Financial Econometrics Coursework, a.y. 2025-2026

1 Part A: Theoretical Analysis and Power Assessment

1.1 Background: The Sup Goldfeld-Quandt (Sup-GQ) Test

The Sup-GQ test is used to detect heteroscedasticity where the variance break point (τ) is unknown. It is defined as:

$$T_{\text{Sup-GQ}} = \sup_{\tau \in (\tau_{\min}, \tau_{\max})} \text{GQ}(\tau)$$

where $F_{\text{GQ}}(\tau)$ is the Goldfeld-Quandt F-statistic computed at the split point τ. Because the asymptotic distribution of $T_{\text{Sup-GQ}}$ is non-standard, critical values are typically found using a **Wild Bootstrap** procedure.

1.2 Task A.1: Proposing a Combined P-Value Alternative

The Sup-GQ test focuses only on the worst-case split. An alternative approach is to combine the information from all splits (τ) into a single test statistic, see for instance the approach of https://doi.org/10.1016/j.jcomm.2018.05.008

Let $\boldsymbol{\tau} = \{\tau_1, \tau_2, \ldots, \tau_M\}$ be the grid of M possible split points (excluding the first and last trim portions). For each τ_m, one can compute the standard asymptotic p-value, p_m, for the $F_{\text{GQ}}(\tau_m)$ test.

1. **Proposal:** Propose a combined test statistic $\mathcal{G}$ that uses the M individual p-values, $p_1, \ldots, p_M$, to test the joint null hypothesis of homoscedasticity across all split points.

2. **Justification:** Explain the theoretical motivation for your choice of combination method (e.g., why your method might offer better power against distributed, subtle changes in variance compared to the localized Sup-GQ test).

3. **Critical Value:** Explain why the asymptotic distribution of your combined statistic $\mathcal{G}$ will likely be non-standard (since the p_m values are not independent) and confirm that its critical values must also be found using a simulation or bootstrap technique.

1.3 Task A.2: Monte Carlo Power Comparison

Design and execute a Monte Carlo experiment to compare, at the 5% significance level, the finite-sample power of i) the Sup-GQ test; ii) your proposed combined p-value test ($\mathcal{G}$) and iii) existing tests for detecting heteroscedasticity .

1. **Test Implementation:**

   - Implement the **Wild Bootstrap procedure** to find the critical values for **both** $T_{\text{Sup-GQ}}$ and your combined statistic $\mathcal{G}$. Use $B = 499$ bootstrap replications for each Monte Carlo run.
   - Use a simple linear model: $y_t = 1 + x_t + \epsilon_t$, where $x_t \sim N(0, 1)$.

2. **Design of Experiment:**

- **Sample Size (N):** Use $N = 100$.
- **Monte Carlo Replications (R):** Use $R = 1000$ overall replications.
- **Trimming:** Set τ_{min} and τ_{max} to ensure a 15% trim on both ends.

3. **Power Calculation:**

- Define the Alternative (H_a): Set a structural break at $\tau_{\text{break}} = 0.5$.
- The error variance σ_t^2 should be defined as:

$$\epsilon_t \sim N(0, \sigma_t^2) \quad \text{where} \quad \sigma_t^2 = 1 + \delta \cdot I(t/N > 0.5)$$

where $I(\cdot)$ is the indicator function. The parameter δ controls the size of the break.

- Compute the empirical **Power** (rejection rate) for both tests (Sup-GQ and $\mathcal{G}$) for three values of δ: $\delta = 0$ (Size Check), $\delta = 1$, and $\delta = 3$.

4. **Report:** Report the empirical size ($\delta = 0$) and power ($\delta = 1, 3$) in a clear table and discuss which test performs better and why.

