



N-gram Language Models

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Agenda

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- Introduction to N-grams
- Estimating N-gram probabilities
- Evaluating language model
- Generalization and zeros
- Smoothing techniques:
 - Add-one (Laplace) smoothing
 - Interpolation, Backoff



Suggested readings

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- Chapter 3. Language Modeling with N-Grams (SLP)
 - <https://web.stanford.edu/~jurafsky/slp3/3.pdf>
- Language Models, by Michael Collins.
 - <http://www.cs.columbia.edu/~mcollins/lm-spring2013.pdf>
- NLP Programming Tutorials, by Graham Neubig
 - <http://www.phontron.com/slides/nlp-programming-en-01-unigramlm.pdf>
 - <http://www.phontron.com/slides/nlp-programming-en-02-bigramlm.pdf>



The language modeling problem

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- Goal: compute the probability of a sentence or sequence of words.

$$P(W) = P(w_1, w_2, \dots, w_n)$$

- E.g., $P(\text{Hôm nay trời đẹp quá}) = P(\text{Hôm, nay, trời, đẹp, quá})$

- Related task: probability of an upcoming word:

$$P(w_4 | w_1, w_2, w_3)$$

- E.g., $P(\text{đẹp} | \text{Hôm, nay, trời})$

- A model that computes either of these: $P(W)$ or $P(w_n | w_1, w_2, \dots, w_{n-1})$ is called a **language model**.



Why language models?

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■ Machine Translation:

□ $P(\text{high winds tonite}) > P(\text{large winds tonite})$

■ Spell Correction

□ The office is about fifteen **minuets** from my house

■ $P(\text{about fifteen minutes from}) > P(\text{about fifteen minuets from})$

■ Speech Recognition

□ $P(\text{I saw a van}) \gg P(\text{eyes awe of an})$



How to compute $P(W)$

- How to compute this joint probability:
 - $P(\text{its, water, is, so, transparent, that})$
- Intuition: let's rely on the Chain Rule of Probability



The Chain Rule

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- Conditional probabilities

$$P(B|A) = P(A, B)/P(A)$$

- Rewriting:

$$P(A, B) = P(A)P(B|A)$$



The Chain Rule cont.

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- More variables:

$$P(A, B, C, D) = ?$$

$$\begin{aligned} P(A, B, C, D) &= P(A, B, C)P(D|A, B, C) \\ &= P(A, B)P(C|A, B)P(D|A, B, C) \\ &= P(A)P(B|A)P(C|A, B)P(D|A, B, C) \end{aligned}$$



The Chain Rule cont.

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- The Chain Rule in general

$$\begin{aligned} &P(x_1, x_2, x_3, \dots, x_n) \\ &= P(x_1)P(x_2|x_1)P(x_3|x_1, x_2) \dots P(x_n|x_1, \dots, x_{n-1}) \end{aligned}$$



Applying the Chain Rule for joint probability

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$$P(w_1 w_2 \dots w_n) = \prod_i P(w_i | w_1 w_2 \dots w_{i-1})$$

$P(\text{"its water is so transparent"}) =$

$P(\text{its}) \times P(\text{water} | \text{its}) \times P(\text{is} | \text{its water})$

$\times P(\text{so} | \text{its water is}) \times P(\text{transparent} | \text{its water is so})$



How to estimate these probabilities

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- Could we just count and divide?

$$P(\text{the l its water is so transparent that}) = \frac{\textit{Count}(\text{its water is so transparent that the})}{\textit{Count}(\text{its water is so transparent that})}$$

- No! Too many possible sentences!
- We'll never see enough data for estimating these



Markov Assumption

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- Simplifying assumption:

$$P(\text{the}|\text{its water is so transparent that}) \approx P(\text{the}|\text{that})$$

- Or maybe:

$$\begin{aligned} &P(\text{the}|\text{its water is so transparent that}) \\ &\approx P(\text{the}|\text{transparent that}) \end{aligned}$$



