Assignment 1 - Quang Tran

January 28, 2019

1 Question 1. Moore's Law

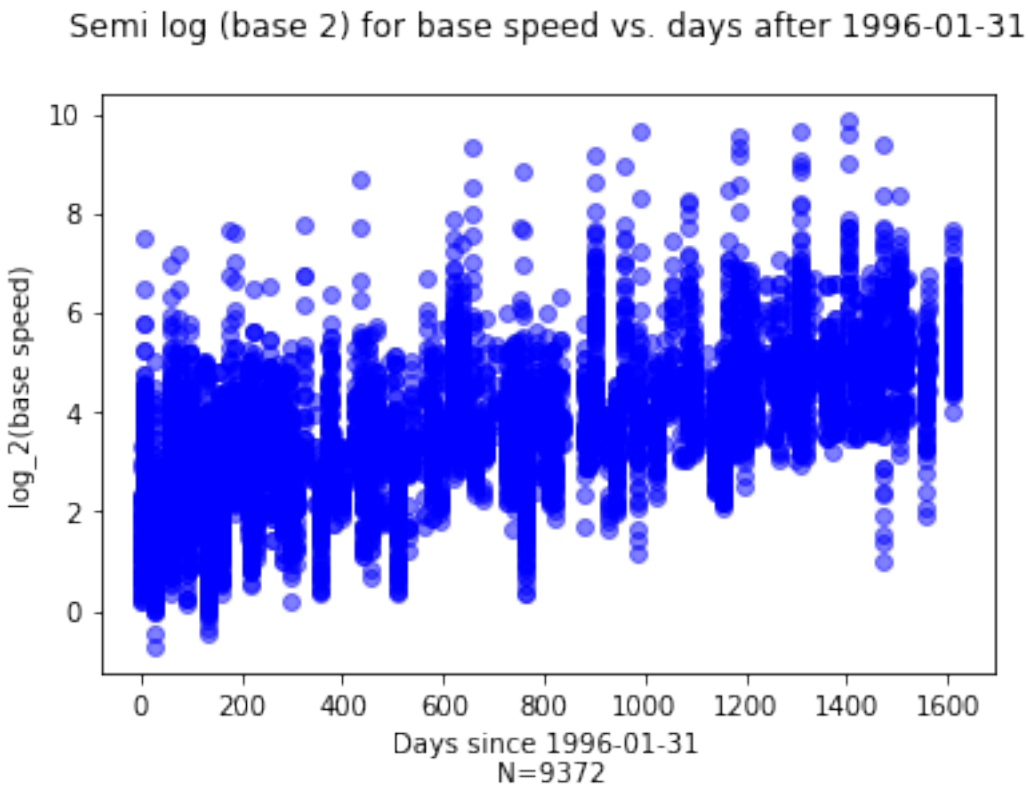
1.1 (a) Extract data and produce a semi-log plot

The benchmark I choose is cpu95. There are N=9372 instances for this benchmark. The y values are the log_2 of the base speeds. Of all the dates recorded for this benchmark, 1996-01-31 is the earliest. All the dates are converted to days after this earliest date.

In [33]:
```
import matplotlib.pyplot as plt
import pandas as pd
import sklearn
from sklearn import datasets, linear_model
from sklearn.metrics import mean_squared_error
from sklearn import model_selection
import numpy as np
import datetime as dt
from scipy import stats

file = open('./specdata20120207/benchmarks.txt', 'r')
i = 0
dates = []
bases = []
for line in file:
    if i==0:  # skip the header
        i += 1
        continue
    testID = line.split(',')[0]
    if not testID.startswith('cpu95'): # only consider benchmark cpu95
        continue
    mask_date = testID.split('-')[1]
    date = ''
    if mask_date.startswith('9'):  # convert, eg, 98 to 1998
        date += '19' + mask_date[:2] + '-' + mask_date[2:4] + '-' + mask_date[4:]
    else:
        date += mask_date[:4] + '-' + mask_date[4:6] + '-' + mask_date[6:]
    dates.append(date)
    bases.append(float(line.split(',')[-2]))
```

```
            i += 1
        dates = [pd.to_datetime(d) for d in dates]
        min_date = min(dates)
        print('Earliest data is:', str(min_date).split(' ')[0])
        dates = np.array(dates)-min_date
        dates = [float(str(timedelta).split(' ')[0]) for timedelta in dates]
        bases = [[base] for base in bases]
        bases = np.log2(bases)
        # plt.semilogy(dates, bases, 'bo', alpha = 0.5)
        plt.plot(dates, bases, 'bo', alpha = 0.5)
        plt.xlabel('Days since 1996-01-31\n N={}'.format(len(dates)))
        plt.ylabel('log_2(base speed)')
        plt.title('Semi log (base 2) for base speed vs. days after 1996-01-31\n')
        plt.show()
```

```
Earliest data is: 1996-01-31
```



1.2 (b) Train a linear model to fit your plot.