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2 Part B: Empirical Application - Quantile Transfer Entropy (QTE)

2.1 Background: Uncertainty and Financial Returns

You are tasked with investigating the non-linear, directional causal impact of economic uncertainty on financial returns using the Quantile Transfer Entropy (QTE) framework. QTE measures the information flow from one time series (Uncertainty, X) to another (Financial Returns, Y) at specific quantiles (τ).

1. **Data Selection:**

- Select one **Economic Uncertainty Index** (e.g., EPU from `https://www.policyuncertainty.com/`) and one major **Financial Market Index** (e.g., S&P 500, FTSE 100).
- Download daily or monthly data for both indices over the longest available common period (at least 5 years).
- Make sure you work with stationary series.

2.2 Task B.2: The QTE Framework

The QTE$_{\tau,k}$ statistic measures the information transfer from X_{t-k} to Y_t, specifically conditioned on the τ-th quantile of the Y process. It is based on comparing the conditional entropy of Y_t given its past, with the conditional entropy of Y_t given its past AND the past of X.

1. **The QTE Statistic (QTE$_{\tau,k}$):** You will calculate the Quantile Transfer Entropy QTE$_{\tau,k}$ statistic (the estimated information flow) for the selected parameters. Note that QTE$_{\tau,k} \geq 0$, and a value significantly greater than zero suggests information transfer from X to Y at quantile τ.

2. **Hypothesis Testing:** We test the null hypothesis of no causality (no information transfer) from X to Y at the τ-th quantile:

$$H_0 : \text{QTE}_{\tau,k} = 0 \quad \text{vs.} \quad H_a : \text{QTE}_{\tau,k} > 0$$

   The distribution of QTE$_{\tau,k}$ under H_0 is non-standard, making a bootstrap procedure necessary for significance testing.

2.3 Task B.3: Bootstrap Implementation and Analysis

You must use a suitable bootstrap method to obtain robust p-values for the QTE$_{\tau,k}$ statistic. Given the time-series nature of the data, a block bootstrap is required to maintain the temporal dependence structure.

1. **Bootstrap Choice:** Use the **Stationary Block Bootstrap** to resample your time series data. Justify your block size selection (l). Crucially, the bootstrap must be implemented to impose the **Null Hypothesis (H_0): X does not cause Y**. This is often done by permuting one series relative to the other or by resampling the joint blocks under the assumption of independence.

2. **Estimation:** Estimate the QTE QTE$_{\tau,k}$ for three different quantiles: $\tau = 0.10$ (low returns/losses), $\tau = 0.50$ (median returns), and $\tau = 0.90$ (high returns/gains).

3. **Fixed Parameter Test:** For the chosen lag k and the three fixed quantiles τ, implement the bootstrap procedure ($B = 499$ replications) to test the significance of QTE$_{\tau,k}$.

   - **Implementation Note:** You are only required to implement the bootstrap test for fixed τ and k. You are **not** required to implement a sup test over τ, k, or both.
   - **Important:** Generate **TWO** stationary bootstrap index samples (one for X and one for Y) to impose the null.

4. **Report:**

   - Present the estimated QTE, QTE$_{\tau,k}$ for $\tau \in \{0.10, 0.50, 0.90\}$ and for reasonable values of k in a table.
   - Report the robust, bootstrap-derived p-values for H_0 : QTE$_{\tau,k} = 0$.
   - Discuss your findings: Does high uncertainty significantly transfer information to different parts of the return distribution? If so, where is the information transfer strongest (e.g., predicting extreme losses or gains)?

Submission Instructions

1. Submit by email your work 3 days (at the latest) before the exam date you choose.
2. The final submission must be a PDF document containing all theoretical explanations, model specifications, tables, and discussion.
3. Include all R code used for both Part A and Part B in an Appendix or as a separate script file. Code must be clearly commented.

Appendices

A Quantile Transfer Entropy

The rationale for the Quantile Transfer Entropy (QTE) method is rooted in extending Granger Causality and Information Theory to specific quantiles of the conditional distribution.