Markov Assumption

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- We approximate each component in the product

$$P(w_i | w_1 w_2 \dots w_{i-1}) \approx P(w_i | w_{i-k} \dots w_{i-1})$$

- So the joint probability of the sequence is

$$P(w_1 w_2 \dots w_n) \approx \prod_i P(w_i | w_{i-k} \dots w_{i-1})$$



Simplest case: Unigram model

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$$P(w_1 w_2 \dots w_n) \approx \prod_i P(w_i)$$

Some automatically generated sentences from a unigram model

fifth, an, of, futures, the, an, incorporated, a,
a, the, inflation, most, dollars, quarter, in, is,
mass

thrift, did, eighty, said, hard, 'm, july, bullish

that, or, limited, the



Bigram model

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- Condition on the previous word:

$$P(w_i | w_1 w_2 \dots w_{i-1}) \approx P(w_i | w_{i-1})$$

texaco, rose, one, in, this, issue, is, pursuing, growth, in,
a, boiler, house, said, mr., gurria, mexico, 's, motion,
control, proposal, without, permission, from, five, hundred,
fifty, five, yen

outside, new, car, parking, lot, of, the, agreement, reached

this, would, be, a, record, november



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Unigram language model

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- Do not use history

$$P(w_1 w_2 \dots w_n) = \prod_i P(w_i)$$

- Estimate $P(w_i)$ by using Maximum Likelihood Estimate (MLE)

$$P(w_i) = \frac{\text{count}(w_i)}{\sum_{w'} \text{count}(w')}$$



Unigram language model: an example

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i live in osaka . </s>

$$P(\text{nara}) = 1/20 = 0.05$$

i am a graduate student . </s>

$$P(i) = 2/20 = 0.1$$

my school is in nara . </s>

$$P(</s>) = 3/20 = 0.15$$

$P(W=i \text{ live in nara . } </s>) =$

$$0.1 * 0.05 * 0.1 * 0.05 * 0.15 * 0.15 = 5.625 * 10^{-7}$$



Bigram language model

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- Condition on the previous word:

$$P(w_i | w_1 w_2 \dots w_{i-1}) \approx P(w_i | w_{i-1})$$

- The Maximum Likelihood Estimate

$$P(w_i | w_{i-1}) = \frac{\text{count}(w_{i-1}, w_i)}{\text{count}(w_{i-1})}$$



Bigram language model: an example

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$$P(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

$$P(I | < s >) = \frac{2}{3} = .67$$

$$P(\text{Sam} | < s >) = \frac{1}{3} = .33$$

$$P(\text{am} | I) = \frac{2}{3} = .67$$

$$P(</s> | \text{Sam}) = \frac{1}{2} = .5$$

$$P(\text{Sam} | \text{am}) = \frac{1}{2} = .5$$

$$P(\text{do} | I) = \frac{1}{3} = .33$$



More examples:

Berkeley Restaurant Project sentences

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- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i'm looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day



Raw bigram counts

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- Out of 9222 sentences

| | i | want | to | eat | chinese | food | lunch | spend |
|---------|----|------|-----|-----|---------|------|-------|-------|
| i | 5 | 827 | 0 | 9 | 0 | 0 | 0 | 2 |
| want | 2 | 0 | 608 | 1 | 6 | 6 | 5 | 1 |
| to | 2 | 0 | 4 | 686 | 2 | 0 | 6 | 211 |
| eat | 0 | 0 | 2 | 0 | 16 | 2 | 42 | 0 |
| chinese | 1 | 0 | 0 | 0 | 0 | 82 | 1 | 0 |
| food | 15 | 0 | 15 | 0 | 1 | 4 | 0 | 0 |
| lunch | 2 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| spend | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |



Raw bigram probabilities

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■ Normalize by unigrams:

| i | want | to | eat | chinese | food | lunch | spend |
|------|------|------|-----|---------|------|-------|-------|
| 2533 | 927 | 2417 | 746 | 158 | 1093 | 341 | 278 |

