Tutorial 7

keywords: level log interpretation, dummy variables, hypothesis test, F-test, t-test, p-value, overall significance, individual significance, multiple linear restrictions, reparameterisation

estimated reading time: 35 minutes

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April 16, 2018

Question 1

EViews workfile: *vote1.wf1*

vote1.wf1 contains data on election outcomes and campaign expenditures for 173 two-party competitive races (Democrats and Republicans) for the House of Representatives in 1988. Information about each two-party competitive race is held in the following variables:

$$\begin{aligned} votea &- \%vote\ received\ by\ Candidate\ A \\ expenda &- Candidate\ A's\ campaign\ expenditure\ (\$'000) \\ expendb &- Candidate\ B's\ campaign\ expenditure\ (\$'000) \\ democa &- 1\ if\ Candidate\ A\ was\ a\ democrat,\ 0\ otherwise\ (dummy\ variable) \end{aligned}$$

Candidate A is chosen to be the candidate whose last name is alphabetically highest.

VOTEA	EXPENDA	EXPENDB	DEMOCA
68	328.296	8.737	1
62	626.377	402.477	0
73	99.607	3.065	1
69	319.69	26.281	0
75	159.221	60.054	0
69	570.155	21.393	1
59	696.748	193.915	0
71	638.688	7.695	1
76	616.936	19.245	1
73	351.687	50.532	1
68	269.887	14.71	1
71	269.51	95.575	1
52	1440.639	1089.57	0
79	252.336	69.563	1
50	1470.674	1548.193	0
64	140.486	100.956	1
72	191.334	15.449	1
68	398.597	15.239	1
60	460.622	382.111	1
67	457.41	20.608	1

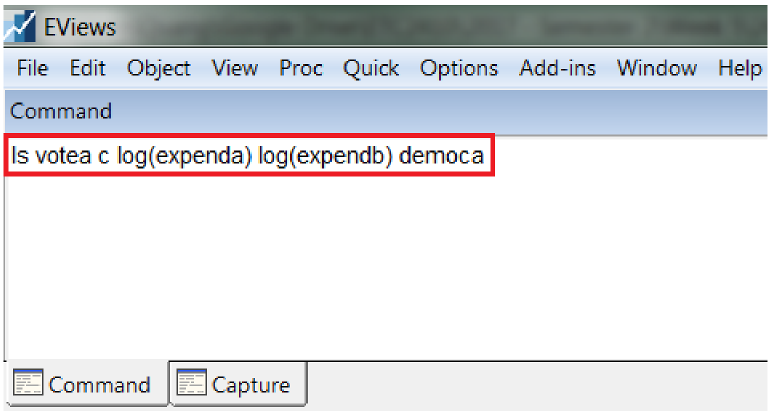
Table 1: Sample of the first 20 two-party competitive race.

Run a regression of *votea* on a constant, $log(expenda)$, $log(expendb)$ and *democa*

$$votea = \beta_0 + \beta_1 log(expenda) + \beta_2 log(expendb) + \beta_3 democa + u$$

To estimate this model from the Command window,

$$ls\ votea\ c\ log(expenda)\ log(expendb)\ democa$$

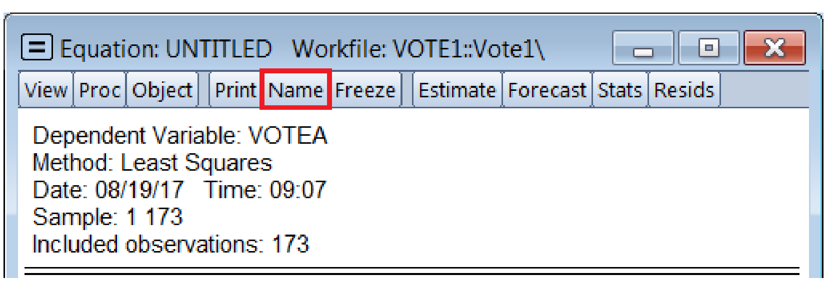


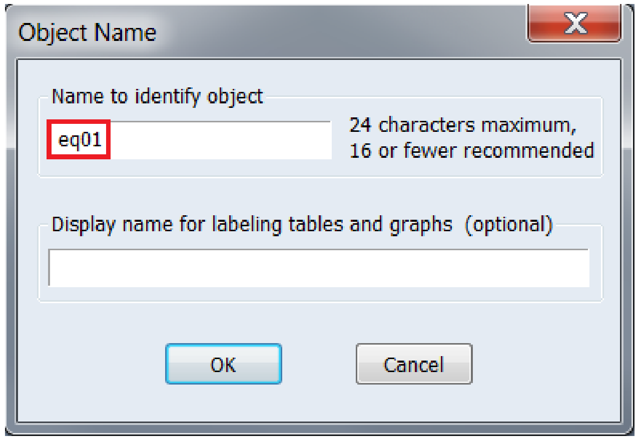
$$(Press\ Enter\ to\ execute\ code)$$

To name (save) the estimated equation,

$$Name \rightarrow Name\ to\ identify\ object: eq01$$

$$(This\ names\ the\ equation\ \boldsymbol{eq01})$$





Dependent Variable: VOTEA
Method: Least Squares
Date: 08/19/17 Time: 09:07
Sample: 1 173
Included observations: 173

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	51.13410	2.903327	17.61224	0.0000
LOG(EXPENDA)	6.299279	0.375274	16.78582	0.0000
LOG(EXPENDB)	−6.666045	0.391187	−17.04054	0.0000
DEMOCA	1.208824	1.241612	0.973593	0.3317

R-squared	0.786385	Mean dependent var	50.50289
Adjusted R-squared	0.782593	S.D. dependent var	16.78476
S.E. of regression	7.826209	Akaike info criterion	6.975683
Sum squared resid	10351.17	Schwarz criterion	7.048592
Log likelihood	−599.3966	Hannan-Quinn criter.	7.005262
F-statistic	207.3816	Durbin-Watson stat	1.652138
Prob(F-statistic)	0.000000		

Table 2: Regression output of *votea* on a constant, *log*(*expenda*), *log*(*expendb*) and *democa*.

$$\widehat{votea} = \underset{(2.9033)}{51.1341} + \underset{(0.3753)}{6.2993}log(expenda) - \underset{(0.3912)}{6.6660}log(expendb) + \underset{(1.2416)}{1.2088}democa$$

$$R^2 = 0.7864$$

(a) Interpreting the regression results when explanatory variables are logarithms of the original variables and also interpreting the coefficient of dummy variables.

