

Answer:

We think of $h(x^{(i)}) = p(y^{(i)} = 1|x^{(i)})$ only as a function of $x^{(i)}$. When $y^{(i)} = 1$, we have that:

$$h(x^{(i)}) = p(t^{(i)} = 1|x^{(i)})p(y^{(i)} = 1|t^{(i)} = 1, x^{(i)}) + p(t^{(i)} = 0|x^{(i)})p(y^{(i)} = 1|t^{(i)} = 0, x^{(i)}) \quad (1)$$

$$= 1 \cdot p(y^{(i)} = 1|t^{(i)} = 1, x^{(i)}) + 0 \cdot p(y^{(i)} = 1|t^{(i)} = 0, x^{(i)}) \quad (2)$$

$$= \alpha \quad (3)$$

In line 2, knowing that $y^{(i)}$ already equals 1, $t^{(i)}$ also takes value 1. As a result, $p(t^{(i)} = 1|x^{(i)}) = 1$ and $p(t^{(i)} = 0|x^{(i)}) = 0$. In the conditional universe given that $y^{(i)} = 1$, $h(x^{(i)})$ always takes value α , therefore its conditional expectation is α . That is, $\mathbb{E}[h(x^{(i)})|y^{(i)} = 1] = \alpha$.