■ Results:

| | i | want | to | eat | chinese | food | lunch | spend |
|---------|---------|------|--------|--------|---------|--------|--------|---------|
| i | 0.002 | 0.33 | 0 | 0.0036 | 0 | 0 | 0 | 0.00079 |
| want | 0.0022 | 0 | 0.66 | 0.0011 | 0.0065 | 0.0065 | 0.0054 | 0.0011 |
| to | 0.00083 | 0 | 0.0017 | 0.28 | 0.00083 | 0 | 0.0025 | 0.087 |
| eat | 0 | 0 | 0.0027 | 0 | 0.021 | 0.0027 | 0.056 | 0 |
| chinese | 0.0063 | 0 | 0 | 0 | 0 | 0.52 | 0.0063 | 0 |
| food | 0.014 | 0 | 0.014 | 0 | 0.00092 | 0.0037 | 0 | 0 |
| lunch | 0.0059 | 0 | 0 | 0 | 0 | 0.0029 | 0 | 0 |
| spend | 0.0036 | 0 | 0.0036 | 0 | 0 | 0 | 0 | 0 |



Bigram estimates of sentence probabilities

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$P(<s> \text{ I want english food } </s>) =$

$P(\text{I} | <s>)$

$\times P(\text{want} | \text{I})$

$\times P(\text{english} | \text{want})$

$\times P(\text{food} | \text{english})$

$\times P(</s> | \text{food})$

$= .000031$



What kinds of knowledge?

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- $P(\text{english} | \text{want}) = .0011$
- $P(\text{chinese} | \text{want}) = .0065$
- $P(\text{to} | \text{want}) = .66$
- $P(\text{eat} | \text{to}) = .28$
- $P(\text{food} | \text{to}) = 0$
- $P(\text{want} | \text{spend}) = 0$
- $P(i | \langle s \rangle) = .25$



Practical Issues

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- We do everything in log space
 - Avoid underflow
 - (also adding is faster than multiplying)

$$P(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$



Language Modeling Toolkits

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- SRILM

- <http://www.speech.sri.com/projects/srilm/>

- KenLM

- <https://kheafield.com/code/kenlm/>



Google N-Gram Release, August 2006

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AUG

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All Our N-gram are Belong to You

Posted by Alex Franz and Thorsten Brants, Google Machine Translation Team

Here at Google Research we have been using word [n-gram models](#) for a variety of R&D projects,

...

That's why we decided to share this enormous dataset with everyone. We processed 1,024,908,267,229 words of running text and are publishing the counts for all 1,176,470,663 five-word sequences that appear at least 40 times. There are 13,588,391 unique words, after discarding words that appear less than 200 times.



Google N-Gram Release

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- serve as the incoming 92
- serve as the incubator 99
- serve as the independent 794
- serve as the index 223
- serve as the indication 72
- serve as the indicator 120
- serve as the indicators 45
- serve as the indispensable 111
- serve as the indispensable 40
- serve as the individual 234

<http://googleresearch.blogspot.com/2006/08/all-our-n-gram-are-belong-to-you.html>



Google Book N-grams

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- <http://storage.googleapis.com/books/ngrams/books/datasetv2.html>



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Evaluation: How good is our model?

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- Does our language model prefer good sentences to bad ones?
 - Assign higher probability to “real” or “frequently observed” sentences
 - than “ungrammatical” or “rarely observed” sentences



Evaluation: How good is our model?

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- We train parameters of our model on a training set.
- We test the model's performance on data we haven't seen
 - A test set is an unseen dataset that is different from our training set, totally unused.
 - An evaluation metric tells us how well our model does on the test set.



Training on the test set

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- We can't allow test sentences into the training set
- We will assign it an artificially high probability when we set it in the test set
- “Training on the test set”
- Bad science!