Explain what each parameter estimate shows.

Background

Logarithms for approximating percentage change

Since,
$$\frac{dlog(x)}{dx} = \frac{1}{x}$$
replacing infinitesimally small change d with finite change Δ gives the approximation of $\frac{1}{x}$,
$$\frac{\Delta log(x)}{\Delta x} \approx \frac{1}{x}$$
and by multiplying Δx on both sides, we have the approximate proportional change in x,
$$\Delta log(x) \approx \frac{\Delta x}{x}$$
Multiplying 100 on both sides gives us the approximate percentage change in x,
$$\begin{aligned} 100\Delta log(x) &\approx 100\frac{\Delta x}{x} \\ &= \%\Delta x \end{aligned}$$

Since the approximation comes from replacing infinitesimally small change d with finite change Δ, this means that when the finite change Δ is large, the approximation will be less precise.

Level-log interpretation

For the following estimated model, which is level in the dependent variable and log in x_1,
$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 log(x_1) + \hat{\beta}_2 x_2$$
the change in $\hat{y}$ depends on change in $log(x_1)$ and x_2,
$$\Delta\hat{y} = \hat{\beta}_1 \Delta log(x_1) + \hat{\beta}_2 \Delta x_2$$
Multiplying $\Delta log(x_1)$ with $\frac{100}{100}$ gives us,
$$\Delta\hat{y} = \hat{\beta}_1 \frac{100}{100} \Delta log(x_1) + \hat{\beta}_2 \Delta x_2$$

$$\Delta \hat{y} = \hat{\beta}_1 \frac{100}{100} \Delta log(x_1) + \hat{\beta}_2 \Delta x_2$$
$$= \frac{\hat{\beta}_1}{100} 100 \Delta log(x_1) + \hat{\beta}_2 \Delta x_2$$
$$= \frac{\hat{\beta}_1}{100} \% \Delta x_1 + \hat{\beta}_2 \Delta x_2$$

$\therefore$ the model estimates that if x_2 is held constant and x_1 increases by 1%,

$$\Delta x_2 = 0$$

$$\% \Delta x_1 = 1$$

on average, y is estimated to change by $\frac{\hat{\beta}_1}{100}$,

$$\Delta \hat{y} = \frac{\hat{\beta}_1}{100} \% \Delta x_1 + \hat{\beta}_2 \Delta x_2$$
$$= \frac{\hat{\beta}_1}{100} \times 1 + \hat{\beta}_2 \times 0$$
$$= \frac{\hat{\beta}_1}{100}$$

(When x_1 increases by 1%, holding x_2 constant, $\hat{y}$ is expected to change by $\frac{\hat{\beta}_1}{100}$ i.e. $\Delta \hat{y} = \frac{\hat{\beta}_1}{100}$.)

Model	Dependent Variable	Independent Variable	Interpretation (population model)	Interpretation (estimated model)
$level - level$	y	x	$\Delta y = \beta_1 \Delta x$	$\Delta \hat{y} = \hat{\beta}_1 \Delta x$
$level - log$	y	$log(x)$	$\Delta y = \left(\frac{\beta_1}{100}\right) \% \Delta x$	$\Delta \hat{y} = \left(\frac{\hat{\beta}_1}{100}\right) \% \Delta x$
$log - level$	$log(y)$	x	$\% \Delta y = (100\beta_1) \Delta x$	$\% \Delta \hat{y} = (100\hat{\beta}_1) \Delta x$
$log - log$	$log(y)$	$log(x)$	$\% \Delta y = \beta_1 \% \Delta x$	$\% \Delta \hat{y} = \hat{\beta}_1 \% \Delta x$

$$\widehat{votea} = \underset{(2.9033)}{51.1341} + \underset{(0.3753)}{6.2993} log(expenda) - \underset{(0.3912)}{6.6660} log(expendb) + \underset{(1.2416)}{1.2088} democa$$

$\hat{\beta}_1 = 6.2993$

The model estimates that regardless of Candidate A's political affiliation (holding *democa* constant) and when there is no change in Candidate B's expenditure (holding *expendb* constant),

$$\Delta log(expendb) = 0$$

$$\Delta democa = 0$$

a 1% increase in Candidate A's expenditure,

$$\%\Delta expenda = 1\%$$

is expected to increase the share of votes received by Candidate A by 0.063 <u>percentage points</u>,

$$\begin{aligned}
\Delta\widehat{votea} &= \hat{\beta}_1 \Delta log(expenda) + \hat{\beta}_2 \Delta log(expendb) + \hat{\beta}_3 \Delta democa \\
&= \hat{\beta}_1 \Delta log(expenda) + \hat{\beta}_2 \times 0 + \hat{\beta}_3 \times 0 \\
&= \hat{\beta}_1 \Delta log(expenda) \\
&= \frac{\hat{\beta}_1}{100} \%\Delta expenda \\
&= \frac{6.2993}{100} \times 1 = 0.063
\end{aligned}$$

$\hat{\beta}_2 = -6.6660$

$$\Delta\widehat{votea} = \frac{\hat{\beta}_2}{100} \%\Delta expendb$$

$$(\textit{holding expendb and democa constant})$$

The model estimates that for a 1% increase in Candidate B's expenditure,

$$\%\Delta expendb = 1\%$$

the share of votes received by Candidate A is expected to decrease by 0.0667 <u>percentage points</u>,

$$\Delta\widehat{votea} = \frac{-6.6660}{100} \times 1 = -0.0667$$

regardless of Candidate A's political affiliation (holding *democa* constant) and when there is no change in Candidate A's expenditure (holding *expenda* constant).