According to Moore's law, the base speed s is expected to double every two years, or every $365.25 \times 2 = 730.5$ days. That means, the Moore's law states that the true relationship between

time and base speed is:

$$s = S_0 \times 2^{t/730.5}$$

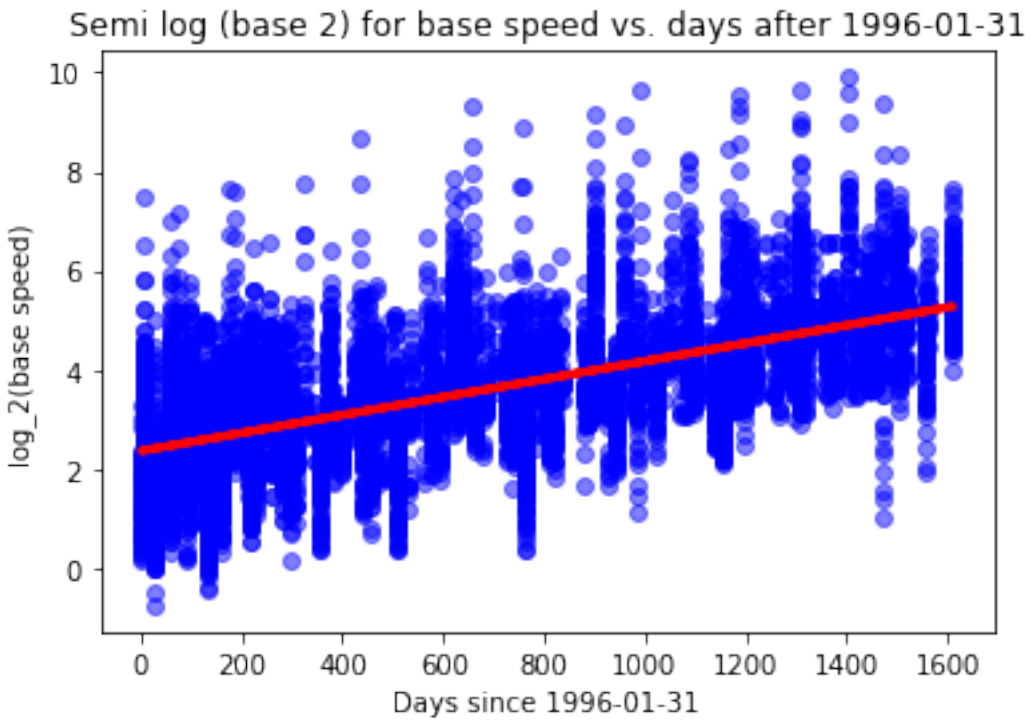
with S_0 being the speed at the initial time $t = 0$ and t is measured in days. Taking the logarithm base 2 of both sides of the above equation gets us:

$$log_2(s) = log_2(S_0 \times 2^{t/730.5}) = log_2(S_0) + \frac{1}{730.5}t$$

It follows that Moore's law concludes a linear model of the relationship between $log_2(s)$ and t. For this reason, checking how well Moore's law holds up is checking whether the relationship between $log_2(s)$ and t follows a linear model. The goal here is to examine the relationship between the predictor t and the response $log_2(s)$, so the interpretive power of the model is of interest. That is why we will use a regression model because such a model is, although not flexible as other models, has good interpretability. Had the prediction accuracy been of primary concern, we would have chosen fancier classification models.