Two evaluation approaches

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- Extrinsic evaluation
 - Compare two language models in downstream tasks
 - e.g., Spelling correction, speech recognition, MT

- Intrinsic evaluation
 - Use some evaluation measures on the test set
 - We will use **perplexity**



Extrinsic evaluation of N-gram models

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- Best evaluation for comparing models A and B
 - Put each model in a task
 - spelling corrector, speech recognizer, MT system
 - Run the task, get an accuracy for A and for B
 - How many misspelled words corrected properly
 - How many words translated correctly
 - Compare accuracy for A and B



Difficulty of extrinsic evaluation of N-gram models

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- Extrinsic evaluation

- Time-consuming; can take days or weeks

- So

- Sometimes use **intrinsic** evaluation: **perplexity**
 - Bad approximation
 - unless the test data looks **just** like the training data
 - So **generally only useful in pilot experiments**
 - But is helpful to think about.



Intuition of Perplexity

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■ The Shannon Game:

- How well can we predict the next word?

I always order pizza with cheese and _____

The 33rd President of the US was _____

I saw a _____

- Unigrams are terrible at this game. (Why?)

mushrooms 0.1

pepperoni 0.1

anchovies 0.01

....

fried rice 0.0001

....

and 1e-100

■ A better model of a text

- is one which assigns a higher probability to the word that actually occurs



Likelihood

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- Likelihood is the probability of some observed data (the test set W_{test}), given the model M

$$P(W_{\text{test}}|M) = \prod_{w \in W_{\text{test}}} P(\mathbf{w}|M)$$



Log Likelihood

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- Likelihood uses very small numbers=underflow
- Taking the log resolves this problem

$$\log P(W_{test}|M) = \sum_{\mathbf{w} \in W_{test}} \log(\mathbf{w}|M)$$



Entropy

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- Entropy H is average negative \log_2 likelihood per word

$$H(W_{test}|M) = \frac{1}{|W_{test}|} \sum_{\mathbf{w} \in W_{test}} -\log_2 P(\mathbf{w}|M)$$

- Perplexity is equal to two to the power of per-word entropy

$$\text{PPL} = 2^H$$

- Lower perplexity = better model



Lower perplexity = better model

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- Training 38 million words, test 1.5 million words, WSJ

| N-gram Order | Unigram | Bigram | Trigram |
|--------------|---------|--------|---------|
| Perplexity | 962 | 170 | 109 |



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Zero probabilities

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■ Training set:

- ... denied the allegations
- ... denied the reports
- ... denied the claims
- ... denied the request

■ Test set

- ... denied the offer
- ... denied the loan

$$P(\text{"offer"} \mid \text{denied the}) = 0$$



Problems with Zero probabilities

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- We underestimate the probability of all sorts of words that might occur
- The entire probability of the test set is 0.
 - So, we cannot calculate perplexity



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Basic idea of smoothing (discounting) techniques

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- When we have sparse statistics:

$P(w \mid \text{denied the})$

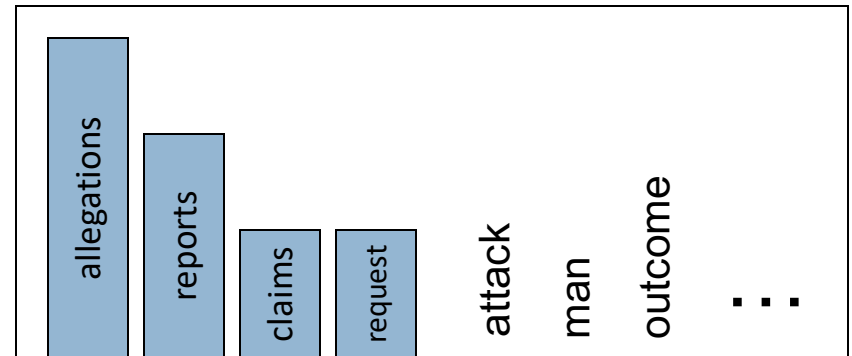
3 allegations

2 reports

1 claims

1 request

7 total



- Steal probability mass to generalize better

$P(w \mid \text{denied the})$

2.5 allegations

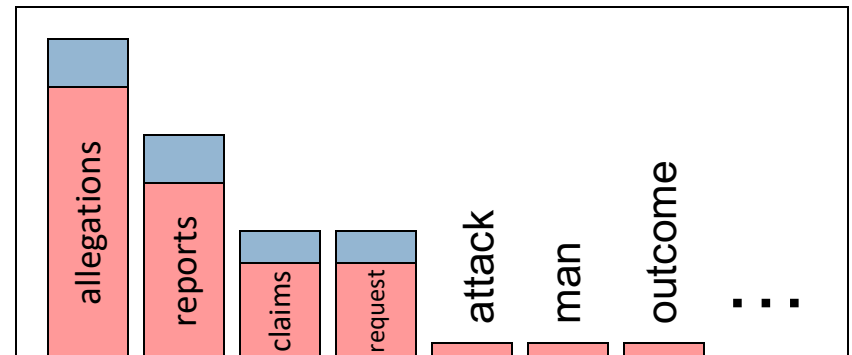
1.5 reports

0.5 claims

0.5 request

2 other

7 total





Laplace smoothing

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- Add one to all the counts
- Pretend we saw each word one more time than we did

- MLE unigram probabilities:

$$P_{ML}(w_i) = \frac{c(w_i)}{N}$$

- Add-1 estimate:

$$P_{\text{Laplace}} = \frac{c(w_i) + 1}{\sum_w (c(w) + 1)} = \frac{c(w_i) + 1}{N + V}$$



Laplace smoothing: unigram model

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i live in osaka . </s>

i am a graduate student . </s>

my school is in nara . </s>

$$P(\text{nara}) = 1/20 = 0.05$$

$$P(i) = 2/20 = 0.1$$

$$P(</s>) = 3/20 = 0.15$$

$$P(\text{kyoto}) = 0/20 = 0$$

Vocab = {i, live, in, osaka, am, graduate, student, my, school, is, nara, </s>}

$$V = 12$$



Laplace smoothing: unigram model

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i live in osaka . </s>

i am a graduate student . </s>

my school is in nara . </s>

Vocab = {i, live, in, osaka, am, gradudate, student, my, school, is, nara, </s>}

$V = 12$

$$P(\text{nara}) = (1+1)/(20+12) = 0.0625$$

$$P(i) = (2+1)/(20+12) = 0.09375$$

$$P(</s>) = (3+1)/(20+12) = 0.125$$

$$P(\text{kyoto}) = (0+1)/(20+12) = 0.03125$$



Laplace smoothing: bigrams

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- MLE estimate:

$$P_{ML}(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Add-1 estimate:

$$P_{\text{Laplace}} = \frac{c(w_{i-1}, w_i) + 1}{\sum_w (c(w_{i-1}w) + 1)} = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$



Berkeley Restaurant Corpus: Laplace smoothed bigram counts

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| | i | want | to | eat | chinese | food | lunch | spend |
|---------|----|------|-----|-----|---------|------|-------|-------|
| i | 6 | 828 | 1 | 10 | 1 | 1 | 1 | 3 |
| want | 3 | 1 | 609 | 2 | 7 | 7 | 6 | 2 |
| to | 3 | 1 | 5 | 687 | 3 | 1 | 7 | 212 |
| eat | 1 | 1 | 3 | 1 | 17 | 3 | 43 | 1 |
| chinese | 2 | 1 | 1 | 1 | 1 | 83 | 2 | 1 |
| food | 16 | 1 | 16 | 1 | 2 | 5 | 1 | 1 |
| lunch | 3 | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| spend | 2 | 1 | 2 | 1 | 1 | 1 | 1 | 1 |



