Tutorial 11

keywords: time series,AR model, GDP, growth rate, residual plot, correlogram, white noise, Breusch-Godfrey test for serial correlation, F-test

estimated reading time: 38 minutes

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October 9, 2018

Question 1

(Complete in class)

Question 2

A dynamically well-specified model is the one with no evidence of serial correlation in its errors.

EViews workfile: *US_gdp.wf1*

US_gdp.wf1 contains quarterly data during the period 1954Q1 and 2017Q2:

$$gdp - U.S.\ real\ Gross\ Domestic\ Product\ (\$\ billions)$$

(a) Create the quarterly GDP annualised growth rate using logarithmic transformation and differencing. Denote the series *dlgdp*.

The **first difference of $\boldsymbol{log(gdp_t)}$**,

$$\Delta log(gdp_t) = log(gdp_t) - log(gdp_{t-1})$$

is the approximate proportional change in quarterly real GDP,

$$\begin{aligned}\Delta log(gdp_t) &= log(gdp_t) - log(gdp_{t-1}) \\ &\approx \frac{gdp_t - gdp_{t-1}}{gdp_{t-1}}\end{aligned}$$

Multiplying 100 to $\Delta log(gdp_t)$ gives the percentage change in quarterly real GDP i.e. quarterly real GDP growth rate,

$$\begin{aligned}100\Delta log(gdp_t) &\approx 100 \times \frac{gdp_t - gdp_{t-1}}{gdp_{t-1}} \\ &= \%\Delta gdp_t\end{aligned}$$

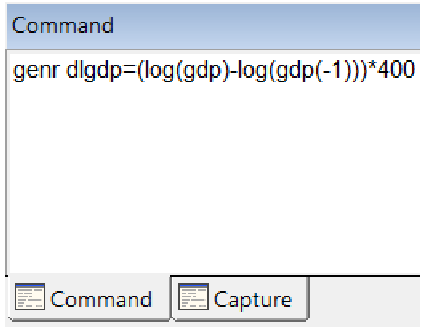
To annualise the quarterly real GDP growth rate, we multiply the quarterly real GDP growth rate by 4,

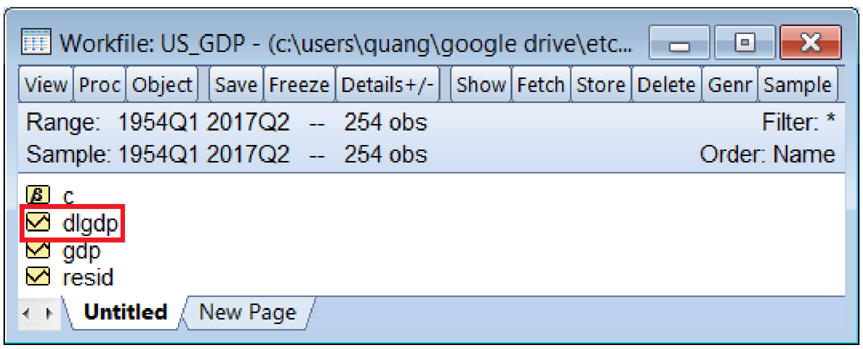
$$\begin{aligned}dlgdp_t &= 4 \times 100\Delta log(gdp_t) \\ &= 400\Delta log(gdp_t)\end{aligned}$$

Note based on Farshid's response on Discussion Forum (2017 Sem2 I think): $100^*\big(log(gdp_t) - log(gdp_{t-4})\big)$ is percentage change in real GDP relative to the same quarter of last year i.e. annual GDP growth. For econometric analysis, we use $400^*\big(log(gdp_t) - log(gdp_{t-1})\big)$ which is one quarter growth rate multiplied by 4 to annualise it when there is no seasonality in that data and $100^*\big(log(gdp_t) - log(gdp_{t-4})\big)$ only when the data is seasonal. Since $log(gdp_t)$ is not seasonal, we use the annualise quarterly real GDP growth. (Outside scope of unit.)

To generate the annualised quarterly real GDP growth rate from the Command window in EViews,

$$genr\ dlgdp = (log(gdp) - log(gdp(-1)))^{*}400$$





	dlgdp
1954Q1	*NA*
1954Q2	0.426987
1954Q3	4.510765
1954Q4	7.723652
⋮	⋮
2016Q4	1.743710
2017Q1	1.227685
2017Q2	2.535842

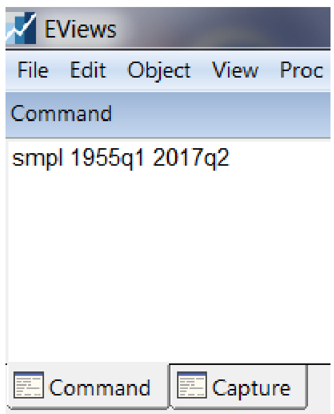
Table 1: Annualised quarterly U.S. GDP growth for the period 1954Q2 to 2017Q2. The first observation is lost because of first differencing.

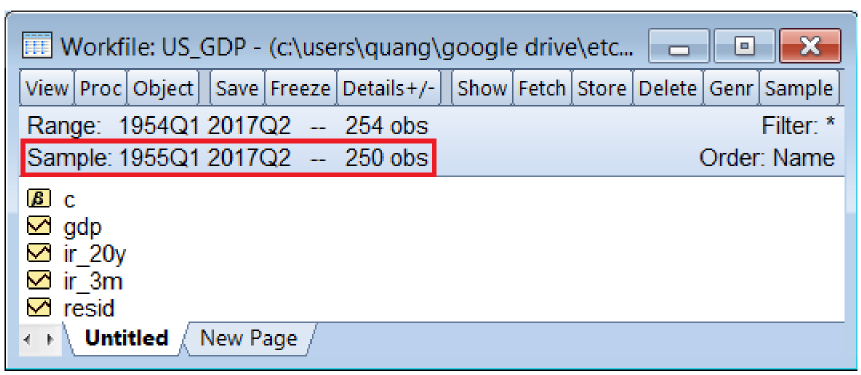
(b) Truncate the sample such that you consider data ranging from 1955Q1 to 2017Q2. Estimate the following OLS regression (which is annualised quarterly U.S. GDP growth modelled with an AR(2) process):

$$dlgdp_t = \varphi_0 + \varphi_1 dlgdp_{t-1} + \varphi_2 dlgdp_{t-2} + u_t$$

To change the sample from the Command window,

$$smpl\ 1955Q1\ 2017Q2$$

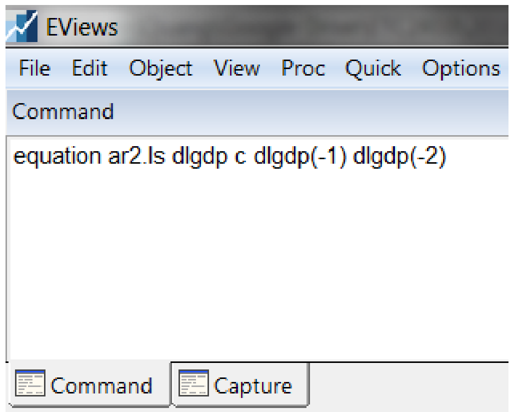




Notice that the sample (but not the range) has changed in the status window. *Do not change the workfile range in the status window.*

