Lab 7
CALCULUS FOR IT 501031

1 Exercises

Exercise 1: Find the specific function values:

(a) $f(x,y) = x^2 + xy^3$ at $f(0,0), f(-1,1), f(2,3), f(-3,-2)$

(b) $f(x,y,z) = \dfrac{x-y}{y^2+z^2}$ at $f(3,-1,2), f(1,1/2,1/4), f(0,-1/3,0), f(2,2,100)$.

Exercise 2: Plot the graph of the functions

(a) $f(x,y) = (cosx)(cosy)e^{-(\sqrt{x^2+y^2})/4}$

(b) $f(x,y) = -\dfrac{xy^2}{x^2+y^2}$

(c) $f(x,y) = \dfrac{xy(x^2-y^2)}{x^2+y^2}$

(d) $f(x,y) = y^2 - y^4 - x^2$

Exercise 3: Find the first-order partial derivatives of the function $f(x,y)$, Then plot the function $f(x,y)$ and the first order partial derivative of $f(x,y)$ with regard to x, and y, respectively.

(a) $f(x,y) = 2x^2 - 3y - 4$

(b) $f(x,y) = (x^2-1)(y+2)$

(c) $f(x,y) = x^2 - xy + y^2$

(d) $f(x,y) = 5xy - 7x^2 - y^2 + 3x - 6y + 2$

(e) $f(x,y) = (xy-1)^2$

(f) $f(x,y) = (2x-3y)^3$

(g) $f(x,y) = \sqrt{x^2+y^2}$

(h) $f(x,y) = (x^3 + \dfrac{y}{2})^{\frac{2}{3}}$

(i) $f(x,y) = \dfrac{1}{x+y}$

(j) $f(x,y) = \dfrac{x}{x^2+y^2}$

(k) $f(x,y) = \dfrac{x+y}{xy-1}$

(l) $f(x,y) = tan^{-1}(\dfrac{y}{x})$

(m) $f(x,y) = e^{x+y+1}$

(n) $f(x,y) = e^{-x}sin(x+y)$

(o) $f(x,y) = ln(x+y)$

Exercise 4: Find all the second-order partial derivatives of the function $f(x,y)$. Then plot the function $f(x,y)$ and the second order partial derivative of $f(x,y)$ with regard to x, and y, respectively.

(a) $f(x,y) = x + y + xy$

(b) $f(x,y) = sin(xy)$

(c) $f(x,y) = x^2y + cosy + ysinx$

(d) $f(x,y) = xe^y + y + 1$

(e) $f(x,y) = ln(x+y)$

(f) $f(x,y) = tan^{-1}(\dfrac{y}{x})$

(g) $f(x,y) = x^2tan(xy)$

(h) $f(x,y) = ye^{x^2-y}$

(i) $f(x,y) = xsinx^2y$

(j) $f(x,y) = \dfrac{x-y}{x^2+y}$

Exercise 5: Verify that $f_{xy} = f_{yx}$ or not.

(a) $f(x,y) = xsiny + ysinx + xy$

(b) $f(x,y) = ln(2x+3y)$

(c) $f(x,y) = xy^2 + x^2y^3 + x^3y^4$

(d) $f(x,y) = e^x + xlny + ylnx$

Exercise 6: Find the fifth-order partial derivative $\dfrac{\partial^5 f}{\partial x^2 \partial y^3}$ of the function following:

(a) $f(x,y) = y^2x^4e^x + 2$

(b) $f(x,y) = y^4 + y(sinx - x^4)$

(c) $f(x,y) = x^5 + 5x^5y^5 + sinx + 7e^x$

(d) $f(x,y) = x^3 e^{\frac{y^4}{2}}$

Exercise 7: Express $\dfrac{dw}{dt}$ as a function of t, both by using the Chain Rule and by expressing w in terms of t and differentiating directly with respect to t. Then evaluate $\dfrac{dw}{dt}$ at the given value of t.

(a) $w = x^2 + y^2, x = cos(t), y = sin(t), t = \pi$

(b) $w = x^2 + y^2, x = cos(t) + sin(t), y = cos(t) - sin(t), t = 0$

(c) $w = \dfrac{x}{z} + \dfrac{y}{z}, x = cos^2(t), y = sin^2(t), z = \dfrac{1}{t}, t = 3$

(d) $w = 2ye^x - lnz, x = ln(t^2+1), y = tan^{-1}t, z = e^t, t = 1$

(e) $w = z - sinxy, x = t, y = ln(t), z = e^{t-1}, t = 1$

Exercise 8: Use the limit definition of partial derivative to compute the partial derivatives of the functions at the specified points

(a) $f(x,y) = 1 - x + y - 3x^2y, \dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}$ at $(1,2)$

(b) $f(x,y) = 4 + 2x - 3y - 3xy^2, \dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}$ at $(-2,1)$

Exercise 9: Let $f(x,y) = 2x + 3y - 4$. Find the slope of plane tangent to this surface at the point $(2,-1)$