Laplace-smoothed bigrams

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$$P^*(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

| | i | want | to | eat | chinese | food | lunch | spend |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| i | 0.0015 | 0.21 | 0.00025 | 0.0025 | 0.00025 | 0.00025 | 0.00025 | 0.00075 |
| want | 0.0013 | 0.00042 | 0.26 | 0.00084 | 0.0029 | 0.0029 | 0.0025 | 0.00084 |
| to | 0.00078 | 0.00026 | 0.0013 | 0.18 | 0.00078 | 0.00026 | 0.0018 | 0.055 |
| eat | 0.00046 | 0.00046 | 0.0014 | 0.00046 | 0.0078 | 0.0014 | 0.02 | 0.00046 |
| chinese | 0.0012 | 0.00062 | 0.00062 | 0.00062 | 0.00062 | 0.052 | 0.0012 | 0.00062 |
| food | 0.0063 | 0.00039 | 0.0063 | 0.00039 | 0.00079 | 0.002 | 0.00039 | 0.00039 |
| lunch | 0.0017 | 0.00056 | 0.00056 | 0.00056 | 0.00056 | 0.0011 | 0.00056 | 0.00056 |
| spend | 0.0012 | 0.00058 | 0.0012 | 0.00058 | 0.00058 | 0.00058 | 0.00058 | 0.00058 |



Reconstituted counts

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$$c^*(w_{i-1}w_i) = \frac{[c(w_{i-1}w_i) + 1] \times c(w_{i-1})}{c(w_{i-1}) + V}$$

| | i | want | to | eat | chinese | food | lunch | spend |
|---------|------|-------|-------|-------|---------|------|-------|-------|
| i | 3.8 | 527 | 0.64 | 6.4 | 0.64 | 0.64 | 0.64 | 1.9 |
| want | 1.2 | 0.39 | 238 | 0.78 | 2.7 | 2.7 | 2.3 | 0.78 |
| to | 1.9 | 0.63 | 3.1 | 430 | 1.9 | 0.63 | 4.4 | 133 |
| eat | 0.34 | 0.34 | 1 | 0.34 | 5.8 | 1 | 15 | 0.34 |
| chinese | 0.2 | 0.098 | 0.098 | 0.098 | 0.098 | 8.2 | 0.2 | 0.098 |
| food | 6.9 | 0.43 | 6.9 | 0.43 | 0.86 | 2.2 | 0.43 | 0.43 |
| lunch | 0.57 | 0.19 | 0.19 | 0.19 | 0.19 | 0.38 | 0.19 | 0.19 |
| spend | 0.32 | 0.16 | 0.32 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 |



Compare with raw bigram counts

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| | i | want | to | eat | chinese | food | lunch | spend |
|---------|----|------|-----|-----|---------|------|-------|-------|
| i | 5 | 827 | 0 | 9 | 0 | 0 | 0 | 2 |
| want | 2 | 0 | 608 | 1 | 6 | 6 | 5 | 1 |
| to | 2 | 0 | 4 | 686 | 2 | 0 | 6 | 211 |
| eat | 0 | 0 | 2 | 0 | 16 | 2 | 42 | 0 |
| chinese | 1 | 0 | 0 | 0 | 0 | 82 | 1 | 0 |
| food | 15 | 0 | 15 | 0 | 1 | 4 | 0 | 0 |
| lunch | 2 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| spend | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |

| | i | want | to | eat | chinese | food | lunch | spend |
|---------|------|-------|-------|-------|---------|------|-------|-------|
| i | 3.8 | 527 | 0.64 | 6.4 | 0.64 | 0.64 | 0.64 | 1.9 |
| want | 1.2 | 0.39 | 238 | 0.78 | 2.7 | 2.7 | 2.3 | 0.78 |
| to | 1.9 | 0.63 | 3.1 | 430 | 1.9 | 0.63 | 4.4 | 133 |
| eat | 0.34 | 0.34 | 1 | 0.34 | 5.8 | 1 | 15 | 0.34 |
| chinese | 0.2 | 0.098 | 0.098 | 0.098 | 0.098 | 8.2 | 0.2 | 0.098 |
| food | 6.9 | 0.43 | 6.9 | 0.43 | 0.86 | 2.2 | 0.43 | 0.43 |
| lunch | 0.57 | 0.19 | 0.19 | 0.19 | 0.19 | 0.38 | 0.19 | 0.19 |
| spend | 0.32 | 0.16 | 0.32 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 |