Background

Dummy variable interpretation

For the following estimated model,

$$\widehat{votea} = \hat{\beta}_0 + \hat{\beta}_1 log(expenda) + \hat{\beta}_2 log(expendb) + \hat{\beta}_3 democa$$

where *democa* is a dummy variable,

$$democa = \begin{cases} 1 & if\ Candidate\ A\ was\ a\ democrat \\ 0 & otherwise \end{cases}$$

The change in $\widehat{votea}$ depends on the change in $log(expenda)$, $log(expendb)$, and *democa*,

$$\Delta\widehat{votea} = \hat{\beta}_1 \Delta log(expenda) + \hat{\beta}_2 \Delta log(expendb) + \hat{\beta}_3 \Delta democa$$

$\therefore$ the model estimates that when Candidate A and B's campaign expenditure are held constant,

$$\Delta log(expenda) = 0$$

$$\Delta log(expenda) = 0$$

the share of votes received by Candidate A if Candidate A is from the Democratic party,

$$\Delta democa = 1$$

is expected to be $\hat{\beta}_3$ percentage points higher than Candidate B,

$$\begin{aligned} \Delta\widehat{votea} &= \hat{\beta}_1 \times 0 + \hat{\beta}_2 \times 0 + \hat{\beta}_3 \times 1 \\ &= \hat{\beta}_3 \end{aligned}$$

(The model estimates that when Candidate A and B's campaign expenditure are held constant, the share of votes received by Candidate A if Candidate A is from the Democratic party is expected to be $\hat{\beta}_3$ percentage points higher than Candidate B.)

$\hat{\beta}_3 = 1.2088$

Controlling for both candidate's campaign expenditure,

(b) Test the overall significance of a regression.

Test the overall significance of the model at the 1% significance level (ignore the fact that EViews produces the F statistic, compute it using R^2). Explain in words the hypothesis that you are testing.

$$votea = \beta_0 + \beta_1 log(expenda) + \beta_2 log(expendb) + \beta_3 democa + u$$

A test of overall significance is a test of whether the regressors in our model jointly help explain the dependent variable. If none of the regressors jointly help to explain *votea* then,

$$\beta_1 = \beta_2 = \beta_3 = 0$$

but if at least one of the regresors helps to explain *votea* then,

$$at\ least\ one\ of\ \beta_1, \beta_2, \beta_3\ is\ not\ 0$$

State the null and alternative hypothesis

The test statistic and its distribution under H_0

Calculate the test statistic

Dependent Variable: VOTEA
Method: Least Squares
Date: 08/19/17 Time: 09:07
Sample: 1 173
Included observations: 173

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	51.13410	2.903327	17.61224	0.0000
LOG(EXPENDA)	6.299279	0.375274	16.78582	0.0000
LOG(EXPENDB)	−6.666045	0.391187	−17.04054	0.0000
DEMOCA	1.208824	1.241612	0.973593	0.3317

R-squared	$\boxed{0.786385}$	Mean dependent var	50.50289
Adjusted R-squared	0.782593	S.D. dependent var	16.78476
S.E. of regression	7.826209	Akaike info criterion	6.975683
Sum squared resid	10351.17	Schwarz criterion	7.048592
Log likelihood	−599.3966	Hannan-Quinn criter.	7.005262
F-statistic	$\boxed{207.3816}$	Durbin-Watson stat	1.652138
Prob(F-statistic)	0.000000		

Critical value and rejection region

$1\%\ significance\ level\ \rightarrow \alpha = 0.01$

To obtain the critical value using the Stats Table, locate the F distribution table at the 1% significance level,

$$Numerator\ d.o.f = 3$$

$$Denominator\ d.o.f = 169$$

Since 169 is not in the table, we take a conservative approach and choose the closest available degrees of freedom less than 169 i.e. $d.o.f = 120$.

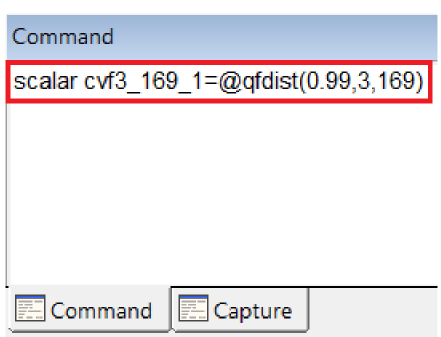
TABLE G.3c 1% Critical Values of the *F* Distribution

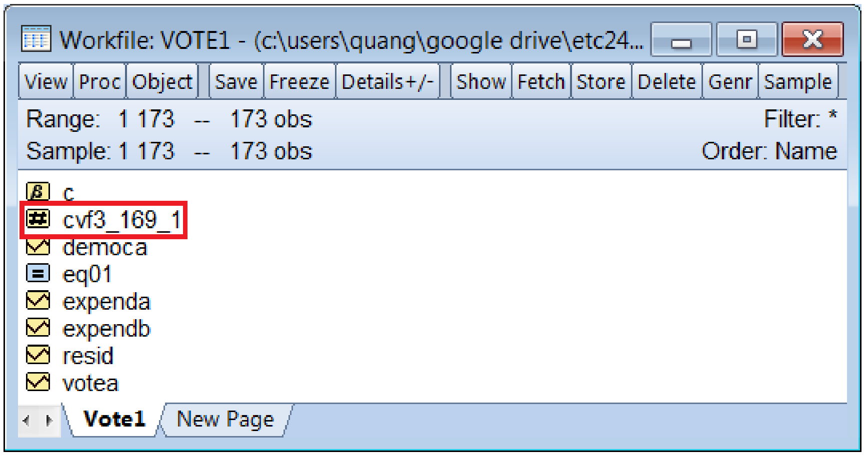
Denominator Degrees of Freedom	Numerator Degrees of Freedom: 1	2	3	4	5	6	7	8	9	10
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
90	6.93	4.85	4.01	3.54	3.23	3.01	2.84	2.72	2.61	2.52
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32

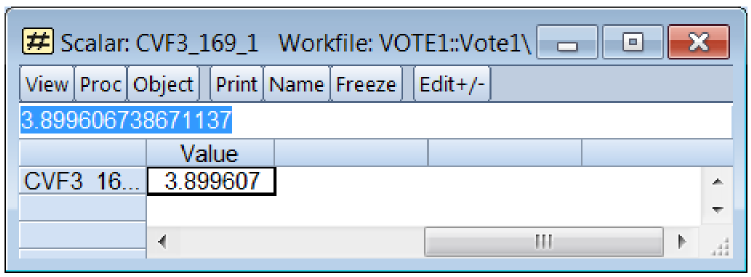
Example: The 1% critical value for numerator $df = 3$ and denominator $df = 60$ is 4.13.
Source: This table was generated using the Stata® function invFtail.

