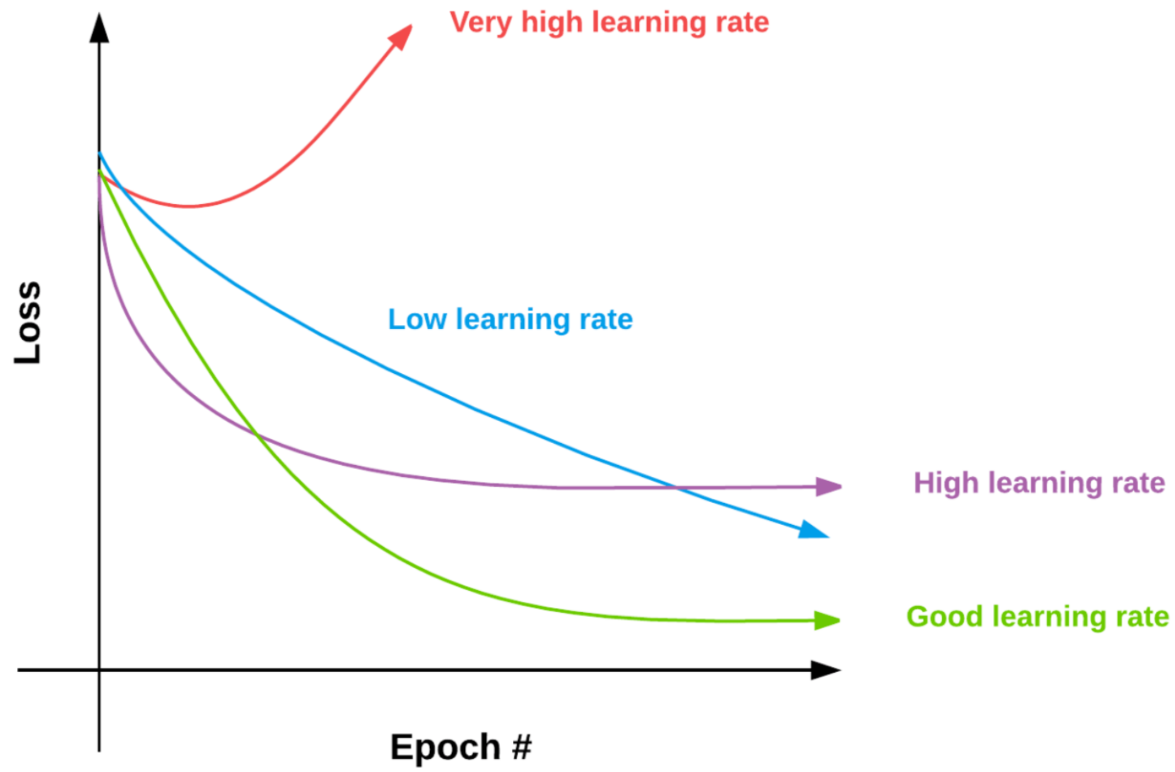


Many times to calculate step 2

Proper times to calculate step 2

Overshoot problem

Effect of Learning Rate Selection



Epoch: Number of times of step 2, Loss: the function to find the minimum value