In [2]:
```
# adapted from https://scikit-learn.org/stable/auto_examples/linear_model/plot_ols.htm
# Create linear regression object
dates = [[date] for date in dates]
regr = linear_model.LinearRegression()

# Train the model using the training set
regr.fit(dates, bases)

# Make predictions on training set
base_preds = regr.predict(dates)

# The coefficients
print('Coefficients: \n', regr.coef_)
print('Intercept:\n', regr.intercept_)
# The mean squared error
print("Mean squared error: %.2f"
      % mean_squared_error(bases, base_preds))

# Plot outputs

plt.scatter(dates, bases,  color='blue', alpha=0.5)
plt.plot(dates,base_preds, color='red', linewidth=3)
plt.xlabel('Days since 1996-01-31')
plt.ylabel('log_2(base speed)')
plt.title('Semi log (base 2) for base speed vs. days after 1996-01-31')

plt.show()
```

```
Coefficients:
 [[0.00180707]]
Intercept:
 [2.36785956]
Mean squared error: 1.12
```



The stats printed above mean that, if we assume a linear relationship between $log_2(s)$ and t ($log_2(s) = log_2(S_0) + S_1 t$), then the estimates for $log_2(S_0)$ and S_1 are:

$$\hat{log_2(S_0)} = 2.36785956$$

$$\hat{S_1} = 0.00180707$$

The true value of S_1, according to Moore's law, is $\frac{1}{730.5} = 0.001368925$. We see that there is a slight difference between the estimate and the claimed true value. So how can we conclude whether the Moore's law is correct? Let us construct the 95% confidence interval for the slope $\hat{S_1}$.

According to ISLR (p.66), the formula for the standard error of the coefficient is:

$$SE(\hat{S_1})^2 = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{RSS/(n-2)}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

where RSS is the residual sum of squares.

```
In [3]: n = len(dates)
        RSS = np.sum((dates - regr.predict(dates))**2)
        mean_x = np.sum(dates)/len(dates)
        SE = (RSS/(n-2))/ np.sum((dates-mean_x)**2)
        upper = regr.coef_ + 2*SE
        lower = regr.coef_ - 2*SE
        print('upper:', upper)
        print('lower:', lower)
```

```
upper: [[0.00244528]]
lower: [[0.00116886]]
```

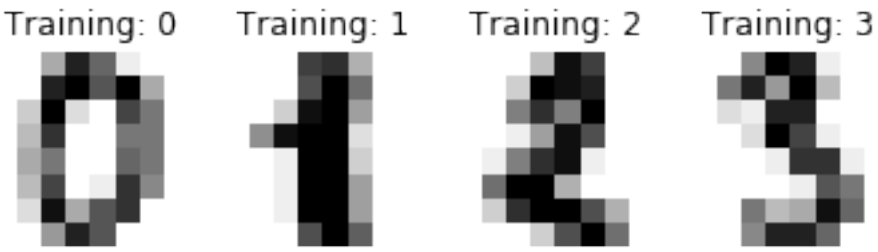
The 95% confidence interval is $[0.00116886, 0.00244528]$. This means that if we construct such an interval repeatedly, 95% of the time the intervals will capture the true coefficient. Because 0.001368925 (1/730.5) lies in the interval we've just constructed, we are confident the Moore's law does hold up. (**applying #confidenceinterval**)

2 Question 2. MNIST

2.1 (a) Load the data and plot some examples

```
In [4]: digits = sklearn.datasets.load_digits()
        print(digits.data.shape)
        # image showing code from https://scikit-learn.org/stable/auto_examples/classification,
        images_and_labels = list(zip(digits.images, digits.target))
        for index, (image, label) in enumerate(images_and_labels[:4]):
            plt.subplot(2, 4, index + 1)
            plt.axis('off')
            plt.imshow(image, cmap=plt.cm.gray_r, interpolation='nearest')
            plt.title('Training: %i' % label)
```

```
(1797, 64)
```



2.2 (b) Choose two digit classes (e.g 7s and 3s) , and train a k-nearest neighbor classifier.