To estimate $dlgdp_t = \varphi_0 + \varphi_1 dlgdp_{t-1} + \varphi_2 dlgdp_{t-2} + u_t$ and name the equation $ar2$ from the Command window in EViews,

$$equation\ ar2.ls\ dlgdp\ c\ dlgdp(\text{-}1to\text{-}2)$$



Dependent Variable: DLGDP
Method: Least Squares
Sample: 1955Q1 2017Q2
Included observations: 250

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.742741	0.300922	5.791343	0.0000
DLGDP(-1)	0.306892	0.063042	4.868049	0.0000
DLGDP(-2)	0.108769	0.063053	1.725045	0.0858

R-squared	0.130079	Mean dependent var	2.999617
Adjusted R-squared	0.123035	S.D. dependent var	3.499670
S.E. of regression	3.277315	Akaike info criterion	5.223854
Sum squared resid	2652.976	Schwarz criterion	5.266111
Log likelihood	−649.9817	Hannan-Quinn criter.	5.240861
F-statistic	18.46692	Durbin-Watson stat	1.999001
Prob(F-statistic)	0.000000		

$$\widehat{dlgdp}_t = \underset{(0.3009)}{1.7427} + \underset{(0.0630)}{0.3069} dlgdp_{t-1} + \underset{(0.0631)}{0.1088} dlgdp_{t-2}$$

(c) Inspect the time plot and the correlogram of the residuals associated with the estimated model. Is there any visual evidence that the dynamics of the model is not specified well?

We want to check if there is evidence of serially correlated errors,

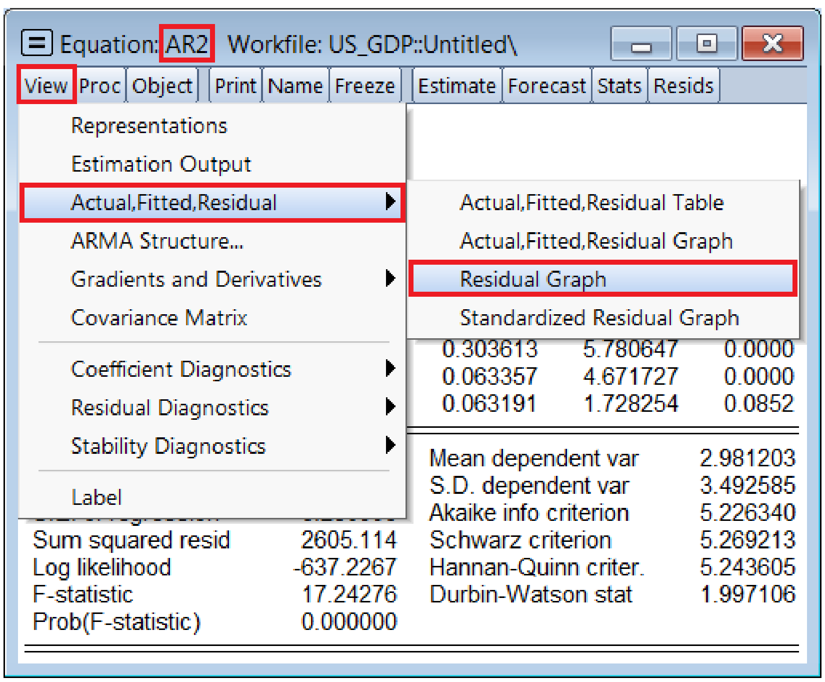
$$cov(u_t, u_{t-j}) \neq 0 \quad for\ j = 1, 2, \ldots$$

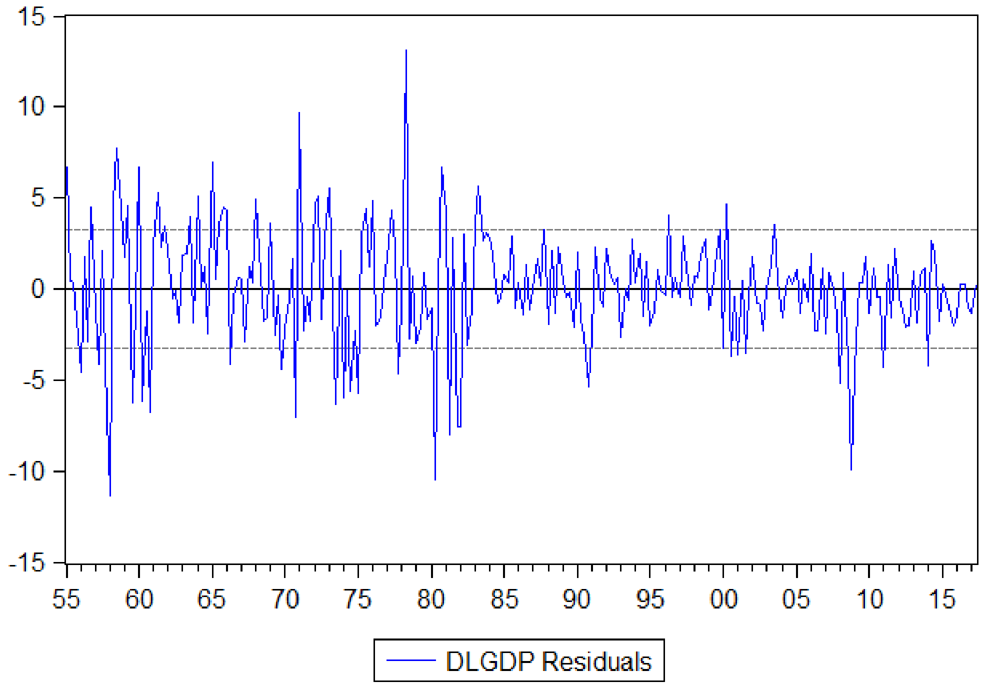
by inspecting the line graph of the OLS residuals.

To obtain a line graph of the residuals from the estimated AR(2) model of annualised quarterly real U.S. GDP growth in EViews,

$$\widehat{dlgdp}_t = \underset{(0.3009)}{1.7427} + \underset{(0.0630)}{0.3069} dlgdp_{t-1} + \underset{(0.0631)}{0.1088} dlgdp_{t-2}$$

