Ver. Aug 9, 2021

University of Hawaii at Manoa
Department of Economics

Mathematics for Economics
ECON 627, Fall 2020
TR 9:00-10:15AM Gartley 103
(AND TR 10:30-11:45AM Gartley 103 during the first part of the semester)

Instructor: Nori Tarui
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Learning Objectives

- Gain understanding about the mathematical underpinnings of economic theory courses– Econ 606, 607, 608 and 609.
    - Understand how to apply optimization methods to solve static problems.
    - Understand how to apply dynamic optimization methods.
- Be able to follow theoretical discussions in economics journal articles.
    - Become familiar with the methods of basic mathematical proofs and other real-analysis tools that are and methods.

In order to develop solid mathematics background necessary for the subsequent coursework and dissertation research, we will seek to translate undergraduate economic theory topics into the language of mathematics, taking considerable care to develop the necessary mathematical. In addition, several increasingly important topics in economic theory are inherently mathematical– notably dynamic optimization models that form the basis of macroeconomic growth theory, natural resources economics and other "dynamic" topics. We will cover both the mathematics and economics of these subjects, though only briefly.

Ideally, we would cover each topic in this course just in time for the theory courses to use that topic. Inevitably, we will miss the timing on at least some topics. If the theory courses arrive at some topic that requires mathematics we have not yet covered in this course, you will nonetheless be expected to read mathematical appendices or other textbooks for that course to obtain at least an intuitive feel for the mathematics. Of course, you are also welcome to request adjustments in our schedule, and I will try to accommodate.

Prerequisites
MATH 203, MATH 215, MATH 241, MATH 251A or equivalent. Familiarity with the topics covered in the summer math cram course (offered by the Department of Economics).

It is strongly recommended that you take Econ 606 (Microeconomic Theory I) concurrently (see "Schedule" below).

Schedule

Econ 627 covers many mathematical concepts and tools that are useful in Econ 606 and 607. For this reason, we plan to use the lecture time for Econ 606 in the first several weeks for lectures in Econ 627—that is, to frontload Econ 627 (using the lecture time for both 606 and 627 in the beginning of the semester) and then focus on Econ 606 later on in the semester. Based on the existing Econ graduate students' experience, we believe that this arrangement would best suit the first-year students' interest. For this lecture arrangement to work, it would be best if an identical set of students take both Econ 606 and 627. **The latest schedule is available on Laulima.**

Here are specific scheduling arrangements for Fall 2021.

(1) Econ 627 will meet on Monday and Wednesday in addition to Tuesady and Thursday in the first week. Thus the schedule for the first week is the following.
Aug 23 (M) 10:30-11:45 online
Aug 24 (T) 9:00-10:15; 10:30-11:45
Aug 23 (M) 10:30-11:45 online
Aug 24 (T) 9:00-10:15; 10:30-11:45

(2)

Grading

Problem Sets	30%
Midterms (2)	40% (20% each)
Final Exam (**Date to be announced**)	30%

There will be periodic problem sets. The problem sets will be mostly analytical but may also include some numerical problems. The latter type may involve the use of *Excel* or *Matlab* on the computer. I encourage you to work together on problem sets but each of you will hand in your own assignment. **Though you are encouraged to work in groups, you must acknowledge your collaborators in your solutions to the problem sets.**

The instructor reserves the right to decide where extra-credit or bonus-point assignments are given, and how much weight the extra-credit or bonus-point assignments would receive for final course grading.

<u>Grading policy for MA and PhD students</u>

1. MA students have two options: (1) to be evaluated on the "MA track" or the "PhD track." Students on the MA track are exempt from completing parts of the assignments—in particular, those related to real analysis and proofs. (I will let you know which assignment questions will be exempt.) Students on the MA track may be assigned an extra (different) set of questions.
2. The students in the PhD Program are graded on the PD track.
3. If you are evaluated on the MA track and decide to transfer to the Ph.D. Program, this class might not count towards the Ph.D. and you might have to repeat this class during your Ph.D. studies unless you can demonstrate proficiency of the Ph.D. material by examination or consent of the Graduate Chair.

4. Each MA students must let the instructor know the choice of the grading option by October 25.

Textbooks

There are two books that we will reference fairly extensively in the course:

(SB) Simon, C. P. and L. Blume (1994) *Mathematics for Economists*, 1st Edition, Norton, New York. (Required) (Both U.S. and the international edition (hard cover or paperback) would be fine.)

Rosenlicht, M. (1986) Introduction to Analysis, Dover Publications, Glenview.

While SB is the main textbook, Rosenlicht's book (at less than $15) will be a useful reference on the terminology and basic real analysis including fundamental methods for mathematical proof.

Other references

Calculus and its applications to economics

***(DLF)** De la Fuente, A. (2000). *Mathematical methods and models for economists*. Cambridge University Press.

***(Sundaram)** Sundaram, R. (1996) *A First Course in Optimization Theory*, Cambridge University Press, Cambridge, UK.

*(DLF and Sundaram are also highly recommended.)

