

Lab 10

APPLIED LINEAR ALGEBRA FOR IT - 501032

1 Exercises

Exercise 1: Find the eigenvalues of

$$A = \begin{pmatrix} -1 & 3.5 & 14 \\ 0 & 5 & -26 \\ 0 & 0 & 2 \end{pmatrix}$$

and

$$B = \begin{pmatrix} -2 & 0 & 0 \\ 99 & 0 & 0 \\ 10 & -4.5 & 10 \end{pmatrix}$$

Find the eigenvalues of

(a)

$$A = \begin{pmatrix} 5 & 5 & 0 & 2 \\ 0 & 2 & -3 & 6 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

(c)

$$C = \begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ -5 & 1 & 0 & 0 & 0 \\ 3 & 8 & 0 & 0 & 0 \\ 0 & -7 & 2 & 1 & 0 \\ -4 & 1 & 9 & -2 & 3 \end{pmatrix}$$

(b)

$$B = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 6 & 2 & 0 & 0 \\ 0 & 3 & 6 & 0 \\ 2 & 3 & 3 & -5 \end{pmatrix}$$

Then, compute the determinant of A, B, C matrix based on the eigenvalues.

Exercise 2: Let

$$A = \begin{pmatrix} -6 & 28 & 21 \\ 4 & -15 & -12 \\ 8 & a & 25 \end{pmatrix}$$

For each a in the set $\{32, 31.9, 31.8, 32.1, 32.2\}$, compute the characteristic of A and the eigenvalues. In each case, create a graph of the characteristic polynomial $p(t) = \det(A - \lambda I)$ for $0 \leq t \leq 3$. If possible, construct all graphs on one coordinate system.

Exercise 3: Let $M = \begin{bmatrix} -3 & -5 & -7 \\ -2 & 1 & 0 \\ 1 & 5 & 5 \end{bmatrix}$

(a) Use any appropriate software to find the eigenvalues of M

- (b) For each eigenvalue λ found above, find the corresponding eigenvector \mathbf{v} of \mathbf{M} by using row reduction to solve $(M - \lambda I)v = 0$
- (c) Construct a matrix \mathbf{P} whose columns are the eigenvectors of \mathbf{M} . Compute the product $\mathbf{D} = \mathbf{P}^{-1}\mathbf{M}\mathbf{P}$ and confirm that \mathbf{D} is diagonal. Compute the determinants of \mathbf{D} and \mathbf{M} , then confirm that they are equal.
- (d) Construct another matrix \mathbf{Q} whose columns are also the eigenvectors of \mathbf{M} but this time place them in a different order than in \mathbf{P} (for example, the first of columns of the eigenvectors of \mathbf{M} is the second column of \mathbf{Q}) and then compute $\mathbf{Q}^{-1}\mathbf{M}\mathbf{Q}$ again. What has changed?

Exercise 4: Let $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

Use any appropriate software to find the eigenvalues and corresponding eigenvectors of A , A^T and A^{-1} . What do you observe ?

Exercise 5: Let $A_1 = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$, $A_3 = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, $A_4 = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$

$$A_5 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

Use any appropriate software to verify the matrices above are diagonalizable or not.

Exercise 6: Find the eigenvectors of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ Using it, construct a matrix \mathbf{P} that diagonalizes \mathbf{A} . Compute $\mathbf{P}^{-1}\mathbf{A}\mathbf{P}$

Exercise 7: Find the singular values of the matrices

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}, A_2 = \begin{bmatrix} -5 & 0 \\ 0 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} \sqrt{6} & 1 \\ 0 & \sqrt{6} \end{bmatrix}, A_4 = \begin{bmatrix} \sqrt{3} & 2 \\ 0 & \sqrt{3} \end{bmatrix}$$

Exercise 8: Compute SVD of each matrix below.

$$B_1 = \begin{bmatrix} -18 & 13 & -4 & 4 \\ 2 & 19 & -4 & 12 \\ -14 & 11 & -12 & 8 \\ -2 & 21 & 4 & 8 \end{bmatrix}, B_2 = \begin{bmatrix} 6 & -8 & -4 & 5 & -4 \\ 2 & 7 & -5 & -6 & 4 \\ 0 & -1 & -8 & 2 & 2 \\ -1 & -2 & 4 & 4 & -8 \end{bmatrix}$$

Exercise 9: Write a program to compress image which input image is given by user.

Exercise 10: Write a program to implement Recommender System. It will be able recommend a item x to user y . Let $M_{n \times m}$ matrix present the relation of users and items

$$M_{n \times m} = \left(\begin{array}{c|ccc} & item1 & item2 & item2 \\ \hline user 1 & \infty & \infty & 3 \\ user 2 & 2 & \infty & 5 \\ user 3 & 5 & 6 & 7 \\ user 4 & \infty & \infty & \infty \end{array} \right)$$

where ∞ means that user y has never rated item x before.