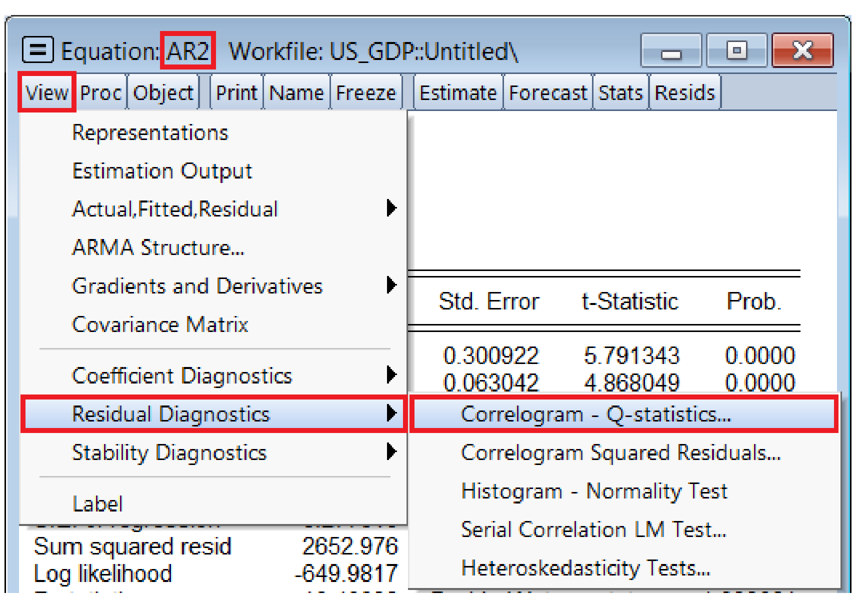
$$AR2 \rightarrow View\ to\ Actual, Fitted, Residual \rightarrow Residual\ Graph$$





We can detect a slight pattern in the evolution of residuals but it does not necessarily persist over time. It is not clear if the errors are serially correlated from visual inspection. The correlogram gives us better information,

$$AR2 \rightarrow View \rightarrow Residual\ Diagnostics \rightarrow Correlogram\ Q - statistics$$



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.008	-0.008	0.0156	0.900
		2	0.006	0.006	0.0250	0.988
		3	-0.009	-0.009	0.0460	0.997
		4	0.008	0.008	0.0633	1.000
		5	-0.094	-0.094	2.3583	0.798
		6	0.033	0.032	2.6359	0.853
		7	-0.012	-0.011	2.6737	0.913
		8	-0.094	-0.097	4.9585	0.762
		9	0.041	0.043	5.4036	0.798
		10	0.083	0.076	7.2014	0.706
		11	0.028	0.032	7.4047	0.765
		12	-0.072	-0.076	8.7784	0.722

and shows no evidence of serial correlation in the errors.

(d) Test for autocorrelation (serially correlation) in the errors of the model,

$$dlgdp_t = \varphi_0 + \varphi_1 dlgdp_{t-1} + \varphi_2 dlgdp_{t-2} + u_t$$

by using the Breusch-Godfrey test (include 8 lags for this test). Discuss your results.

Since our time series data is quarterly, testing up to 8 lags covers two years.

We are testing for serial correlation in u_t of order 8, so we specify a model of $dlgdp$ with errors that follow an AR(8) process.

$$\begin{aligned} dlgdp_t &= \varphi_0 + \varphi_1 dlgdp_{t-1} + \varphi_2 dlgdp_{t-2} + u_t \\ u_t &= \rho_1 u_{t-1} + \rho_2 u_{t-2} + \rho_3 u_{t-3} + \cdots + \rho_7 u_{t-7} + \rho_8 u_{t-8} + e_t \end{aligned}$$

$$\{e_t, t = 1, 2, \ldots, n \sim i.i.d(0, \sigma^2)\}$$

where e_t for $t = 1, 2, \ldots, n$ are independent and identically distributed with mean 0 and variance σ^2

$$\begin{aligned} &H_0 : \rho_1 = \rho_2 = \rho_3 = \cdots = \rho_8 = 0 \\ &H_1 : at\ least\ one\ of\ the\ above\ \rho \neq 0 \end{aligned}$$

To perform the Breusch-Godfrey test for serial correlation in the errors of model of $dlgdp$ at any lag up to and include lag 8 (8th order serial correlation),

- Estimate the model

$$dlgdp_t = \varphi_0 + \varphi_1 dlgdp_{t-1} + \varphi_2 dlgdp_{t-2} + u_t$$

- Save the residuals from the above estimated model

- Then estimate the following *auxiliary regression* ...

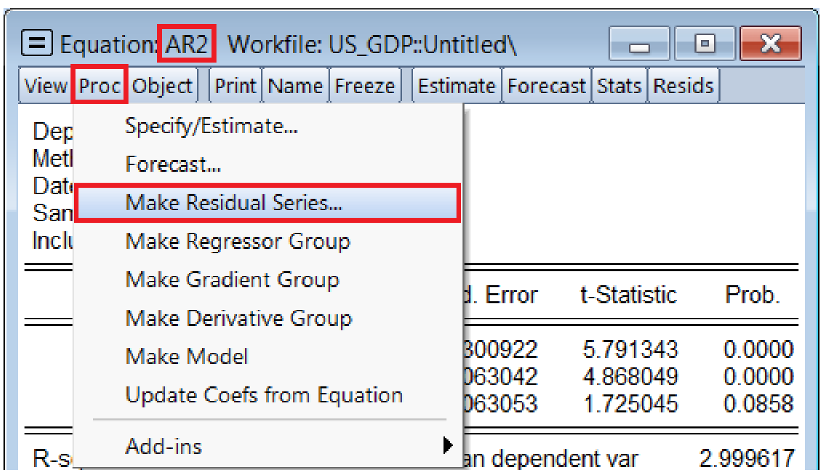
$$\hat{u}_t = \alpha_0 + \alpha_1 dlgdp_{t-1} + \alpha_2 dlgdp_{t-2} + \rho_1 \hat{u}_{t-1} + \rho_2 \hat{u}_{t-2} + \cdots + \rho_8 \hat{u}_{t-8} + v_t$$

- We're testing the null hypothesis that there is no serial correlation in the errors at any lag up to and include lag 8, against the alternative hypothesis that there is serial correlation in the errors in at least one lag up to and including lag 8.

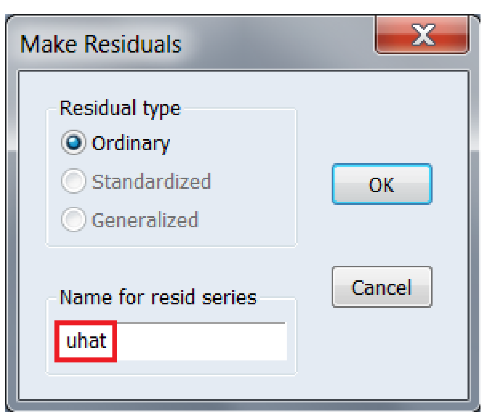
- Compare the calculated test statistics with the critical value and conclude if there is evidence of serial correlation in the errors in at least one lag up to and including lag 8.