To obtain the critical value using EViews,

$$Command\ window:\ \ scalar\ cvf3\_169\_1 = @qfdist(0.99, 3, 169)$$







$$F_{crit}\ (from\ Stat\ Table) = 3.95$$

$$F_{crit}\ (from\ EViews) = 3.8996$$

Rejection rule:

Comparing the calculated test statistic with the critical value, we reject H_0 if,

$$F_{calc} > F_{crit}$$

Conclusion

(c) Test of significance of an explanatory variable.

Test the hypothesis that controlling for campaign expenditure, being a democratic candidate is not significant in predicting the % of vote received in competitive races at the 5% level of significance. Perform that test by two methods: (i) comparing the t statistic with an appropriate critical value, and (ii) using the p-value.

If after controlling for campaign expenditure, being a democratic candidate is not significant in predicting the % of vote received then,

$$votea = \beta_0 + \beta_1 log(expenda) + \beta_2 log(expendb) + \cancel{\beta_3 democa} + u$$

that is,

$$\beta_3 = 0$$

but if it does then,

$$\beta_3 \neq 0$$

State the null and alternative hypothesis

$$H_0 : \beta_3 = 0$$
$$H_1 : \beta_3 \neq 0$$

The test statistic and its distribution under H_0

$$t = \frac{\hat{\beta}_3 - \beta_3}{se(\hat{\beta}_3)} = \frac{\hat{\beta}_3}{se(\hat{\beta}_3)} \sim t_{n-k-1} \quad under\ H_0$$

$$n = sample\ size = 173$$
$$k = number\ of\ regressors\ in\ the\ model = 3$$

Calculate the test statistic

$$t_{calc} = \frac{1.208824}{1.241612} = 0.9736$$

Dependent Variable: VOTEA
Method: Least Squares
Date: 08/19/17 Time: 09:07
Sample: 1 173
Included observations: 173

Variable	Coefficient	Std. Error	t-Statistic	Prob.
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Sum squared resid	10351.17	Schwarz criterion	7.048592
Log likelihood	−599.3966	Hannan-Quinn criter.	7.005262
F-statistic	207.3816	Durbin-Watson stat	1.652138
Prob(F-statistic)	0.000000		

Note: The $t - Statistics$ from the EViews regression output are t_{calc} values for a two-sided t-test to test if a regressor has a statistically significant on the dependent variable, holding the other regressors constant.

p-value from regression output

Dependent Variable: VOTEA
Method: Least Squares
Date: 08/19/17 Time: 09:07
Sample: 1 173
Included observations: 173

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	51.13410	2.903327	17.61224	0.0000
LOG(EXPENDA)	6.299279	0.375274	16.78582	0.0000
LOG(EXPENDB)	−6.666045	0.391187	−17.04054	0.0000
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Log likelihood	−599.3966	Hannan-Quinn criter.	7.005262
F-statistic	207.3816	Durbin-Watson stat	1.652138
Prob(F-statistic)	0.000000		

$$p - value = 0.3317$$

Note: The *Prob.* values from the EViews regression output are p-values for a two-sided t-test to test if a regressor has a statistically significant on the dependent variable, holding the other regressors constant.

Critical value and rejection region

$5\%\ significance\ level\ \rightarrow \alpha = 0.05$

To obtain the critical value using the Stats Table, locate the t distribution table,

$$degrees\ of\ freedom = 169$$

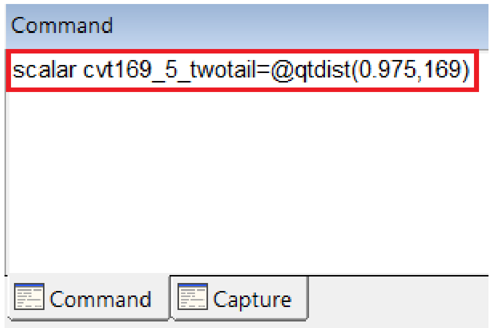
Since 169 is not in the table, we take a conservative approach and choose the closest available degrees of freedom less than 169 i.e. $d.o.f = 120$.

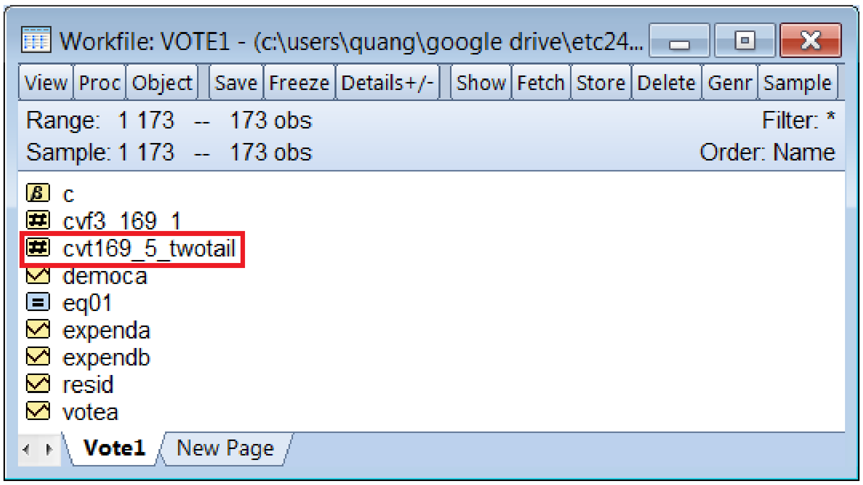
TABLE G.2 Critical Values of the t Distribution

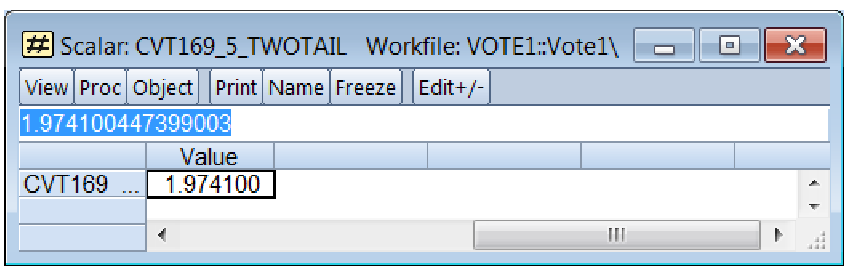
	Significance Level					
1-Tailed:	.10	.05	.025	.01	.005
2-Tailed:	.20	.10	$\boxed{.05}$	.02	.01
Degrees of Freedom					
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
90	1.291	1.662	1.987	2.368	2.632
$\boxed{120}$	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