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Linear interpolation

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- Mix trigrams, bigrams and unigrams

$$\begin{aligned} P(w_i | w_{i-2}, w_{i-1}) \\ = \lambda_1 \times P_{ML}(w_i | w_{i-2}, w_{i-1}) + \lambda_2 \times P_{ML}(w_i | w_{i-1}) \\ + \lambda_3 \times P_{ML}(w_i) \end{aligned}$$

where $\lambda_1 + \lambda_2 + \lambda_3 = 1$, and $\lambda_i \geq 0$ for all i

- Sometimes, all trigrams, bigrams, unigrams do not exist

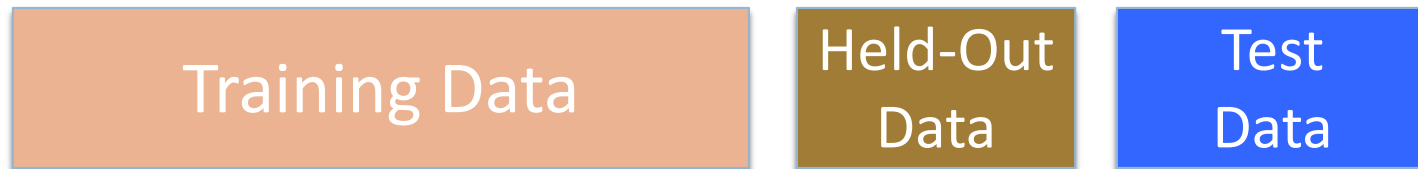
□ Recall: $P(w_i) = \lambda \times P_{ML}(w_i) + (1 - \lambda) \times \frac{1}{N}$



How to set lambdas?

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- Use a **held-out** corpus



- Choose λ s to maximize the probability of held-out data:
 - Fix the N-gram probabilities (on the training data)
 - Then search for λ s that give largest probability to held-out set:

$$\log P(w_1 \dots w_n \mid M(\lambda_1 \dots \lambda_k)) = \sum_i \log P_{M(\lambda_1 \dots \lambda_k)}(w_i \mid w_{i-1})$$



Example: bigrams

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i live in osaka . </s>

i am a graduate student . </s>

my school is in nara . </s>

■ Maximum-likelihood estimation:

☐ $P(\text{osaka} \mid \text{in}) = c(\text{in osaka})/c(\text{in}) = 1/2 = 0.5$

☐ $P(\text{nara} \mid \text{in}) = c(\text{in nara})/c(\text{in}) = 1/2 = 0.5$

☐ $P(\text{school} \mid \text{in}) = c(\text{in school})/c(\text{in}) = 0/2 = 0$



Example: interpolation

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i live in osaka . </s>

i am a graduate student . </s>

my school is in nara . </s>

■ Using interpolation

$$\square P(\text{school} \mid \text{in}) = \lambda_2 P_{ML}(\text{school} \mid \text{in}) + (1 - \lambda_2) P(\text{school})$$

$$\begin{aligned} \square P(\text{school}) &= \lambda_1 P_{ML}(\text{school}) + (1 - \lambda_1) \frac{1}{N} \\ &= \lambda_1 \times \frac{1}{20} + (1 - \lambda_1) \times \frac{1}{N} \end{aligned}$$



Unknown words: open vs. closed vocabulary

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- If we know all the words in advanced
 - ☐ Vocabulary V is fixed
 - ☐ Closed vocabulary task

- Often we don't know this
 - ☐ Out Of Vocabulary = OOV words
 - ☐ Open vocabulary task



Unknown words: open vs. closed vocabulary

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- Instead: create an unknown word token <UNK>
 - Training of <UNK> probabilities
 - Create a fixed lexicon L of size V
 - At text normalization phase, any training word not in L changed to <UNK>
 - Now we train its probabilities like a normal word
- At decoding time
 - If text input: Use UNK probabilities for any word not in training



Advanced smoothing methods

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- Kneser-Ney Smoothing
- Good-Turing Smoothing
- Katz's back-off