Generate the residuals from the estimated model of *dlgdp*,

$$AR2 \rightarrow Proc \rightarrow Make\ Residual\ Series$$



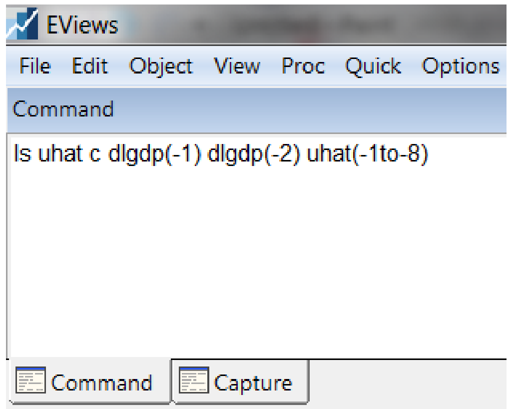
and name the residual series *uhat*,



then estimated the following auxiliary regression,

$$\hat{u}_t = \alpha_0 + \alpha_1 dlgdp_{t-1} + \alpha_2 dlgdp_{t-2} + \rho_1 \hat{u}_{t-1} + \rho_2 \hat{u}_{t-2} + \cdots + \rho_7 \hat{u}_{t-7} + \rho_8 \hat{u}_{t-8} + v_t$$

$$ls\ uhat\ c\ dlgdp(-1)\ dlgdp(-2)\ uhat(-1to-8)$$



Dependent Variable: UHAT
Sample (adjusted): 1957Q1 2017Q2
Included observations: 242 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	12162.92	36065.18	0.337248	0.7362
DLGDP(-1)	−8468.594	25065.25	−0.337862	0.7358
DLGDP(-2)	4390.371	12972.63	0.338433	0.7353
UHAT(-1)	8468.599	25065.24	0.337862	0.7358
UHAT(-2)	−1791.427	5280.311	−0.339265	0.7347
UHAT(-3)	371.3373	1105.837	0.335797	0.7373
UHAT(-4)	−80.86302	234.9714	−0.344140	0.7311
UHAT(-5)	15.46959	48.18367	0.321055	0.7485
UHAT(-6)	−3.979231	10.78031	−0.369120	0.7124
UHAT(-7)	0.436618	1.947329	0.224214	0.8228
UHAT(-8)	−0.392500	0.591407	−0.663671	0.5076

R-squared	0.026798	Mean dependent var	−0.018619
Adjusted R-squared	−0.015331	S.D. dependent var	3.254370
S.E. of regression	3.279223	Akaike info criterion	5.257479
Sum squared resid	2484.012	Schwarz criterion	5.416067
Log likelihood	−625.1549	Hannan-Quinn criter.	5.321364
F-statistic	0.636091	Durbin-Watson stat	1.994378
Prob(F-statistic)	0.782138		

$$(Type\ out\ estimated\ aux)$$

The BG test statistic under the null,

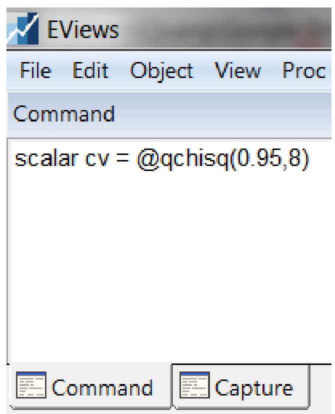
$$BG = (n-8) \times R^2_{\hat{u}} \overset{asy}{\sim} \chi^2_8 \quad under\ H_0$$

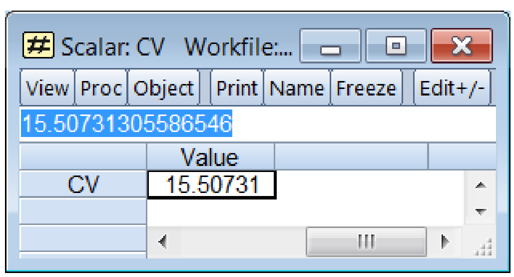
The calculated BG test statistic,

$$BG_{calc} = (250-8) \times 0.02680 = 6.4851$$

The critical value,

$$scalar\ cv = @qchisq(0.95, 8)$$





$$BG_{crit} = \chi^2_{0.95,8} = 15.5073$$

Since $BG_{calc} = 6.4851 < BG_{crit} = 15.5073$ we fail to reject the null and conclude that there is insufficient evidence from the sample to suggest that there is serial correlation in the errors in at least one lag up to and include the 8th lag.

Note: The EViews inbuilt BG test produces different values as it sets the missing values of the lagged residuals to 0 so that there are no missing observations and q becomes 0. The official tutorial solution has used the inbuilt BG test which gives $BG_{calc} = 8.972$.

Question 3

We can use usual econometric tools in a dynamically well-specified model.

Early warning systems are very useful for policy makers. In economics, variables which can give us advanced warning that the economy may be slowing down in 3 months to a year ahead are very useful. Such variables are called "leading indicators". The "interest rate spread",

$$spread = ir\_20y - ir\_3m$$

which is the difference between the long term and short interest rates, is believed to be a leading indicator. When the spread becomes very small or even negative, it means that confidence in the long-term prospects of the economy is low, which warns of a possible low growth period or even a recession ahead.

The data file used in the previous section *US_gdp.wf1* contains quarterly observations on:

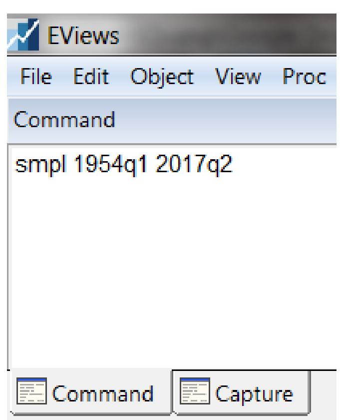
$$gdp - U.S.\ real\ Gross\ Domestic\ Product\ (\$\ billions)$$
$$ir\_3m - U.S.\ 3\text{-}month\ Treasury\ bill\ interest\ rates\ (\%)$$
$$ir\_20y - U.S.\ 20\text{-}year\ Government\ bond\ yields\ (\%)$$

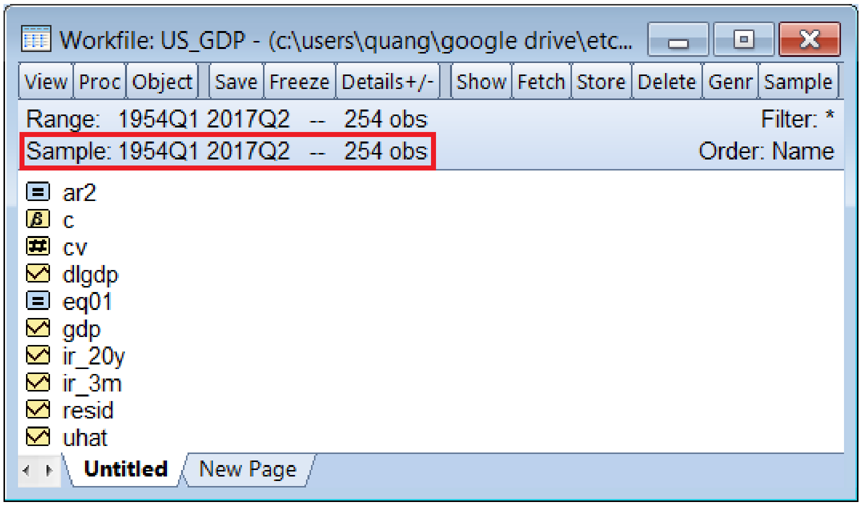
You have already created *dlgdp*.

a) Switch the sample back to 1954Q1 and 2017Q2 and generate a new variable called $spread = ir\_20y - ir\_3m$. Is *spread* white noise? Is it mean-reverting? Does its correlogram suggest that *spread* is stationary?