To obtain the critical value using EViews,

$$Command\ window:\ \ scalar\ cvt169\_5\_twotail = @qtdist(0.975, 169)$$







From Stat Table:

$$+t_{crit} = 1.980$$
$$-t_{crit} = -1.980$$

From EViews:

$$+t_{crit} = 1.9751$$
$$-t_{crit} = -1.9751$$

Rejection rule:

Comparing the calculated test statistic with the critical value, we reject H_0 if,

$$t_{calc} > +t_{crit}$$

$$or$$

$$t_{calc} < -t_{crit}$$

Comparing the p-value with the significance level, we reject H_0 if,

$$p - value < \alpha = 0.05$$

Conclusion

Since $p - value = 0.3317 > \alpha = 0.05$, we do not reject the null at the 5% significance level and conclude that there is insufficient evidence from our sample to suggest that Candidate A's political affiliation is statistically significant in explaining the share of votes received by Candidate A, holding campaign expenditure constant.

(d) Joint test of multiple linear restrictions.

Test the joint hypothesis that controlling for campaign expenditure, being a democratic candidate does not contribute to the % vote received and that the effect (on *votea*) of every percentage increase in campaign expenditure by Candidate A can be offset exactly by the same percentage increase in the opponent's campaign expenditure. Perform this test at the 5% level of significance.

From the statement,

> "..controlling for campaign expenditure, being a democratic candidate does not contribute to the % of vote received.."

we have our first linear restriction,

$$\beta_3 = 0$$

From the statement,

> "..the effect (on *votea*) of every percentage increase in campaign expenditure by Candidate A can be offset exactly by the same percentage increase in the opponent's campaign expenditure.."

we have our second linear restriction,

$$\beta_1 + \beta_2 = 0$$

$$\beta_2 = -\beta_1$$

It tells us that the effect on *votea* when both Candidate's campaign expenditure increases by the same percentage equals to 0.

Unrestricted model (the model before imposing restrictions):

$$votea = \beta_0 + \beta_1 log(expenda) + \beta_2 log(expendb) + \beta_3 democa + u$$

Restricted model (the model after imposing restrictions):

$$votea = \beta_0 + \beta_1 log(expenda) - \beta_1 log(expendb) + 0 \times democa + u$$
$$votea = \beta_0 + \beta_1 (log(expenda) - log(expendb)) + u$$

State the null and alternative hypothesis

The test statistic and its distribution under H_0

$$\begin{aligned}
n &= sample\ size = 173 \\
k &= number\ of\ regressors\ in\ the\ unrestricted\ model = 3 \\
q &= number\ of\ restrictions\ \ = 2 \\
SSR_r &= sum\ of\ squared\ residuals\ from\ estimated\ restricted\ model \\
SSR_{ur} &= sum\ of\ squared\ residuals\ from\ estimated\ unrestricted\ model
\end{aligned}$$

Calculate the test statistic

From the regression output of the estimated unrestricted model,

$$SSR_{ur} = 10351.17$$

Dependent Variable: VOTEA
Method: Least Squares
Date: 08/19/17 Time: 09:07
Sample: 1 173
Included observations: 173

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	51.13410	2.903327	17.61224	0.0000
LOG(EXPENDA)	6.299279	0.375274	16.78582	0.0000
LOG(EXPENDB)	−6.666045	0.391187	−17.04054	0.0000
DEMOCA	1.208824	1.241612	0.973593	0.3317

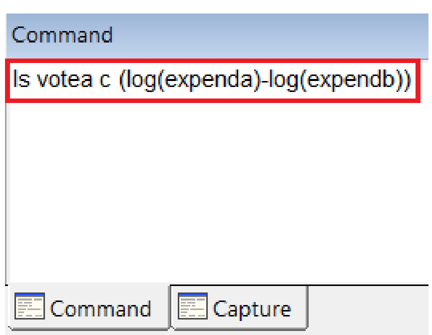
R-squared	0.786385	Mean dependent var	50.50289
Adjusted R-squared	0.782593	S.D. dependent var	16.78476
S.E. of regression	7.826209	Akaike info criterion	6.975683
Sum squared resid	10351.17	Schwarz criterion	7.048592
Log likelihood	−599.3966	Hannan-Quinn criter.	7.005262
F-statistic	207.3816	Durbin-Watson stat	1.652138
Prob(F-statistic)	0.000000		