Chiang, A.C. and K. Wainwright (2005) *Fundamental Methods of Mathematical Economics*, 4th Edition, McGraw-Hill, New York.

Silberberg, E. and W. Suen (2000) *The Structure of Economics: A Mathematical Analysis*, 3rd Edition, McGraw-Hill/Irwin, New York.

Takayama, A. (1993) *Analytical Methods in Economics*, University of Michigan Press, Ann Arbor.

Dynamic programming

(SLP) Stokey, N. L. and R. E. Lucas (with E. C. Prescott) (1989) *Recursive Methods in Economic Dynamics*, Harvard University Press, Cambridge, MA.

Ljungqvist, L. and T. J. Sargent (2004) *Recursive Macroeconomic Theory*, 2nd edition, MIT Press, Cambridge, MA.

Optimal control theory

Chiang, A. C. (1992) *Elements of Dynamic Optimization*, Waveland Press, Illinois. (Highly recommended)

Seierstad, A., Sydsæter, K. (1987) Optimal Control Theory with Economic Applications, North-Holland, Amsterdam.

Analysis

Bartle, R. G. (1997) *The Elements of Real Analysis*, 2nd Edition, John Wiley & Sons, New York.

Rudin, W. (1976) *Principles of Mathematical Analysis*, 3rd Edition, McGraw-Hill, New York.

Ok, E. A. (2007) *Real Analysis with Economic Applications*, Princeton University Press, Princeton, NJ.

Numerical methods

Miranda, M. and P. Fackler (2002) *Applied Computational Economics and Finance*, MIT Press, Cambridge, MA.

Judd, K. (1998) *Numerical Methods in Economics*, MIT Press, Cambridge, MA.

Topics covered (See Laulima for the updated list)

1. Preliminaries

Elements of set theory, logic, and proof

*SB Appendix A1
Sundaram Appendices A and B

Properties of Euclidean space and metric space

Sequence, convergence and limits, Cauchy sequence, vector space, norm, metric space, complete metric space, Banach space

*SB Ch 10 and 12.1, 12.2
*SLP Ch 3.1
Sundaram, Ch 1.1 and 1.2
Sundaram, Appendix C

Topology of Euclidean spaces

Open, closed, bounded, compact sets
*SB Ch 12.3-12.6, Ch 29.

2. Multivariate calculus

Functions: basic concepts

Basic terminology of functions, continuity, monotonicity, partial and total derivatives, chain rule, higher order derivatives

*SB Ch 13, 14.1-14.9
Sundaram, Ch 1.4

Functions: applications

Intermediate and mean value theorems, Taylor's Theorem, inverse and implicit function theorem
*SB Ch 15, 30
Sundaram, Ch 1.5 and 1.6

Existence of solutions to optimization
Weierstrass Theorem
*SB Ch 30.1

Sundaram, Ch 3

MIDTERM I

3. Static optimization

Unconstrained optimization

Quadratic forms, first order necessary conditions, second order sufficient conditions

*SB 16.1, 16.2, 17

Sundaram, Ch 4

Constrained optimization

Lagrangian method, first order necessary conditions, second order sufficient conditions, equality and inequality constraints, Kuhn-Tucker Theorem, interpretation of Lagrangian multipliers, envelope theorems, concave and quasiconcave functions, concave programming

*SB Ch 18, 19.1-19.5, 21.1-21.5

Sundaram, Ch 5, 6, 7

Economic applications of static optimization

Utility maximization and expenditure minimization, Roy's identity, Shephard's lemma, Slutsky matrix

*SB Ch 22

MIDTERM II

4. Dynamic optimization

Difference and differential equations

*Difference equation: SB 23.1, 23.2

*Differential equation: SB 24.1-24.4, 25.1-25.5

Dynamic optimization: Calculus of variations, optimal control theory

The Calculus of Variations: Chiang Ch 2, 3, 6

The Hamiltonian Function: Chiang Ch. 7

More on Optimal Control: Chiang Ch 8

Infinite-Horizon Problems: Chiang Ch 5, 9

*Dorfman, R. (1969) An Economic Interpretation of Optimal Control Theory, *American Economic Review*, 59(5) 817-831.

Plourde, C. G. (1970) A Simple Model of Replenishable Natural Resource Exploitation, *American Economic Review*, 60(3) 518-522.

Appendices A.1 and A.3 of Barro and Sala-i-Martin (2003) *Economic Growth*, 3rd Edition, MIT Press, Cambridge, MA.

5. Other possible topics

Maximum Theorem, Fixed Point Theorem, …

Disability Access

If you feel you need reasonable accommodations because of the impact of a disability, please: (1) contact the KOKUA Program (V/T) at 956-7511 or 956-7612 in room 013 of the QLCSS (Queen Lili'uokalani DCenter for Student Services); (2) speak with me privately to discuss your specific needs. I will be happy to work with you and the KOKUA Program to meet access needs related to a documented disability.