The QTE measures the extent to which the past values of one or more series, $(x_{1,t-k}, \ldots, x_{n,t-k})$, improve the prediction of the current value of the τ-quantile of the y series, $Q_\tau(y_t)$.

The core rationale is based on comparing the prediction error from two quantile regression models both with y_t as response variable. The first of the two quantile regressions is the null or restricted model that predicts the τ-quantile of y_t using its own lag y_{t-k} only:

$$Q_\tau(y_t|y_{t-k}) = \alpha_0 + \alpha_1 y_{t-k}.$$

The alternative or unrestricted model predicts the τ-quantile of y_t using its own lag y_{t-k} and the lag of all the predictors $x_1, x_2, \ldots, x_n$:

$$Q_\tau(y_t|y_{t-k}, x_{1,t-k}, \ldots, x_{n,t-k}) = \beta_0 + \beta_1 y_{t-k} + \sum_{j=1}^{n} \beta_{j+1} x_{j,t-k}.$$

Denote by $\hat{u}_{1,t}$ and $\hat{u}_{2,t}$ the time-t residual of the restricted and unrestricted model, respectively. Let the check function for the tau-quantile be denoted by:

$$\rho_\tau(u) = u \cdot (\tau - I(u < 0))$$

where u is the residual and $I(\cdot)$ is the indicator function. The total quantile loss, i.e. the quantity that quantile regression is designed to minimize, is the sum of the check function applied to the residuals. The QTE is calculated as the log-ratio of the total quantile losses:

$$\text{QTE} = \log\left(\sum_t \rho_\tau(\hat{u}_{1,t})\right) - \log\left(\sum_t \rho_\tau(\hat{u}_{2,t})\right) = \log\left(\frac{\sum_t \rho_\tau(\hat{u}_{1,t})}{\sum_t \rho_\tau(\hat{u}_{2,t})}\right).$$

Same as in Transfer Entropy, The QTE is the log of the ratio of the uncertainty (loss) in the restricted model to the uncertainty (loss) in the unrestricted model. If past lags of $x_1, x_2, \ldots, x_n$ provide no predictive information for y_t's τth quantile, the ratio is ≈ 1, and QTE ≈ 0. If instead past lags of $x_1, x_2, \ldots, x_n$ have some predictive power for τ-quantile of y_t, the denominator, $\sum_t \rho_\tau(\hat{u}_{2,t})$, is smaller than the numerator, $\sum_t \rho_\tau(\hat{u}_{1,t})$, and therefore the ratio is > 1, resulting in a positive QTE.

A.1 Testing QTE Significance via the Stationary (block) Bootstrap

These tests utilize the Centered/Pivotal Stationary Bootstrap (SB) method to test the null hypothesis of no causality (H_0 : QTE $= 0$) against the one-sided alternative of positive causality (H_1 : QTE > 0).

The bootstrap P-value is calculated as the proportion of centered bootstrap maximal statistics that are greater than or equal to the observed maximal statistic. Let $\widehat{\text{QTE}}(\tau, k)$ and $\widehat{\text{QTE}}^{*b}(\tau, k)$ be the observed QTE statistic for quantile τ and lag k, and the corresponding QTE statistic of the b-th SB replication, respectively.

To test the causality at one specific quantile and one specific lag, you should fix both k and τ and compute the centered bootstrap statistic as the difference between the bootstrap and the observed QTE:

$$\widehat{Z}^{*b} = \widehat{\text{QTE}}^{*b}(\tau, k) - \widehat{\text{QTE}}(\tau, k), \quad \text{for } b = 1, \dots, B$$

where B be the number of bootstrap replication.

The P-value is the probability that the error term Z^{*b} is as extreme as the observed QTE:

$$\text{P-value}(\tau, k) = \frac{1}{B} \sum_{b=1}^{B} I\left(\widehat{Z}^{*b} \geq \widehat{\text{QTE}}(\tau, k)\right)$$