Introductory Econometrics

Tutorial 12

<u>PART A:</u> To be done before you attend the tutorial. The solutions will be made available at the end of the week.

1. Suppose time series $\{y_t,\ t = 1, 2, \ldots\}$ has a linear trend and also a unit root. Differencing this time series (i.e. generating $\triangle y_t = y_t - y_{t-1}$) gets rid of

   (a) the linear trend only

   (b) the unit root only

   (c) both the linear trend and the unit root

   (d) neither the linear trend nor the unit root

2. To verify your answer to the first question, consider the time series y_t generated by

$$\begin{aligned} y_t &= \alpha + \beta\ t + u_t \\ u_t &= u_{t-1} + e_t \end{aligned}$$

   where e_t is white noise. This series is the sum of a linear trend $(\alpha + \beta\ t)$ and a random walk u_t, which has a unit root. Write what y_{t-1} is, and then form $\triangle y_t$ by subtracting y_{t-1} from y_t. Does $\triangle y_t$ have a linear trend? Does it have a unit root? If this does not accord with your answer to the previous question, you need to change your answer to the previous question.

3. Define each of the following and provide an example:

   (a) an estimator

   (b) an unbiased estimator

   (c) the best linear unbiased estimator

   (d) a consistent estimator

   (e) an asymptotically normal estimator

Do not forget to bring your answers to PART A and a copy of the tutorial questions to your tutorial.

<u>Part B:</u> This part will be covered in the tutorial. It is still a good idea to attempt these questions before the tutorial.

1. Assume that y_t is a **stationary** AR(1) process given by

$$y_t = c + \varphi_1 y_{t-1} + e_t, \tag{1}$$

   where

$$e_t \sim i.i.d.(0, \sigma^2).$$

   (a) Show that

$$E(y_t) = \mu = \frac{c}{(1 - \phi_1)},\ \forall t.$$

   (b) Show that

$$Var(y_t) = \gamma_0 = \frac{\sigma^2}{(1 - \varphi_1^2)},\ \forall t.$$

(c) Show that (1) may be written in mean deviation form as

$$y_t - \mu = \varphi_1(y_{t-1} - \mu) + e_t. \tag{2}$$

(d) Use (2) to show that

$$\gamma_1 = Cov(y_t, y_{t-1}) \;=\; \frac{\sigma^2}{(1-\varphi_1^2)}\varphi_1.$$

(e) Show that

$$\rho_1 = Corr\,(y_t, y_{t-1}) = \varphi_1.$$

2. **This is based on a question from S1, 2018 final exam:** A researcher wants to test the Efficient Market Hypothesis (EMH) using weekly percentage returns, denoted by r_t, on the New York Stock Exchange composite index. In its strict form the EMH states that information observable to the market prior to week t should not help to predict the return during week t. If we use only past information on r, the EMH is stated as

$$E\left(r_t | r_{t-1}, r_{t-2}, \ldots\right) = E\left(r_t\right). \tag{3}$$

One simple way to test that (3) holds is to specify the following alternative AR(1) model to describe r_t:

$$r_t = \beta_0 + \beta_1 r_{t-1} + u_t, \tag{4}$$

where $E\left(u_t | r_{t-1}, r_{t-2}, \ldots\right) = 0$ and $Var\left(u_t | r_{t-1}, r_{t-2}, \ldots\right) = \sigma^2$. Using data from the first week of January 2004 to the third week of April 2018, estimation of (4) gives:

$$\begin{aligned} \hat{r}_t &= \underset{(0.096)}{0.086} - \underset{(0.038)}{0.059} r_{t-1}, \qquad (5) \\ n &= 689,\ R^2 = 0.0035,\ \bar{R}^2 = 0.0020. \end{aligned}$$

(standard errors are reported in parentheses underneath the parameter estimates).

(a) i. How would you formulate the null hypothesis that the EMH holds based on (4)? Briefly explain your intuition behind your choice of H_0.

ii. Given the OLS regression results in (5) do you reject or not reject H_0 from (i)? Briefly explain.