To obtain SSR_r, we need to estimate the restricted model,

$$votea = \beta_0 + \beta_1(log(expenda) - log(expendb)) + u$$

To estimate this model from the Command window,

$$ls\ votea\ c\ (log(expenda) - log(expendb))$$

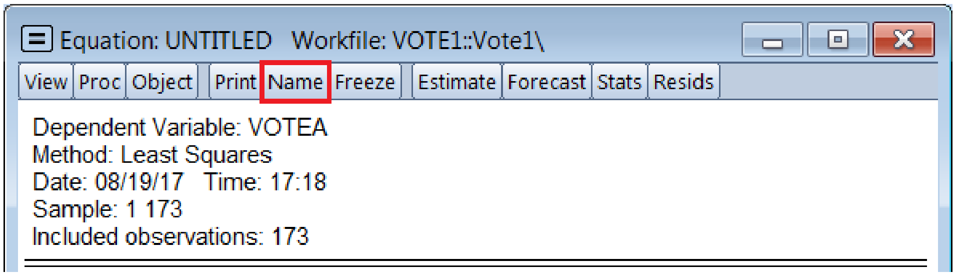


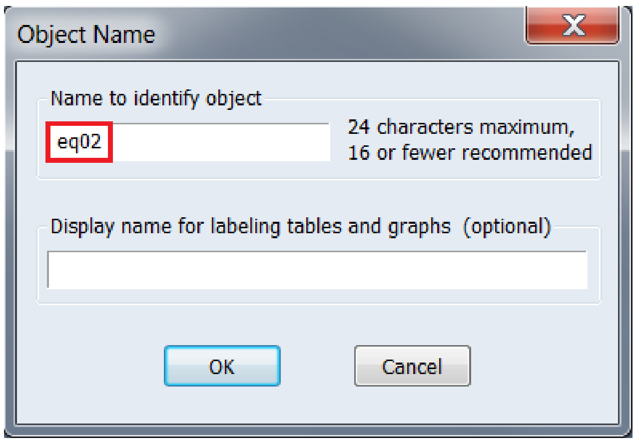
$(Press\ Enter\ to\ execute\ code)$

To name (save) the estimated equation,

$$Name \rightarrow Name\ to\ identify\ object : eq02$$

$$(This\ names\ the\ equation\ \boldsymbol{eq02})$$





Dependent Variable: VOTEA
Method: Least Squares
Date: 08/19/17 Time: 17:18
Sample: 1 173
Included observations: 173

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	49.97148	0.594598	84.04253	0.0000
LOG(EXPENDA)-LOG(EXPENDB)	6.545465	0.262391	24.94545	0.0000
R-squared	0.784438	Mean dependent var		50.50289
Adjusted R-squared	0.783178	S.D. dependent var		16.78476
S.E. of regression	7.815690	Akaike info criterion		6.961637
Sum squared resid	10445.54	Schwarz criterion		6.998091
Log likelihood	−600.1816	Hannan-Quinn criter.		6.976426
F-statistic	622.2757	Durbin-Watson stat		1.657779
Prob(F-statistic)	0.000000			

Table 3: Regression output of $votea$ on a constant and $log(expenda) - log(expendb)$.

$$SSR_r = 10445.54$$

$$F_{calc} =$$

Critical value and rejection region

$5\%\ significance\ level\ \rightarrow \alpha = 0.05$

To obtain the critical value using the Stats Table, locate the F distribution table at the 5% significance level,

$$Numerator\ d.o.f = 2$$

$$Denominator\ d.o.f = 169$$

Since 169 is not in the table, we take a conservative approach and choose the closest available degrees of freedom less than 169 i.e. $d.o.f = 120$.

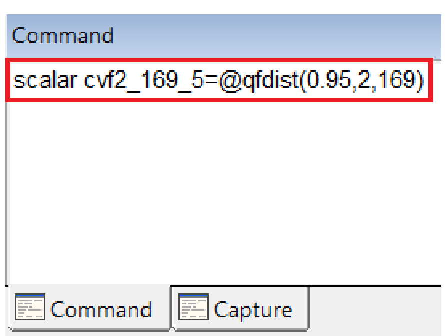
TABLE G.3b 5% Critical Values of the F Distribution

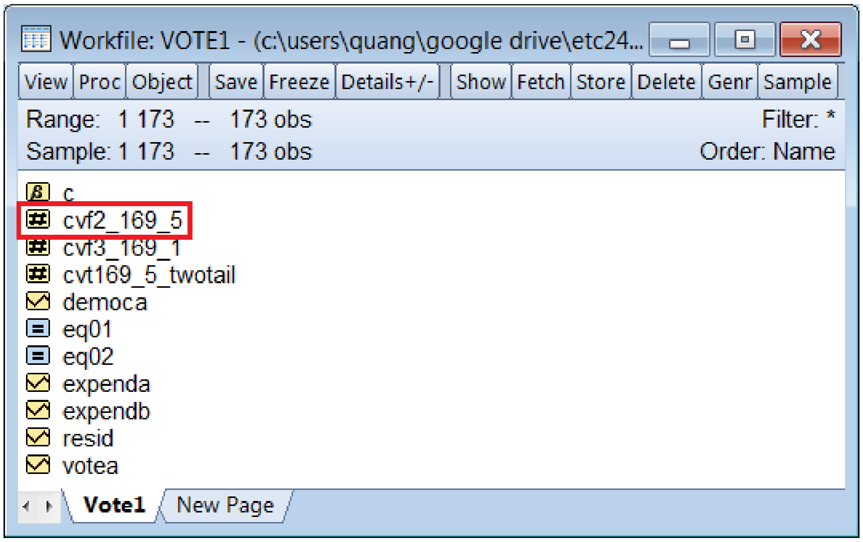
Denominator Degrees of Freedom	Numerator Degrees of Freedom: 1	2	3	4	5	6	7	8	9	10
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

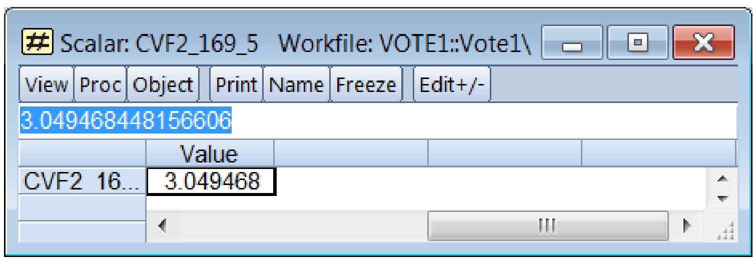
Example: The 5% critical value for numerator $df = 4$ and large denominator $df(\infty)$ is 2.37.
Source: This table was generated using the Stata® function invFtail.

To obtain the critical value using EViews,

$$Command\ window:\ \ scalar\ cvf2\_169\_5 = @qfdist(0.95, 2, 169)$$







$$F_{crit}\ (from\ Stat\ Table) = 3.07$$

$$F_{crit}\ (from\ EViews) = 3.049$$

Rejection rule:

Comparing the calculated test statistic with the critical value, we reject H_0 if,

$$F_{calc} > F_{crit}$$

Conclusion

Since $F_{calc} = 0.77 < F_{crit} = 3.95$, we do not reject the null at the 5% significance level and conclude that there is insufficient evidence to reject the assumption that being a democratic candidate has no impact on share of votes holding campaign expenditure constant **and** that the effect on share of votes received by Candidate A from a percentage increase in Candidate A's campaign expenditure can be offset exactly by the same percentage increase in Candidate B's campaign expenditure.

(e) Testing a single hypothesis about a linear combination of parameters.

