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HOMEWORK 3

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Problem 1: Consider one-dimensional Kalman Filter without process noise to solve the following problem.
Assume that we would like to estimate the height of a building using an imprecise altimeter. We know that building height doesn't change over time, at least during the short measurement process. Thus, the dynamic model is given by

$$\hat{x}_{n+1,n} = \hat{x}_{n,n},$$
$$p_{n+1,n} = p_{n,n}.$$

The true building height is 50 meters. The altimeter measurement error (standard deviation) is 5 meters. The ten measurements are: 49.03m, 48.44m, 55.21m, 49.98m, 50.6m, 52.61m, 45.87m, 42.64m, 48.26m, 55.84m.
Given the initial guess as follows:

$$\hat{x}_{0,0} = 60m,$$
$$p_{0,0} = 225.$$

(a) Write a MATLAB program to find the state estimates and estimate uncertainties.

```
true_height = 50;                    % True building height (meters)
measurement_std_dev = 5;             % alimeter measurement error (standard deviation)
(meters)
measurements = [49.03, 48.44, 55.21, 49.98, 50.6, 52.61, 45.87, 42.64, 48.26, 55.84];

% Kalman Filter initialization with given initial guess
num_measurements = length(measurements);
x_est = 60;                          % Initial estimate of the building height
p_est = 225;                         % Initial uncertainty in the estimate

% Measurement noise (variance) from standard deviation
r = measurement_std_dev^2;

% Initialize arrays to store estimates
x_estimates = zeros(1, num_measurements);
uncertainties = zeros(1, num_measurements);

% Kalman Filter loop
for n = 1:num_measurements
   % Measurement updat
   Kn = p_est / (p_est + r);          % Calculate Kalman Gain
```

```
    x_est = x_est + Kn * (measurements(n) - x_est); % Update estimate with measurement
    p_est = (1 - Kn) * p_est;                  % Update uncertainty

    % results
    x_estimates(n) = x_est;
    uncertainties(n) = p_est;
end

% Display the results
disp('State estimates and estimate uncertainties after each measurement:');
disp(table((1:num_measurements)', measurements', x_estimates', uncertainties', ...
    'VariableNames', {'Measurement_No', 'Measurement', 'Estimate', 'Uncertainty'}));
```

Results in MATLAB:

```
Command Window
>> q_hw3
State estimates and uncertainties after each measurement:
    Measurement_No    Measurement    Estimate    Uncertainty
    ______________    ___________    ________    ___________

          1              49.03        50.127         22.5
          2              48.44        49.328       11.842
          3              55.21        51.219       8.0357
          4              49.98        50.917       6.0811
          5               50.6        50.855       4.8913
          6              52.61        51.142       4.0909
          7              45.87        50.401       3.5156
          8              42.64        49.444       3.0822
          9              48.26        49.314       2.7439
         10              55.84         49.96       2.4725

fx >>
```

(b) Fill in the state estimates and estimate uncertainties in the following table.

n	1	2	3	4	5	6	7	8	9	10
z_n	49.03m	48.44m	55.21m	49.98m	50.6m	52.61m	45.87m	42.64m	48.26m	55.84m
$\hat{x}_{n,n}$	50.127	49.328	51.219	50.917	50.855	51.142	50.401	49.444	49.314	49.96
$p_{n,n}$	22.5	11.842	8.0357	6.0811	4.8913	4.0909	3.5156	3.0822	2.7439	2.4725