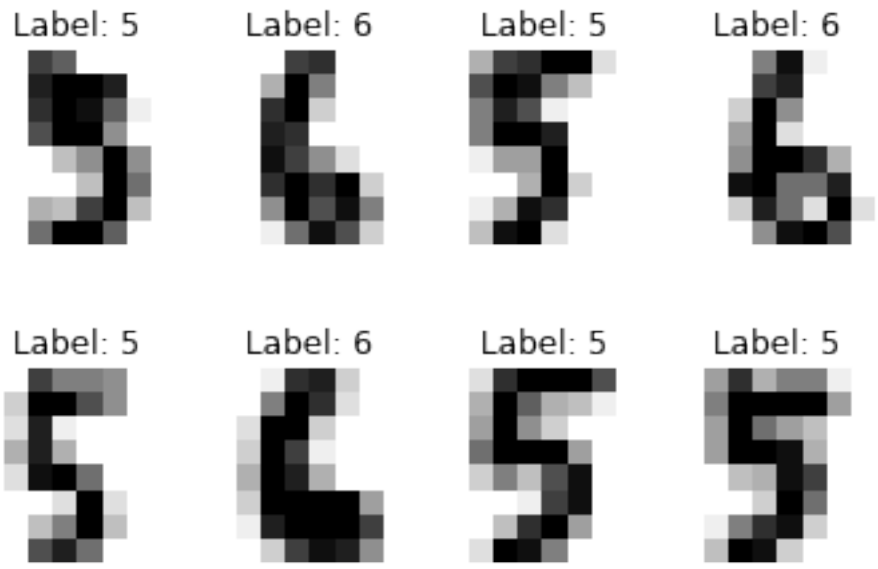
In this section we will choose 5 and 6 as the two classes. We will follow the KNN algorithm specified in Barber (2012). That means in the `sklearn.neighbors.KNeighborsClassifier` instantiation, `weights` is set to `'uniform'`, as the algorithm in Barber (2012) assigns equal weights to the k neighbors. There are several things to decide about the KNN: 1. The number of neighbors k 2. The dissimilarity function. We will investigate two metrics: L1 and L2 distances. L1 distance is the sum of the pixel-wise absolute differences, while L2 is the vanilla Euclidean distance (sum of squared differences). To do this, we set the argument `metric=minkowski`, which calculates the

distances according to the formula `sum(|x - y|^p)^(1/p)`. If we the further set `p=1`, it is the L1 distance; if `p=2`, it is the L2 distance.

To tune k and decide on the dissimilarity function, we will use cross validation (leave one out). Leave one out is chosen over k-fold because we have little data (~360), and only part of it is training data. If we make a fixed validation set, then the training data is further shrunken.

```
In [16]: # get 5's and 6's
         def get_data(X, y, digit_list):
             mask = []
             for label in y:
                 to_append = False
                 for digit in digit_list:
                     if label == digit:
                         to_append = True
                 mask.append(to_append)
             return X[mask,:,:], y[mask]

         X, y = get_data(digits.images,digits.target, [5,6])
         print('Data size:', len(X))

         # plotting some examples
         print('\nPlotting some examples ...')
         images_and_labels = list(zip(X,y))
         for index, (image, label) in enumerate(images_and_labels[:8]):
             plt.subplot(2, 4, index + 1)
             plt.axis('off')
             plt.imshow(image, cmap=plt.cm.gray_r, interpolation='nearest')
             plt.title('Label: %i' % label)
```

```
Data size: 363

Plotting some examples ...
```



In [8]:
```
import pandas as pd
from sklearn import datasets, linear_model
# adapted from : https://machinelearningmastery.com/evaluate-performance-machine-learn
# adapted from: https://scikit-learn.org/stable/modules/cross_validation.html#cross-va

import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap
from sklearn import neighbors, datasets

X_train, X_test, y_train, y_test = model_selection.train_test_split(X, y, test_size=.33
X_train = np.reshape(X_train, (X_train.shape[0],-1))
X_test = np.reshape(X_test, (X_test.shape[0],-1))
print('Shape of training features:', X_train.shape)
print('Shape of training labels:', y_train.shape)
print('Shape of testing features:', X_test.shape)
print('Shape of testing labels:', y_test.shape)
def run_knn_selection(X_train, y_train, cv_type='loo', verbose=False):
    i = 0
    max_k = 10
    best_accuracy = -1
    best_params = {'k': -1, 'p': -1 }
    print('Running ...')
    for k in range(1, max_k+1):
        for p in [1,2]:
            clf = neighbors.KNeighborsClassifier(n_neighbors=k,
                                                 weights='uniform',
```

```
                                                    p=p,
                                                    metric='minkowski')
                    cv = model_selection.LeaveOneOut()
                    scores = model_selection.cross_val_score(clf, X_train, y_train, cv=cv)
                    if np.mean(scores) > best_accuracy:
                        best_accuracy = np.mean(scores)
                        best_params['k'], best_params['p'] = k, p
                    i += 1
                    if verbose:
                        if i%5==0:
                            print('Finished %d/%d'%(i,max_k*2))
            print('Best k:', best_params['k'])
            print('Best distance type:', best_params['p'])
            print('Val Accuracy:', best_accuracy)
            return best_params, best_accuracy
        best_params, best_accuracy = run_knn_selection(X_train, y_train, verbose=True)
```

```
Shape of training features: (243, 64)
Shape of training labels: (243,)
Shape of testing features: (120, 64)
Shape of testing labels: (120,)
Running ...
Finished 5/20
Finished 10/20
Finished 15/20
Finished 20/20
Best k: 8
Best distance type: 1
Val Accuracy: 1.0
```