To change the sample from the Command window,

$$smpl\ 1954Q1\ 2017Q2$$

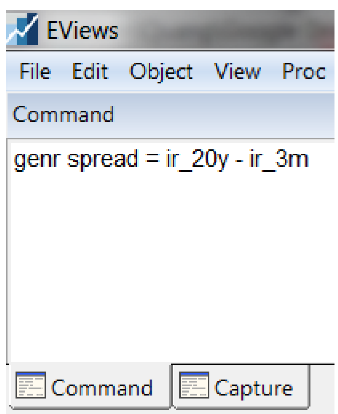


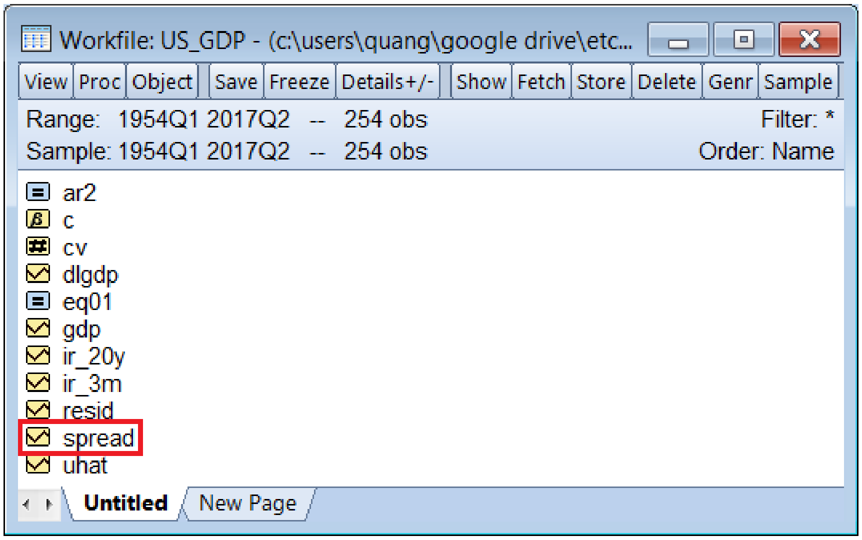


Notice that the sample (but not the range) has changed in the status window. *Do not change the workfile range in the status window.*

To generate the *spread* variable from the Command window,

$$genr\ spread = ir\_20y - ir\_3m$$

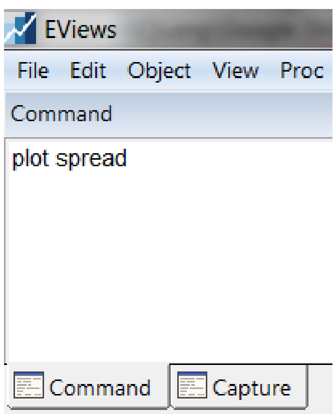


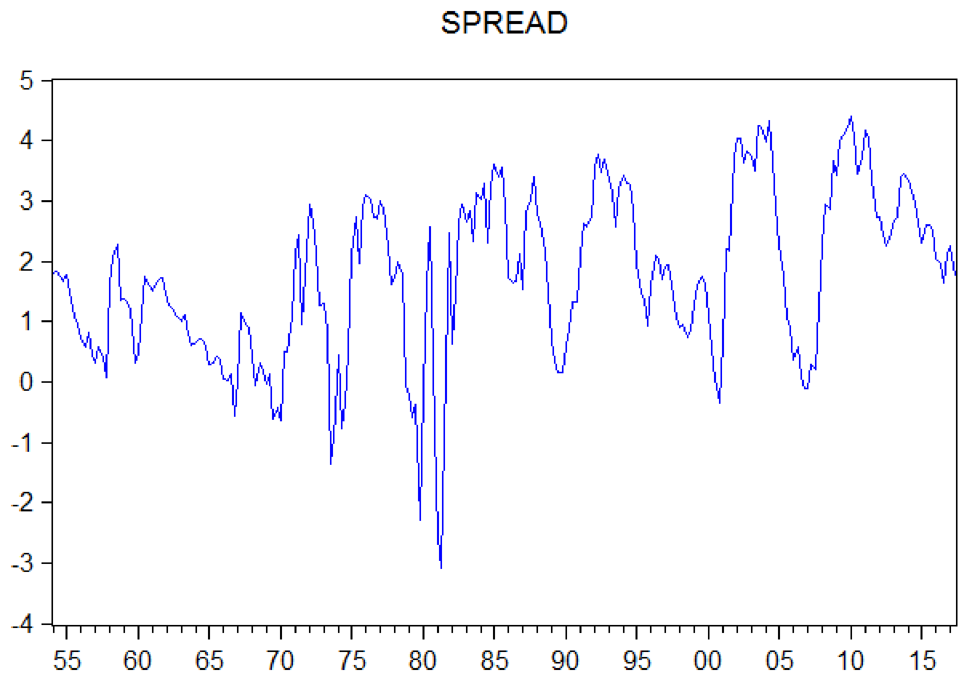


Is *spread* white noise? Is it mean-reverting?

Inspecting the line graph of *spread* can help us determine whether it is white noise and if it is mean-reverting. To obtain a line graph of *spread* from the Command window,

plot spread

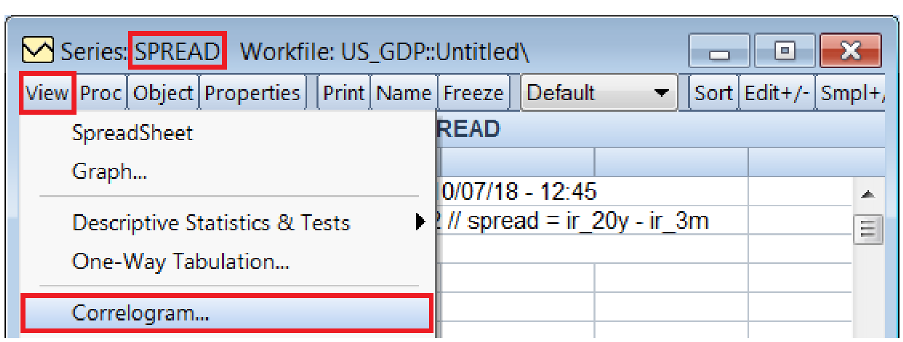


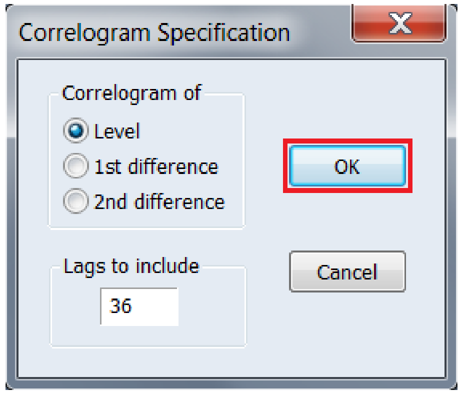


We can see long swings in the term spread which would suggest that it is not white noise. Although the term spread tends to revert back to its mean, it has some persistence (autocorrelation).

