University of Technology - HCMC Faculty of Applied Mathematics	**Numerical Method Project** **Semester 2 (2021 - 2022)**

Instruction

Students use Matlab or Python to solve the following problems and write a report.

The report must have 3 parts:

i) The theory and algorithm;
ii) The Matlab or Python commands;
iii) The results and conclusion.

Project 1

Problem 1. Given the data:

x_i	0.2	0.3	0.6	0.9	1.1	1.3	1.4	1.6
y_i	0.050446	0.098426	0.33277	0.72660	1.0972	1.5697	1.8487	2.5015

a. Construct the least squares polynomial of degree 1 and compute the error.

b. Construct the least squares polynomial of degree 2 and compute the error.

c. Construct the least squares polynomial of degree 3 and compute the error.

d. Construct the least squares approximation of the form be^{ax}, and compute the error.

e. Construct the least squares approximation of the form bx^a, and compute the error.

Problem 2. Let R be the rectangle $[0;2] \times [1;4]$.

(a) Let $f(x;y) = x\cos(x^2 + y)$. Calculate the integral $\iint\limits_R f(x,y)dA$.

(b) Study the Simpson formula. Develop a function to estimate the integral in R using composite trapezoidal formula.

(c) Let n and m be the number of sub-interval in x and y components, respectively. Estimate the integral with $[n,m] = [40,60]$ and $[n,m] = [80,120]$ and estimate the errors.

Problem 3. Heat is conducted along a metal rod positioned between two fixed temperature walls. Aside from conduction, heat is transferred between the rod and the surrounding air by convection. Based on a heat balance, the distribution of temperature along the rod is described by the following second-order differential equation

$$0 = \frac{d^2T}{dx^2} + h(T_\infty - T)$$

where T = temperature (K), h = a bulk heat transfer coefficient reflecting the relative importance of convection to conduction m^{-2}, x =distance along the rod (m), and T_∞ = temperature of the surrounding fluid (K).

(a) Convert this differential equation to a equivalent system of simultaneous algebraic equations using a centered difference approximation for the second derivative.

(b) Develop a function to solve these equations from $x = 0$ to L and return the resulting distances and temperatures, in which, the algebraic equations must be solved by tridiagonal matrix.

(c) Develop a script that invokes this function and then plots the results.

(d) Test your script for the following parameters: $h = 0.0425\ \mathrm{m}^{-2}$, $L = 12$ m, $T_\infty = 220$ K, $T(0) = 320$ K, $T(L) = 450$ K, and $\Delta x = 0.5$ m.

Project 2

Problem 1. Given the data:

x_i	4.0	4.2	4.5	4.7	5.1	5.5	5.9	6.3	6.8
y_i	102.56	113.18	130.11	142.05	167.53	195.14	224.87	256.73	299.50

a. Construct the least squares polynomial of degree 1 and compute the error.

b. Construct the least squares polynomial of degree 2 and compute the error.

c. Construct the least squares polynomial of degree 3 and compute the error.

d. Construct the least squares approximation of the form be^{ax}, and compute the error.

e. Construct the least squares approximation of the form bx^a, and compute the error.

Problem 2. Let R be the rectangle $[0;2] \times [1;4]$.

(a) Let $f(x;y) = x\cos(x^2+y)$. Calculate the integral $\iint\limits_R f(x,y)dA$.

(b) Study the Simpson formula. Develop a function to estimate the integral in R using Simpson formula.

(c) Let n and m be the number of sub-interval in x and y components, respectively. Estimate the integral with $[n,m] = [40,60]$ and $[n,m] = [80,120]$ and estimate the errors.

Problem 3. The following differential equation results from a steady-state mass balance for a chemical in a one dimensional canal,

$$0 = D\frac{d^2c}{dx^2} - U\frac{dc}{dx} - kc$$

where x = distance along the canal (m), c = concentration, t = time, x = distance, D = diffusion coefficient, U = fluid velocity, and k = a first - order decay rate.

a. Convert this differential equation to an equivalent system of simultaneous algebraic equations using centered difference approximations for the derivatives.

b. Develop a function to solve these equations from x = 0 to L and return the resulting distances and concentrations.

c. Develop a script that invokes this function and then plots the results.

d. Test your script for the following parameters: L = 10 m, ∇x = 0.5m, $D = 2\ m^2/d$, U = 1 m/d, k = 0.2/d, c(0) = 80 mg/L, and c(10) = 20 mg/L.