

Different views of matrix multiplication

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October 10, 2021

Matrix multiplication $A \in R^{m \times n}, B \in R^{n \times k}, C = AB \in R^{m \times k}$

$$C_{ij} = \sum_{k=1}^n a_{ik} b_{kj}$$

For example:

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \times 1 + 2 \times 1 + 3 \times 2 & 1 \times 0 + 2 \times 1 + 3 \times 1 \\ 1 \times 1 + 0 \times 1 + 1 \times 2 & 1 \times 0 + 0 \times 1 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ 3 & 1 \end{bmatrix}$$

Column multiplication:

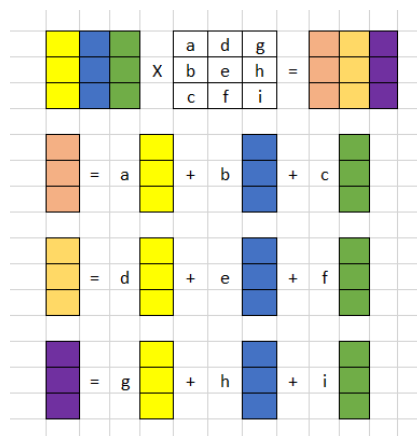


Figure 1: Column multiplication

For example:

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ 3 & 1 \end{bmatrix}$$

Row multiplication:

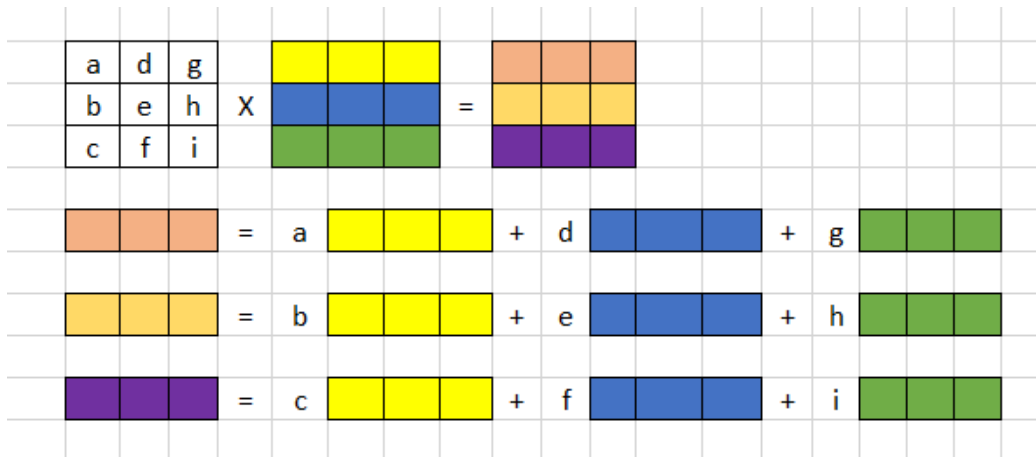


Figure 2: Row multiplication

For example:

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} = 1 \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} + 2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} + 3 \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} = 1 \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} + 0 \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} + 1 \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ 3 & 1 \end{bmatrix}$$

Row and column multiplication:

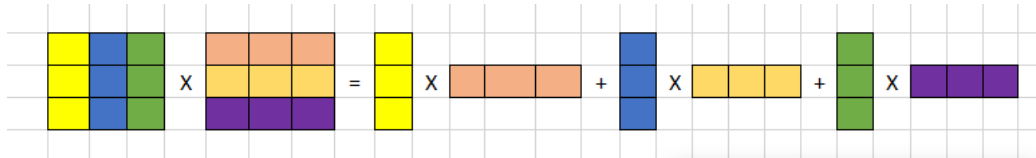


Figure 3: Row and column multiplication

For example: $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} +$

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ 3 & 1 \end{bmatrix}$$