The results show that the optimal k is 8 with L1 distance used. We will train the data using these parameters:

```
In [9]: best_k, best_p = best_params.values()
        clf = neighbors.KNeighborsClassifier(n_neighbors=best_k,
                                             weights='uniform',
                                             p=best_p,
                                             metric='minkowski')
        clf.fit(X_train, y_train)
```

```
Out[9]: KNeighborsClassifier(algorithm='auto', leaf_size=30, metric='minkowski',
                    metric_params=None, n_jobs=1, n_neighbors=8, p=1,
                    weights='uniform')
```

2.3 (c) Report your error rates on a held out part of the data.

```
In [10]: test_acc = clf.score(X_test, y_test)
         print('Test accuracy:', test_acc)
```

```
Test accuracy: 0.9916666666666667
```

2.4 (d) Train for 10 classes on the full data set

Loading the data:

```
In [23]: # ! pip install python-mnist
         from mnist import MNIST

         mndata = MNIST('samples')
         X_train, y_train = mndata.load_training()

         X_test, y_test = mndata.load_testing()
```

As the dataset is large, we can designate a fixed part of the training data set as the validation set (no need for cross-validation).

```
In [24]: X_train = np.array(X_train)
         y_train = np.array(y_train)
         X_test = np.array(X_test)
         y_test = np.array(y_test)
         X_train, X_val, y_train, y_val = model_selection.train_test_split(X_train,
                                                                           y_train,
                                                                           test_size=.33)

         print('Shape of training features:', X_train.shape)
         print('Shape of training labels:', y_train.shape)
         print('Shape of val features:', X_val.shape)
         print('Shape of val labels:', y_val.shape)
         print('Shape of testing features:', X_test.shape)
         print('Shape of testing labels:', y_test.shape)
```

```
Shape of training features: (40200, 784)
Shape of training labels: (40200,)
Shape of val features: (19800, 784)
Shape of val labels: (19800,)
Shape of testing features: (10000, 784)
Shape of testing labels: (10000,)
```

I found that training using the library is extremely time-consuming, while this should not be the case: all the KNN does in the training phase is memorizing data and the distances between each of the testing/validating point to all the training data points should be computed only once. I coded the KNN from scratch below (adapted from Assignment 1 of CS1231n) and the implementation was feasible.

```
In [27]: class KNearestNeighbor(object):
           """ a kNN classifier with Eucledian distance """
```

```
    def __init__(self):
      pass

    def train(self, X, y):
      self.X_train = X
      self.y_train = y

    def compute_distances(self, X):
      """
      Compute the distance between each test point in X and each training point
      in self.X_train using no explicit loops.

      Input / Output: Same as compute_distances_two_loops
      """
      num_test = X.shape[0]
      num_train = self.X_train.shape[0]
      dists = np.zeros((num_test, num_train))
      dists = np.sqrt((np.sum(X**2, axis=1)).reshape(num_test,1) + np.sum(self.X_train*
      return dists

    def predict_labels(self, dists, k=1):
      """
      Given a matrix of distances between test points and training points,
      predict a label for each test point.
      """
      num_test = dists.shape[0]
      y_pred = np.zeros(num_test)
      for i in range(num_test):
        # A list of length k storing the labels of the k nearest neighbors to
        # the ith test point.
        closest_y = []
        closest_y = self.y_train[np.argsort(dists[i,:])[0:k]]
        y_pred[i] = stats.mode(closest_y)[0]
      return y_pred
```

In [28]:
```
# training
classifier = KNearestNeighbor()
classifier.train(X_train, y_train)
dists = classifier.compute_distances(X_val)
```

We assumed the L2 distance metric and only tune the parameter k.