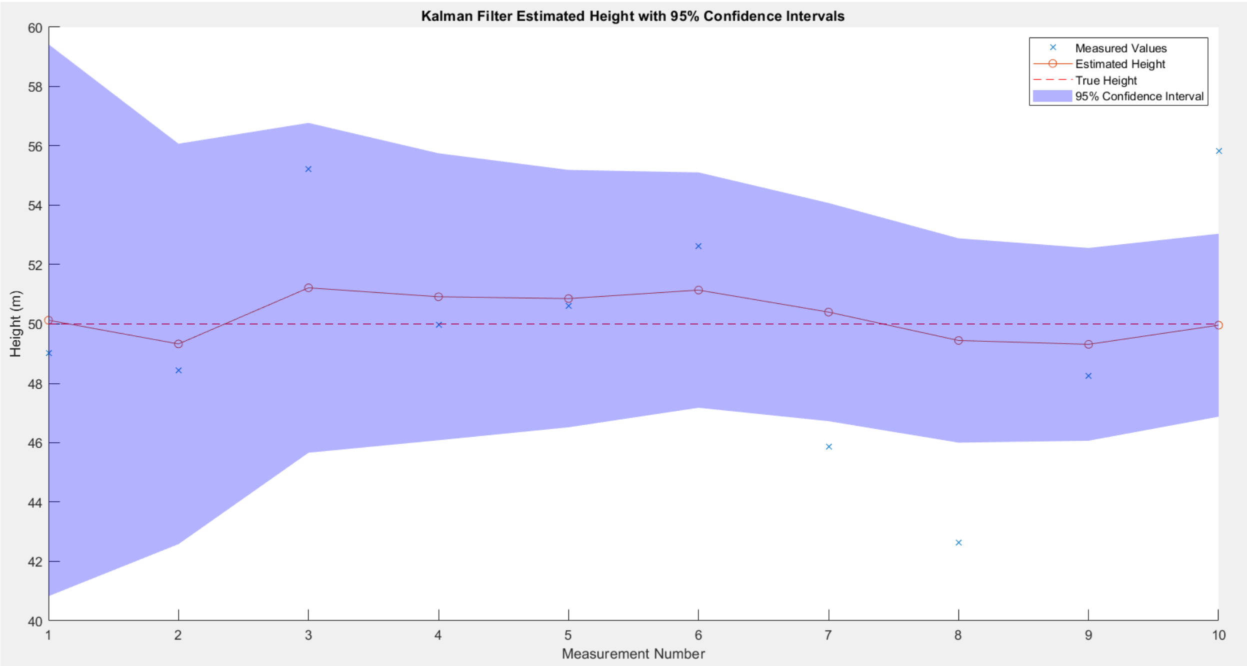
(c) Plot the true value, measured values, estimates, and 95% confidence intervals.

Code in MATLAB:

```
true_height = 50;                    % True building height (meters)
measurement_std_dev = 5;               % alimeter measurement error (standard deviation)
(meters)
measurements = [49.03, 48.44, 55.21, 49.98, 50.6, 52.61, 45.87, 42.64, 48.26, 55.84];

% Kalman Filter initialization with given initial guess
num_measurements = length(measurements);
x_est = 60;                           % Initial estimate of the building height
p_est = 225;                          % Initial uncertainty in the estimate

% Measurement noise (variance) from standard deviation
r = measurement_std_dev^2;

% Initialize arrays to store estimates
x_estimates = zeros(1, num_measurements);
uncertainties = zeros(1, num_measurements);
confidence_interval = zeros(1, num_measurements);

% Kalman Filter loop
for n = 1:num_measurements
   % Measurement update
   Kn = p_est / (p_est + r);                % Calculate Kalman Gain
   x_est = x_est + Kn * (measurements(n) - x_est); % Update estimate with measurement
   p_est = (1 - Kn) * p_est;                % Update uncertainty

   % results
   x_estimates(n) = x_est;
   uncertainties(n) = p_est;
   confidence_interval(n) = 1.96 * sqrt(p_est);        % 95% confidence interval
end

% Display the results
disp('State estimates and uncertainties after each measurement:');
disp(table((1:num_measurements)', measurements', x_estimates', uncertainties', ...
   'VariableNames', {'Measurement_No', 'Measurement', 'Estimate', 'Uncertainty'}));

% Plot the results
figure;
hold on;
plot(1:num_measurements, measurements, 'x', 'DisplayName', 'Measured Values'); %
Measured values
plot(1:num_measurements, x_estimates, '-o', 'DisplayName', 'Estimated Height'); % Estimates
```

```
plot(1:num_measurements, true_height * ones(1, num_measurements), '--r', 'DisplayName', 'True Height'); % True height

% Plot 95% confidence intervals
fill([1:num_measurements, num_measurements:-1:1], ...
    [x_estimates + confidence_interval, fliplr(x_estimates - confidence_interval)], ...
    'blue', 'FaceAlpha', 0.3, 'EdgeColor', 'none', 'DisplayName', '95% Confidence Interval');

xlabel('Measurement Number');
ylabel('Height (m)');
title('Kalman Filter Estimated Height with 95% Confidence Intervals');
legend;
hold off;
```

Plot results in MATLAB:



Phân tích:

- Hội tụ: Ban đầu, giá trị ước lượng (Initial estimate) của chiều cao khá xa giá trị thực initial uncertainty in the estimate. Tuy nhiên, khi có thêm phép đo, giá trị ước lượng hội tụ gần hơn với chiều cao thực.
- Độ rộng của khoảng tin cậy: Khoảng tin cậy 95% rộng nhất ở lúc đầu, do giá trị initial uncertainty in the estimate cao. Nó thu hẹp dần theo thời gian khi bộ lọc Kalman xử lý thêm các phép đo và giảm mức không chắc chắn (uncertainty).
- Ảnh hưởng của nhiễu trong phép đo: Một số phép đo (như khoảng phép đo 7 và 8) lệch khá xa so với chiều cao thực. Tuy nhiên, Bộ lọc Kalman không theo sát các giá trị bất thường này, cho thấy nó có khả năng làm mịn nhiễu để duy trì ước lượng hợp lý.