Drop *democa* from the model.

$$votea = \beta_0 + \beta_1 log(expenda) + \beta_2 log(expendb) + u$$

In close races each candidate believes that he or she needs to increase their campaign expenditure by more than 1% to offset the effect of a 1% increase in their opponent's expenditure. The null hypothesis is,

$$H_0 : \beta_1 + \beta_2 = 0$$

and although it involves two parameters, it tests only one restriction. The alternative is,

$$H_0 : \beta_1 + \beta_2 < 0$$

That is, the impact on the share of votes received by Candidate A for a 1% increase in Candidate A's campaign expenditure is not enough to offset the impact from a 1% increase in Candidate B's campaign expenditure on the share of votes received by Candidate A. Candidate A needs to increase their campaign expenditure by more than 1% increase to offset the effect of a 1% increase in Candidate B's campaign expenditure.

so we cannot use the F test because F test provides inference against $\beta_1 + \beta_2 \neq 0$. In such cases that we have only one restriction about a linear combination, we use a reparameterisation trick:

$$Define\ \delta = \beta_1 + \beta_2 \implies \beta_2 = \delta - \beta_1$$

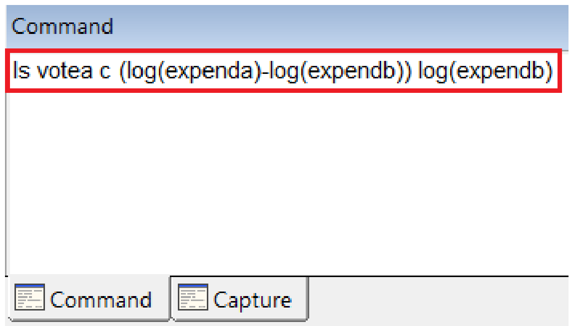
Subsitute for β_2 in the population model and re-arrange, you will see that δ becomes the coefficient of one of the explanatory variables in the reparameterised model. You can see that testing $\delta = 0$ against $\delta < 0$ can be performed with a simple t test in this reparameterised model.

Reparameterised model

$$\begin{aligned} votea &= \beta_0 + \beta_1 log(expenda) + \beta_2 log(expendb) + u \\ &= \beta_0 + \beta_1 log(expenda) + (\delta - \beta_1) log(expendb) + u \\ &= \beta_0 + \beta_1 log(expenda) + \delta log(expendb) - \beta_1 log(expendb) + u \\ &= \beta_0 + \beta_1 (log(expenda) - log(expendb)) + \delta log(expendb) + u \end{aligned}$$

To estimate the reparameterised model from the Command window in EViews,

$$ls\ votea\ c\ (log(expenda) - log(expendb))\ log(expendb)$$



(*Press Enter to execute code*)

Dependent Variable: VOTEA
Method: Least Squares
Date: 08/19/17 Time: 18:42
Sample: 1 173
Included observations: 173

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	52.03893	2.750137	18.92231	0.0000
LOG(EXPENDA)-LOG(EXPENDB)	6.341950	0.372649	17.01858	0.0000
LOG(EXPENDB)	−0.414801	0.538688	−0.770019	0.4424

R-squared	0.785187	Mean dependent var	50.50289
Adjusted R-squared	0.782660	S.D. dependent var	16.78476
S.E. of regression	7.825009	Akaike info criterion	6.969716
Sum squared resid	10409.23	Schwarz criterion	7.024397
Log likelihood	−599.8804	Hannan-Quinn criter.	6.991900
F-statistic	310.6936	Durbin-Watson stat	1.662940
Prob(F-statistic)	0.000000		

Table 4: Regression output of *votea* on a constant, $log(expenda) - log(expendb)$ and $log(expendb)$.

$$\widehat{votea} = \underset{(se(\hat{\beta}_0))}{\hat{\beta}_0} + \underset{(se(\hat{\beta}_1))}{\hat{\beta}_1}(log(expenda) - log(expendb)) + \underset{(se(\hat{\delta}))}{\hat{\delta}} log(expendb)$$

$$\widehat{votea} = \underset{(2.7501))}{52.0389} + \underset{(0.3726))}{6.3420}(log(expenda) - log(expendb)) - \underset{(0.5387)}{0.4148}log(expendb)$$

State the null and alternative hypothesis

$$H_0 : \delta = 0 \quad (same\ as\ \beta_1 + \beta_2 = 0)$$
$$H_1 : \delta < 0 \quad (same\ as\ \beta_1 + \beta_2 < 0)$$

$$(one - sided\ t\ test)$$

The test statistic and its distribution under H_0

$$t = \frac{\hat{\delta} - \delta}{se(\hat{\delta})} = \frac{\hat{\delta}}{se(\hat{\delta})} \sim t_{n-k-1} \quad under\ H_0$$

$$n = sample\ size = 173$$
$$k = number\ of\ regressors\ in\ the\ model = 2$$

Calculate the test statistic

$$t_{calc} = \frac{-0.414801}{0.538688} = -0.77$$

p-value

$$p - value = \frac{0.4424}{2} = 0.2212$$

Note: The *Prob.* values from the EViews regression output are p-values for a two-sided t-tests of individual significance so to calculate the p-value for our one-sided t-test, we divided the two-sided p-value by 2.

Dependent Variable: VOTEA
Method: Least Squares
Date: 08/19/17 Time: 18:42
Sample: 1 173
Included observations: 173

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	52.03893	2.750137	18.92231	0.0000
LOG(EXPENDA)-LOG(EXPENDB)	6.341950	0.372649	17.01858	0.0000
LOG(EXPENDB)	−0.414801	0.538688	−0.770019	0.4424

R-squared	0.785187	Mean dependent var	50.50289
Adjusted R-squared	0.782660	S.D. dependent var	16.78476
S.E. of regression	7.825009	Akaike info criterion	6.969716
Sum squared resid	10409.23	Schwarz criterion	7.024397
Log likelihood	−599.8804	Hannan-Quinn criter.	6.991900
F-statistic	310.6936	Durbin-Watson stat	1.662940
Prob(F-statistic)	0.000000		

Critical value and rejection region

$5\%\ significance\ level\ \rightarrow \alpha = 0.05$

To obtain the critical value using the Stats Table, locate the t distribution table,

$$degrees\ of\ freedom = 170$$

Since 170 is not in the table, we take a conservative approach and choose the closest available degrees of freedom less than 170 i.e. $d.o.f = 120$.