(4 marks)

(b) The alternative AR(1) model in (4) does not preclude that potentially there could be dependence between returns that are more than one week apart.

i. If (4) were the correct specification for describing returns, what type of process would you expect u_t to follow? Provide the properties of this process.

ii. If the researcher suspects that returns 3 and 4 weeks apart individually add power to the prediction of r_t, what problem do you think that he would be worried about with regard to the behaviour of u_t? Briefly explain.

(5 marks)

(c) The researcher is also interested in the behaviour of the squared residuals from regression (5) because he is concerned that the variance given past information might not be constant. For this purpose he runs a regression of $\hat{u}_t^2$ on r_{t-1} and obtains the following results:

$$\begin{aligned} \hat{u}_t^2 &= \underset{(0.43)}{4.66} - \underset{(0.201)}{1.104} r_{t-1} + \hat{v}_t, \qquad (6) \\ n &= 689,\ R^2 = 0.042. \end{aligned}$$

i. Which problem is the researcher worried about in this case? Define the problem and set up a formal test that makes use of the goodness-of-fit of (6). Clearly state the steps involved in the implementation of this test, the null and alternative hypotheses of the test, the statistic(s) of interest and corresponding distribution(s).

ii. What advice would you give this researcher based on your analysis of c.(i)? Briefly explain your answer.

(6 marks)

3. **This is based on a question from S2, 2016 final exam:**

(a) A researcher who wished to study the behavior of a stationary time series $\{y_t\}$ estimated both an AR(2) model,

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + u_t,$$

and an ADL(2,1) model,

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \beta_1 x_{t-1} + u_t.$$

The researcher obtained the results reported in Table 1 below (standard errors are reported below the estimated coefficients). Based on the information in Table 1, which model do you prefer? Briefly explain.

Table 1		
	AR(2)	ADL(2,1)
$\widehat{c}$	1.28 (0.53)	1.30 (0.44)
$\widehat{\phi}_1$	-0.31 (0.09)	-0.42 (0.08)
$\widehat{\phi}_2$	-0.39 (0.08)	-0.37 (0.08)
$\widehat{\beta}_1$	–	-2.64 (0.46)
$\overline{R}^2$	0.55	0.71
SSR	475	462
AIC	1.08	1.04
BIC	1.09	1.11
n	200	200

(4 marks)

(b) When the researcher estimated the model

$$y_t = c + \beta_0 D_t + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \beta_1 x_{t-1} + \beta_2 (D_t x_{t-1}) + u_t \tag{2}$$

by OLS, where

$$D_t = \begin{cases} 1 \text{ for } t = 1, 2, ...100 \\ 0 \text{ for } t = 101, 102, ...200 \end{cases},$$

he obtained the results reported in (3) below (standard errors are reported in parentheses):

$$\begin{aligned} \widehat{y}_t &= \underset{(0.44)}{1.32} + \underset{(0.04)}{0.30} D_t - \underset{(0.07)}{0.39} y_{t-1} - \underset{(0.06)}{0.31} y_{t-2} + \underset{(0.41)}{2.15} x_{t-1} + \underset{(0.07)}{0.15} (D_t x_{t-1}), \\ SSR &= 450, \overline{R}^2 = 0.69, n = 200. \end{aligned} \tag{3}$$

i. What is

$$\widehat{E}(y_t | y_{t-1}, y_{t-2}, x_{t-1})$$

for different values of t? Briefly explain.

ii. What are the estimated immediate and long run effect of a one unit increase in x on y before and after time $t = 100$? Briefly explain.

iii. Test the null **hypothesis** that there is no structural break in either the intercept or the coefficient attached to x_{t-1} in the ADL(2,1) model. State the null and the alternative hypothesis, the form and asymptotic distribution of the test statistic under the null, the sample value and critical value of the test statistic and you conclusion.

(7 marks)

(c) Carefully describe the steps involved in performing a **Breush-Godfrey test** for autocorrelation up to order two in the error term in the ADL(2,1) model

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \beta_1 x_{t-1} + u_t. \tag{4}$$

Make sure you specify the form and distribution of the test statistic.

(5 marks)