In [30]:
```
best_acc = -1
i = 0
for k in range(1,11):
    y_val_pred = classifier.predict_labels(dists, k=k)
    # Compute val accuracy
    num_correct = np.sum(y_val_pred == y_val)
```

```
            accuracy = float(num_correct) / len(y_val)
            if accuracy > best_acc:
                best_acc = accuracy
                best_k = k
            i += 1
            print('Finished %d/10'%i)
        print('-'*50)
        print('Optimal k is:', best_k)
```

```
Finished 1/10
Finished 2/10
Finished 3/10
Finished 4/10
Finished 5/10
Finished 6/10
Finished 7/10
Finished 8/10
Finished 9/10
Finished 10/10
--------------------------------------------------
Optimal k is: 1
```

```
In [31]: # testing
         dists = classifier.compute_distances(X_test)
         y_test_pred = classifier.predict_labels(dists, k=best_k)
         # Compute test accuracy
         num_correct = np.sum(y_test_pred == y_test)
         accuracy = float(num_correct) / len(y_test)
         print('Testing accuracy:', accuracy)
```

```
Testing accuracy: 0.9661
```

Textbook questions

1.14

There are 9 total numbers, and each choice consists of 4 numbers. Since the order of the chosen numbers is irrelevant, there are $9C4 = \frac{9!}{4!\times(9-4)!} = 126$ choices. The probability that any number comes up is, therefore, $\frac{1}{126}$. Because 1 in every 100 players will pick 1, 3, 5, 7, the number of people having that choice is $\frac{1}{100} \times 1,000,000 = 10,000$. For a person choosing 1, 3, 5, 7, the utility he gets when he wins the lottery of the week is $U(win,\ \{1,3,5,7\}) = \frac{1,000,000}{10,000} - 1 = \$99$ (the winning prize is split equally among the 10,000 winners and the entry costs \$1. The utility he gets when he loses that week is $U(lose,\ \{1,3,5,7\}) = -\$1,$ which is the entry cost.
The expected money a person playing 1, 3, 5, 7 gets in the week is:

$$E(U(\{1,3,5,7\}))$$
$$= p(outcome = \{1,3,5,7\}) \times U(win, \{1,3,5,7\}) + p(outcome \neq \{1,3,5,7\}) \times U(lose, \{1,3,5,7\})$$
$$= \frac{1}{126} \times 99 + \left(1 - \frac{1}{126}\right) \times (-1) = -\$0.206$$

The number of people picking 1, 2, 3, 4 is $\frac{1}{10,000} \times 1,000,000 = 100$. The utility a person picking 1, 2, 3, 4 gets if he wins is $U(win,\ \{1,2,3,4\}) = \frac{1,000,000}{100} - 1 = \$9999,$ and the utility he gets when he loses that week is $U(lose,\ \{1,2,3,4\}) = -\$1$. The expected money a person playing 1, 2, 3, 4 gets in the week is:

$$E(U(\{1,2,3,4\}))$$
$$= p(outcome = \{1,2,3,4\}) \times U(win, \{1,2,3,4\}) + p(outcome \neq \{1,2,3,4\}) \times U(lose, \{1,2,3,4\})$$
$$= \frac{1}{126} \times 9999 + \left(1 - \frac{1}{126}\right) \times (-1) = \$78.365$$

We see that opting for a less common choice in lottery yields higher expected gain. However, from a game-theory perspective, this decision of choosing least common choice is not stable, as other rational agents will also picking this least common choice, and the choice is not least common any more.[1]

3.15.

A model that can achieve 100% test-set accuracy on all current learning problem is certainly impressive. Assuming that in training the neural network WowCo only used the training sets and

[1] **#gametheory:** frame the lottery as a game where players are rational and want to maximize their utilities and analyze one of its potential solutions

never looked at the testing set until the testing phase, this is quite an accomplishment. However, several points that need considerations include:

1. A trillion hidden units mean large memory requirement for storing a huge number of parameters.
2. It also means feeding forward takes a huge amount of time to get one prediction.
3. After all, the claim is only about “known problems". What about future, unknown problems? The claim only means that the neural network learns very well, if not perfectly, the underlying distribution of current data of known problems. There is no guarantee as to how it will performs when presented with data with a different distribution.