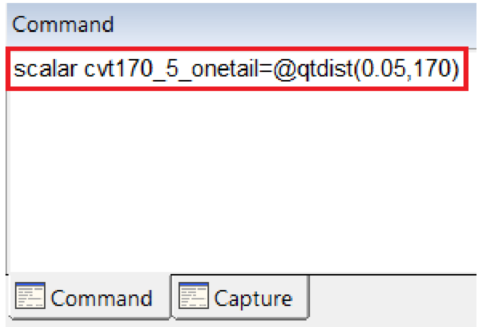
TABLE G.2 Critical Values of the *t* Distribution

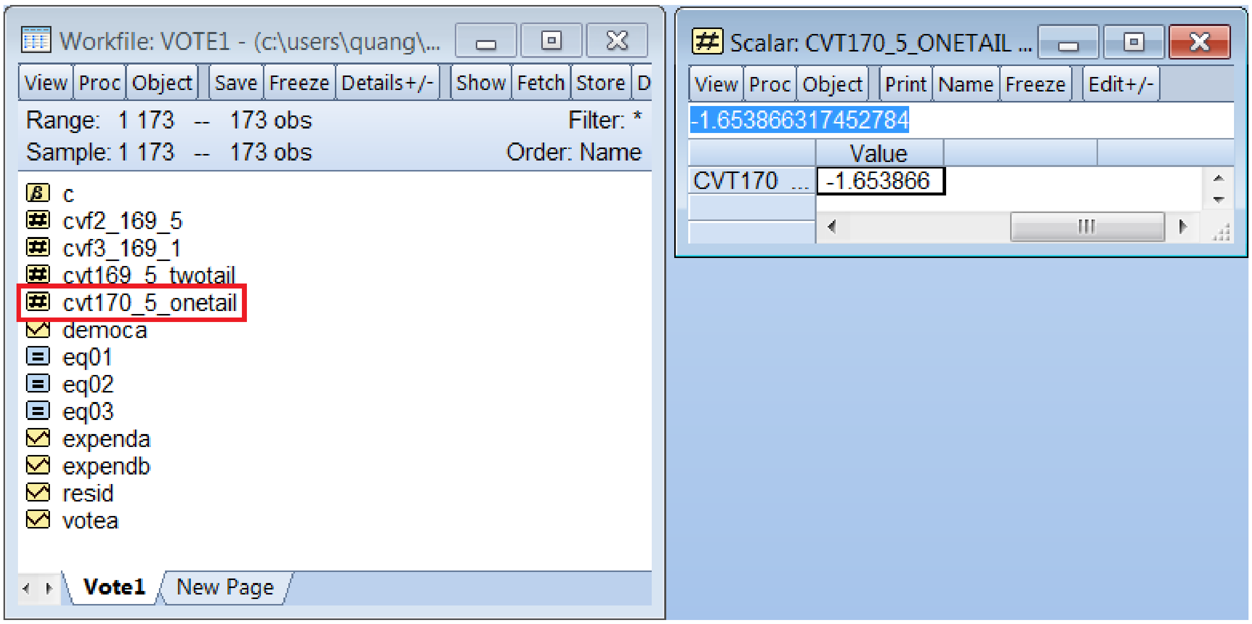
		Significance Level				
1-Tailed:		.10	.05	.025	.01	.005
2-Tailed:		.20	.10	.05	.02	.01
Degrees of Freedom	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	5	1.476	2.015	2.571	3.365	4.032
	6	1.440	1.943	2.447	3.143	3.707
	7	1.415	1.895	2.365	2.998	3.499
	8	1.397	1.860	2.306	2.896	3.355
	9	1.383	1.833	2.262	2.821	3.250
	10	1.372	1.812	2.228	2.764	3.169
	11	1.363	1.796	2.201	2.718	3.106
	12	1.356	1.782	2.179	2.681	3.055
	13	1.350	1.771	2.160	2.650	3.012
	14	1.345	1.761	2.145	2.624	2.977
	15	1.341	1.753	2.131	2.602	2.947
	16	1.337	1.746	2.120	2.583	2.921
	17	1.333	1.740	2.110	2.567	2.898
	18	1.330	1.734	2.101	2.552	2.878
	19	1.328	1.729	2.093	2.539	2.861
	20	1.325	1.725	2.086	2.528	2.845
	21	1.323	1.721	2.080	2.518	2.831
	22	1.321	1.717	2.074	2.508	2.819
	23	1.319	1.714	2.069	2.500	2.807
	24	1.318	1.711	2.064	2.492	2.797
	25	1.316	1.708	2.060	2.485	2.787
	26	1.315	1.706	2.056	2.479	2.779
	27	1.314	1.703	2.052	2.473	2.771
	28	1.313	1.701	2.048	2.467	2.763
	29	1.311	1.699	2.045	2.462	2.756
	30	1.310	1.697	2.042	2.457	2.750
	40	1.303	1.684	2.021	2.423	2.704
	60	1.296	1.671	2.000	2.390	2.660
	90	1.291	1.662	1.987	2.368	2.632
	120	1.289	1.658	1.980	2.358	2.617
	∞	1.282	1.645	1.960	2.326	2.576

$$Note: This\ gives\ us\ +t_{crit}\ but\ we\ need\ -t_{crit}\ for\ this\ test$$

To obtain the critical value using EViews,

$$Command\ window:\ \ scalar\ cvt170\_5\_onetail = @qtdist(0.05, 170)$$





From Stat Table:

$$-t_{crit} = -1.658$$

From EViews:

$$-t_{crit} = -1.6539$$

Rejection rule:

Comparing the calculated test statistic with the critical value, we reject H_0 if,

$$t_{calc} < -t_{crit}$$

Comparing the p-value with the significance level, we reject H_0 if,

$$p - value < \alpha = 0.05$$

Conclusion

Since $t_{calc} = -0.77 > -t_{crit} = -1.658$, we do not reject the null at the 5% significance level and conclude that there is insufficient evidence from our sample to suggest that Candidate A needs a larger than 1% increase in campaign expenditure to offset the effect of a 1% increase in Candidate B's campaign expenditure on share of votes received by Candidate A.