From the correlogram of *spread*

$$spread \rightarrow Correlogram$$





Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.875	0.875	196.93	0.000
		2	0.723	-0.183	331.99	0.000
		3	0.606	0.074	427.04	0.000
		4	0.522	0.044	498.00	0.000
		5	0.419	-0.160	543.76	0.000
		6	0.316	-0.013	569.99	0.000
		7	0.236	0.008	584.61	0.000
		8	0.184	0.025	593.61	0.000
		9	0.117	-0.119	597.26	0.000
		10	0.036	-0.074	597.62	0.000
		11	-0.028	0.006	597.83	0.000
		12	-0.059	0.026	598.77	0.000

we find that the sample autocorrelation coefficients of *spread* is statistically significantly different from 0 (up to and including the 8th lag, the sample autocorrelation coefficients lie outside the 2 standard error bands) which indicates that it is not white noise.

Does the correlogram suggest that *spread* is stationary?

Background

Stationary time series process

A stationary time series process has the following properties,

$$E(y_t) = \mu \quad \textit{for all } t \ \Rightarrow \textit{the mean is time invariant}$$
$$Var(y_t) = \gamma_0 \quad \textit{for all } t \ \Rightarrow \textit{the variance is time invariant}$$
$$Cov(y_t, y_{t-j}) = \gamma_j \quad \textit{for all } t \textit{ and } j$$
$$\Downarrow$$

the covariance of y_t and y_{t-j} only depends on the time interval separating them i.e. only j and not t

If any of these stationary properties/assumptions are violated then we have a non-stationary time series.

Random Walk process

A random walk process is an AR(1) process with a unit root i.e. a special case of a unit root process. If the long-term interest rate ir_20y is a random walk process then it will assume the following model,

$$ir\_20y_t = ir\_20y_{t-1} + u_t$$

$$where\ u_t \sim i.i.d(0, \sigma^2)$$

As we can see, it has the same form as an AR(1) model (without a constant) where $\varphi_1 = 1$

$$ir\_20y_t = c + \varphi_1 ir\_20y_{t-1} + u_t$$

$$\downarrow$$

$$ir\_20y_t = \cancel{c} + 1 \times ir\_20y_{t-1} + u_t$$

$$\downarrow$$

$$ir\_20y_t = ir\_20y_{t-1} + u_t$$

If a time series process is a random walk process then it is non-stationary.

From the correlogram of *spread*, we find that the sample autocorrelation coefficient declines exponentially which indicates that *spread* is stationary. If *spread* declines very slowly, then *spread* is said to be a unit root process (a unit root process is non-stationary and has high persistence).

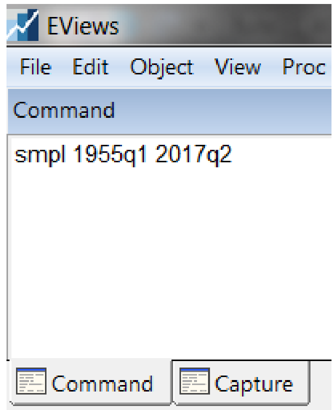
b) In order to investigate the leading indicator property of *spread*, use data from *1955q1 to 2017q2* to estimate the following model (ARDL(2,2)):

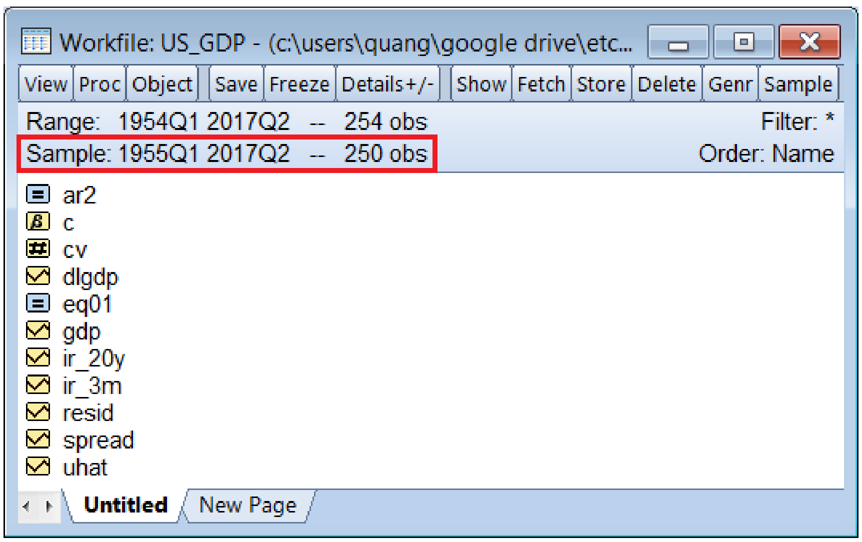
$$dlgdp_t = \beta_0 + \beta_1 dlgdp_{t-1} + \beta_2 dlgdp_{t-2} + \beta_3 spread_{t-1} + \beta_4 spread_{t-2} + u_t$$

and test the joint significance of $spread_{t-1}$ and $spread_{t-2}$ at the 5% level of significance. What are the null and alternative hypothesis? What is the restricted model? What is the test statistics and its distribution under the null? Perform the test and state your conclusion.

To change the sample from the Command window,

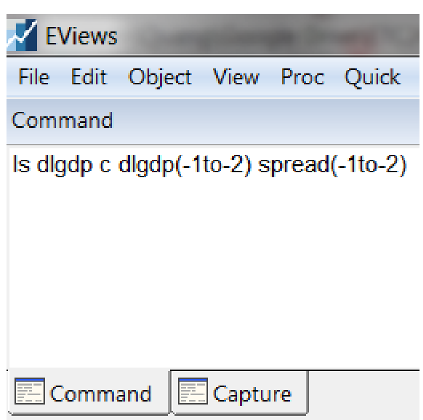
$$smpl\ 1955Q1\ 2017Q2$$





To estimate the ARDL(2,2) model of *dlgdp* from the Command window,

$$ls\ c\ dlgdp(-1to-2)\ spread(-1to-2)$$



Dependent Variable: DLGDP
Method: Least Squares
Sample: 1955Q1 2017Q2
Included observations: 250

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.103416	0.394741	2.795293	0.0056
DLGDP(-1)	0.277960	0.063335	4.388718	0.0000
DLGDP(-2)	0.107821	0.062999	1.711453	0.0883
SPREAD(-1)	−0.122029	0.315459	−0.386831	0.6992
SPREAD(-2)	0.547081	0.317431	1.723466	0.0861

R-squared	0.160183	Mean dependent var	2.999617
Adjusted R-squared	0.146472	S.D. dependent var	3.499670
S.E. of regression	3.233226	Akaike info criterion	5.204635
Sum squared resid	2561.168	Schwarz criterion	5.275064
Log likelihood	−645.5794	Hannan-Quinn criter.	5.232981
F-statistic	11.68257	Durbin-Watson stat	1.987428
Prob(F-statistic)	0.000000		

$$SSR_{ur} = 2561.168$$

The null and alternative hypothesis:

$$H_0 : \beta_3 = \beta_4 = 0$$
$$H_1 : at\ least\ one\ of\ the\ above\ \beta' s\ is\ not\ 0$$

After imposing the restrictions $\beta_3 = \beta_4 = 0$ on the unrestricted model,

$$dlgdp_t = \beta_0 + \beta_1 dlgdp_{t-1} + \beta_2 dlgdp_{t-2} + \beta_3 spread_{t-1} + \beta_4 spread_{t-2} + u_t$$

We get the following restricted model,

$$dlgdp_t = \beta_0 + \beta_1 dlgdp_{t-1} + \beta_2 dlgdp_{t-2} + u_t$$

The estimated restricted model:

Dependent Variable: DLGDP
Method: Least Squares
Sample: 1955Q1 2017Q2
Included observations: 250

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.742741	0.300922	5.791343	0.0000
DLGDP(-1)	0.306892	0.063042	4.868049	0.0000
DLGDP(-2)	0.108769	0.063053	1.725045	0.0858

R-squared	0.130079	Mean dependent var	2.999617
Adjusted R-squared	0.123035	S.D. dependent var	3.499670
S.E. of regression	3.277315	Akaike info criterion	5.223854
Sum squared resid	2652.976	Schwarz criterion	5.266111
Log likelihood	−649.9817	Hannan-Quinn criter.	5.240861
F-statistic	18.46692	Durbin-Watson stat	1.999001
Prob(F-statistic)	0.000000		

$$\widehat{dlgdp}_t = \underset{(0.3009)}{1.7427} + \underset{(0.0630)}{0.3069} dlgdp_{t-1} + \underset{(0.0631)}{0.1088} dlgdp_{t-2} \qquad SSR_r = 2652.976$$

The test statistic and its distribution under H_0

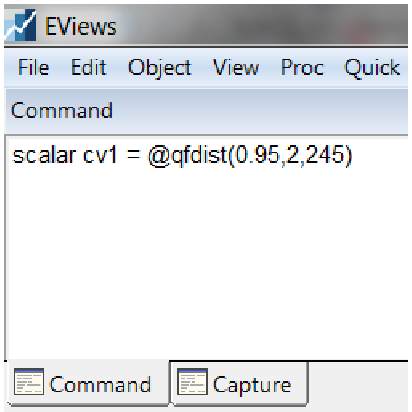
$$F = \frac{(SSR_r - SSR_{ur})/2}{SSR_{ur}/(250-4-1)} \sim F_{2,250-4-1} \quad under\ H_0$$

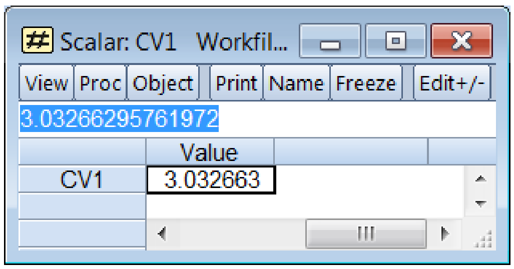
Calculate the test statistic:

$$F_{calc} = \frac{(2652.976 - 2561.168)/2}{2561.168/(245)} = 4.391$$

To obtain the critical value from the Command window in EViews,

$$scalar\ cv1\ = @qfdist(0.95, 2, 245)$$





$$F_{crit} = 3.0327$$

Since $F_{calc} = 4.391 > F_{crit} = 3.0327$ we reject the null at the 5% significance level and conclude that at least one of the *spread* lags is significant in explaining *dlgdp*.

(c) Drop the lag of *spread* that is least significant

Dependent Variable: DLGDP
Method: Least Squares
Sample: 1955Q1 2017Q2
Included observations: 250

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.103416	0.394741	2.795293	0.0056
DLGDP(-1)	0.277960	0.063335	4.388718	0.0000
DLGDP(-2)	0.107821	0.062999	1.711453	0.0883
SPREAD(-1)	−0.122029	0.315459	−0.386831	0.6992
SPREAD(-2)	0.547081	0.317431	1.723466	0.0861

R-squared	0.160183	Mean dependent var	2.999617
Adjusted R-squared	0.146472	S.D. dependent var	3.499670
S.E. of regression	3.233226	Akaike info criterion	5.204635
Sum squared resid	2561.168	Schwarz criterion	5.275064
Log likelihood	−645.5794	Hannan-Quinn criter.	5.232981
F-statistic	11.68257	Durbin-Watson stat	1.987428
Prob(F-statistic)	0.000000		

and re-estimate the equation. Using this estimated equation, explain the dynamic effect of a 1 percentage point *decrease* in the spread between long-term and short-term interest rates on the growth rate. In particular, how long does it take for this change to start affecting the growth rate, and what is the long-run effect of this change on the growth rate?

The least significant lag of spread is $spread_{t-1}$ so we remove this from the model,

$$dlgdp_t = \beta_0 + \beta_1 dlgdp_{t-1} + \beta_2 dlgdp_{t-2} + \cancel{\beta_3 spread_{t-1}} + \beta_4 spread_{t-2} + u_t$$

and re-estimate,

$$ls\ dlgdp\ c\ dlgdp(-1to-2)\ spread(-2)$$

Dependent Variable: DLGDP
Method: Least Squares
Date: 10/07/18 Time: 17:01
Sample: 1955Q1 2017Q2
Included observations: 250

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.058540	0.376656	2.810361	0.0053
DLGDP(-1)	0.281108	0.062701	4.483279	0.0000
DLGDP(-2)	0.111661	0.062105	1.797950	0.0734
SPREAD(-2)	0.438723	0.149062	2.943227	0.0036

R-squared	0.159670	Mean dependent var	2.999617
Adjusted R-squared	0.149422	S.D. dependent var	3.499670
S.E. of regression	3.227633	Akaike info criterion	5.197246
Sum squared resid	2562.733	Schwarz criterion	5.253589
Log likelihood	−645.6557	Hannan-Quinn criter.	5.219922
F-statistic	15.58073	Durbin-Watson stat	1.991805
Prob(F-statistic)	0.000000		

$$\widehat{dlgdp}_t = \underset{(0.3767)}{1.0585} + \underset{(0.0627)}{0.2811}dlgdp_{t-1} + \underset{(0.0621)}{0.1117}dlgdp_{t-2} + \underset{(0.1491)}{0.4387}spread_{t-2}$$

Using this estimated equation, explain the dynamic effect of a 1 percentage point *decrease* in the spread between long-term and short-term interest rates on the growth rate. In particular, how long does it take for this change to start affecting the growth rate, and what is the long-run effect of this change on the growth rate?

Since $spread_t$ is not included in the model of growth rate, the term spread does not have an immediate effect on the US GDP growth rate. Instead, $spread_{t-2}$ is included, so according to our model, the effect of a change in the term spread on the US GDP growth rate is delayed by 2 periods e.g. it takes 2 quarters before a 1 percentage point decrease in the term spread will affect the US GDP growth rate and when it does, GDP growth is expected to decrease by 0.44 percentage points.

In general, the long-run effect on y of a unit increase in x at time t

$$= \frac{sum\ of\ the\ coefficients\ x_t\ and\ its\ lags}{1 - sum\ of\ the\ coefficients\ of\ lags\ of\ y_t}$$

$\therefore$ the *estimated* long-run effect on $dlgdp$ of a 1 percentage point decrease in the term spread (which is a 1 unit decrease since the term spread is a variable whose unit of measurement is in percentage point)

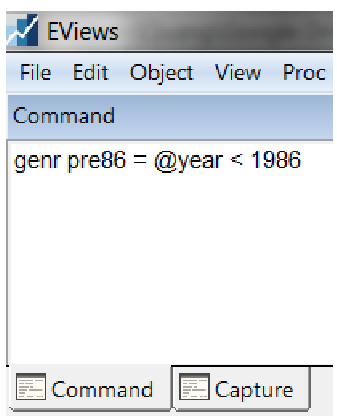
$$
\begin{aligned}
&= \frac{sum\ of\ the\ estimated\ coefficients\ of\ x_t\ and\ its\ lags}{1 - sum\ of\ the\ estimated\ coefficients\ of\ lags\ of\ y_t} \\
&= \frac{\hat{\beta}_{spread_{t-2}}}{1 - (\hat{\beta}_{dlgdp_{t-1}} + \hat{\beta}_{dlgdp_{t-2}})} \\
&= \frac{0.4387}{1 - (0.2811 + 0.1117)} \\
&= 0.723\ percentage\ points
\end{aligned}
$$

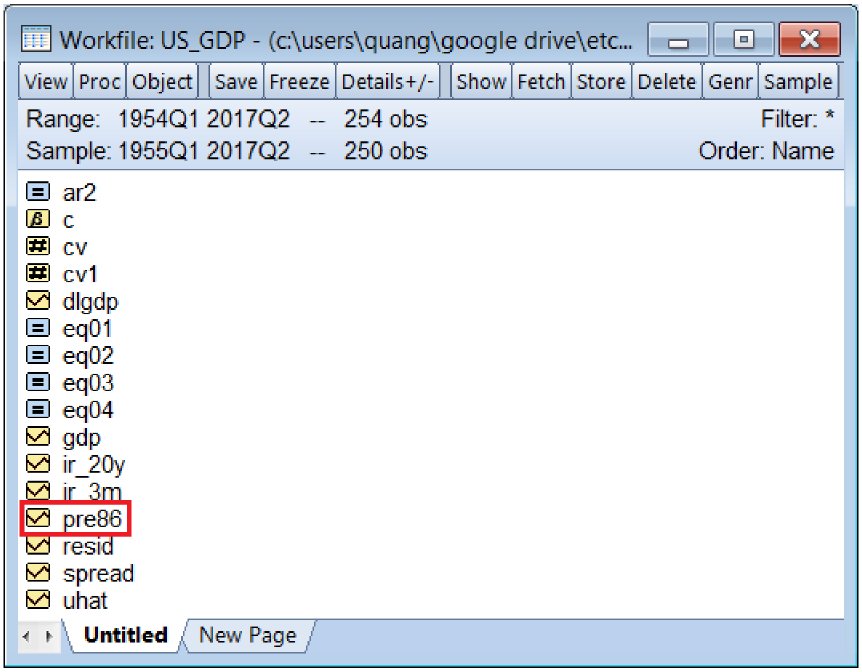
(d) Some economists believe that because Central Banks (the Federal Reserve Bank in the case of the US) have become more sophisticated in implementing monetary policy after the mid-1980s, and the informativeness of the *spread* as a leading indicator has faded. Create an appropriate dummy variable to help you determine that the lag of *spread* has become insignificant since 1986Q1.

To test if $spread_{t-2}$ has become insignificant since 1986Q1, we need to construct the following dummy variable:

$$
pre86 = \begin{cases} 1 & if\ observation\ is\ before\ 1986 \\ 0 & if\ observation\ is\ on\ or\ after\ 1986 \end{cases}
$$

$$
genr\ pre86 = @year < 1986
$$





and then estimate the following model:

$$dlgdp_t = \beta_0 + \beta_1 dlgdp_{t-1} + \beta_2 dlgdp_{t-2} + \beta_3 pre86^* spread_{t-2} + \beta_4 (1 - pre86)^* spread_{t-2} + u_t$$

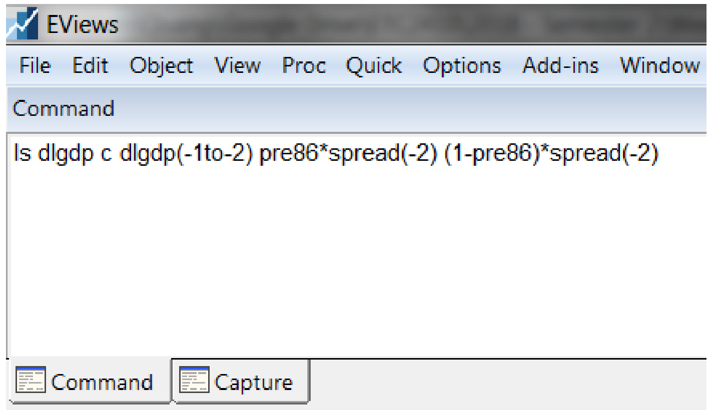
with $spread_{t-2}$ once interacted with $pre86$ and once with $(1 - pre86)$.

- $pre86^* spread_{t-2}$ contains data on the (2nd lag) term spread prior to 1986Q1. i.e. from 1986Q1 onwards, this variable takes on a value of 0, so it only captures information about the term spread pre-1986Q1
- $(1 - pre86)^* spread_{t-2}$ contains data on the (2nd lag) term spread from 1986Q1 onwards i.e. prior to 1986Q1, this variable takes on a value of 0, so it captures information about the term spread post-1986Q1

If the leading indicator of power *spread* has deteriorate then we would expect:

The $pre86^* spread$ to be statistically significant in explaining the US GDP growth rate but $(1 - pre86)^* spread$ to be statistically insignificant in explain the US GDP growth rate.

$$ls\ dlgdp\ c\ dlgdp(-1to-2)\ pre86^* spread(-2)\ (1 - pre86)^* spread(-2)$$



Dependent Variable: DLGDP
Method: Least Squares
Date: 10/07/18 Time: 18:38
Sample: 1955Q1 2017Q2
Included observations: 250

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.211209	0.362702	3.339408	0.0010
DLGDP(-1)	0.203281	0.062346	3.260521	0.0013
DLGDP(-2)	0.078950	0.059966	1.316575	0.1892
PRE86*SPREAD(-2)	1.213964	0.217398	5.584054	0.0000
(1-PRE86)*SPREAD(-2)	0.228862	0.149687	1.528941	0.1276

R-squared	0.230086	Mean dependent var	2.999617
Adjusted R-squared	0.217516	S.D. dependent var	3.499670
S.E. of regression	3.095743	Akaike info criterion	5.117730
Sum squared resid	2347.988	Schwarz criterion	5.188159
Log likelihood	−634.7163	Hannan-Quinn criter.	5.146076
F-statistic	18.30431	Durbin-Watson stat	1.999856
Prob(F-statistic)	0.000000		

As we can see from the regression output, the term spread has become much less relevant as a leading indicator since monetary policies implementation have been more sophisticated.